1 Examples of multivariable derivative

Compute the gradient $\nabla f(x)$ of each of the following functions.

1. $f_1(x) = \sin(x_1)$ where $x \in R^2$.
   Answer:
   $$\nabla f(x) = \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \end{bmatrix} = \begin{bmatrix} cos(x_1) \\ 0 \end{bmatrix}$$

2. $f(x) = \exp(x_1 x_2)$ where $x \in R^2$.
   Answer:
   $$\nabla f(x) = \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \end{bmatrix} = \begin{bmatrix} x_2 \exp(x_1 x_2) \\ x_1 \exp(x_1 x_2) \end{bmatrix} = \begin{bmatrix} x_2 f(x) \\ x_1 f(x) \end{bmatrix}$$

3. $f(x) = x_1^2 + \exp(x_2)$ where $x \in R^2$.
   Answer:
   $$\nabla f(x) = \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 2x_1 \\ exp(x_2) \end{bmatrix}$$

4. $f(x) = \frac{1}{2}x^2$ where $x \in R^d$.
   Answer:
   $$\nabla f(x) = \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \\ \vdots \\ \frac{\partial}{\partial x_d} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix} = x$$