efficiency_correlation_report

November 30, 2024

1 Lap Efficiency Correlation Report

Date: November 21, 2024

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1.1 Overview

Following our participation in the Formula Sun Grand Prix in July 2024, we present an analysis the factors correlating to our efficiency throughout the race.

1.1.1 Motivation

See Lap Efficiency Correlation DR0.

- Quantitatively investigate which factors affect efficiency at FSGP using:
  - Telemetry data
  - Timing spreadsheet records
  - Weather data
- Purpose: understanding efficiency can help us optimize performance by operating as close as possible to our most efficient conditions

1.1.2 Vocabulary

- Lap Energy
  - The net electrical energy consumed by the motor (accounting for regen) between the lap start and end time recorded in our FSGP Timing Spreadsheet.
- Practical Efficiency
  - The energy per unit distance (J/m) computed as Lap Energy / 5070m, where 5070m is the given length of the NCM Motorsports Park track.
- Real Efficiency
  - The energy per unit distance (J/m) computed as Lap Energy / Distance Travelled where Distance Travelled is obtained as an integral of speed over the lap.

Why motor energy? - LVS & Array Power are largely independent of driving behaviour, so they not relevant to our optimization of speed & driving style.

1.2 Imports

[2]:
```
from data_tools.query import DBClient
from data_tools.collections import FSGPDayLaps
import datetime
import numpy as np
import pandas as pd

# Plotting
import matplotlib.pyplot as plt
from sklearn.preprocessing import PolynomialFeatures
from sklearn.linear_model import LinearRegression

# Open Meteo API
import openmeteo_requests
import requests_cache
from retry_requests import retry

FSGP_TRACK_LEN_M = 5_070

driver_colours = {
    "Alex": "red",
    "Bryan": "orange",
    "Diego": "green",
    "Phoebe": "blue"
}

data_client = DBClient("can_log_prod")
```

```
Creating client with API Token: s4Z9_S6_O09kDzYn1KZcs7LVoCA2cVK9_ObY44vR4xMh-
wYLSWBkypSOSOZHQgBvEV2A5LgvQ1IKr8byHes2LA==
Creating client with Org: 8a0b66d77a331e96
```

1.3 Load Data

See `correlation_df.py` for querying and derivation of data. Since this querying requires a connection to our UBC Solar Tailnet and takes a few minutes, we have stored our derived data for this analysis in a `lap_data.csv`.

[3]:
```
df = pd.read_csv("./lap_data.csv")
df.head(10)
```

[3]:

	lap_distance_(m)	lap_energy_(J)	lap_energy_(kJ)	energy_regen_(J)	\
0	5422.621218	845440.759250	845.440759	19444.581795	
1	5110.366020	742762.677311	742.762677	24433.210032	
2	5090.330928	622935.357656	622.935358	8203.242310	
3	5171.397776	628939.849032	628.939849	5167.430689	
4	5116.421189	632982.270880	632.982271	1826.344625	

5	5147.460343	650760.347944	650.760348	2754.900281
6	5269.859661	665763.351505	665.763352	2868.443349
7	5151.753050	650773.678031	650.773678	6558.633813
8	5106.233113	627637.125102	627.637125	8121.477653
9	5152.105993	668908.144054	668.908144	8188.121412

	energy_regen_(kJ)	speed_variance_(mph^2)	motor_power_variance_(W^2)	\
0	19.444582	10.706944	4.299373e+06	
1	24.433210	2.699048	4.754985e+06	
2	8.203242	1.182816	1.788749e+06	
3	5.167431	1.897055	2.097405e+06	
4	1.826345	1.927264	2.787216e+06	
5	2.754900	1.752659	2.993137e+06	
6	2.868443	9.733535	2.921008e+06	
7	6.558634	1.587772	2.751867e+06	
8	8.121478	1.341076	2.897804e+06	
9	8.188121	1.224626	3.412304e+06	

	motor_current_variance_(A^2)	acceleration_variance_(m^2/s^4)	\
0	285.122210	0.003677	
1	318.214644	0.003228	
2	118.544287	0.001757	
3	140.488556	0.002137	
4	189.245535	0.002160	
5	205.296704	0.002312	
6	200.933250	0.002285	
7	191.864322	0.002384	
8	205.418751	0.002357	
9	246.431553	0.002805	

	accelerator_variance	…	battery_temp_avg_(C)	pack_current_avg_(A)	\
0	1330.364989	…	29.816664	15.096574	
1	1492.935963	…	30.666005	15.107015	
2	575.607814	…	31.000000	10.042580	
3	706.120444	…	31.618644	10.421670	
4	893.573462	…	32.000000	10.687180	
5	942.051720	…	32.211091	11.190644	
6	981.239839	…	33.000000	9.809870	
7	890.992444	…	32.498828	11.374014	
8	969.874765	…	32.000000	11.014715	
9	1195.662138	…	32.000000	12.117389	

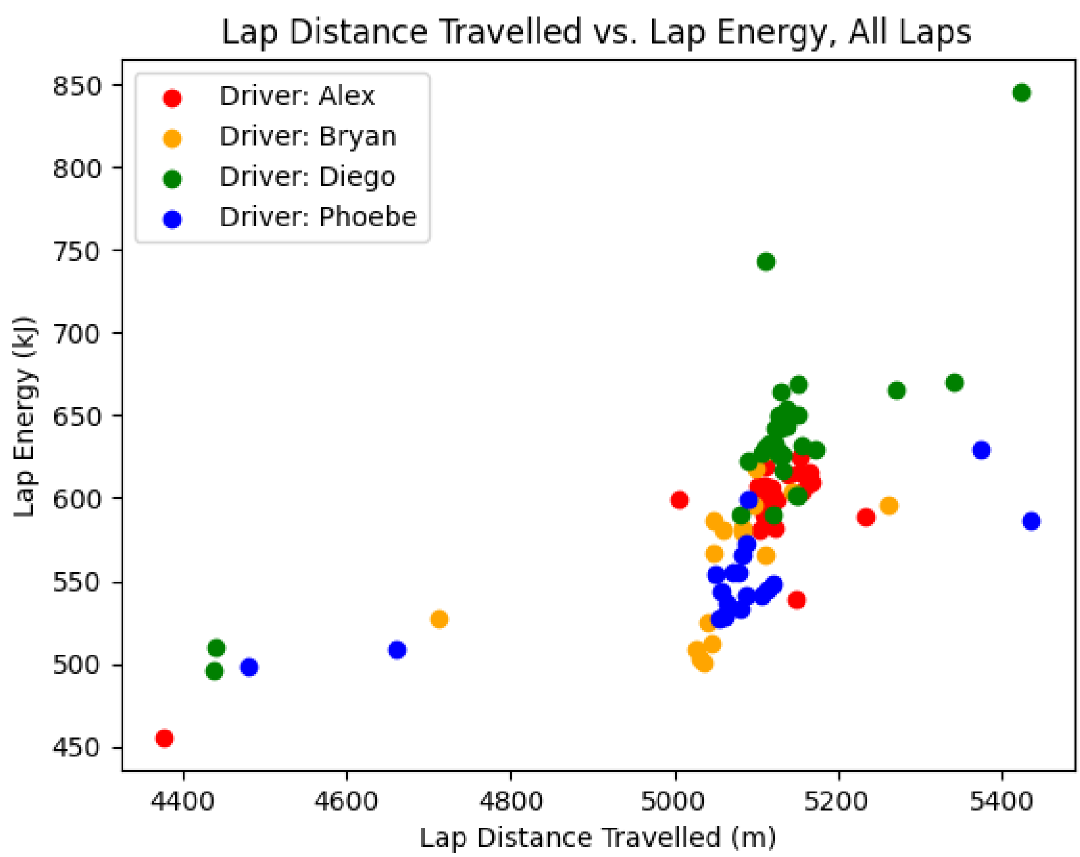
	lap_index	lap_number	lap_end_time	day	driver	\
0	0	1	2024-07-16 15:07:04+00:00	1	Diego	
1	1	2	2024-07-16 15:13:09+00:00	1	Diego	
2	2	3	2024-07-16 15:20:19+00:00	1	Diego	
3	3	4	2024-07-16 15:27:21+00:00	1	Diego	

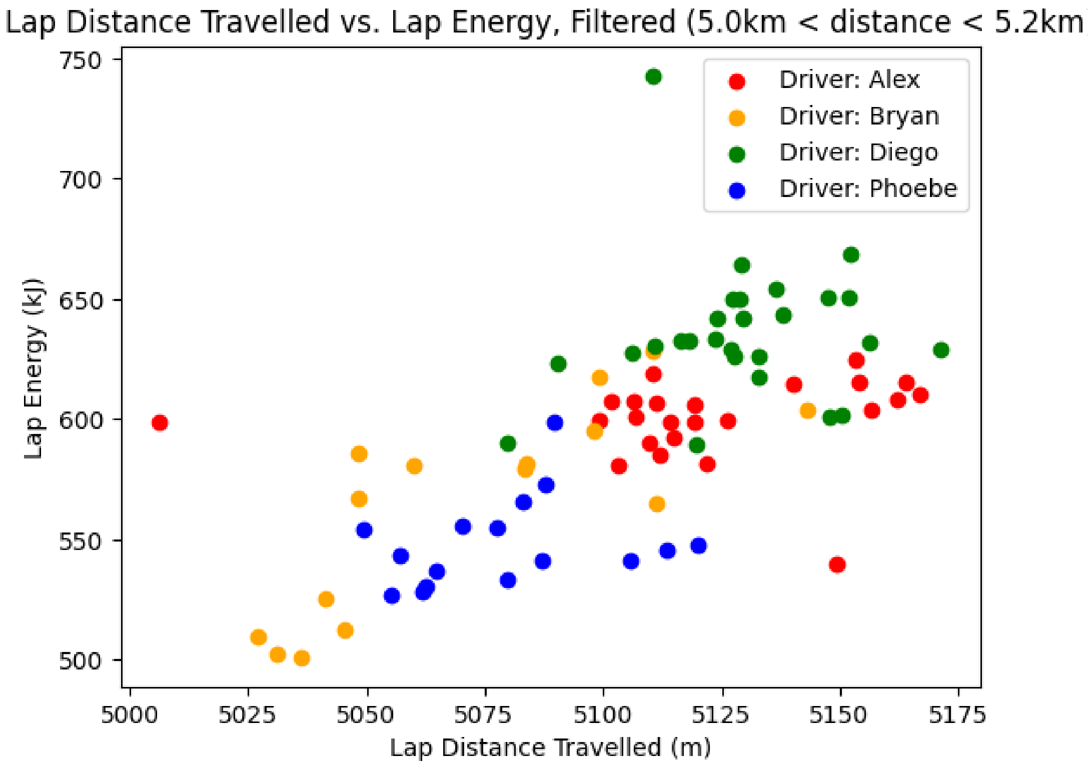
```
4           4           5  2024-07-16 15:33:59+00:00   1   Diego
5           5           6  2024-07-16 15:40:21+00:00   1   Diego
6           6           7  2024-07-16 15:47:45+00:00   1   Diego
7           7           8  2024-07-16 15:54:10+00:00   1   Diego
8           8           9  2024-07-16 16:00:39+00:00   1   Diego
9           9          10  2024-07-16 16:07:05+00:00   1   Diego

  speed_avg_(mph)  efficiency_practical_(J/m)  efficiency_real_(J/m)
0          26.745                  166.753601             155.909979
1          31.068                  146.501514             145.344321
2          26.372                  122.866934             122.376200
3          26.872                  124.051252             121.618927
4          28.492                  124.848574             123.715825
5          29.686                  128.355098             126.423577
6          25.541                  131.314271             126.334171
7          29.455                  128.357727             126.320822
8          29.152                  123.794305             122.915878
9          29.378                  131.934545             129.831984

[10 rows x 21 columns]
```

Our data contains several outlier laps due to various competition conditions: pitting to switch out a driver or stopping due to an accident on the track, for example. This leads to anomalous data points with energy values that do not reflect car performance. By filtering out values with distances outside the typical range, we can remove such outliers and provide better analysis. Here is a plot of distance vs efficiency (explored in more detail later) to demonstrate the filter.

[4]:
```python
distance_filter = np.logical_and(df["lap_distance_(m)"] > 5000,␣
 ↪df["lap_distance_(m)"] < 5200)
filtered_df = df[distance_filter]

for driver, colour in driver_colours.items():
    plt.scatter(df["lap_distance_(m)"][df["driver"] == driver],
                df["lap_energy_(kJ)"][df["driver"] == driver],
                c=colour,
                label=f"Driver: {driver}")
plt.xlabel("Lap Distance Travelled (m)")
plt.ylabel("Lap Energy (kJ)")
plt.legend()
plt.title(f"Lap Distance Travelled vs. Lap Energy, All Laps")
plt.show()

for driver, colour in driver_colours.items():
    combined_filter = np.logical_and(distance_filter, df["driver"] == driver)
    plt.scatter(df["lap_distance_(m)"][combined_filter],
                df["lap_energy_(kJ)"][combined_filter],
                c=colour,
```

```
                label=f"Driver: {driver}")
plt.xlabel("Lap Distance Travelled (m)")
plt.ylabel("Lap Energy (kJ)")
plt.legend()
plt.title(f"Lap Distance Travelled vs. Lap Energy, Filtered (5.0km < distance <␣
  ↪5.2km)")
plt.show()
```



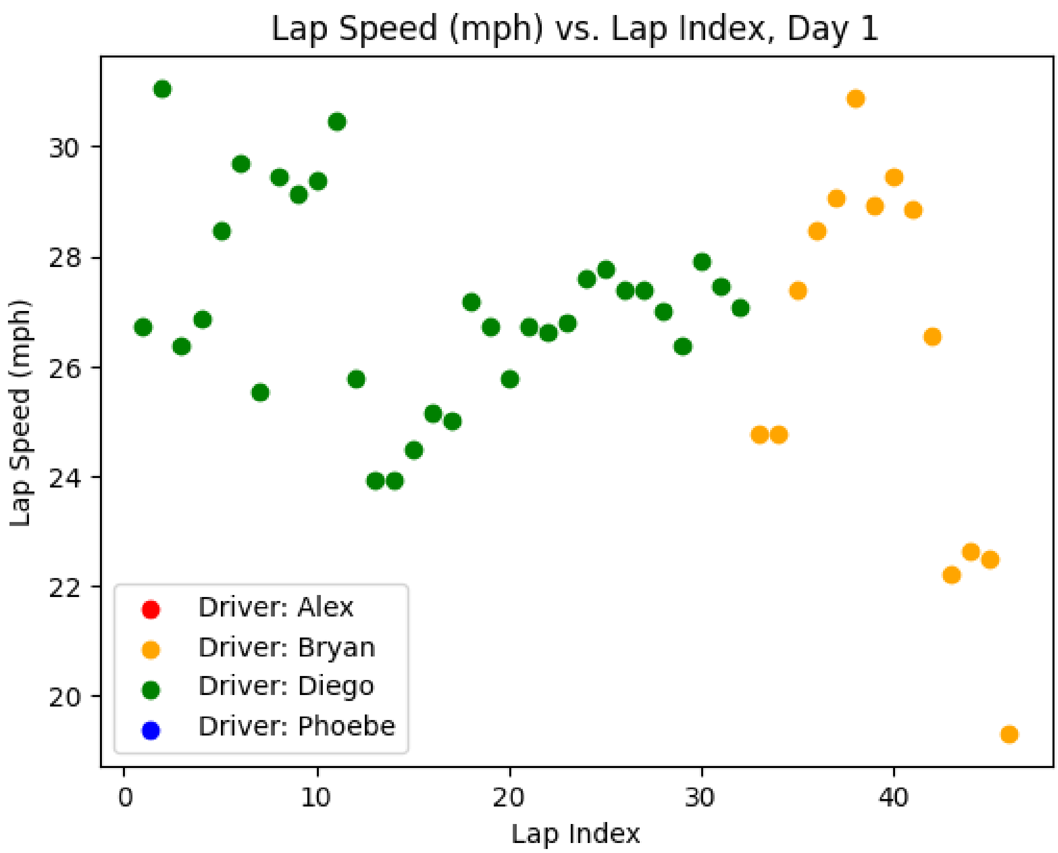


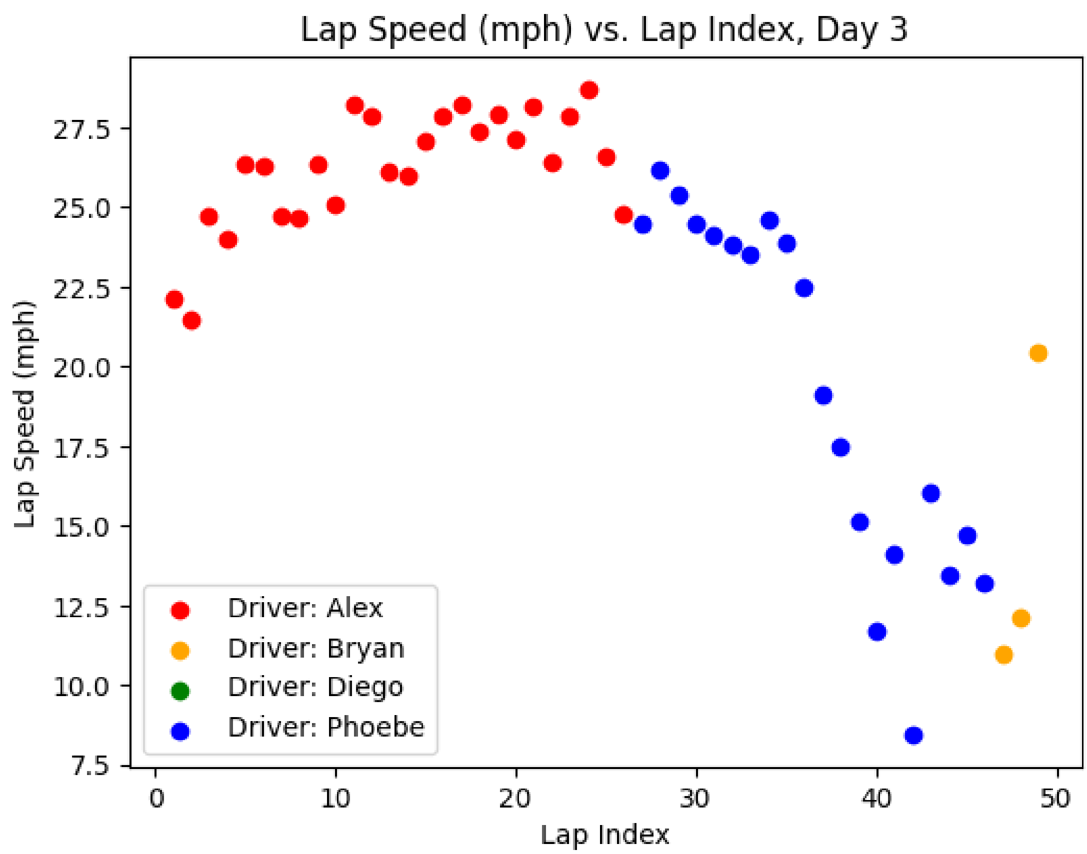
1.4 Context

The plots below show the speeds that we drove at for each lap in FSGP 2024, excluding day 2. We began quickly, aiming to qualify in one day because of concerns of poor weather on day 2. After running out of battery on day 1, we were made aware of the possibility of a provisional qualification which led us to adapt our strategy to instead demonstrate endurance. We drove three slow laps (just enough to qualify Bryan) in heavy rain on day 2 and spent the rest of the day charging as much as possible for day 3. On day 3, we aimed to spend as long as possible on the track, and succeeded in racing all day long. To maintain SoC, we had to dramatically reduce our speed near the end of the day as can be seen with Phoebe's slow laps.

[5]:
```
laps1 = FSGPDayLaps(1)
day_1_laps = laps1.get_lap_count()
day_1_df = df[:day_1_laps]
for driver, colour in driver_colours.items():
    plt.scatter(day_1_df["lap_number"][day_1_df["driver"] == driver],
                day_1_df["speed_avg_(mph)"][day_1_df["driver"] == driver],
                c=colour,
                label=f"Driver: {driver}")
plt.xlabel("Lap Index")
plt.ylabel("Lap Speed (mph)")
plt.legend(loc="lower left")
```

```
plt.title(f"Lap Speed (mph) vs. Lap Index, Day 1")
plt.show()

day_3_df = df[day_1_laps:]
for driver, colour in driver_colours.items():
    plt.scatter(day_3_df["lap_number"][day_3_df["driver"] == driver],
                day_3_df["speed_avg_(mph)"][day_3_df["driver"] == driver],
                c=colour,
                label=f"Driver: {driver}")
plt.xlabel("Lap Index")
plt.ylabel("Lap Speed (mph)")
plt.legend(loc="lower left")
plt.title(f"Lap Speed (mph) vs. Lap Index, Day 3")
plt.show()
```





1.5 Results

The below function simplifies plotting correlation. We then analyze several factors that we believe may have a correlation with lap energy.

[6]:
```
def plot_relationship(df, feature_col, target_col='lap_energy_(kJ)',␣
 ↪poly_degree=2, color_by_driver=False, show_fit=True):
    """
    Plot the relationship between a feature and the target variable.

    Parameters:
    df (pandas.DataFrame): Input DataFrame
    feature_col (str): Name of the feature column
    target_col (str): Name of the target column
    poly_degree (int): Degree of polynomial fit (default: 2)
    color_by_driver (bool): If True, points will be colored by driver (default:␣
 ↪False)
    show_fit (bool): If True, shows polynomial fit line (default: True)
    """
    import matplotlib.pyplot as plt
```

```python
import matplotlib.dates as mdates

plt.figure(figsize=(12, 6))

# Convert datetime to numbers for plotting if necessary
if pd.api.types.is_datetime64_any_dtype(df[feature_col]):
    x = mdates.date2num(df[feature_col])
    is_datetime = True
else:
    x = df[feature_col].values
    is_datetime = False

y = df[target_col].values

if color_by_driver and 'driver' in df.columns:
    # Plot points for each driver with their assigned color
    for driver, color in driver_colours.items():
        mask = df['driver'] == driver
        if mask.any():  # Only plot if driver exists in the data
            plt.scatter(df[feature_col][mask], y[mask], alpha=0.5, color=color, label=driver)
else:
    # Original single-color scatter plot
    plt.scatter(df[feature_col], y, alpha=0.5)

if show_fit and not is_datetime:  # Only show fit for non-datetime x values
    # Fit polynomial regression
    x_reshape = x.reshape(-1, 1)
    poly_features = PolynomialFeatures(degree=poly_degree)
    x_poly = poly_features.fit_transform(x_reshape)
    model = LinearRegression()
    model.fit(x_poly, y)

    # Sort points for smooth curve
    sort_idx = np.argsort(x.ravel())
    x_sorted = x_reshape[sort_idx]
    y_pred = model.predict(poly_features.transform(x_sorted))

    plt.plot(x_sorted, y_pred, 'r--', label='Polynomial fit')

plt.xlabel(feature_col)
plt.ylabel(target_col)
plt.title(f'Relationship between {feature_col} and {target_col}')

if is_datetime:
    # Format datetime axis
    plt.gcf().autofmt_xdate()
```

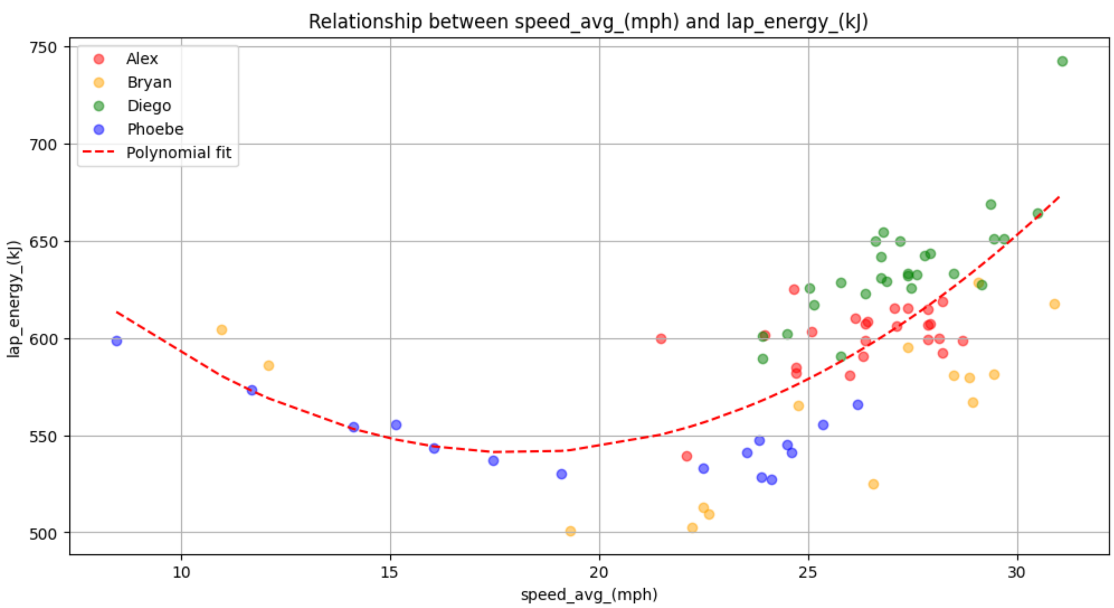
```
    if color_by_driver and 'driver' in df.columns: plt.legend()
    plt.grid(True)
    plt.show()
```

1.6 Speed Factors

1.6.1 Average Speed

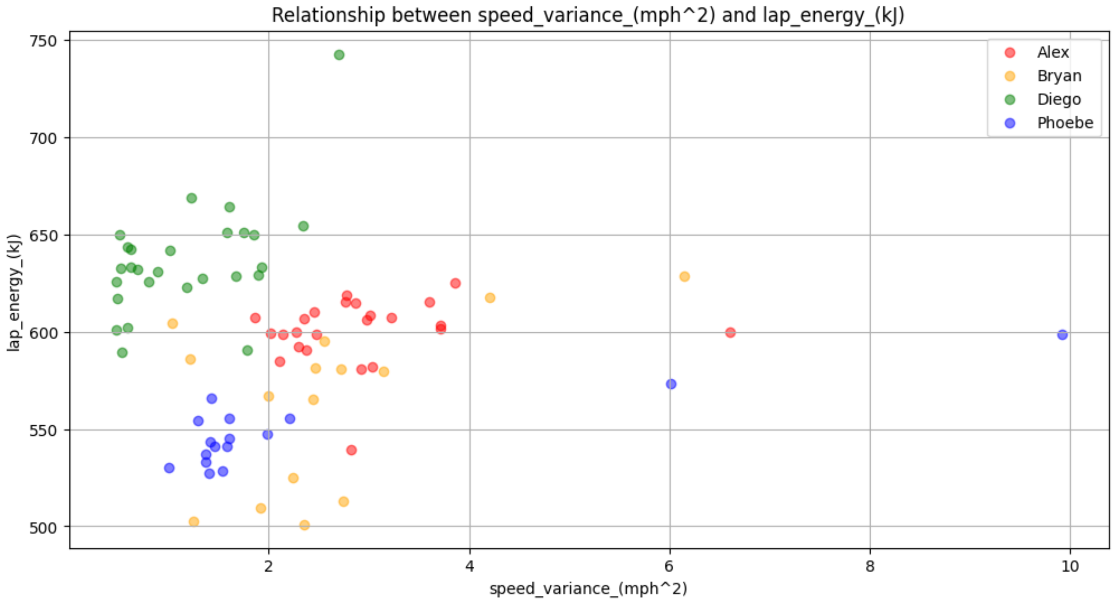
From the average speed plot we see that there seems to be a quadratic relationship between lap energy usage / efficiency and the average speed of a race. The ideal speed to drive that maximized efficiency seems to be around 20 mph. We hypothesize that this optimum exists because aerodynamic drag dominates losses at high speeds ($F_d = C_d A \frac{1}{2} \rho V^2$) and because our motor efficiency is not efficient at low speeds.

[7]:
```
plot_relationship(filtered_df, "speed_avg_(mph)", poly_degree=2,␣
 ↪color_by_driver=True)
```



Below we also have the speed variance per lap. There aren't any clear trends to correlate minimizing speed variance with maximizing efficiency

[8]:
```
plot_relationship(filtered_df, "speed_variance_(mph^2)", show_fit=False,␣
 ↪color_by_driver=True)
```

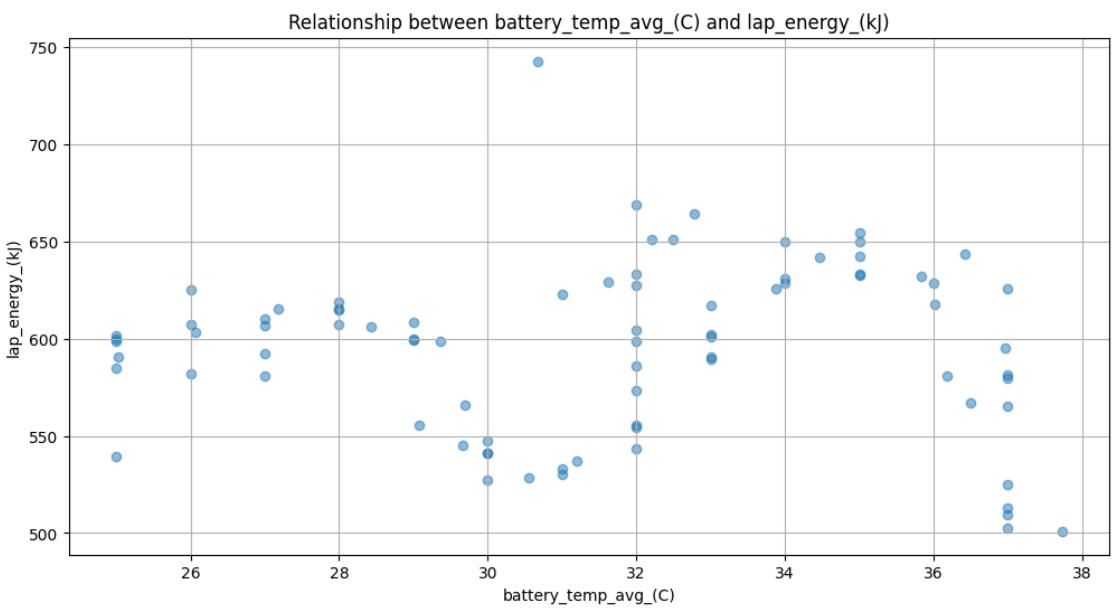


1.7 Battery and Motor

1.7.1 Average Battery Temperature

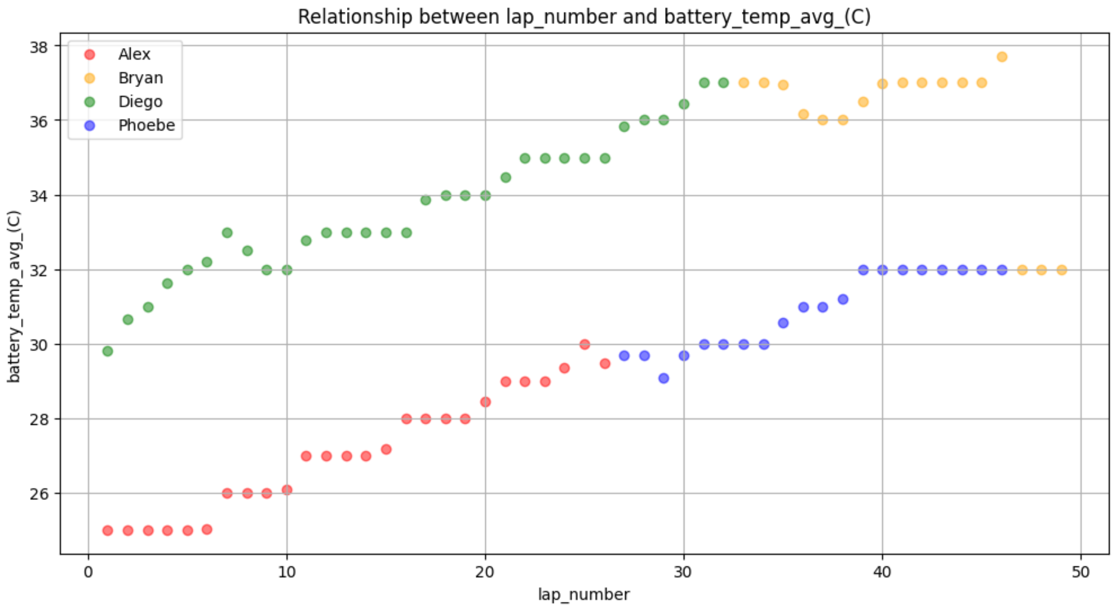
From the average battery temperature plots, we don't see much of a correlation between it and efficiency but, we do get an idea of how the battery warms throughout a race day.

[9]:
```
plot_relationship(filtered_df, "battery_temp_avg_(C)", show_fit=False)
```



[10]:

```
plot_relationship(df, target_col="battery_temp_avg_(C)",␣
↪feature_col="lap_number", show_fit=False, color_by_driver=True)
```



Below, we can watch see how the temperatures creep up in the day and how this relates to ambient temperature. We see that the ambient temperature increases as we continue to race, but also that the temperature of the battery also increases relative to ambient. This makes sense because we continue to output heat into the car and this can slowly climb as the cooling struggles to keep up.

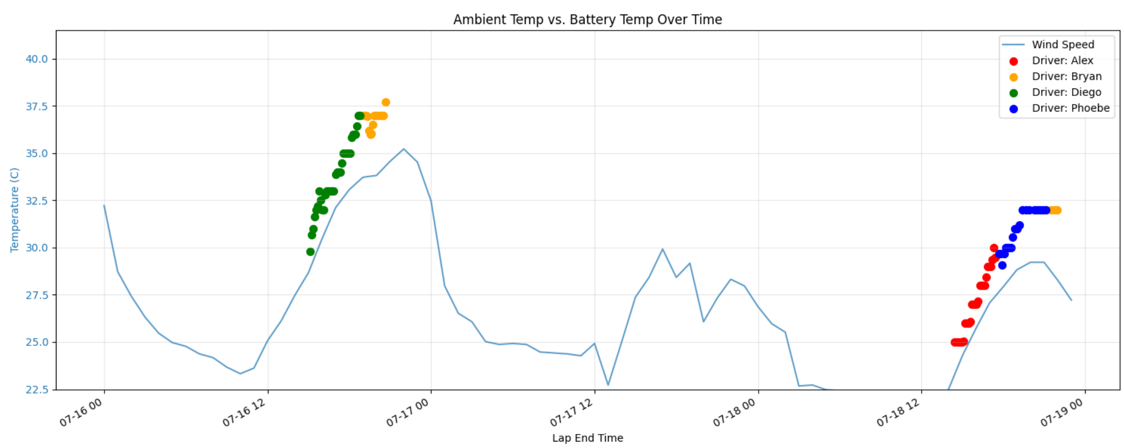
Note that the ambient temperature peaks at 5pm on day 1 (similarly on later days) - I initially thought this was a time zone error, but I have double-checked using the open-meteo GUI and confirmed this to be true. It is possible that the large amount of concrete continued to absorb heat throughout the day resulting in a delayed peak temperature.

[59]:

```
# Setup the Open-Meteo API client with cache and retry on error
cache_session = requests_cache.CachedSession('.cache', expire_after = -1)
retry_session = retry(cache_session, retries = 5, backoff_factor = 0.2)
openmeteo = openmeteo_requests.Client(session = retry_session)

def fetch_temp_data(latitude, longitude, start_date, end_date):
    """
    Fetch hourly wind speed data from Open-Meteo API
    """
    url = "https://archive-api.open-meteo.com/v1/archive"
    params = {
        "latitude": latitude,
        "longitude": longitude,
        "start_date": start_date,
        "end_date": end_date,
```

```
        "hourly": ["temperature_2m"],
    }

    responses = openmeteo.weather_api(url, params=params)
    response = responses[0]

    # Process hourly data
    hourly = response.Hourly()
    hourly_data = {
        "date": pd.date_range(
            start = pd.to_datetime(hourly.Time(), unit = "s", utc = True),
            end = pd.to_datetime(hourly.TimeEnd(), unit = "s", utc = True),
            freq = pd.Timedelta(seconds = hourly.Interval()),
            inclusive = "left"
        ),
        "temperature_2m": hourly.Variables(0).ValuesAsNumpy(),
    }

    return pd.DataFrame(data = hourly_data)

def plot_temp_analysis(df, lap_end_times, battery_temps, lap_drivers):
    """
    Create a combined plot of ambient temperature vs battery temperature using␣
 ↪lap end times.
    """
    fig, ax = plt.subplots(figsize=(15, 6))

    # Plot wind data on primary y-axis (left)
    ax.plot(df['date'], df['temperature_2m'], label='Wind Speed', color='tab:
 ↪blue', alpha=0.7)
    ax.set_ylabel('Temperature (C)', color='tab:blue')
    ax.tick_params(axis='y', labelcolor='tab:blue')
    ax.grid(True, alpha=0.3)

    # Set y-axis limits to fit lap_efficiencies data range
    if not battery_temps.empty:
        ax.set_ylim([battery_temps.min() * 0.9, battery_temps.max() * 1.1])

    # Plot each driver's efficiencies on secondary y-axis using lap end times
    for driver, color in driver_colours.items():
        mask = np.array(lap_drivers) == driver
        if np.any(mask):
            ax.scatter(
                lap_end_times[mask],
                battery_temps[mask],
                color=color,
                label=f"Driver: {driver}",
```

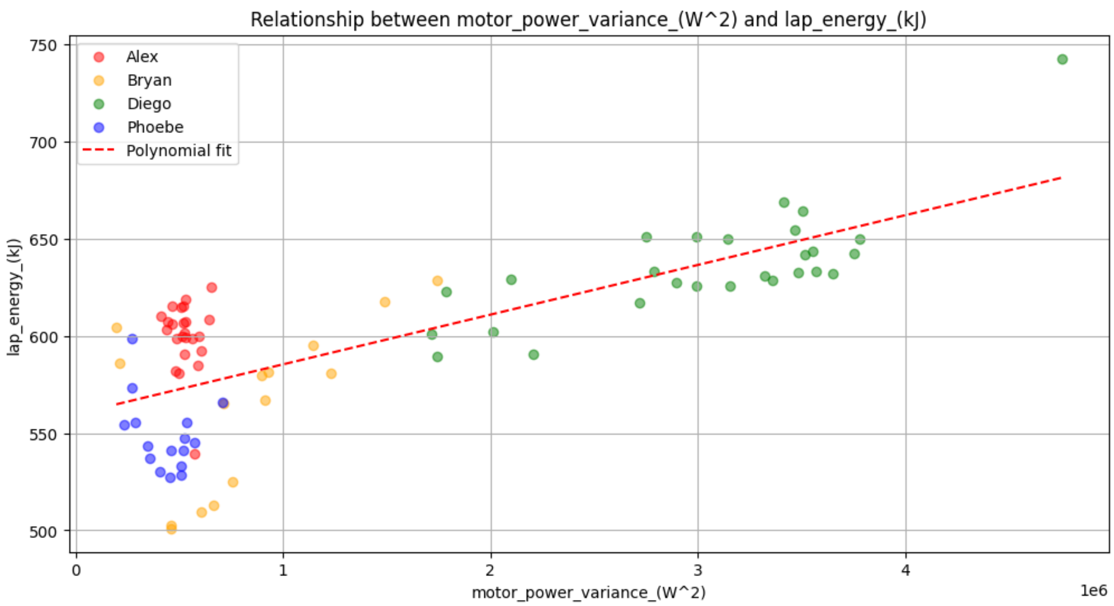
```python
                alpha=1,
                s=50  # Increase marker size for visibility
            )

    ax.legend(loc='upper right')

    # Format x-axis to show dates nicely
    plt.title('Ambient Temp vs. Battery Temp Over Time')
    plt.xlabel('Lap End Time')
    plt.gcf().autofmt_xdate()

    plt.tight_layout()
    plt.show()

# lat lon for center of track
latitude = 37.00272354871939
longitude = -86.36671627935802
start_date = "2024-07-16"  # FSGP Day 1
end_date = "2024-07-18"    # FSGP Day 3

temp_df = fetch_temp_data(latitude, longitude, start_date, end_date)
lap_end_timestamps = df["lap_end_time"]
lap_end_times = np.array(
    [datetime.datetime.strptime(ts, "%Y-%m-%d %H:%M:%S%z").
 ↪replace(tzinfo=datetime.timezone.utc) for ts in lap_end_timestamps]

)
plot_temp_analysis(temp_df, lap_end_times, df["battery_temp_avg_(C)"],␣
 ↪lap_drivers=df["driver"])
```



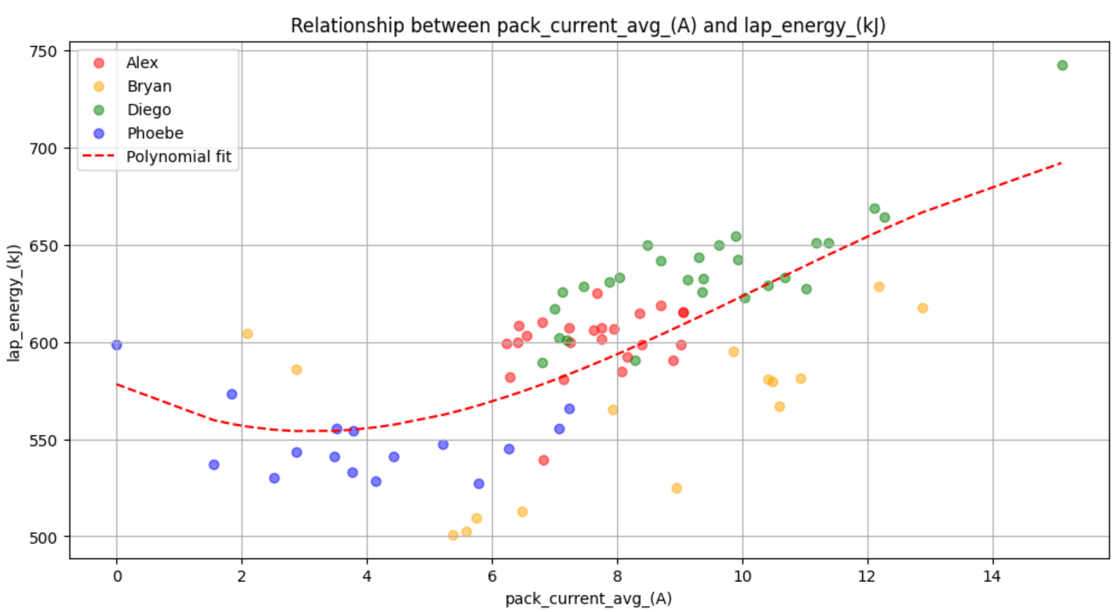
1.7.2 Power & Current

Below are plots to show how our power/current draw from our motor and battery relates to our total energy usage.

[11]:
```
plot_relationship(filtered_df, feature_col="motor_power_variance_(W^2)",␣
↪poly_degree=1, color_by_driver=True)
```



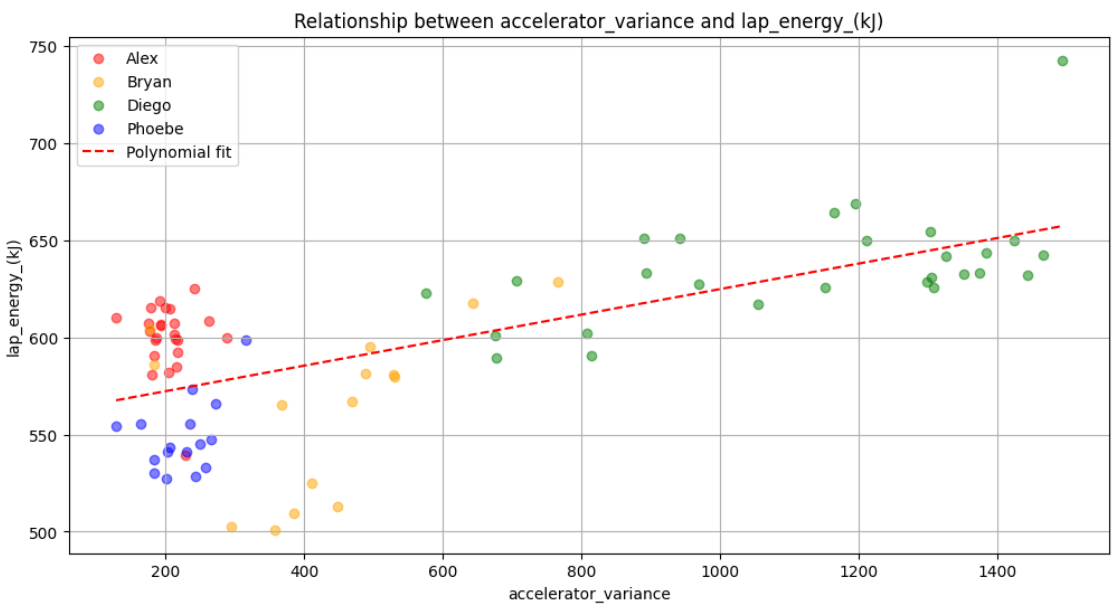
[12]:
```
plot_relationship(filtered_df, "pack_current_avg_(A)", poly_degree=3,␣
↪color_by_driver=True)
```



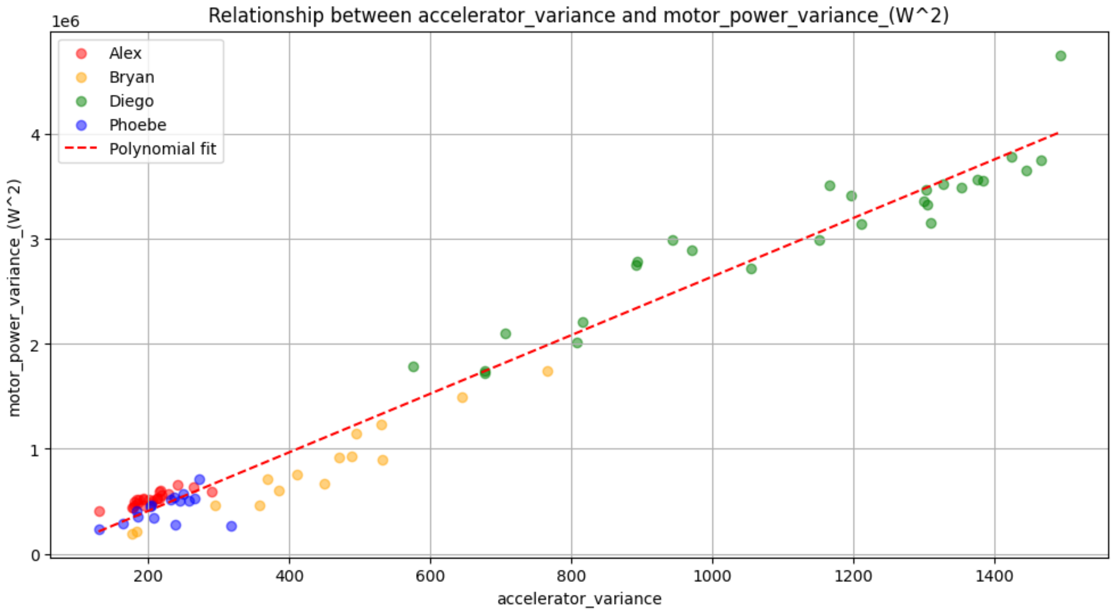
1.8 Accelerator

Below are plots that relate how the driver steps on the accelerator with what happens in the rest of the car. Note that we are comparing variance here, which quantifies how chaotic/aggressive the driver is with the accelerator pedal

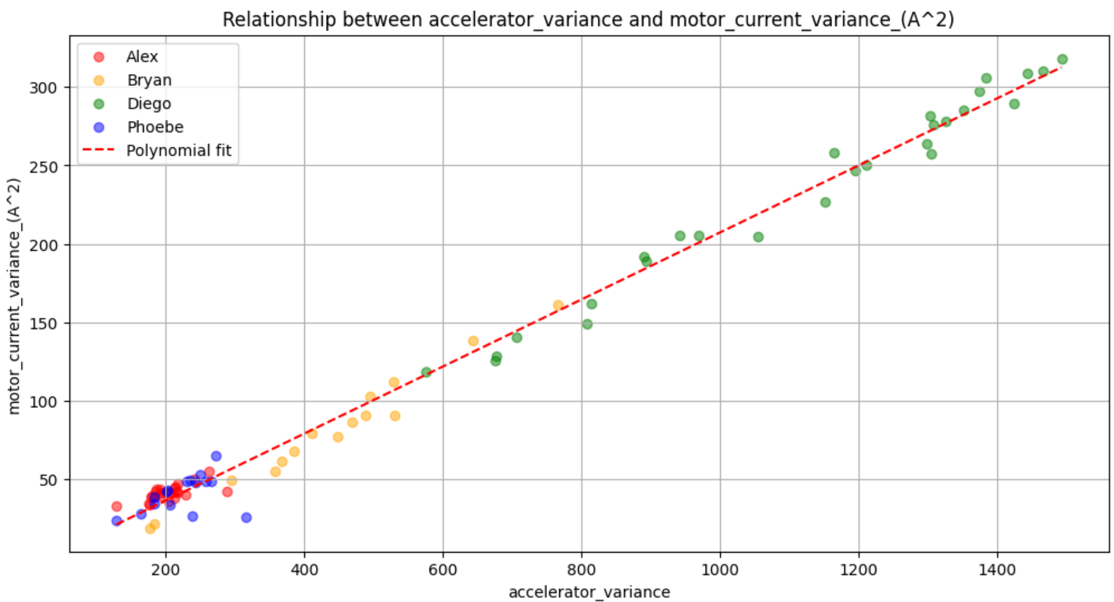
[13]:
```
plot_relationship(filtered_df, feature_col="accelerator_variance",␣
    ↪poly_degree=1, color_by_driver=True)
```



[14]:
```
plot_relationship(filtered_df, feature_col="accelerator_variance",␣
    ↪target_col="motor_power_variance_(W^2)", poly_degree=1, color_by_driver=True)
```



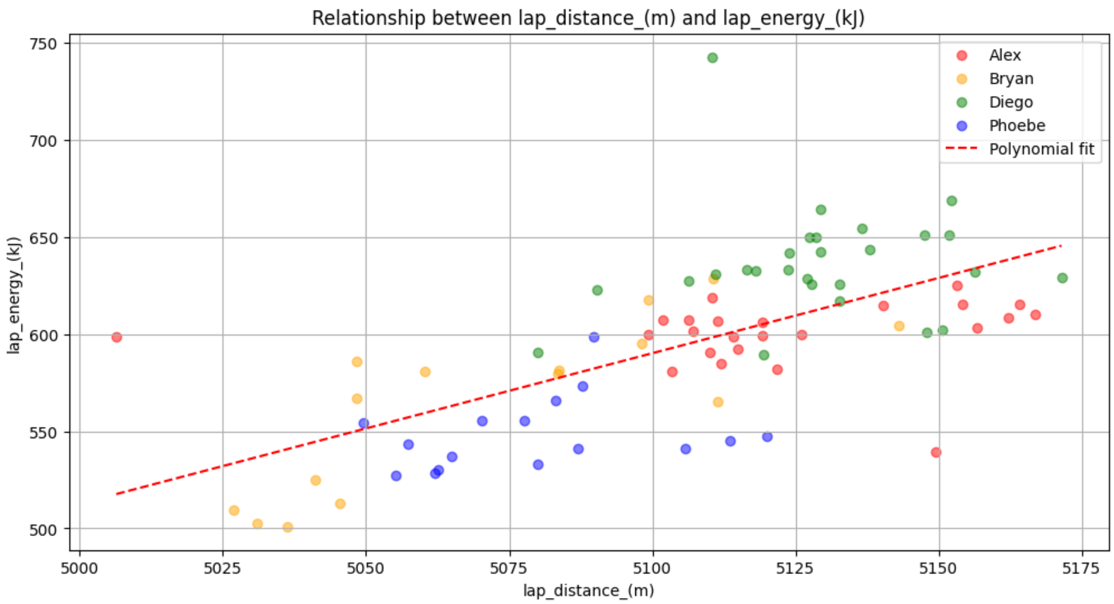
[15]:
```
plot_relationship(filtered_df, feature_col="accelerator_variance",␣
↪target_col="motor_current_variance_(A^2)", poly_degree=1,␣
↪color_by_driver=True)
```



1.9 Distance

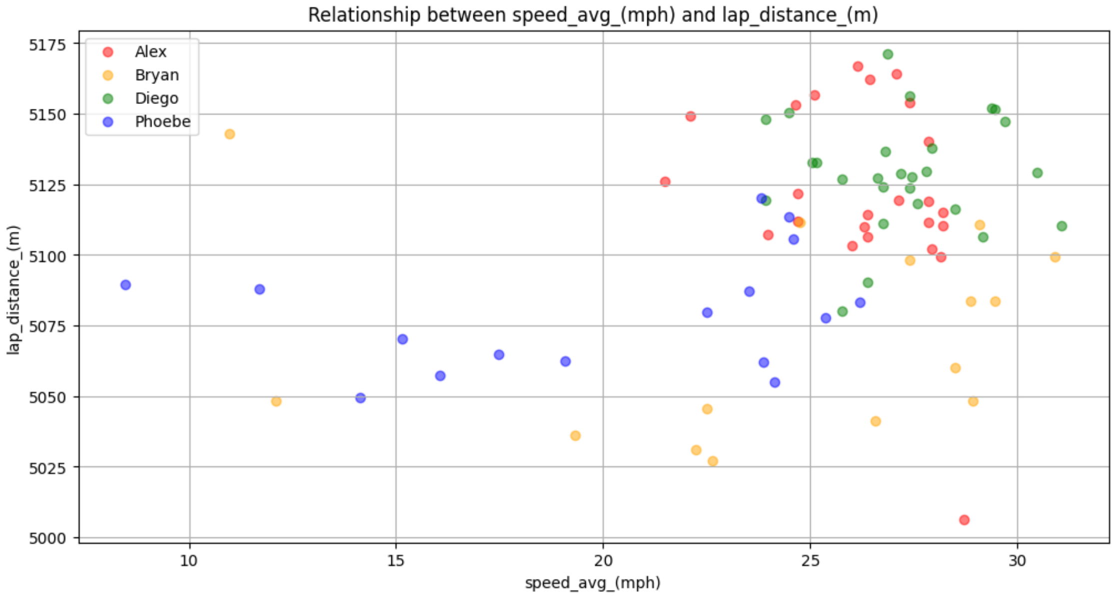
Below is a plot of the actual distance traveled by the car to complete a lap. It suggests a linear trend and intuitive trend that by travelling less of a distance would also reduce energy usage. This suggests that optimizing race lines can be a good strategy. We do see however that lower distance is strongly correlated with slower laps, which is likely because it is easier to take tight corners at lower speed.

[16]:
```
plot_relationship(filtered_df, feature_col="lap_distance_(m)", poly_degree=1,␣
↪color_by_driver=True)
```



To verify the hypothesis that lower speeds makes it easier to take tighter turns and thus reduce lap distance, we can check for a positive correlation between speed and lap distance. We do see a very slight trend, but this is certainly not strong enough to be the cause for the correlation seen above. The ability of a driver to minimize distance and how this is affected by speed also seems to vary person-to-person. For example, Bryan (orange) seemed to be very good at minimizing lap distance when driving at a reduced speed.

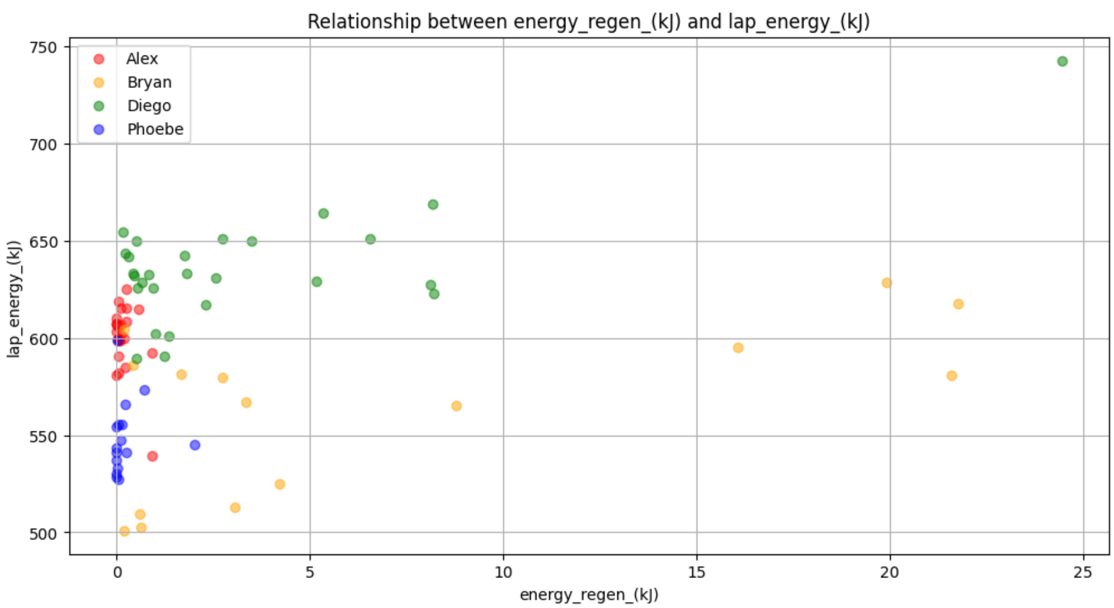
[17]:
```
plot_relationship(filtered_df, feature_col="speed_avg_(mph)",␣
↪target_col="lap_distance_(m)", show_fit=False, color_by_driver=True)
```



1.10 Regen

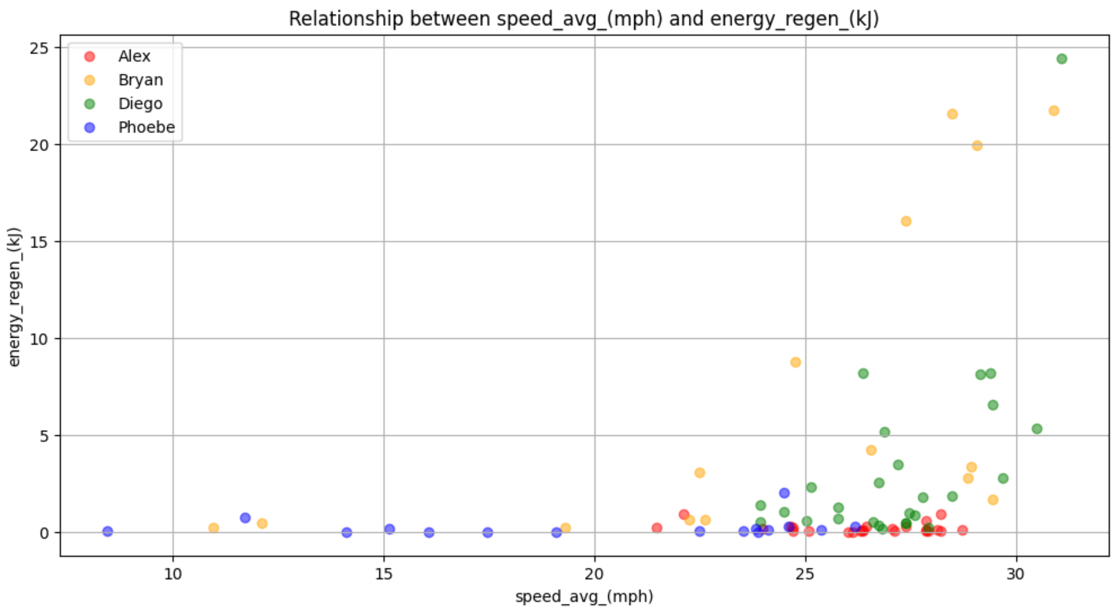
From a regened energy plot alone, we don't see a direct trend with efficiency, but it seems that regen with a specific driver style may optimize efficiency.

[18]:
```
plot_relationship(filtered_df, "energy_regen_(kJ)", show_fit=False,␣
↪color_by_driver=True)
```



It's also work checking if we get more regened energy at higher average speeds. We see below that this is indeed true - we see more regen as we go faster. Interestingly, there seems to be a cutoff speed at 20mph below which we do not get any regen. When considering the magnitude of regened energy, we note that we never got over 25kJ in a lap – less than five percent of the average energy in a lap. This minimal energy recovery explains why our most efficient speed (~20mph) is a speed where we get almost no regen. This data could point to something working incorrectly with our regen, or it may simply highlight that regen is simply not very effective for our car. It could be worth comparing this with other teams to see if it is possible to improve our yield.

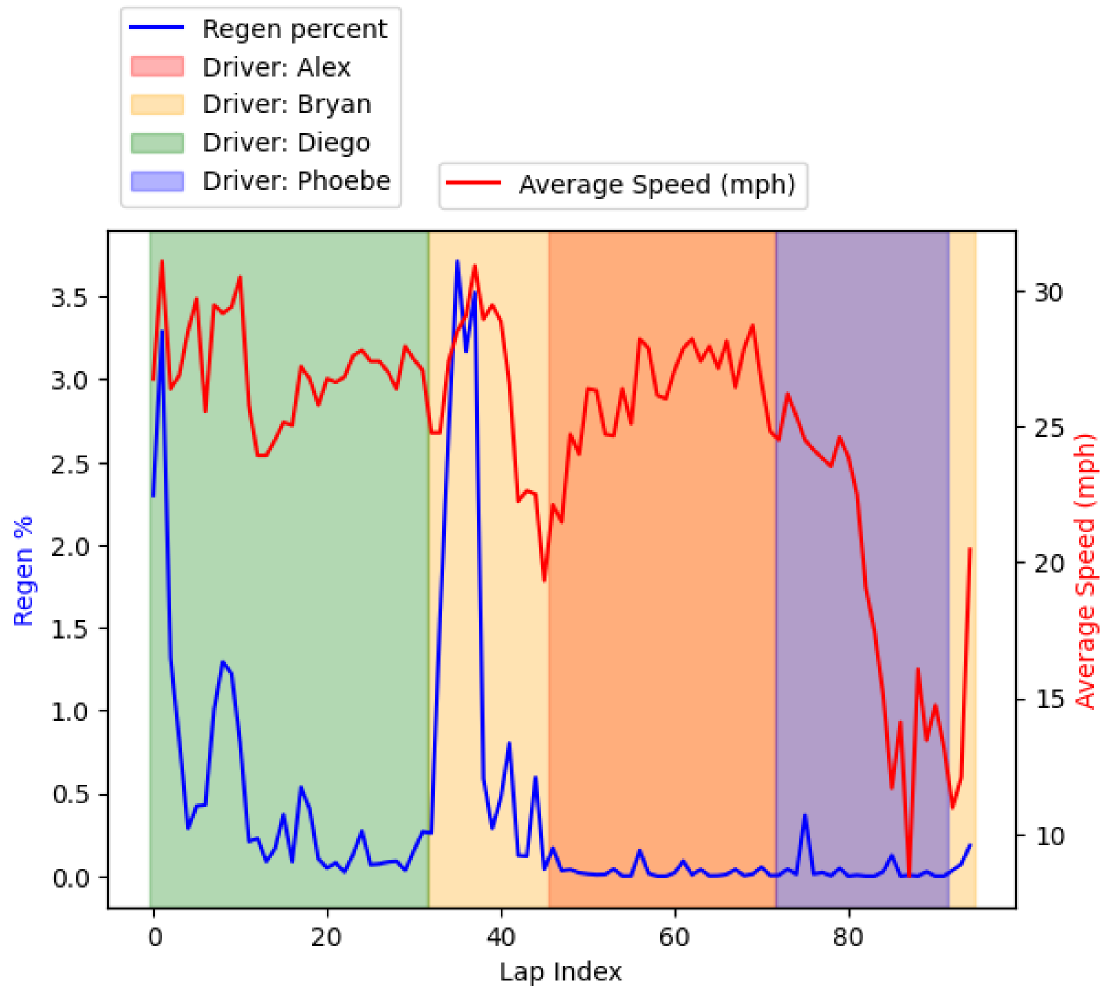
[19]:
```
plot_relationship(filtered_df, feature_col="speed_avg_(mph)",␣
↪target_col="energy_regen_(kJ)", show_fit=False, color_by_driver=True)
```



To confirm the regened percentegas, we can plot regen percent as the ratio of regened energy to net energy used for each lap. Below we observe that the maximum value is around 3.6%. We can also see that Bryan seemed go be able to get the most regen energy during his faster laps.

[52]:
```
fig, ax1 = plt.subplots()

regen_percent = df["energy_regen_(J)"] / df["lap_energy_(J)"] * 100
ax1.plot(regen_percent, "b-", label='Regen percent')
ax1.set_xlabel("Lap Index")
ax1.set_ylabel("Regen %", color='b')

ax2 = ax1.twinx()
ax2.plot(df["speed_avg_(mph)"], "r-", label='Average Speed (mph)')
ax2.set_ylabel("Average Speed (mph)", color='r')
```

```
for driver, colour in driver_colours.items():
    driver_indices = df[df["driver"] == driver].index
    ax1.axvspan(driver_indices[0]-0.5, driver_indices[-1]+0.5, color=colour,␣
 ↪alpha=0.3, label=f"Driver: {driver}")

ax1.legend(loc="upper left", bbox_to_anchor=(0, 1.35))
ax2.legend(loc="upper left", bbox_to_anchor=(0.35, 1.12))

plt.show()
```



1.11 Wind

Below is a plot of wind speeds throughout the data and lap efficiencies. We hypothesized that high wind speeds would reduce efficiency by increasing aerodynamic drag (even though we might also benefit from tailwinds, the quadratic relationship makes higher wind speeds more punishing). However, the plot below does not show any clear relationship, at least not without decoupling efficiency from the many other more important factors.

[20]:
```
# Setup the Open-Meteo API client with cache and retry on error
cache_session = requests_cache.CachedSession('.cache', expire_after = -1)
retry_session = retry(cache_session, retries = 5, backoff_factor = 0.2)
openmeteo = openmeteo_requests.Client(session = retry_session)

def fetch_wind_data(latitude, longitude, start_date, end_date):
    """
    Fetch hourly wind speed data from Open-Meteo API
    """
    url = "https://archive-api.open-meteo.com/v1/archive"
    params = {
        "latitude": latitude,
        "longitude": longitude,
        "start_date": start_date,
        "end_date": end_date,
        "hourly": ["wind_speed_10m", "wind_gusts_10m"],
        "wind_speed_unit": "ms"  # Using m/s for scientific analysis
    }

    responses = openmeteo.weather_api(url, params=params)
    response = responses[0]

    # Process hourly data
    hourly = response.Hourly()
    hourly_data = {
        "date": pd.date_range(
            start = pd.to_datetime(hourly.Time(), unit = "s", utc = True),
            end = pd.to_datetime(hourly.TimeEnd(), unit = "s", utc = True),
            freq = pd.Timedelta(seconds = hourly.Interval()),
            inclusive = "left"
        ),
        "wind_speed": hourly.Variables(0).ValuesAsNumpy(),
        "wind_gusts": hourly.Variables(1).ValuesAsNumpy(),
    }

    return pd.DataFrame(data = hourly_data)

def plot_wind_analysis(df, lap_end_times, lap_efficiencies, lap_drivers):
    """
    Create a combined plot of wind data and driver efficiencies using lap end␣
↪times.
    """
    fig, ax1 = plt.subplots(figsize=(15, 6))

    # Plot wind data on primary y-axis (left)
```

```python
    ax1.plot(df['date'], df['wind_speed'], label='Wind Speed', color='tab:
↪blue', alpha=0.7)
    ax1.plot(df['date'], df['wind_gusts'], label='Wind Gusts', color='tab:
↪orange', linestyle='--', alpha=0.7)
    ax1.set_ylabel('Wind Speed (m/s)', color='tab:blue')
    ax1.tick_params(axis='y', labelcolor='tab:blue')
    ax1.grid(True, alpha=0.3)

    # Set up secondary y-axis for lap efficiencies (right)
    ax2 = ax1.twinx()
    ax2.set_ylabel('Lap Energy (kJ)', color='tab:red')
    ax2.tick_params(axis='y', labelcolor='tab:red')

    # Set y-axis limits to fit lap_efficiencies data range
    if not lap_efficiencies.empty:
        ax2.set_ylim([lap_efficiencies.min() * 0.9, lap_efficiencies.max() * 1.
↪1])

    # Plot each driver's efficiencies on secondary y-axis using lap end times
    for driver, color in driver_colours.items():
        mask = np.array(lap_drivers) == driver
        if np.any(mask):
            ax2.scatter(
                lap_end_times[mask],
                lap_efficiencies[mask],
                color=color,
                label=f"Driver: {driver}",
                alpha=1,
                s=50  # Increase marker size for visibility
            )

    # Combine legends from both axes
    lines, labels = ax1.get_legend_handles_labels()
    lines2, labels2 = ax2.get_legend_handles_labels()
    ax2.legend(lines + lines2, labels + labels2, loc='upper right')

    # Format x-axis to show dates nicely
    plt.title('Wind Speed and Driver Efficiency Over Time')
    plt.xlabel('Lap End Time')
    plt.gcf().autofmt_xdate()

    plt.tight_layout()
    plt.show()

# lat lon for center of track
latitude = 37.00272354871939
longitude = -86.36671627935802
```

```
start_date = "2024-07-16"   # FSGP Day 1
end_date = "2024-07-18"     # FSGP Day 3

wind_df = fetch_wind_data(latitude, longitude, start_date, end_date)
lap_end_timestamps = df["lap_end_time"]
lap_end_times = np.array(
    [datetime.datetime.strptime(ts, "%Y-%m-%d %H:%M:%S%z") for ts in␣
 ↪lap_end_timestamps]

)
plot_wind_analysis(wind_df, lap_end_times, df["lap_energy_(kJ)"],␣
 ↪lap_drivers=df["driver"])
```