1 Induction (Optional)

We are not going to go into too much detail about induction since we think its better left to the instructors who are qualified to teach it. If you take MATH 220, this will be covered in that course.

Generally, induction is a mathematical proof method where you try to prove some general case (n), by proving the next most general case (n+1). Generally, this proof methods involves using two steps:

1. Verifying the base case for the smallest value of n (which we will denote as n_0)
2. Proving our inductive hypothesis by showing that if the formula holds for n, then it will also hold for n+1
   (a) This step is called the inductive step

1.1 Example

Woah, we're in LaTeX. We are using LaTeX since doing math text was easier with this typeset. Anyways, suppose we are trying to prove the formula below.

$$S(n) = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

According to the induction method of proof, we must prove our smallest case n. n in this case is 1, thus:

$$S(1) = \sum_{i=1}^{1} i = \frac{1(1+1)}{2} = \frac{2}{2} = 1$$

Now our next step is to prove that the $n+1$ not only exists but is equal to $S(n) + (n+1)$:

$$S(n+1) = \sum_{i=1}^{n+1} i = \frac{(n+1)((n+1)+1)}{2} = \frac{(n+1)(n+2)}{2}$$

We have to prove that:

$$S(n) + (n+1) = S(n+1)$$

Recall that the summation function has the following property:

$$\sum_{i=1}^{n+1} i = \sum_{i=1}^{n} i + (n+1)$$

Now, with a little bit of algebra:

$$S(n+1) = \frac{n(n+1)}{2} + (n+1) = \frac{n^2+n+2n+2}{2} = \frac{(n+1)(n+2)}{2}$$

This formula is exactly the formula that we wanted when we first tried to prove $S(n) + (n+1) = S(n+1)$. Thus we are done, time for the fancy square/QED symbol (this denotes the end of a proof).

$\square$

1.2 Resources

If you are still interested in learning more about induction please check out the following resources:

1. UBC Math Wiki - Induction
2. Book of Proof 3ed
   (a) This is the textbook that MATH 220 uses, the book is free from the authro
   (b) We want to emphasize that this is not necessary for CPEN 221/ CPEN 223 but simply an ancillary resource for those who are interested.
3. Khan academy - Proof by Induction