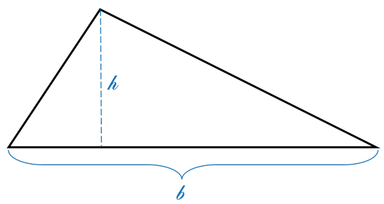
Mathematical Tools Used in FRE 460

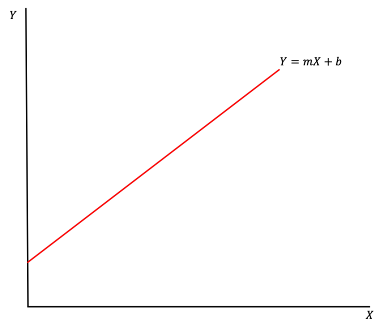
Tools you are assumed to have at the ready

Graphing, including

- the area of a triangle: $\frac{1}{2}bh$ where b is the length of the base and h is the height.



- formula for a line: $y = mx + b$ where m is the slope and b is the y-intercept.



Tools to be reviewed during the course

Discounting and present-discounted value

Example: Joi deposits \$100 into the bank and leaves it there for 4 years. The bank pays out 2% interest, compounded annually. How much does Joi have at the end of 4 years (including the principal)?

Answer: $(\$100)(1.02)^4 = \$108.243216$.

Example: K is promised \$1000 dollars to be paid 6 years from today. K's rate of time preference is 5%. What is K's present discounted value for the promised sum? Answer: $\frac{\$1000}{(1.05)^6} \approx \$746.22$.

Present discounted value of a perpetuity

If, starting next year, you are paid $\$x$ every year for ever more, and the discount rate is given by r, then the present discounted value *today* of the perpetuity is $\frac{\$x}{r}$.

Present Discounted Utility

Example: Juan's instantaneous utility from drinking a cup of coffee in the morning is 2 utils if he drank coffee the day before, and 4 utils if he did not drink coffee the previous day. Juan's daily discount rate is

10%. What is Juan's present discounted utility if he drinks coffee today, tomorrow, and every day after until the end of eternity? Assume Juan did not drink coffee yesterday. Answer: Juan's present discounted utlity (PDU) from drinking coffee every day, ever more is

$$
\begin{aligned}
PDU &= 4 + \frac{2}{1+.10} + \frac{2}{(1+.10)^2} + \frac{2}{(1+.10)^3} + \frac{2}{(1+.10)^4} + \dots \\
&= 4 + \sum_{t=1}^{\infty} \frac{2}{(1+.10)^t} \\
&= 4 + \frac{2}{.10} = 24
\end{aligned}
$$

Quasi-hyperbolic discounting

Example: Esther also likes coffee but engages in quasi-hyperbolic discounting. That is, consumption that isn't enjoyed right now gets an extra discount $\beta \in (0,1)$. Specifically, her present discounted utility from the consumption path $\{c_0, c_1, c_2, c_3, c_4, ...\}$ is given by

$$PDU = u(c_0) + \beta \sum_{t=1}^{\infty} \frac{u(c_t)}{(1+r)^t}$$

where $u(c)$ measures her instantaneous utility from consumption level c while r is her intertemporal discount factor. Suppose $u(c) = 10$ for $c > 1$ while $u(0) = 0$. That is, Esther's instantaneous utility from drinking coffee-drinking is 0 if she doesn't drink any coffee, and 10 if she drinks a positive amount. What's Esther's PDU from eschewing coffee at $t = 0$ and drinking it every day afterward? Let $\beta = 0.5$ and $r = .07$. Answer:

$$
\begin{aligned}
PDU &= 0 + 0.5 \left[\sum_{t=1}^{\infty} \frac{10}{(1.07)^t} \right] \\
&= 0 + 0.5 \left[\frac{10}{.07} \right] \\
&= 71.4285714
\end{aligned}
$$

Expected value of a gamble

Suppose Gamble A pays out \$100 with probability $p = .75$ and pays out \$30 with probability $1 - p = 0.25$. What's the expected value of the gamble? Answer: Expected Value of Gamble A $= .75 * 100 + .25 * 30 = 82.5$.

Expected utility

Joan has no money to start with. Her utility from money y is given by $u(y) = y^{\frac{1}{2}}$. What is Joan's expected utility from gamble A? Answer:

$$
\begin{aligned}
EU(\text{Gamble A}) &= 0.75\ u(100) + 0.25\ u(30) = 0.75(100^{\frac{1}{2}}) + 0.25(30^{\frac{1}{2}}) \\
&= 8.8693064.
\end{aligned}
$$

Notice that Joan's expected utility from Gamble A is less than her utility from the expected value of that gamble where

$$u(\text{Expected Value of Gamble A}) = u(82.5) = 82.5^{\frac{1}{2}} = 9.0829511$$

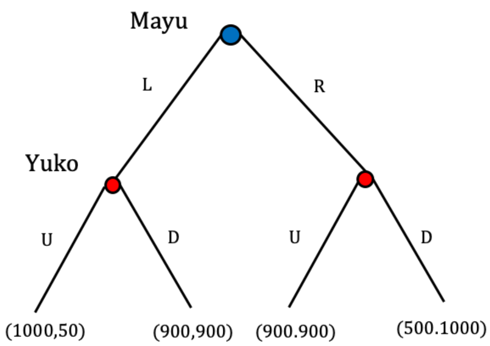
Extensive-form games

Mayu and Yuko are playing a game. Mayu gets to move first. Mayu can move either Left or Right. Yuko can move either Up or Down. If Mayu moves Left while Yuko moves Up, then Mayu receives a payoff of $\pi_m = 1000$ while Yuko receives a payoff of $\pi_y = 500$; we can represent these payoffs using the following tuple: $(\pi_m, \pi_y) = (1000, 500)$. If Mayu moves Right while Yuko moves Down, their respective payoffs are $(500, 1000)$, i.e. 500 for Mayu and 1000 for Yuko. Otherwise, both players receive 900 each.

We can represent their payoffs using a "Normal-form" as follows:

		Yuko	
		U	**D**
Mayu	**L**	1000,500	900,900
	R	900,900	500,1000

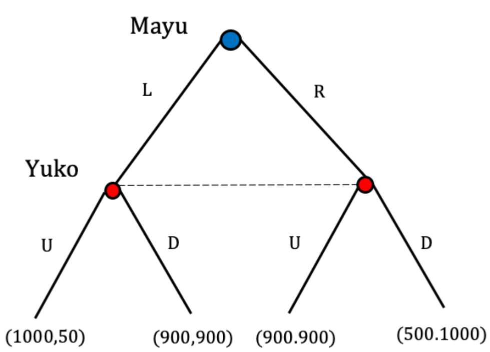
Or we can depict the game using an "Extensive-form":



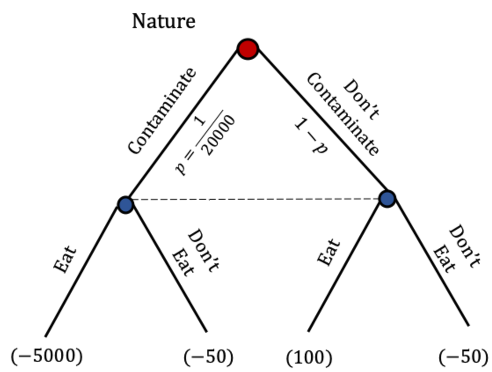
One advantage of depicting a game using the extensive-form is that it is obvious who moves first.

Extensive-form games with uncertainty

Sometimes a player does not know which branch they are on when it is their turn to act. If, for example, Yuko didn't know which way Mayu had moved, we would adjust the Extensive-form depiction by drawing a dashed line connecting the decision node associated with Mayu's move; the dashed line depicts uncertainty:



Extensive-forms are useful for thinking about actions in the presence of uncertainty. Suppose the first mover is Nature, and Juyun is the second mover, deciding whether or not to eat a raw egg. There's a 1 in 20,000 chance that a randomly selected raw egg contains enough salmonella bacteria to make her sick. Suppose that, if she eats the raw egg and then gets sick, her utility is -5000. If she eats the raw egg and doesn't get sick, her utility is 100. If she doesn't eat the egg, she can't get sick and gets utility -50. The extensive-form depiction of this "game" is as follows:



Notice that, because Nature doesn't get a "payoff" in this game, we only showed Juyun's payoffs at the bottom of each branch.

Drawing on the tool of expected utility, Juyun's expected utility from eating the egg is

$$\frac{1}{20000} * (-5000) + \left(1 - \frac{1}{20000}\right) * 100 = 99.745$$

while if she doesn't eat the egg her expected utility is -50.

p-values

p-values are commonly reported in the context of statistical analyses. Wikipedia offers the following definition of a p-value: "In statistical hypothesis testing, the p-value ... is the probability for a given statistical model that, when the null hypothesis is true, the statistical summary (such as the sample mean difference between two compared groups) would be the same as or of greater magnitude than the actual observed results." (Accessed January 2, 2018 at https://en.wikipedia.org/wiki/P-value). This is accurate but somewhat abstruse. The following (crude and incomplete) definition might be more accessible: 'the probability of estimating such a big coefficient when its real value is zero.'