Shigley's Mechanical Engineering Design | (10th Edition)

Chapter 13, Problem 24P

Problem

A gearbox is to be designed with a compound reverted gear train that transmits 25 horsepower with an input speed of 2500 rev/min. The output should deliver the power at a rotational speed in the range of 280 to 300 rev/min. Spur gears with 20 pressure angle are to be used. Determine suitable numbers of teeth for each gear, to minimize the gearbox size while providing an output speed within the specified range. Be sure to avoid an interference problem in the teeth.

Step-by-step solution

Step 1 of 6

Write the governing equation for a compound reverted gear train.

$$N_2 + N_3 = N_4 + N_5$$

Here, N_2 is the teeth on gear 2, N_3 is the teeth on gear 3, N_4 is the teeth on gear 4, and N_5 is the teeth on gear 5.

Write the equation for the output speed.

$$280 \text{ rev/min} \le \omega_o \le 300 \text{ rev/min}$$

Assume the output speed of the gear train to minimize the gear size as,

$$\omega_o = 300 \text{ rev/min}$$

Calculate the ratio of output speed to the input speed $\left(\frac{\omega_o}{\omega_i}\right)$ of the compound reverted gear train.

Here, ω_i is the input speed of the compound reverted gear train.

Substitute 300 rev/min for ω_o and 2500 rev/min for ω_i.

$$\left(\frac{\omega_o}{\omega_i}\right) = \frac{300}{2500}$$

$$\left(\frac{\omega_o}{\omega_i}\right) = \frac{1}{8.333}$$

Comment

Step 2 of 6

Write the equation of the ratio of the output speed of the gear train to the input speed.

$$\left(\frac{\omega_o}{\omega_i}\right) = \frac{N_2}{N_3}\frac{N_4}{N_5}$$

Here, N is the number of teeth.

Assume $\frac{N_2}{N_3} = \frac{N_4}{N_5}$

Substitute $\frac{N_2}{N_3}$ for $\frac{N_4}{N_5}$ and $\frac{1}{8.333}$ for $\left(\frac{\omega_o}{\omega_i}\right)$.

$$\left(\frac{N_2}{N_3}\right)^2 = \frac{1}{8.333}$$

$$\left(\frac{N_2}{N_3}\right) = \frac{1}{\sqrt{8.333}}$$

$$\left(\frac{N_2}{N_3}\right) = \frac{1}{2.887}$$

Comment

Step 3 of 6

Calculate the smallest number of the pinion without interference.

$$N_P = \frac{2k}{(1+2m)\ \sin^2\phi}\left(m+\sqrt{m^2+(1+2m)\sin^2\phi}\right)$$

Substitute 1 for k, 2.887 for m and $20°$ for ϕ.

$$N_P = \frac{2(1)}{(1+2(2.887))\ (\sin 20°)^2}\left((2.887)+\sqrt{(2.887)^2+(1+2(2.887))(\sin 20°)^2}\right)$$

$$N_P = 14.91$$

$$N_P \approx 15 \text{ teeth}$$

Comment

Step 4 of 6

Assume,

$$N_2 = N_4 = 15 \text{ teeth}$$

Calculate the number of teeth on gear 3.

$$\left(\frac{N_2}{N_3}\right) = \frac{1}{2.887}$$

Substitute 15 for N_2.

$$\left(\frac{15}{N_3}\right) = \frac{1}{2.887}$$

$$N_3 = 43.3 \text{ teeth}$$

Comment

Step 5 of 6

Calculate the number of teeth on gear 5.

$$\left(\frac{N_4}{N_5}\right) = \frac{1}{2.887}$$

Substitute 15 for N_4.

$$\left(\frac{15}{N_5}\right) = \frac{1}{2.887}$$

$$N_5 = 43.3 \text{ teeth}$$

Consider,

$$N_3 = N_5 = 43 \text{ teeth}$$

$\left(\frac{\omega_o}{\omega_i}\right) = \frac{N_2}{N_3}\frac{N_4}{N_5}$

Substitute 15 for N_2, 15 for N_4, 43 for N_3, 43 for N_5 and 2500 rev/min for ω_i.

$$\left(\frac{\omega_o}{2500}\right) = \left(\frac{15}{43}\right)\left(\frac{15}{43}\right)$$

$$\omega_o = 304.2 \text{ rev/min}$$

Comment

Step 6 of 6

Here, by using the above assumption the output speed ω_o is greater than 300 rev/min,

Which does not fall within limits.

Assume,

$$N_3 = N_5 = 44 \text{ teeth}$$

Calculate the output speed.

$$\left(\frac{\omega_o}{\omega_i}\right) = \frac{N_2}{N_3}\frac{N_4}{N_5}$$

Substitute 15 for N_2, 15 for N_4, 44 for N_3, 44 for N_5 and 2500 rev/min for ω_i.

$$\left(\frac{\omega_o}{2500}\right) = \left(\frac{15}{44}\right)\left(\frac{15}{44}\right)$$

$$\omega_o = 290.5 \text{ rev/min}$$

Here, the output speed falls within the limits that is,

$$280 \text{ rev/min} \le 290.5 \text{ rev/min} \le 300 \text{ rev/min}$$

So the assumption is correct.

$$N_2 = N_4 = 15$$

And,

$$N_3 = N_5 = 44 \text{ teeth}$$

So, the number of teeth of the compound reverted gear train are $\boxed{N_2 = N_4 = 15 \text{ teeth}}$ and $\boxed{N_3 = N_5 = 44 \text{ teeth}}$.

Comment

Was this solution helpful? 37 0

Recommended solutions for you in Chapter 13

Chapter 13, Problem 38P

For the countershaft in Prob. 3?72, p. 138, assume the gear ratio from gear B to its mating gearis 2 to 1.(a) Determine the minimum number of teeth that can be used on gear B without an interference problem in the teeth.(b) Using the number of...

See solution

Chapter 13, Problem 7P

A parallel helical gearset consists of a 19-tooth pinion driving a 57-tooth gear. The pinion has a left-hand helix angle of 30º, a normal pressure angle of 20º, and a normal module of 2.5 mm. Find:(a) The normal, transversa and axial circular...

See solution