DSCI 572: Supervised Learning II

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Computing With Gates

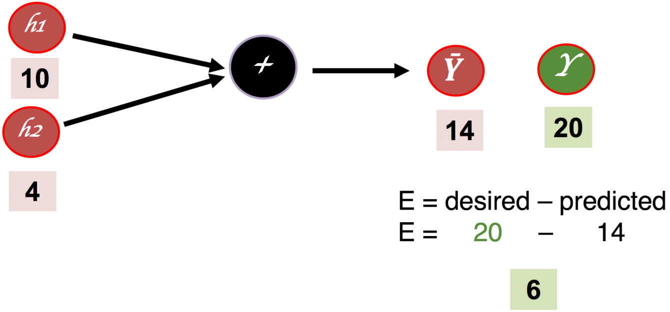


Figure: An add gate operating over units from a hidden layer **l**. The error **E** tells us how far the predicted outcome $\hat{y}$ is from the gold target y. In practice, we use cross-entropy and so **E** here is an oversimplification.

A Change in Input, Causes a Change in Output

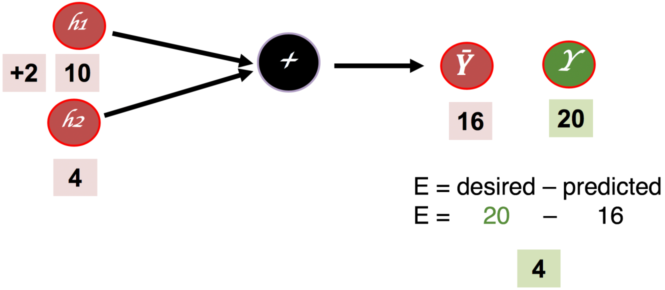
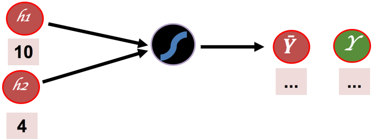


Figure: By increasing the value of h_1, we can decrease the error **E**.

Generalizing to Other Functions, Across Layers



Changing Weights, Across Layers

- We can use other functions, like **sigmoid**.
- We change the weights by **identifying the derivative**.
- Since we want to minimize our cost function, we **move in the direction opposite to the derivative**.
- We can **change weights in any layer except input**. (why?)

Pre-Activation in One Unit

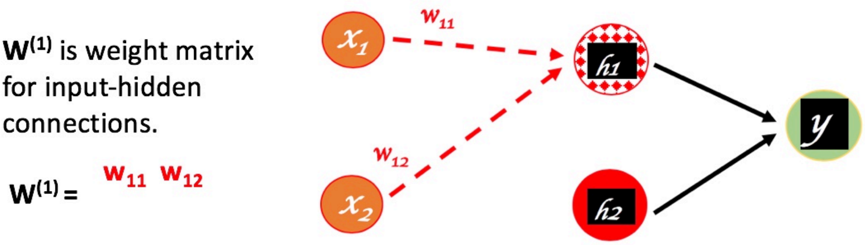


Figure: Connection weights from input to hidden in the 2 x 2 matrix **W** are indexed with j (hidden unit index) and i (input unit index). We refer to the matrix as $\mathbf{W}^{(1)}$ to denote it is the first layer matrix (input-to-hidden). For simplification, bias is dropped.

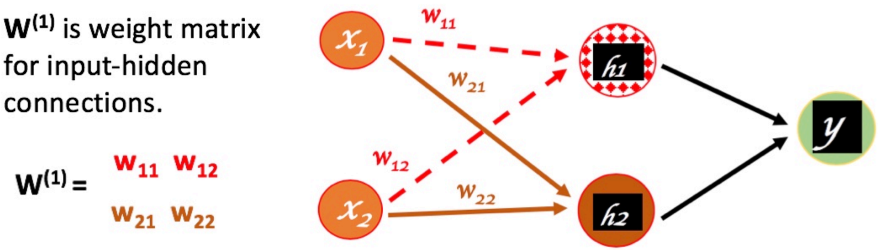


Figure: Connection weights from input to hidden in the 2 x 2 matrix **W** are indexed with j (hidden unit index) and i (input unit index). We refer to the matrix as $\mathbf{W}^{(1)}$ to denote it is the first layer matrix (input-to-hidden).

Two Hidden Layers Network, With One Output

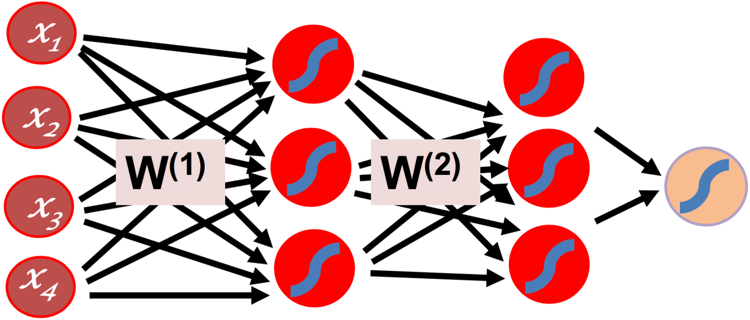


Figure: A network with two hidden layers. Now we have **two connection weight matrixes** $\mathbf{W}^{(1)}$ and $\mathbf{W}^{(2)}$. The network has only one output unit (e.g., performing binary classification).

Two Hidden Layers Network, With Multiple Outputs

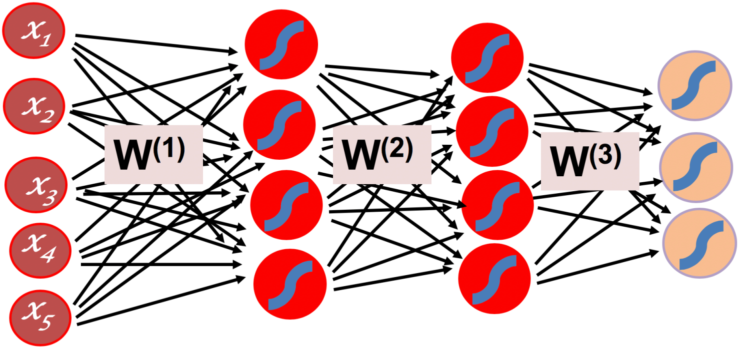


Figure: A network with two hidden layers. Now we have **three connection weight matrixes** $\mathbf{W}^{(1)}$, $\mathbf{W}^{(2)}$, and $\mathbf{W}^{(3)}$. The network has multiple output units (e.g., performing multi-class classification).

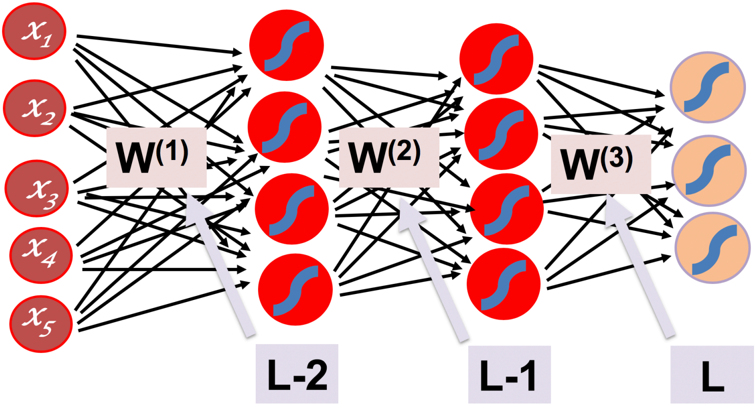


Figure: We also index our **hidden layers** as ℓ =1, 2, L-1, . . . L.

Re-Indexing Connection Weights With Hidden Layer Labels

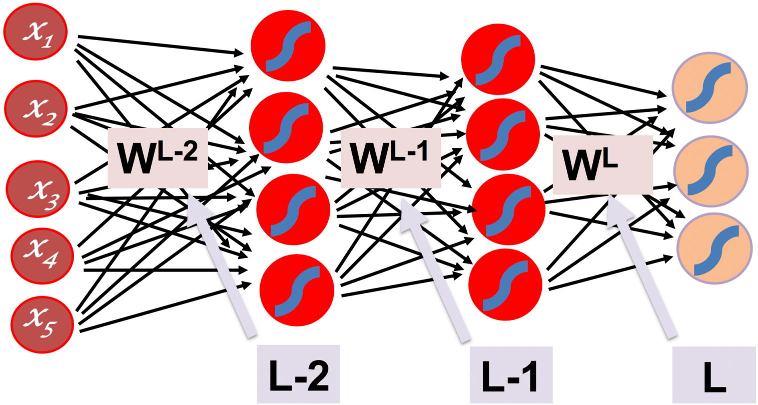


Figure: We re-index our **connection weights with hidden layer labels**, to simplify.

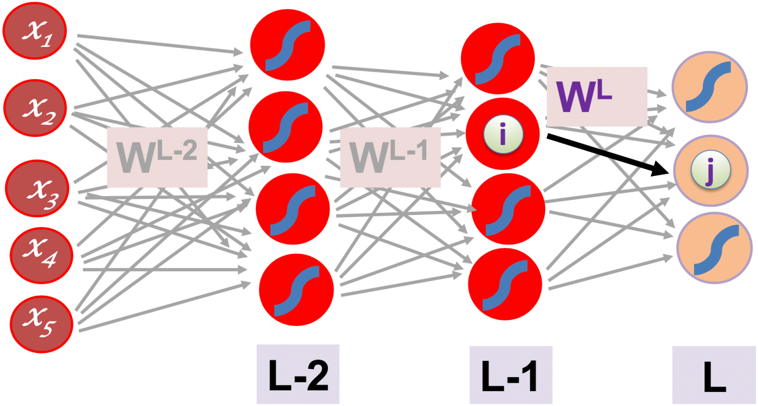


Figure: We re-index our **connection weights with hidden layer labels**, to simplify.

Node Connections (From Connection Weights Matrix W)

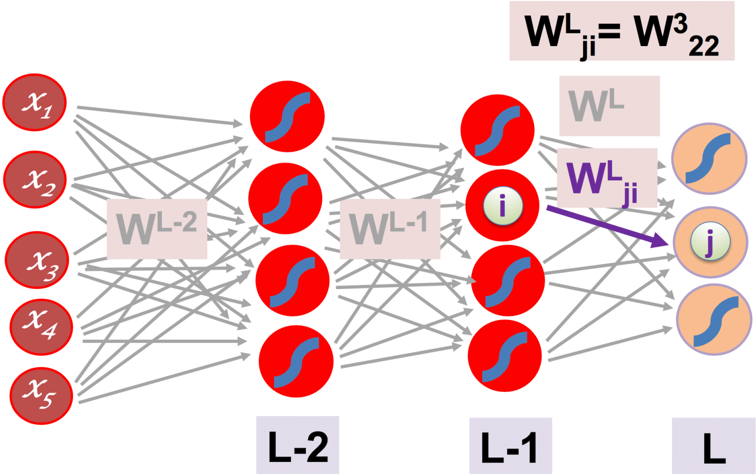


Figure: Weight of the connection from **node i** (in layer L-1) to **node j** (in layer L).

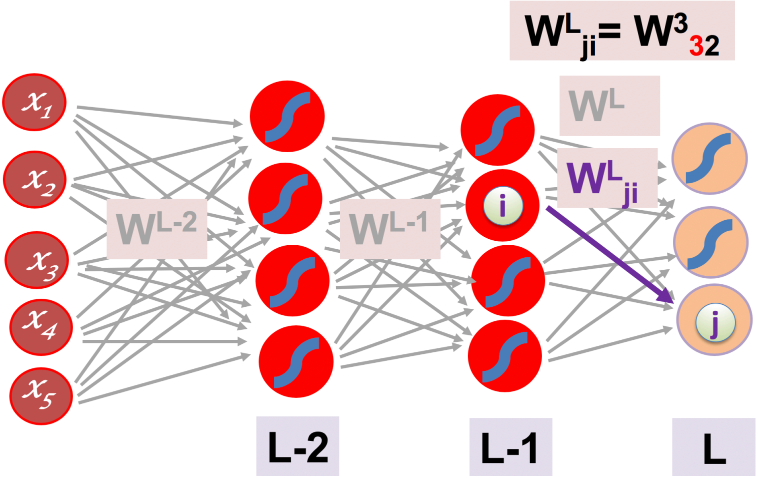


Figure: Weight of the connection from **node i=2** (in layer L-1 → **not indexed**) to **node j=3** (in layer L → **indexed in superscript**).

The Vector of Weights $W^L_{j,:}$

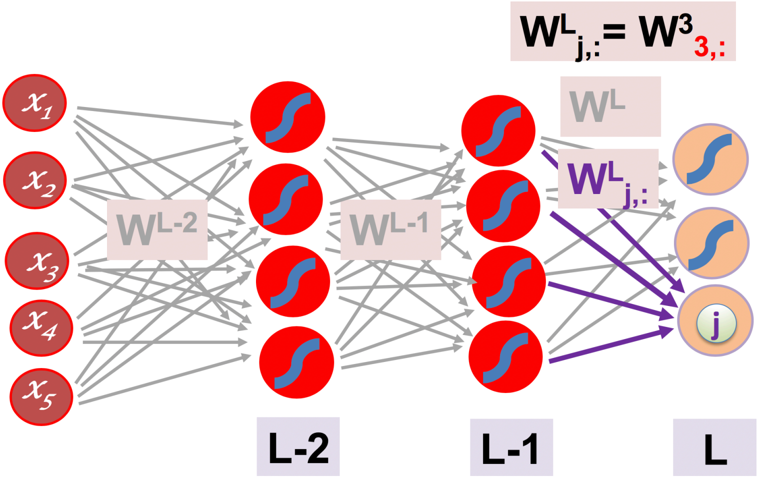


Figure: **Vector of weights** from **each node in layer L-1** to **node j in Layer L**.

The Vector of Inputs $Z^L_{j,:}$

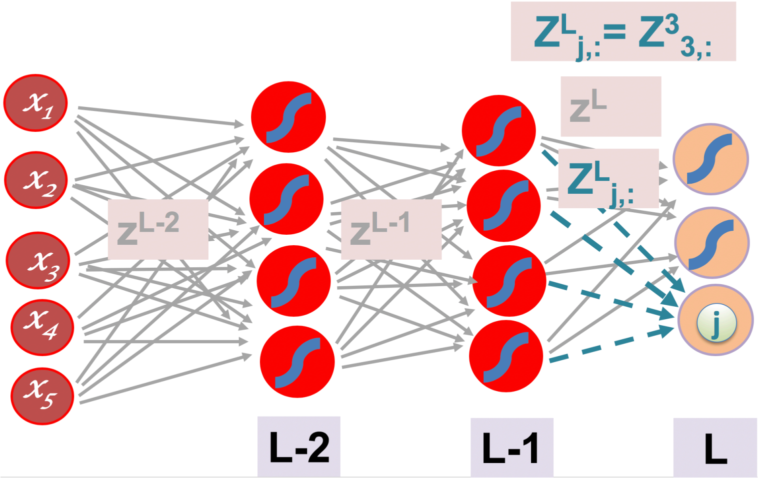


Figure: **Vector of inputs** from **each node in layer L-1** to **node j in Layer L**.

The Vector of Activations $a_:^{L-1}$

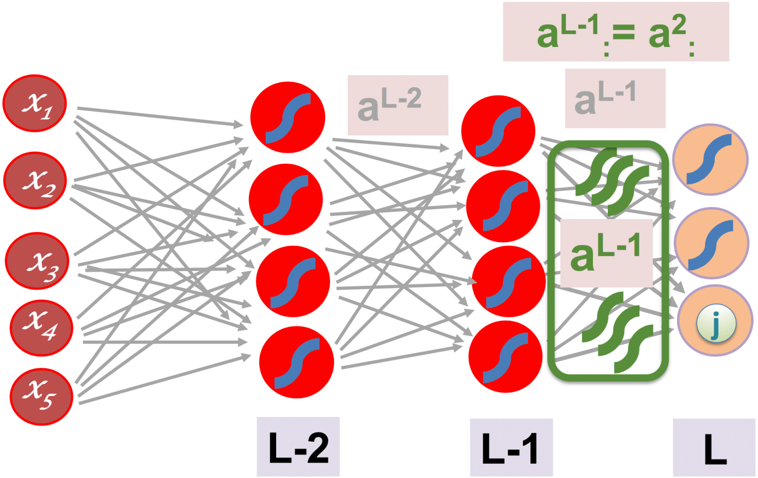


Figure: **Vector of activations** from **each node in layer L-1** to **node j in Layer L**.

What's in a Cell?

$$z_j^l = \sum_{i=0}^{n-1} w_{ji}^l a_i^{l-1}$$

$$a_j^l = g^l(z_j^l)$$

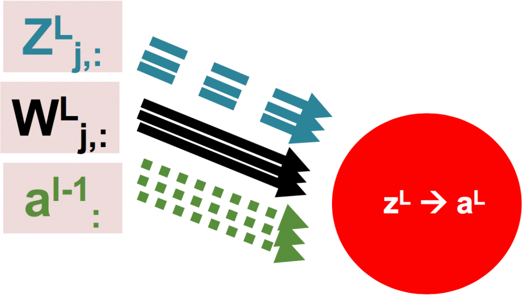


Figure: **Processing in a cell.**

The Loss C

- For a single data point, a cost C can be expressed as squared difference, summing over each node j in output layer L

1: Cost in each activation node j at Layer L

$$C = \sum_{j=0}^{n-1} \left(a_j^L - y_j\right)^2$$

Cross-Entropy

- **Note:** In deep learning, we **usually use cross-entropy as the loss function (see here)**, but for simplicity we use the squared difference here. This does not change the way backpropagation works.

Input For Node j in layer l

- Input for node j in layer l is the weighted sum of the activation outputs from the previous layer $\ell - 1$.
- An individual term is as follows:

2: Individual term of sum of input for node j in layer $\ell - 1$

$$w_{ji}^{l} a_{i}^{l-1}$$

- Input sum:

3: Input for node j in layer $\ell - 1$ summed

$$z_{j}^{l} = \sum_{i=0}^{n-1} w_{ji}^{l} a_{i}^{l-1}$$

Activation Output

- Activation output of a given node j in layer l is the result of passing z_j^l to an activation function g (e.g., sigmoid).

4: Individual term of sum of input for node j in layer $\ell - 1$

$$a_j^l = g^l(z_j^l)$$

Cost C as Composition of Functions

- We can express cost as a function of activation output a.

5: Cost as a Function of a

$$a_j^l = g^l(z_j^l)$$

$$C_j = \left(a_j^l - y_j\right)^2$$

- Total loss for a single input is a sum over all losses of all output nodes:

6: Cost as a Function of a

$$C = \sum_{j=0}^{n-1} C_j$$

Changing Weights, Across Layers

- **Loss is a composition of functions** and will use the **chain rule of calculus to differentiate** this function of functions:
- This means we will **take the derivative of the loss with respect to** θ (the weights and biases).

7: Cost as a Function of a

$$C_j = (a^l_j(g^l(z^l_j)) - y_j)^2$$

P.S. y_j is a constant parameter that we only use to define the cost function, and so **C can be thought of as just a function of output** a.

Derivative of Loss C With Respect to Weight W^2_{32}

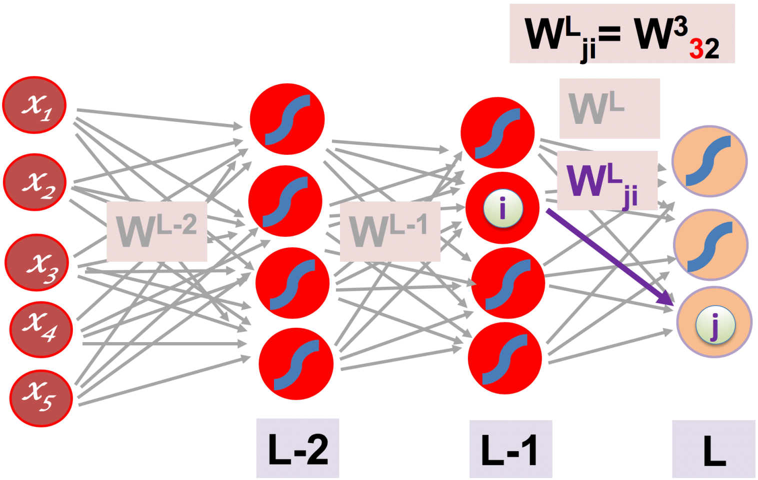
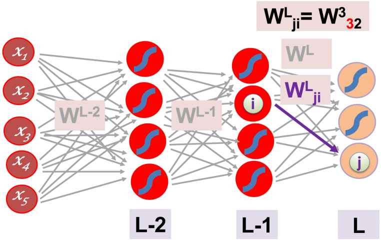


Figure: Derivative of C with respect to W^3_{32} is denoted as $\frac{\partial C}{\partial W^3_{32}}$.



- Derivative of C with respect to W_{32}^3 is denoted as $\frac{\partial C}{\partial W_{32}^3}$
- **Loss C is a function of output activation** a, which in turn is a function of input z, which itself is a function of weights w (all indexes dropped for simplification).

Putting it All Together

8: The C, a, and z functions

z is a function of w:

$$z_j^l = \sum_{j=0}^{n-1} w_{ji}^l a_i^{l-1}$$

a is a function of z:

$$a_j^l = g^l(z_j^l)$$

C is a function of a:

$$C_j = a_j^l(g^l(z_j^l))$$

We take the product of the derivatives of compositional function C:

$$\frac{\partial C}{\partial W_{32}^3} = \left(\frac{\partial C}{\partial a_3^3}\right)\left(\frac{\partial a_3^3}{\partial z_3^3}\right)\left(\frac{\partial z_3^3}{\partial W_{32}^3}\right)$$