Mechanics of Materials Formula and Data Sheet

Some Engineering Material Properties:

Property	Steel	Aluminum	Brass
Young's Modulus, **E**	210 GPa (30 psi x 10^6)	70 GPa (10 psi x 10^6)	105 GPa (15 psi x 10^6)
Shear Modulus, **G**	81 GPa (11.6 psi x 10^6)	26 GPa (3.7 psi x 10^6)	39 GPa (5.6 psi x 10^6)
Poisson's Ratio, ν	0.30	0.33	0.35
Thermal Expansion, α	11×10^{-6} /°C (6×10^{-6} °/F)	22×10^{-6} /°C (12×10^{-6} °/F)	20×10^{-6} /°C (11×10^{-6} °/F)
Density, ρ	7850 kg/m^3 (0.28 lb/in^3)	2720 kg/m^3 (0.10 lb/in^3)	8410 kg/m^3 (0.30 lb/in^3)
Yield stress, σ_Y	250 MPa (36 x 10^3 psi)	230 MPa (33 x 10^3 psi)	210 MPa (30 x 10^3 psi)

Moments of Area for Common Shapes:

Shape	Cross-section A	I_x	I_y	$J = I_x + I_y$
solid circle	πc^2	$\pi c^4 / 4$	$\pi c^4 / 4$	$\pi c^4 / 2$
thin-wall circle	$2 \pi t c$	$\pi t c^3$	$\pi t c^3$	$2 \pi t c^3$
solid rectangle	$b h$	$b h^3 / 12$	$h b^3 / 12$	$b h (b^2 + h^2) / 12$

Elastic relationships:

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E} + \alpha \Delta T \qquad \gamma = \frac{\tau}{G} \qquad G = \frac{E}{2(1+\nu)} \qquad K = \frac{E}{3(1-2\nu)}$$

Failure Criteria:

(for principal stresses $\sigma_1 \geq \sigma_2 \geq \sigma_3$)

Tresca: $\sigma_1 - \sigma_3 = \sigma_Y$ **Mises**: $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_Y^2$ **Mohr**: $\sigma_1/\sigma_{UT} - \sigma_3/\sigma_{UC} = 1$

Thin-Walled Pressure Vessels:

$$\text{Cylinder:} \quad \sigma_\theta = \frac{pR}{t} \quad \sigma_a = \frac{pR}{2t} \quad \sigma_r \approx 0 \qquad \text{Sphere:} \quad \sigma_\theta = \sigma_a = \frac{pR}{2t} \quad \sigma_r \approx 0$$

Shaft Torsion:

$$\frac{\tau}{r} = \frac{T}{J} = \frac{G\phi}{L}$$

Beam Bending:

$$\frac{\sigma}{-Y} = \frac{M}{I} = \frac{E}{R}$$

Parallel Axis Theorem:

$$I(d) = I_0 + A d^2$$

Power Transmission:

$$P = \frac{2\pi f J \tau_{max}}{c} \text{ watts } \textit{(metric units)} \qquad P = \frac{15.87 \times 10^{-6} \text{ rpm } J \tau_{max}}{c} \text{ h.p. } \textit{(lb.in units)}$$

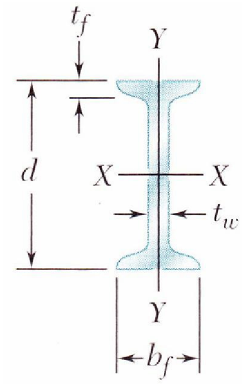
Beam Formulas:

$$V = \frac{dM}{dx} \qquad w = \frac{dV}{dx} \qquad \frac{d^2y}{dx^2} = \frac{M}{EI} \qquad q = \frac{VA\bar{y}}{I} = \frac{VQ}{I} \text{ where } Q = \int y\, dA$$

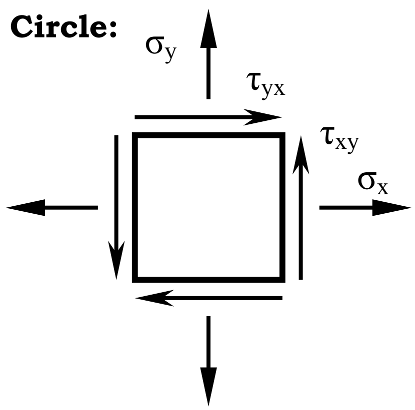
$$\langle x - a \rangle = (x - a) \text{ if } x - a > 0, \quad = 0 \text{ if } x - a \leq 0$$

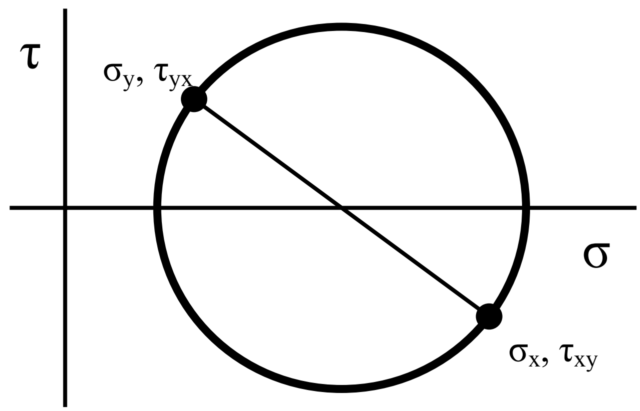
Some Typical I-Beams:

Designation†	Area A, mm²	Depth d, mm	Flange Width b_f, mm	Flange Thickness t_f, mm	Web Thickness t_w, mm	Axis X-X I_x 10^6 mm⁴	Axis X-X S_x 10^3 mm³	Axis X-X r_x mm	Axis Y-Y I_y 10^6 mm⁴	Axis Y-Y S_y 10^3 mm³	Axis Y-Y r_y mm
S150 × 25.7	3260	152	90.7	9.12	11.8	10.9	143	57.9	0.953	21.0	17.1
18.6	2360	152	84.6	9.12	5.89	9.16	120	62.2	0.749	17.7	17.8
S130 × 15	1890	127	76.2	8.28	5.44	5.12	80.3	52.1	0.495	13.0	16.2
S100 × 14.1	1800	102	71.1	7.44	8.28	2.81	55.4	39.6	0.369	10.4	14.3
11.5	1460	102	67.6	7.44	4.90	2.52	49.7	41.7	0.311	9.21	14.6
S75 × 11.2	1420	76.2	63.8	6.60	8.86	1.21	31.8	29.2	0.241	7.55	13.0
8.5	1070	76.2	59.2	6.60	4.32	1.04	27.4	31.2	0.186	6.28	13.2



Mohr's Circle:





Use sign convention "***in the kitchen, the <u>clock is above</u> and the <u>counter is below</u>***" for the shear stresses. Then, rotations in the Mohr's circle have the same direction and double the rotation angle of the physical stresses. For Mohr's circle of strain, use **ε** and **γ/2** in place of **σ** and **τ**.

Column Buckling:

$$P_{CR} = \frac{\pi^2 EI}{L_e^{\,2}} = \frac{\pi^2 EA}{(L_e/r)^2}$$

where: $L_e = L$ (pinned-pinned) $\qquad L_e = 2\,L$ (free-fixed)
$L_e = 0.7\,L$ (pinned-fixed) $\qquad L_e = 0.5\,L$ (sliding-fixed)

$$\text{Eccentric load: } y_{max} = e\left(\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{CR}}}\right) - 1\right) \qquad \text{Initial curvature: } y_{max} = \frac{a\,P}{P_{CR} - P}$$

$$I = A\,r^2 \qquad \text{For "short" steel columns (Johnson): } \frac{L_e}{r} < \sqrt{\frac{2\pi^2 E}{\sigma_Y}} \qquad \sigma_{CR} = \sigma_Y - \left(\frac{\sigma_Y^{\,2}}{4\pi^2 E}\right)\left(\frac{L_e}{r}\right)^2$$

Strain Energy:

$$\text{rod: } U = \int \frac{P^2}{2EA}\,dx \qquad \text{beam: } U = \int \frac{M^2}{2EI}\,dx \qquad \text{shaft: } U = \int \frac{T^2}{2GJ}\,dx$$

Castigliano's Theorem:

$$\delta_j = \frac{\partial U}{\partial P_j} \qquad \theta_j = \frac{\partial U}{\partial M_j} \qquad \text{rod: } \delta_j = \int \frac{P}{EA}\frac{\partial P}{\partial P_j}\,dx \qquad \text{beam: } \delta_j = \int \frac{M}{EI}\frac{\partial M}{\partial P_j}\,dx$$