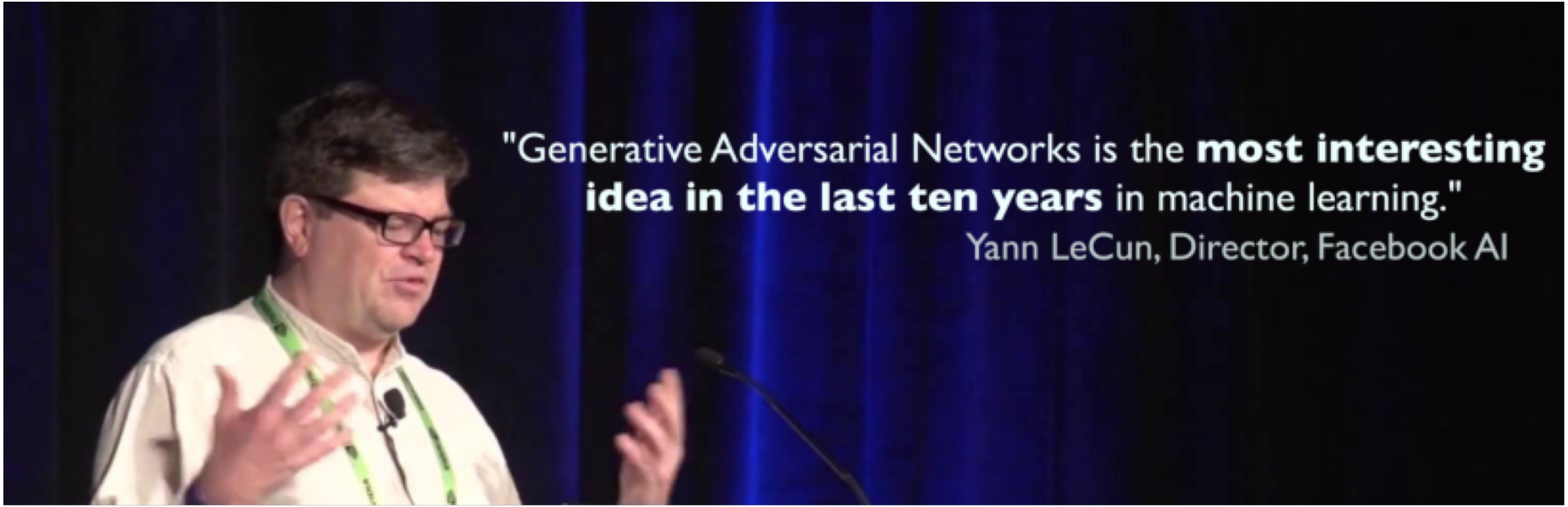
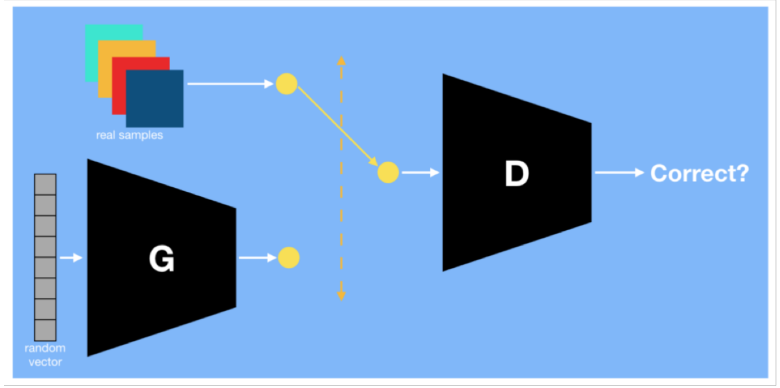
Introduction to Generative Adversarial Network (GAN)

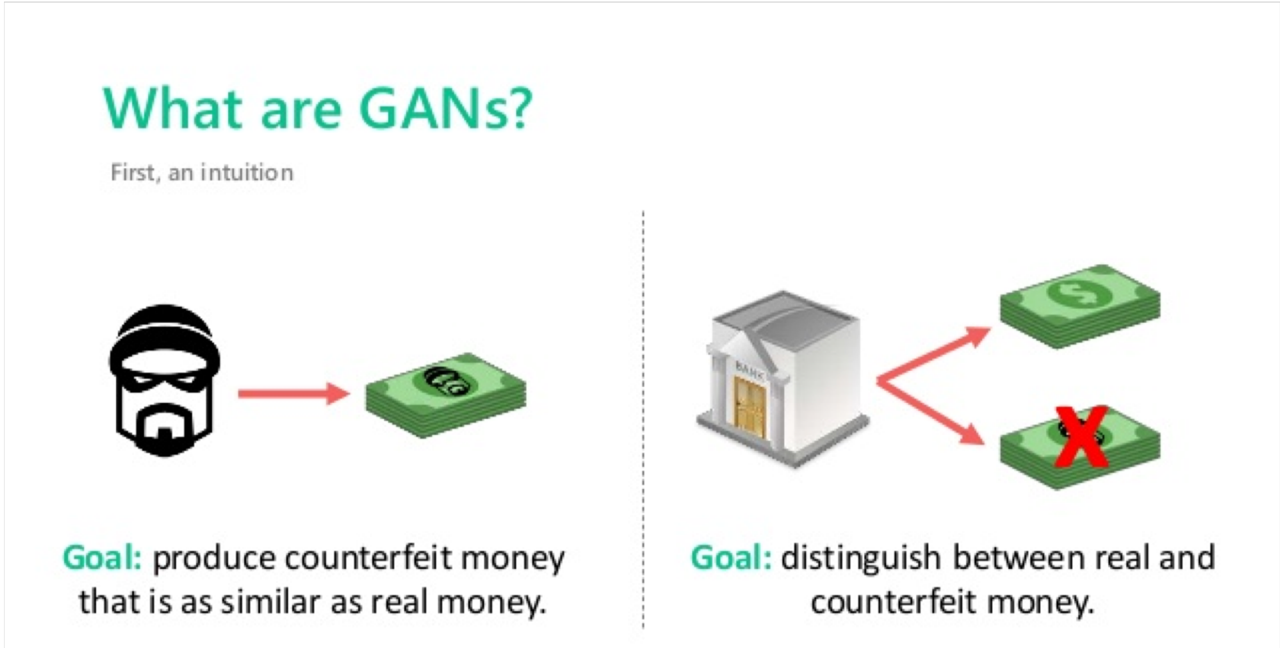
Pramit Saha

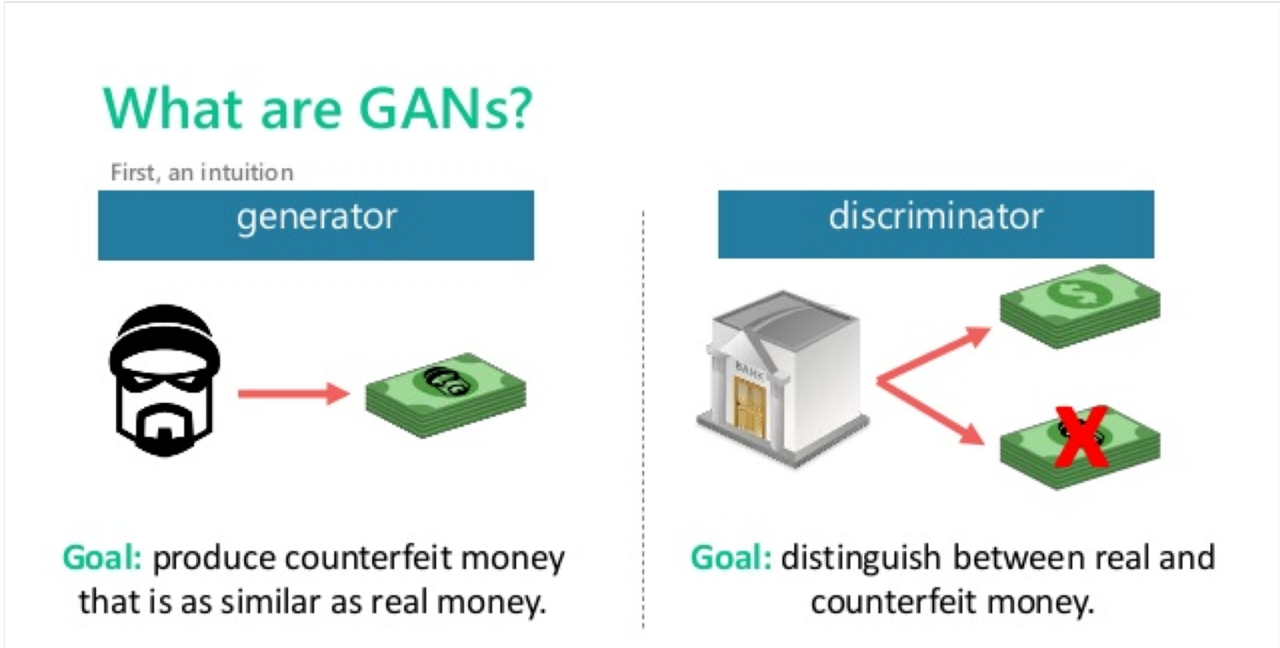

"Generative Adversarial Networks is the most interesting idea in the last ten years in machine learning."
Yann LeCun, Director, Facebook AI


real samples
random vector
G
D
Correct?

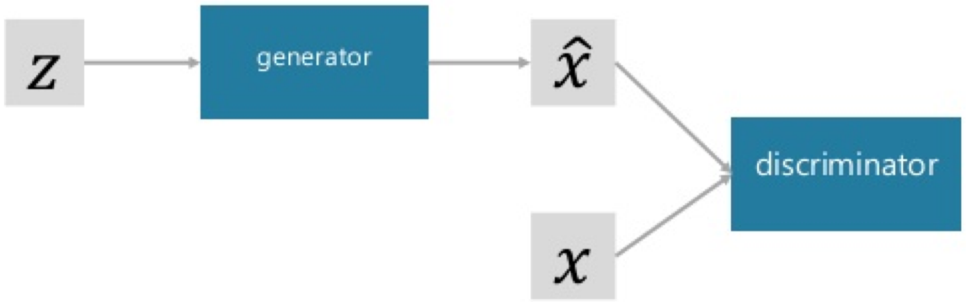
Generative Adversarial Networks

- Created by Ian Goodfellow (OpenAI);
- Two neural networks compete (*minmax game*)
  - Discriminative network tries to distinguish between real and fake data
  - Generative network tries to generate samples to fool the discriminative one
- Use latent code (z).

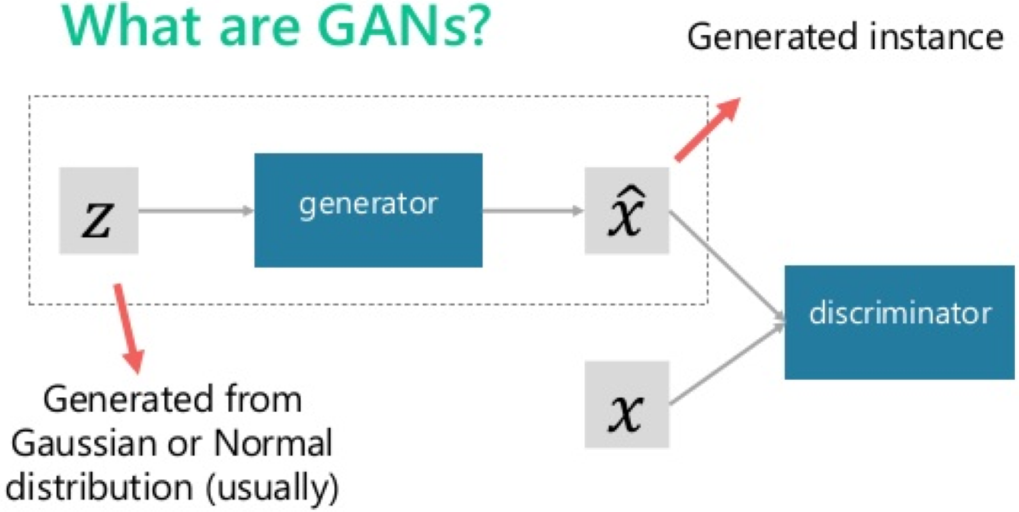

What are GANs?
First, an intuition
Goal: produce counterfeit money that is as similar as real money.
Goal: distinguish between real and counterfeit money.

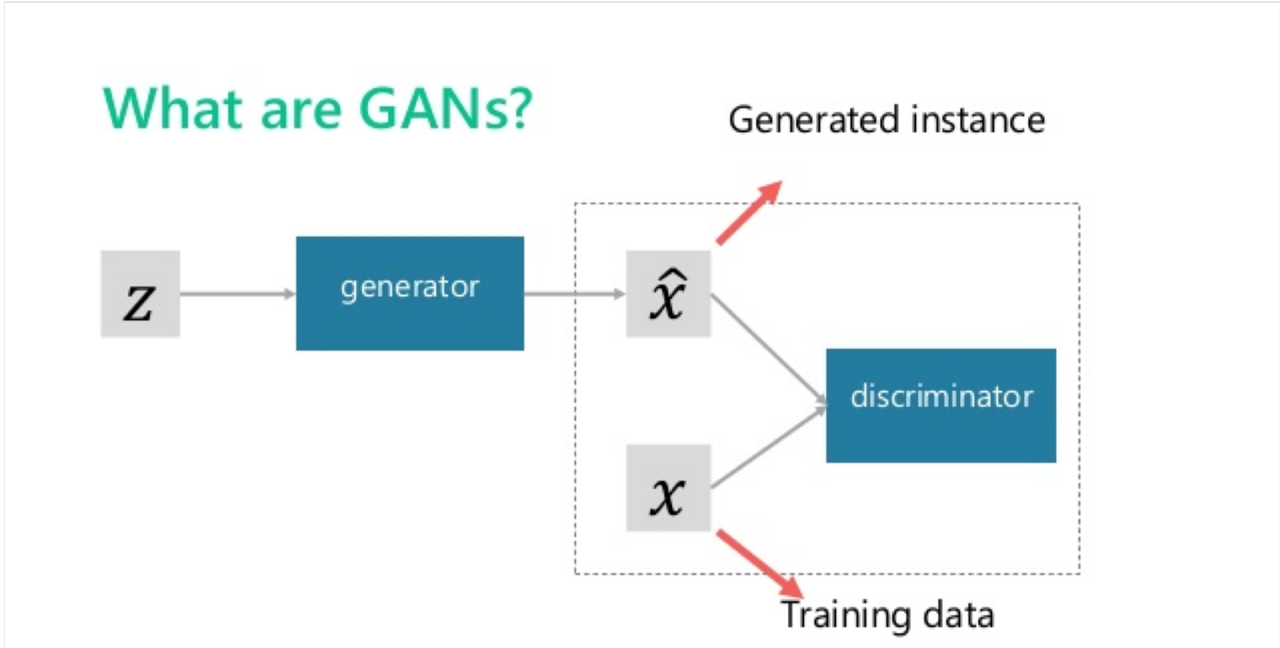

What are GANs?
First, an intuition
generator
discriminator
Goal: produce counterfeit money that is as similar as real money.
Goal: distinguish between real and counterfeit money.

What are GANs?

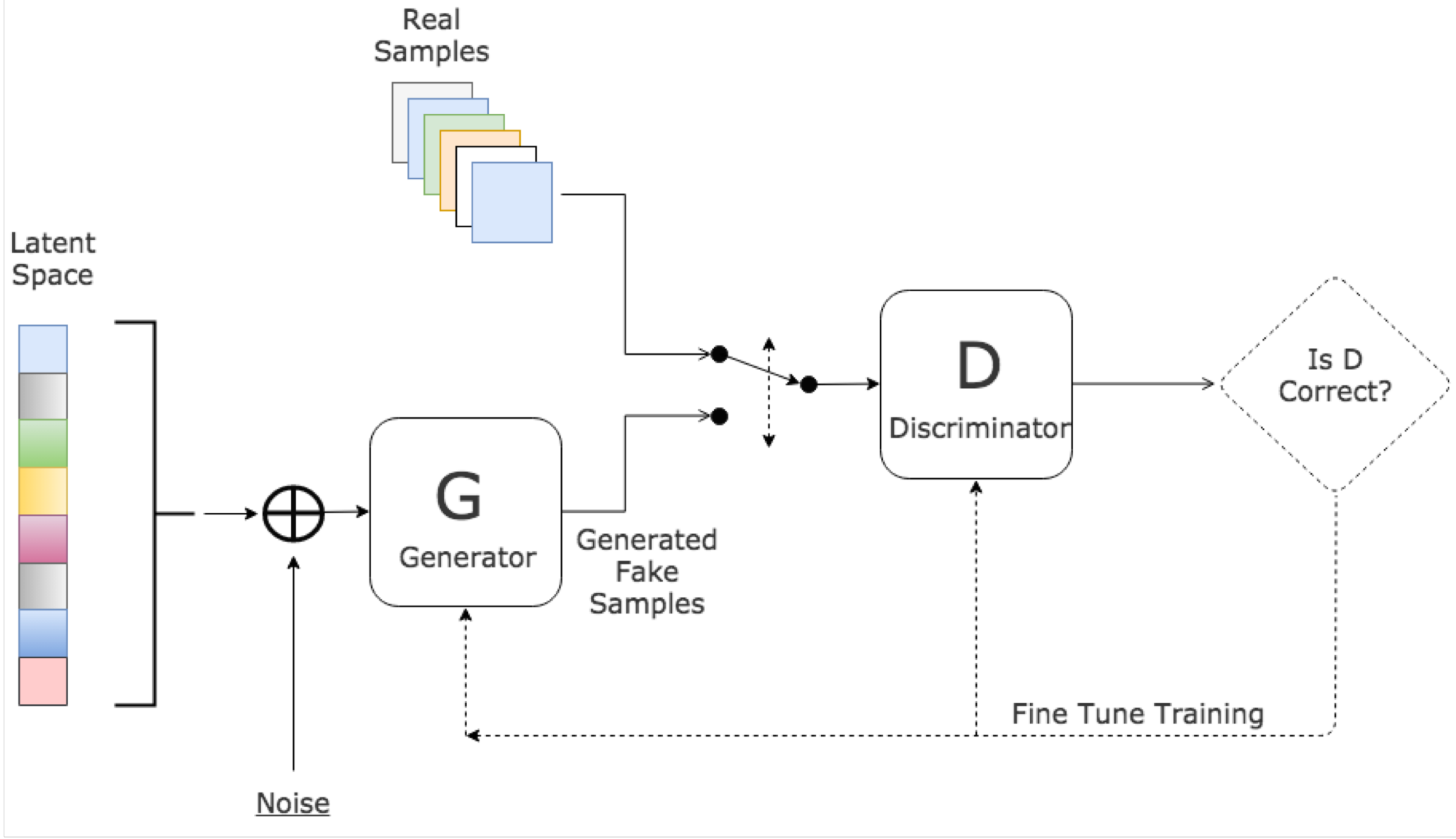


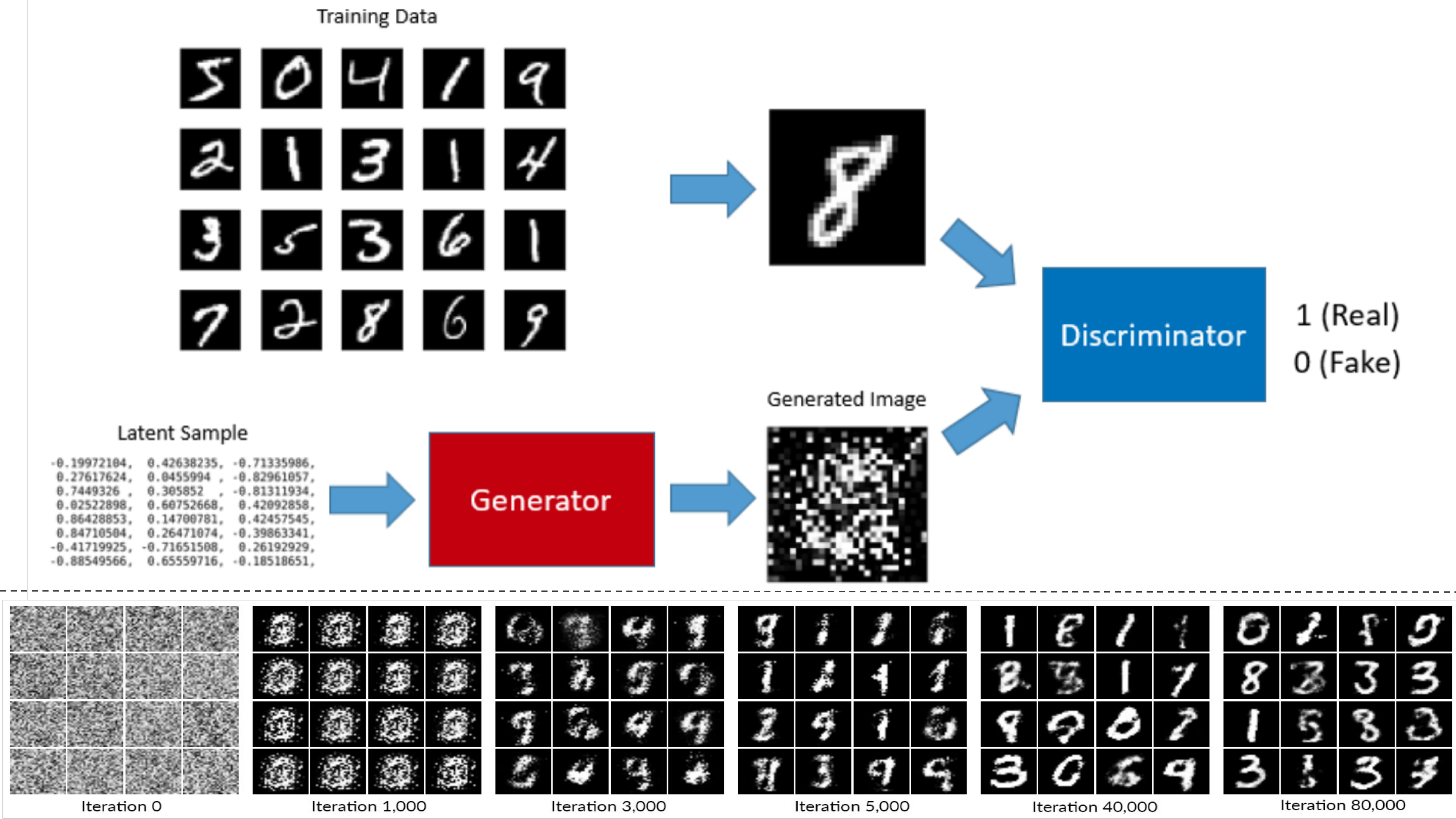
What are GANs?

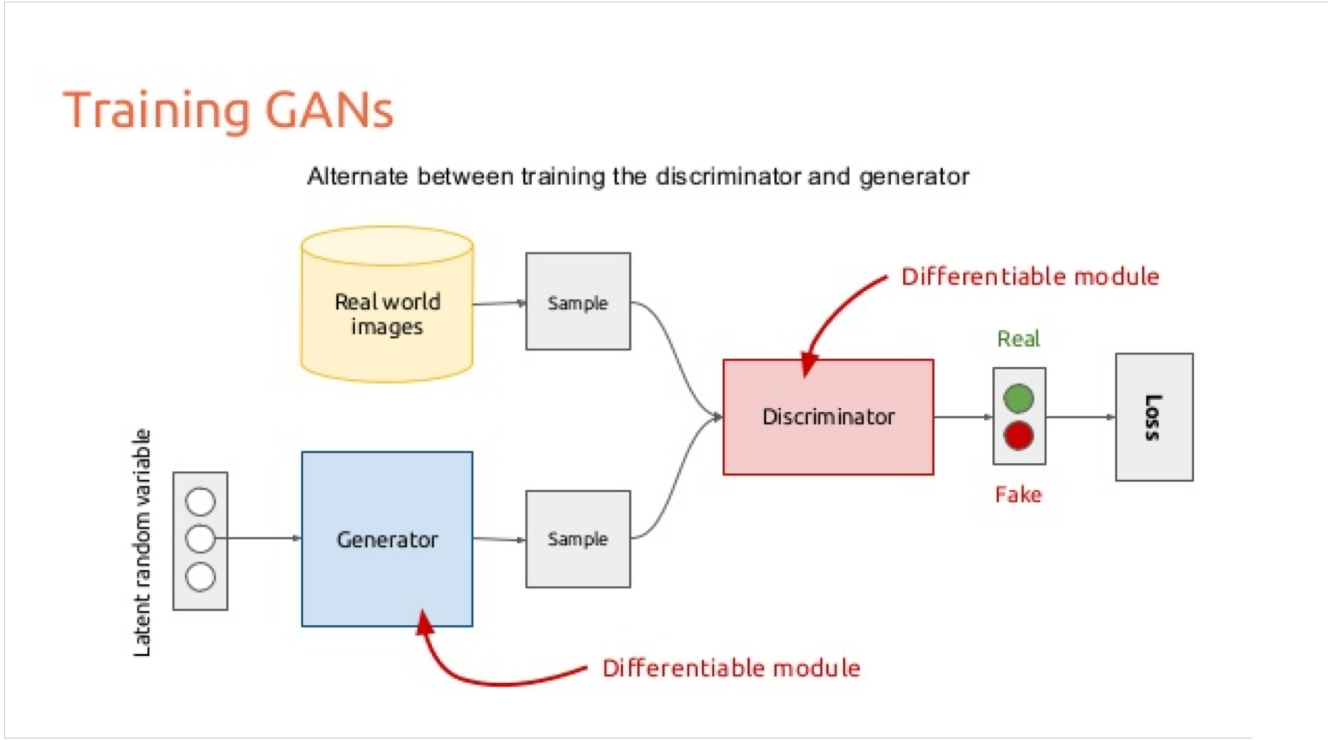



What are GANs?
Generated instance
z
generator
$\hat{x}$
discriminator
x
Training data

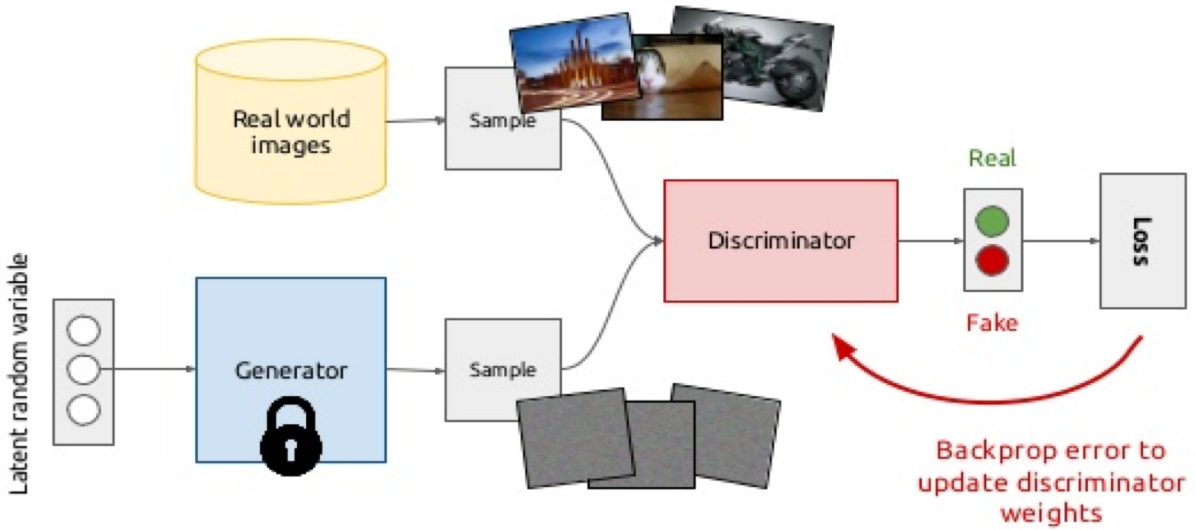
Generative Adversarial Network



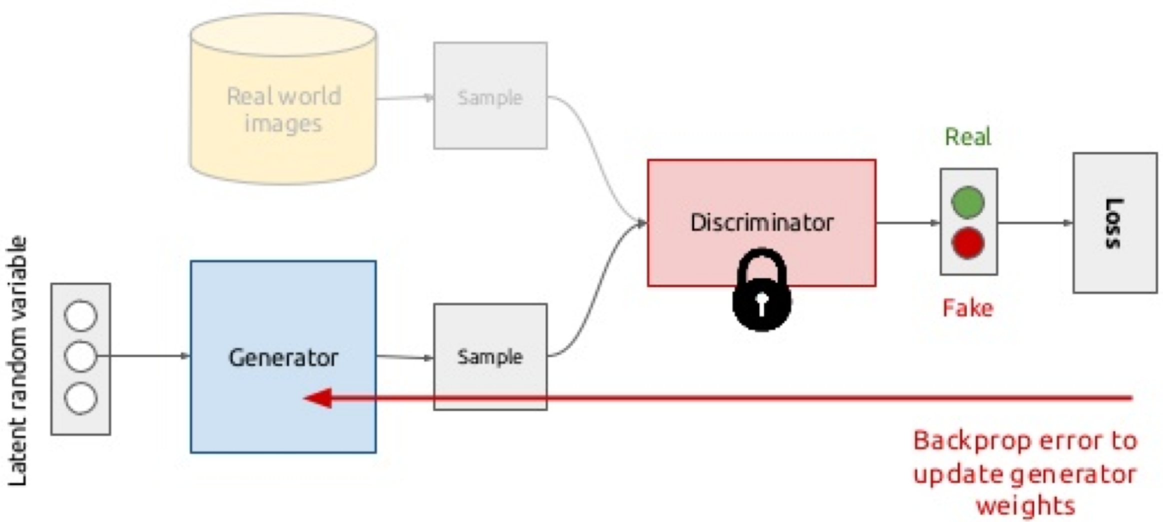

Training Data
Discriminator
1 (Real)
0 (Fake)
Generated Image
Latent Sample
-0.19972104, 0.42638235, -0.71335986,
0.27617624, 0.0455994 , -0.82961057,
0.7449326 , 0.305852 , -0.81311934,
0.02522898, 0.60752668, 0.42092858,
0.86428853, 0.14700781, 0.42457545,
0.84710504, 0.26471074, -0.39863341,
-0.41719925, -0.71651508, 0.26192929,
-0.88549566, 0.65559716, -0.18518651,
Generator
Iteration 0
Iteration 1,000
Iteration 3,000
Iteration 5,000
Iteration 40,000
Iteration 80,000


Training GANs
Alternate between training the discriminator and generator
Real world images
Sample
Differentiable module
Real
Discriminator
Loss
Fake
Latent random variable
Generator
Sample
Differentiable module

1. Fix generator weights, draw samples from both real world and generated images
2. Train discriminator to distinguish between real world and generated images



1. Fix discriminator weights
2. Sample from generator
3. Backprop error through discriminator to update generator weights

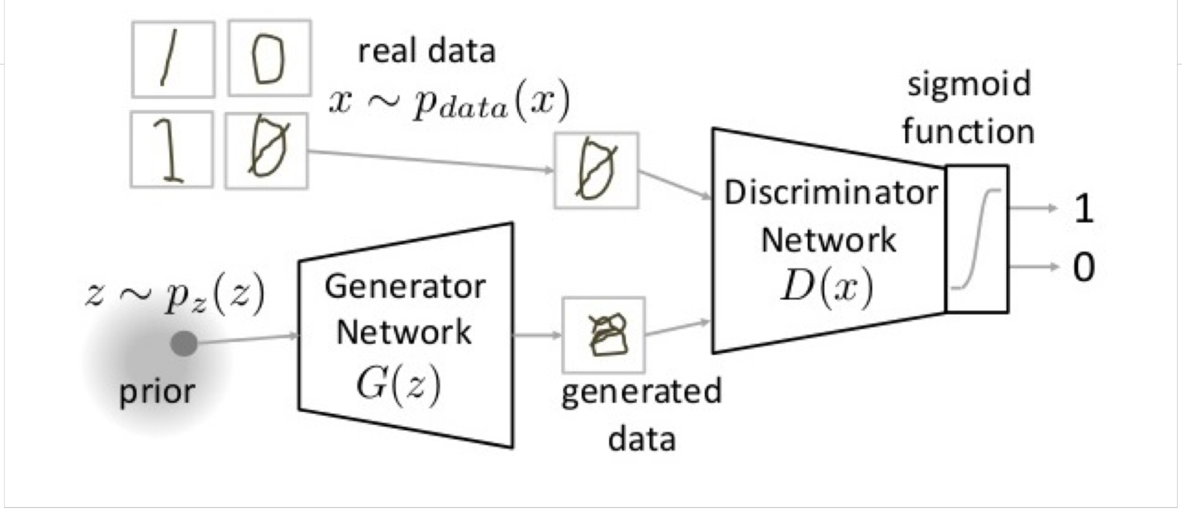


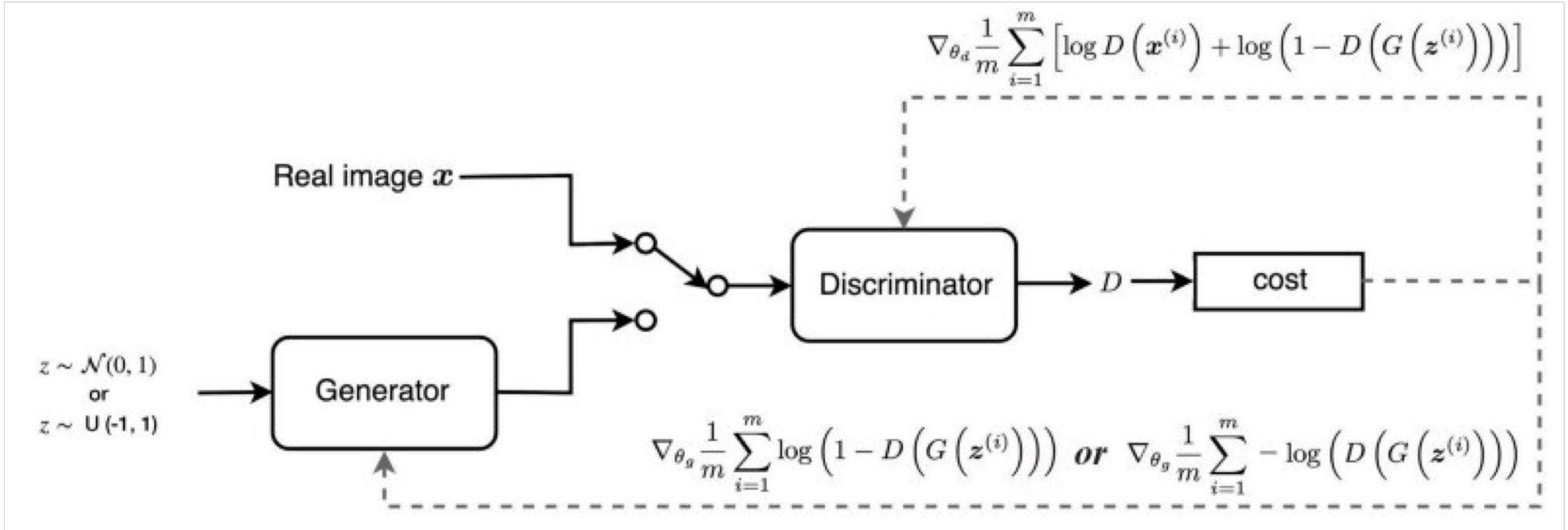
Mathematical Formulation

Two-player minimax game with value function V (D,G)

$$\min_G \max_D V(D, G)$$

$$V(D,G) = \mathbb{E}_{x \sim p_{data}(x)}[\log D(x)] + \mathbb{E}_{z \sim p_z(z)}[\log(1 - D(G(z)))]$$





$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^{m} \left[\log D\left(\boldsymbol{x}^{(i)}\right) + \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right)\right]$
Real image $\boldsymbol{x}$
Discriminator
D
cost
$z \sim \mathcal{N}(0,1)$
or
$z \sim$ U (-1, 1)
Generator
$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^{m} \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right)$ or $\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^{m} -\log\left(D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right)$

Algorithm 1 Minibatch stochastic gradient descent training of generative adversarial nets. The number of steps to apply to the discriminator, k, is a hyperparameter. We used $k = 1$, the least expensive option, in our experiments.

for number of training iterations **do**

  for k steps **do**

   • Sample minibatch of m noise samples $\{\boldsymbol{z}^{(1)}, \dots, \boldsymbol{z}^{(m)}\}$ from noise prior $p_g(\boldsymbol{z})$.

   • Sample minibatch of m examples $\{\boldsymbol{x}^{(1)}, \dots, \boldsymbol{x}^{(m)}\}$ from data generating distribution $p_{\text{data}}(\boldsymbol{x})$.

   • Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^{m} \left[\log D\left(\boldsymbol{x}^{(i)}\right) + \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right)\right].$$

  end for

  • Sample minibatch of m noise samples $\{\boldsymbol{z}^{(1)}, \dots, \boldsymbol{z}^{(m)}\}$ from noise prior $p_g(\boldsymbol{z})$.

  • Update the generator by descending its stochastic gradient:

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^{m} \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right).$$

end for

The gradient-based updates can use any standard gradient-based learning rule. We used momentum in our experiments.

$$\min_G \max_D V(D,G)$$

$$V(D,G) = \mathbb{E}_{x \sim p_{data}(x)}[\log D(x)] + \mathbb{E}_{z \sim p_z(z)}[\log(1 - D(G(z)))]$$

$$\begin{aligned} V(G,D) &= \int_{\boldsymbol{x}} p_{\text{data}}(\boldsymbol{x}) \log(D(\boldsymbol{x}))dx + \int_z p_{\boldsymbol{z}}(\boldsymbol{z}) \log(1 - D(g(\boldsymbol{z})))dz \\ &= \int_{\boldsymbol{x}} p_{\text{data}}(\boldsymbol{x}) \log(D(\boldsymbol{x})) + p_g(\boldsymbol{x}) \log(1 - D(\boldsymbol{x}))dx \end{aligned} \tag{3}$$

For any $(a,b) \in \mathbb{R}^2 \setminus \{0,0\}$, the function $y \to a\log(y) + b\log(1-y)$ achieves its maximum in $[0,1]$ at $\frac{a}{a+b}$. The discriminator does not need to be defined outside of $Supp(p_{\text{data}}) \cup Supp(p_g)$, concluding the proof. □

Note that the training objective for D can be interpreted as maximizing the log-likelihood for estimating the conditional probability $P(Y = y|\boldsymbol{x})$, where Y indicates whether $\boldsymbol{x}$ comes from p_{data} (with $y = 1$) or from p_g (with $y = 0$). The minimax game in Eq. 1 can now be reformulated as:

$$\begin{aligned} C(G) &= \max_D V(G,D) \\ &= \mathbb{E}_{\boldsymbol{x} \sim p_{\text{data}}}[\log D_G^*(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{z} \sim p_{\boldsymbol{z}}}[\log(1 - D_G^*(G(\boldsymbol{z})))] \\ &= \mathbb{E}_{\boldsymbol{x} \sim p_{\text{data}}}[\log D_G^*(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{x} \sim p_g}[\log(1 - D_G^*(\boldsymbol{x}))] \\ &= \mathbb{E}_{\boldsymbol{x} \sim p_{\text{data}}}\left[\log \frac{p_{\text{data}}(\boldsymbol{x})}{P_{\text{data}}(\boldsymbol{x}) + p_g(\boldsymbol{x})}\right] + \mathbb{E}_{\boldsymbol{x} \sim p_g}\left[\log \frac{p_g(\boldsymbol{x})}{p_{\text{data}}(\boldsymbol{x}) + p_g(\boldsymbol{x})}\right] \end{aligned} \tag{4}$$

Theorem 1. *The global minimum of the virtual training criterion $C(G)$ is achieved if and only if $p_g = p_{data}$. At that point, $C(G)$ achieves the value $-\log 4$.*

Proof. For $p_g = p_{\text{data}}$, $D^*_G(\boldsymbol{x}) = \frac{1}{2}$, (consider Eq. 2). Hence, by inspecting Eq. 4 at $D^*_G(\boldsymbol{x}) = \frac{1}{2}$, we find $C(G) = \log \frac{1}{2} + \log \frac{1}{2} = -\log 4$. To see that this is the best possible value of $C(G)$, reached only for $p_g = p_{\text{data}}$, observe that

$$\mathbb{E}_{\boldsymbol{x}\sim p_{\text{data}}}\left[-\log 2\right] + \mathbb{E}_{\boldsymbol{x}\sim p_g}\left[-\log 2\right] = -\log 4$$

and that by subtracting this expression from $C(G) = V(D^*_G, G)$, we obtain:

$$C(G) = -\log(4) + KL\left(p_{\text{data}} \left\| \frac{p_{\text{data}} + p_g}{2}\right.\right) + KL\left(p_g \left\| \frac{p_{\text{data}} + p_g}{2}\right.\right) \tag{5}$$

where KL is the Kullback–Leibler divergence. We recognize in the previous expression the Jensen–Shannon divergence between the model's distribution and the data generating process:

$$C(G) = -\log(4) + 2 \cdot JSD\left(p_{\text{data}} \| p_g\right) \tag{6}$$

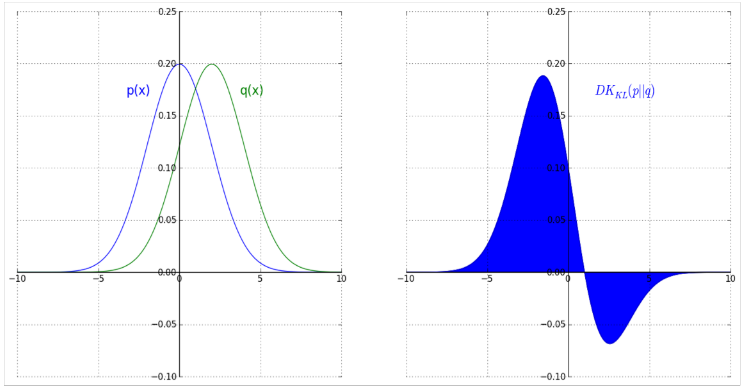
Since the Jensen–Shannon divergence between two distributions is always non-negative, and zero iff they are equal, we have shown that $C^* = -\log(4)$ is the global minimum of $C(G)$ and that the only solution is $p_g = p_{\text{data}}$, i.e., the generative model perfectly replicating the data distribution. $\square$

KL and JS Divergence

$$D_{\mathrm{KL}}(P \parallel Q) = -\sum_i P(i) \log\left(\frac{Q(i)}{P(i)}\right),$$

which is equivalent to

$$D_{\mathrm{KL}}(P \parallel Q) = \sum_i P(i) \log\left(\frac{P(i)}{Q(i)}\right).$$



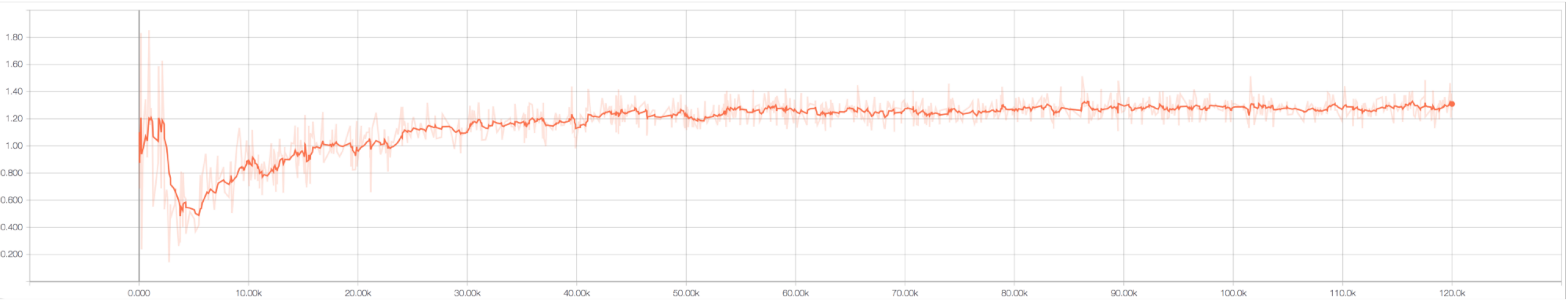
The Jensen–Shannon divergence (JSD) $M_+^1(A) \times M_+^1(A) \to [0, \infty)$ is a symmetrized and smoothed version of the Kullback–Leibler divergence $D(P \parallel Q)$. It is defined by

$$\mathrm{JSD}(P \parallel Q) = \frac{1}{2}D(P \parallel M) + \frac{1}{2}D(Q \parallel M)$$

where $M = \frac{1}{2}(P + Q)$

GENERATOR ERROR

DISCRIMINATOR ERROR



Performance Evaluation

Model	MNIST	TFD
DBN [3]	138 ± 2	1909 ± 66
Stacked CAE [3]	121 ± 1.6	$\mathbf{2110 \pm 50}$
Deep GSN [5]	214 ± 1.1	1890 ± 29
Adversarial nets	$\mathbf{225 \pm 2}$	$\mathbf{2057 \pm 26}$

Table 1: Parzen window-based log-likelihood estimates. The reported numbers on MNIST are the mean log-likelihood of samples on test set, with the standard error of the mean computed across examples. On TFD, we computed the standard error across folds of the dataset, with a different σ chosen using the validation set of each fold. On TFD, σ was cross validated on each fold and mean log-likelihood on each fold were computed. For MNIST we compare against other models of the real-valued (rather than binary) version of dataset.