Shigley's Mechanical Engineering Design | (10th Edition)

Chapter 13, Problem 36P | 3 Bookmarks | Show all steps: ON

Problem

Continue Prob. 13?24 by finding the following information, assuming a diametral pitch of 6 teeth/in.

(*a*) Determine pitch diameters for each of the gears.

(*b*) Determine the pitch line velocities (in ft/min) for each set of gears.

(*c*) Determine the magnitudes of the tangential, radial, and total forces transmitted between each set of gears.

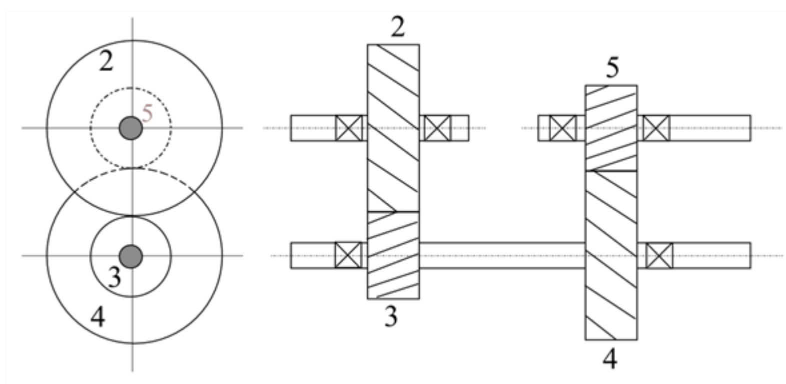
(*d*) Determine the input torque.

(*e*) Determine the output torque, neglecting frictional losses.

Step-by-step solution

Step 1 of 20

Draw the schematic of the compound reverted gear train.



Comment

Step 2 of 20

To minimize gear size for minimal gear ratio let $\omega_o = 300\,\text{rev/min}$.

Calculate the ratio of the rotational speed.

$$\frac{\omega_o}{\omega_i} = \frac{N_2}{N_3}\frac{N_4}{N_5} \quad \text{...... (1)}$$

Here, the input speed is ω_i, the output speed is ω_o, the number of teeth on gear 2 is N_2, the number of teeth on gear 3 is N_3, the number of teeth on gear 4 is N_4, and the number of teeth on gear 5 is N_5.

Substitute $300\,\text{rev/min}$ for ω_o and $2500\,\text{rev/min}$ for ω_i.

$$\frac{N_2}{N_3}\frac{N_4}{N_5} = \frac{300}{2500}$$
$$\frac{N_2}{N_3}\frac{N_4}{N_5} = \frac{1}{8.333} \quad \text{...... (2)}$$

Step 3 of 20

Let $\frac{N_2}{N_3}=\frac{N_4}{N_5}$

Substitute $\frac{N_2}{N_3}$ for $\frac{N_4}{N_5}$ in equation (2).

$$\frac{N_2}{N_3}\frac{N_4}{N_5}=\frac{1}{8.333}$$

$$\frac{N_2}{N_3}\left(\frac{N_2}{N_3}\right)=\frac{1}{8.333}$$

$$\left(\frac{N_2}{N_3}\right)^2=\frac{1}{8.333} \quad \text{...... (3)}$$

$$\frac{N_2}{N_3}=\frac{1}{2.887}$$

Comment

Step 4 of 20

Calculate the minimum number of teeth on smallest gear to avoid interference.

$$N_p \geq \frac{2k}{(1+2m)\sin^2\phi}\left(m+\sqrt{m^2+(1+2m)\sin^2\phi}\right)$$

Here, the gear ratio is m, the constant is k and the pressure angle is ϕ.

Substitute 1 for $k(\text{full depth teeth})$, 2.887 for m, and $20°$ for ϕ.

$$\begin{aligned} N_p &\geq \frac{2(1)}{(1+2(2.887))\sin^2(20°)}\left(2.887+\sqrt{2.887^2+(1+2(2.887))\sin^2(20°)}\right) \\ &\geq 14.9 \end{aligned}$$

Number of teeth should be an integer value. Therefore, consider the nearest integer.

$$N_p = 15$$

Let,

$$N_2 = 15$$
$$N_4 = 15$$

Substitute 15 for N_2 in equation (3).

$$\frac{N_2}{N_3}=\frac{1}{2.887}$$

$$\frac{15}{N_3}=\frac{1}{2.887}$$

$$\begin{aligned} N_3 &= 2.887(15) \\ &= 43.31\,\text{teeth} \end{aligned}$$

Comment

Step 5 of 20

Try,

$$N_3 = 43\,\text{teeth}$$
$$N_5 = 43\,\text{teeth}$$

Substitute $2500\,\text{rev/min}$ for ω_i, $15\,\text{teeth}$ for N_2, N_4, and $43\,\text{teeth}$ for N_3, N_5 in equation (1).

$$\omega_o = \left(\frac{15}{43}\right)\left(\frac{15}{43}\right)(2500)$$
$$= 304.2\,\text{rev/min}$$

Since, the output speed is not in the specified range.

So now try,

$N_3 = 44\,\text{teeth}$
$N_5 = 44\,\text{teeth}$

Substitute $2500\,\text{rev/min}$ for ω_i, $15\,\text{teeth}$ for N_2, N_4, and $44\,\text{teeth}$ for N_3, N_5 in equation (1).

$$\frac{\omega_o}{\omega_i} = \frac{N_2}{N_3}\frac{N_4}{N_5}$$
$$\omega_o = \left(\frac{15}{44}\right)\left(\frac{15}{44}\right)(2500)$$
$$= 290.55\,\text{rev/min}$$

Therefore, the out speed is within range for N_2, N_4 with $15\,\text{teeth}$ and N_3, N_5 with $44\,\text{teeth}$.

Comment

Step 6 of 20

(a)

Calculate the pitch diameter for gear 2 using the following equation:

$$d_2 = \frac{N_2}{P}$$

Here, the diametric pitch is P.

Substitute $15\,\text{teeth}$ for N_2 and $6\,\text{teeth/in}$ for P.

$$d_2 = \frac{15}{6}$$
$$= 2.5\,\text{in}$$

Therefore, the pitch diameter for gear 2 is $\boxed{2.5\,\text{in}}$.

Comment

Step 7 of 20

Calculate the pitch diameter for gear 4 using the following equation:

$$d_4 = \frac{N_4}{P}$$

Substitute $15\,\text{teeth}$ for N_4 and $6\,\text{teeth/in}$ for P.

$$d_4 = \frac{15}{6}$$
$$= 2.5\,\text{in}$$

Therefore, the pitch diameter for gear 4 is $\boxed{2.5\,\text{in}}$.

Comment

Step 8 of 20

Calculate the pitch diameter for gear 3 using the following equation:

$$d_3 = \frac{N_3}{P}$$

Substitute $44\,\text{teeth}$ for N_3 and $6\,\text{teeth/in}$ for P.

Therefore, the pitch diameter for gear 3 is $\boxed{7.33\,\text{in}}$.

Comment

Step 9 of 20

Calculate the pitch diameter for gear 5 using the following equation:

$$d_5 = \frac{N_5}{p}$$

Substitute $44\,\text{teeth}$ for N_5 and $6\,\text{teeth/in}$ for P.

$$d_5 = \frac{44}{6} = 7.33\,\text{in}$$

Therefore, the pitch diameter for 5 is $\boxed{7.33\,\text{in}}$.

Comment

Step 10 of 20

(b)

Calculate the pitch line velocities for the gear set 2,3 using the following equation:

$$V_i = \frac{\pi d_2 n_2}{12}$$

Here, the rotational speed of gear 2 is n_2.

Substitute $2.5\,\text{in}$ for d_2 and $2500\,\text{rev/min}$ for n_2.

$$V_i = \frac{\pi(2.5)(2500)}{12} = 1636\,\text{ft/min}$$

Therefore, the pitch line velocity for the gear set 2,3 is $\boxed{1636\,\text{ft/min}}$.

Comment

Step 11 of 20

Calculate the following relation for the mating gears 2-3.

$$n_3 = \frac{N_2}{N_3}(n_2)$$

Since gears 3 and 4 lie on the same shaft. So, the rotational speeds of gears 3 and 4 are same.

$$n_4 = n_3$$

$$n_4 = \frac{N_2}{N_3}(n_2)$$

Substitute $15\,\text{teeth}$ for N_2, $44\,\text{teeth}$ for N_3, and $2500\,\text{rev/min}$ for n_2.

$$n_4 = \frac{15}{44}(2500) = 852.27\,\text{rev/min}$$

Comment

Calculate the pitch line velocities for the gear set 4,5 using the following equation:

$$V_o = \frac{\pi d_4 n_4}{12}$$

Substitute $2.5\,\text{in}$ for d_4 and $852.27\,\text{rev/min}$ for n_4.

$$\begin{aligned} V_o &= \frac{\pi(2.5)(852.27)}{12} \\ &= 557.8\,\text{ft/min} \end{aligned}$$

Therefore, the pitch line velocity for the gear set 4,5 is $\boxed{557.8\,\text{ft/min}}$.

Comment

Step 13 of 20

(c)

Input gears:

Calculate the tangential force transmitted by the input gears using the following relation:

$$W_{ti} = 33000\frac{H}{V_i}$$

Here, the power transmitted is H.

Substitute $25\,\text{hp}$ for H and $1636\,\text{ft/min}$ for V_i.

$$\begin{aligned} W_{ti} &= 33000\frac{(25)}{1636} \\ &= 504.3\,\text{lbf} \end{aligned}$$

Therefore, the tangential force transmitted by input gears is $\boxed{504.3\,\text{lbf}}$.

Comment

Step 14 of 20

Calculate the radial force transmitted by the input gears using the following relation:

$$W_{ri} = W_{ti}\tan\phi$$

Substitute $504.3\,\text{lbf}$ for W_{ti} and $20°$ for ϕ.

$$\begin{aligned} W_{ri} &= 504.3\tan(20°) \\ &= 184\,\text{lbf} \end{aligned}$$

Therefore, the radial force transmitted by input gears is $\boxed{184\,\text{lbf}}$.

Comment

Step 15 of 20

Calculate the total force transmitted by the input gears using the following relation:

$$W_i = \frac{W_{ti}}{\cos\phi}$$

Substitute $504.3\,\text{lbf}$ for W_{ti} and $20°$ for ϕ.

$$\begin{aligned} W_i &= \frac{504.3}{\cos(20°)} \\ &= 537\,\text{lbf} \end{aligned}$$

Therefore, the total force transmitted by input gears is $\boxed{537\,\text{lbf}}$.

Step 16 of 20

Output gears:

Calculate the tangential force transmitted by the output gears using the following relation:

$$W_{to} = 33000\frac{H}{V_o}$$

Substitute $25\,\text{hp}$ for H and $558\,\text{ft/min}$ for V_o.

$$\begin{aligned} W_{to} &= 33000\frac{(25)}{558} \\ &= 1478\,\text{lbf} \end{aligned}$$

Therefore, the tangential force transmitted by output gears is $\boxed{1478\,\text{lbf}}$.

Comment

Step 17 of 20

Calculate the radial force transmitted by the output gears using the following relation:

$$W_{ro} = W_{to}\tan\phi$$

Substitute $1478\,\text{lbf}$ for W_{to} and $20°$ for ϕ.

$$\begin{aligned} W_{ro} &= 1478\tan(20°) \\ &= 538\,\text{lbf} \end{aligned}$$

Therefore, the radial force transmitted by output gears is $\boxed{538\,\text{lbf}}$.

Comment

Step 18 of 20

Calculate the total force transmitted by the output gears using the following relation:

$$W_o = \frac{W_{to}}{\cos\phi}$$

Substitute $1478\,\text{lbf}$ for W_{to} and $20°$ for ϕ.

$$\begin{aligned} W_i &= \frac{1478}{\cos(20°)} \\ &= 1573\,\text{lbf} \end{aligned}$$

Therefore, the total force transmitted by output gears is $\boxed{1573\,\text{lbf}}$.

Comment

Step 19 of 20

(d)

Calculate the input torque using the following relation:

$$T_i = W_{ti}\left(\frac{d_2}{2}\right)$$

Substitute $504.3\,\text{lbf}$ for W_{ti} and $2.5\,\text{in}$ for d_2.

$$\begin{aligned} T_i &= 504.3\left(\frac{2.5}{2}\right) \\ &= 630\,\text{lbf}\cdot\text{in} \end{aligned}$$

Comment

Step **20** of 20

(e)

Calculate the output torque using the following relation:

$$T_i\omega_i = T_o\omega_o$$

$$\frac{T_i}{T_o} = \frac{\omega_o}{\omega_i}$$

$$\frac{T_i}{T_o} = \frac{N_2}{N_3}\frac{N_4}{N_5}$$

Let $\frac{N_2}{N_3} = \frac{N_4}{N_5}$

$$\frac{T_i}{T_o} = \frac{N_2}{N_3}\frac{N_2}{N_3}$$

$$\frac{T_i}{T_o} = \left(\frac{N_2}{N_3}\right)^2$$

$$T_o = T_i\left(\frac{N_3}{N_2}\right)^2$$

Substitute $15\,\text{teeth}$ for N_2, $44\,\text{teeth}$ for N_3, and $630\,\text{lbf}\cdot\text{in}$ for T_i.

$$T_o = 630\left(\frac{44}{15}\right)^2$$
$$= 5420\,\text{lbf}\cdot\text{in}$$

Therefore, the output torque is $\boxed{5420\,\text{lbf}\cdot\text{in}}$.

Comment

Was this solution helpful? 16 0

Recommended solutions for you in Chapter 13

Chapter 13, Problem 38P

For the countershaft in Prob. 3?72, p. 138, assume the gear ratio from gear B to its mating gearis 2 to 1.(a) Determine the minimum number of teeth that can be used on gear B without an interference problem in the teeth.(b) Using the number of...

See solution

Chapter 13, Problem 7P

A parallel helical gearset consists of a 19-tooth pinion driving a 57-tooth gear. The pinion has a left-hand helix angle of 30º, a normal pressure angle of 20º, and a normal module of 2.5 mm. Find:(a) The normal, transversa and axial circular...

See solution