Shigley's Mechanical Engineering Design | (10th Edition)

Chapter 14, Problem 22P | 6 Bookmarks | Show all steps: ON

Problem

A spur gearset has 17 teeth on the pinion and 51 teeth on the gear. The pressure angle is 20 and the overload factor*k0* = 1. The diametral pitch is 6 teeth/in and the face width is 2 in. The pinion speed is 1120 rev/min and its cycle life is to be 10^8 revolutions at a reliability*R* = 0.99. The quality number is 5. The material is a through-hardened steel, grade 1, with Brinell hardnesses of 232 core and case of both gears. For a design factor of 2, rate the gearset for these conditions using the AGMA method.

Step-by-step solution

Step 1 of 17

Obtain the following properties for pinion from the figure 14-6 "spur gear geometry factors *J*".

$$N_P = 17T,\ J_P = 0.292$$

Obtain the following properties for gear from the figure 14-6 "spur gear geometry factors *J*".

$$N_G = 51T,\ J_G = 0.396$$

Comment

Step 2 of 17

Find the diameter of pinion using the equation.

$$d_P = \frac{N_P}{P_d}$$

Here, N_P is the pinion tooth and P_d is the diametral pitch.

Substitute $6\,\text{teeth/in}$ for P_d and 17 for N_P.

$$\begin{aligned} d_P &= \frac{17}{6} \\ &= 2.833\ \text{in} \end{aligned}$$

Find the diameter of gear using the equation.

$$d_G = \frac{N_G}{P_d}$$

Here, N_G is the gear tooth and P_d is the diametral pitch.

Substitute $6\,\text{teeth/in}$ for P_d and 51 for N_G.

$$\begin{aligned} d_G &= \frac{51}{6} \\ &= 8.5\ \text{in} \end{aligned}$$

Comment

Step 3 of 17

Find the velocity using the equation.

Substitute $1120\,\text{rev/min}$ for n and 2.833 in for d_p.

$$\begin{aligned}V &= \pi \times 2.833 \times 1120\\ &= 9968.147\text{ in/min}\\ &= 830.7\text{ ft/min}\end{aligned}$$

Comment

Step 4 of 17

Assume constant thickness of the gears.

$$K_B = 1$$

Assume uniform loading.

$$k_o = 1$$

Find the value of *B* using the equation.

$$\begin{aligned}B &= 0.25\left(12 - Q_v\right)^{\frac{2}{3}}\\ &= 0.25\left(12-5\right)^{2/3}\\ &= 0.9148\end{aligned}$$

Here, Q_v is quality number.

Find the value of *A* using the equation.

$$\begin{aligned}A &= 50 + 56\left(1 - B\right)\\ &= 50 + 56\left(1 - 0.9148\right)\\ &= 54.77\end{aligned}$$

Comment

Step 5 of 17

Find the dynamic factor using the equation.

$$K_v = \left(\frac{A + \sqrt{V}}{A}\right)^B \text{ when, } V \text{ in ft/min}$$

Substitute 0.9148 for B and 54.77 for A.

$$\begin{aligned}K_v &= \left(\frac{54.77 + \sqrt{830.7}}{54.77}\right)^{0.9148}\\ &= 1.472\end{aligned}$$

From elastic coefficient C_p

$$C_p = 2300\sqrt{\text{psi}}$$

Comment

Step 6 of 17

Consider pinion bending

Obtain SI equation from figure of 14-3 “allowable bending stresses number through hardened steels” for grade-1.

The SI equations is

$$\begin{aligned}S_t &= 77.3H_B + 12800\\ &= 77.3\left(232\right) + 12800\\ &= 30734\text{ psi}\end{aligned}$$

Stress-cycle factor for bending,

$$\begin{aligned} Y_N &= 1.6831(N)^{-0.0323} \\ &= 1.6831(10^8)^{-0.0323} \\ &= 0.928 \end{aligned}$$

Comment

Step 7 of 17

Find the allowable stress

$$\sigma_{all} = \frac{S_t}{S_F}\frac{Y_N}{K_T K_R}$$

Here, S_F is the AGMA factor of safety, K_T is temperature factor and K_R is the reliability factor.

Substitute 0.928 for Y_N, 30734 psi for S_t, 2 for S_F, 1 for K_T and 1 for K_R.

$$\begin{aligned} \sigma_{all} &= \frac{30734(0.928)}{2(1)(1)} \\ &= 14261\text{ psi} \end{aligned}$$

Comment

Step 8 of 17

Obtain the Lewis form factor for pinion from the table of 14-2 “values of the Lewis Form Factor *Y* (These values are for a normal pressure Angle of $20°$, Full depth Teeth and a diametral Pitch of unity in the Plane of Rotation)” at number of teeth of pinion is 17.

Lewis form factor for pinion, $Y_P = 0.303$

Obtain the Lewis form factor for gear from the table of 14-2 “values of the Lewis form factor *Y* (These values are for a normal pressure Angle of $20°$, Full depth Teeth and a diametral Pitch of unity in the Plane of Rotation)” at number of teeth of gear is 51.

Lewis form factor for gear, $Y_G = 0.4103$

Comment

Step 9 of 17

Find the size factor for pinion using the equation.

$$(K_s)_p = 1.192\left(\frac{F\sqrt{Y_p}}{p}\right)^{0.0535}$$

Here, *F* is the face width.

Substitute 2 in for *F* and 0.303 for Y_p.

$$\begin{aligned} (K_s)_p &= 1.192\left(\frac{2\sqrt{0.296}}{6}\right)^{0.0535} \\ &= 1.088 \end{aligned}$$

Find the size factor for gear using the equation.

$$\begin{aligned} (K_s)_G &= 1.192\left(\frac{F\sqrt{Y_G}}{p}\right)^{0.0535} \\ &= 1.192\left(\frac{2\sqrt{0.4103}}{6}\right)^{0.0535} \\ &= 1.097 \end{aligned}$$

Step 10 of 17

Find the load distribution factor using the equation.

$$K_m = C_{mf} = 1 + C_{mc}\left(C_{pf}C_{pm} + C_{ma}C_e\right)$$

Here,

C_{mf} is load distribution factor

$$C_{mc} = \begin{Bmatrix} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{Bmatrix}$$

$$C_{pf} = \begin{Bmatrix} \dfrac{F}{10d} - 0.025 & F \le 1\,\text{in} \\ \dfrac{F}{10d} - 0.0375 + 0.0125F & 1 \le F \le 17\,\text{in} \\ \dfrac{F}{10d} - 0.1109 + 0.0207F - 0.000228F^2 & 17 \le F \le 40\,\text{in} \end{Bmatrix}$$

$$C_{pm} = \begin{Bmatrix} 1 & \text{for straddle mounted pinion with S1/S<0.175} \\ 1.1 & \text{for straddle mounted pinion with S1/S} \ge 0.175 \end{Bmatrix}$$

$C_{ma} = A + BF + CF^2$ Where A, B, C are Empirical constants

$$C_e = \begin{Bmatrix} 0.8 & \text{for gearing adjusted assembly} \\ 1 & \text{for all other conditions} \end{Bmatrix}$$

Comment

Step 11 of 17

Find the following values.

$$\begin{aligned} C_{pf} &= \frac{F}{10d} - 0.0375 + 0.0125F \\ &= \frac{2}{10(2.833)} - 0.0375 + 0.0125(2) \\ &= 0.0581 \end{aligned}$$

$C_{mc} = 1$

$C_{pm} = 1$

$$\begin{aligned} C_{ma} &= A + BF + CF^2 \\ &= 0.127 + 0.0158(2) - 0.093(10^{-4})(2^2) \\ &= 0.1586 \end{aligned}$$

$C_e = 1$

Comment

Step 12 of 17

Substitute 0.1586 for C_{ma}, 1 for C_{pm}, 1 for C_{mc}, and 1 for C_e in equation (1).

$$\begin{aligned} K_m &= C_{mf} \\ &= 1 + C_{mc}\left(C_{pf}C_{pm} + C_{ma}C_e\right) \\ &= 1 + 1\left[0.0581(1) + 0.1586(1)\right] \\ &= 1.2167 \end{aligned}$$

Comment

Find the load acting using the equation.

$$W^t = \frac{FJ_P\sigma_{all}}{K_oK_v(K_s)_pP_dK_mK_B}$$

Substitute 1 for K_B, 1 for K_o, 1.2167 for K_m, 2 in for *F*, 0.292 for J_P, 14261 psi for σ_{all}, 1.088 for $(K_s)_p$.

$$W^t = \frac{2(0.292)(14261)}{1(1.472)(1.088)(6)(1.2167)(1)} = 712 \text{ lbf}$$

Find the horse power required using the equation.

$$H = \frac{W^tV}{33000}$$

Substitute 712 lbf for W^t and 830.7 ft/min.

$$H = \frac{775(830.7)}{33000} = 17.92 \text{ hp}$$

Comment

Step 14 of 17

Consider the pinion wear

Obtain the stress cycle factor from figure of 14-15 "pitting resistance stress cycle" for 10^8.

$$Z_N = 2.466N^{-0.056} = 2.466(10^8)^{-0.056} = 0.879$$

Find the gear ratio using the equation.

$$m_G = \frac{N_G}{N_P} = \frac{51}{17} = 3$$

Comment

Step 15 of 17

Find the geometry factor using the equation.

$$I = \frac{\cos\phi_t \sin\phi_t}{2m_N}\left(\frac{m_G}{m_G + 1}\right)$$

Here, ϕ_t is the pressure angle.

Substitute 1 for m_N, and 3 for m_G.

$$I = \left[\frac{\sin 20° \cos 20°}{2(1)}\right]\left(\frac{3}{3+1}\right) = 0.1205$$

Comment

Step 16 of 17

$$\frac{H_{BP}}{H_{BG}} = C_H$$
$$= 1$$

Obtain the SI equation from figure of 14-5 "contact fatigue strength at 10^7 cycles and 0.99 reliability for steel gears" for grade 1.

$$\begin{aligned}(S_c) &= 322(H_B) + 29100\,\text{psi} \\ &= 322(232) + 29100 \\ &= 103804\ \text{psi}\end{aligned}$$

Comment

Step 17 of 17

Find the allowable contact stress using the equation.

$$\sigma_{c,all} = \frac{S_c(Z_N)}{S_H(C_H)(1)}$$

Substitute $103804\ \text{psi}$ for S_c, 0.879 for Z_N, 1 for C_H and $\sqrt{2}$ for S_H.

$$\begin{aligned}\sigma_{c,all} &= \frac{103804(0.879)}{\sqrt{2}(1)(1)} \\ &= 64519\ \text{psi}\end{aligned}$$

Find the load acting using the equation.

$$W^t = \left(\frac{\sigma_{c,all}}{C_p}\right)^2 \frac{Fd_p\, I}{K_o K_v K_s K_m C_f}$$

Substitute 0.1205 for I, $64519\ \text{psi}$ for $\sigma_{c,all}$, $2300\sqrt{\text{psi}}$ for C_p and 1 for C_f.

$$\begin{aligned}W^t &= \left(\frac{64519}{2300}\right)^2 \left[\frac{2(2.833)(0.1205)}{1(1.472)(1.088)(1.2167)(1)}\right] \\ &= 275.71\ \text{lbf}\end{aligned}$$

Find the horse power required using the equation.

$$H = \frac{W^t V}{33000}$$

Substitute $275.71\ \text{lbf}$ for W^t and $830.7\ \text{ft/min}$.

$$\begin{aligned}H &= \frac{W^t V}{33000} \\ &= \frac{275.71 \times 830.7}{33000} \\ &= 6.94\ \text{hp}\end{aligned}$$

From the values of rated horse power is minimum value controls the pinion.

$$H_{rated} = 6.94\ \text{hp}$$

Therefore, required rated horse power is $\boxed{6.94\ \text{hp}}$.

Comment

Was this solution helpful? 14 0

Recommended solutions for you in Chapter 14

Chapter 14, Problem 11P

Chapter 14, Problem 16P