A New Model for Control of Systems with Friction

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***Abstract*—In this paper we propose a new dynamic model for friction. The model captures most of the friction behavior that has been observed experimentally. This includes the Stribeck effect, hysteresis, spring-like characteristics for stiction, and varying break-away force. Properties of the model that are relevant to control design are investigated by analysis and simulation. New control strategies, including a friction observer, are explored, and stability results are presented.**

I. Introduction

FRICTION is an important aspect of many control systems both for high quality servo mechanisms and simple pneumatic and hydraulic systems. Friction can lead to tracking errors, limit cycles, and undesired stick-slip motion. Control strategies that attempt to compensate for the effects of friction, without resorting to high gain control loops, inherently require a suitable friction model to predict and to compensate for the friction. These types of schemes are therefore named model-based friction compensation techniques. A good friction model is also necessary to analyze stability, predict limit cycles, find controller gains, perform simulations, etc. Most of the existing model-based friction compensation schemes use classical friction models, such as Coulomb and viscous friction. In applications with high precision positioning and with low velocity tracking, the results are not always satisfactory. A better description of the friction phenomena for low velocities and especially when crossing zero velocity is necessary. Friction is a natural phenomenon that is quite hard to model, and it is not yet completely understood. The classical friction models used are described by static maps between velocity and friction force. Typical examples are different combinations of Coulomb friction, viscous friction, and Stribeck effect [1]. The latter is recognized to produce a destabilizing effect at very low velocities. The classical models explain neither hysteretic behavior when studying friction for nonstationary velocities nor variations in the break-away force with the experimental condition nor small displacements that occur at the contact interface during stiction. The latter very much resembles that of a connection with a stiff spring with damper and is sometimes referred to as the Dahl effect. Later studies (see, e.g., [1], [2]) have shown that a friction model involving dynamics is necessary to describe the friction phenomena accurately.

A dynamic model describing the spring-like behavior during stiction was proposed by Dahl [3]. The Dahl model is essentially Coulomb friction with a lag in the change of friction force when the direction of motion is changed. The model has many nice features, and it is also well understood theoretically. Questions such as existence and uniqueness of solutions and hysteresis effects were studied in an interesting paper by Bliman [4]. The Dahl model does not, however, include the Stribeck effect. An attempt to incorporate this into the Dahl model was done in [5] where the authors introduced a second-order Dahl model using linear space invariant descriptions. The Stribeck effect in this model is only transient, however, after a velocity reversal and is not present in the steady-state friction characteristics. The Dahl model has been used for adaptive friction compensation [6], [7], with improved performance as the result. There are also other models for dynamic friction. Armstrong-Hélouvry proposed a seven parameter model in [1]. This model does not combine the different friction phenomena but is in fact one model for stiction and another for sliding friction. Another dynamic model suggested by Rice and Ruina [8] has been used in connection with control by Dupont [9]. This model is not defined at zero velocity. In this paper we will propose a new dynamic friction model that combines the stiction behavior, i.e., the Dahl effect, with arbitrary steady-state friction characteristics which can include the Stribeck effect. We also show that this model is useful for various control tasks.

II. A New Friction Model

The qualitative mechanisms of friction are fairly well understood (see, e.g., [1]). Surfaces are very irregular at the microscopic level and two surfaces therefore make contact at a number of asperities. We visualize this as two rigid bodies that make contact through elastic bristles. When a tangential force is applied, the bristles will deflect like springs which gives rise to the friction force; see Fig. 1.

If the force is sufficiently large some of the bristles deflect so much that they will slip. The phenomenon is highly random due to the irregular forms of the surfaces. Haessig and Friedland [10] proposed a bristle model where the random behavior was captured and a simpler reset-integrator model which describes the aggregated behavior of the bristles. The model we propose is also based on the average behavior of

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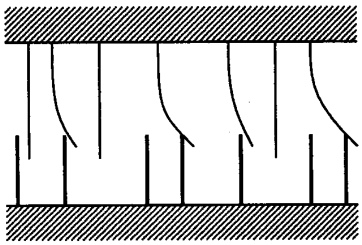

Fig. 1. The friction interface between two surfaces is thought of as a contact between bristles. For simplicity the bristles on the lower part are shown as being rigid.

the bristles. The average deflection of the bristles is denoted by z and is modeled by

$$\frac{dz}{dt} = v - \frac{|v|}{g(v)}z \tag{1}$$

where v is the relative velocity between the two surfaces. The first term gives a deflection that is proportional to the integral of the relative velocity. The second term asserts that the deflection z approaches the value

$$z_{ss} = \frac{v}{|v|}g(v) = g(v)\mathrm{sgn}(v) \tag{2}$$

in steady state, i.e., when v is constant. The function g is positive and depends on many factors such as material properties, lubrication, temperature. It need not be symmetrical. Direction dependent behavior can therefore be captured. For typical bearing friction, $g(v)$ will decrease monotonically from $g(0)$ when v increases. This corresponds to the Stribeck effect. The friction force generated from the bending of the bristles is described as

$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt}$$

where σ_0 is the stiffness and σ_1 a damping coefficient. A term proportional to the relative velocity could be added to the friction force to account for viscous friction so that

$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v. \tag{3}$$

The model given by (1) and (3) is characterized by the function g and the parameters σ_0, σ_1 and σ_2. The function $\sigma_0 g(v) + \sigma_2 v$ can be determined by measuring the steady-state friction force when the velocity is held constant. A parameterization of g that has been proposed to describe the Stribeck effect is

$$\sigma_0 g(v) = F_C + (F_S - F_C)e^{-(v/v_s)^2} \tag{4}$$

where F_C is the Coulomb friction level, F_S is the level of the stiction force, and v_s is the Stribeck velocity; see [1]. With this description the model is characterized by six parameters σ_0, σ_1, σ_2, F_C, F_S, and v_s. It follows from (2)–(4) that for steady-state motion the relation between velocity and friction force is given by

$$\begin{aligned} F_{ss}(v) &= \sigma_0 g(v)\mathrm{sgn}(v) + \sigma_2 v \\ &= F_C\mathrm{sgn}(v) + (F_S - F_C)e^{-(v/v_s)^2}\mathrm{sgn}(v) + \sigma_2 v. \end{aligned}$$

Note, however, that when velocity is not constant, the dynamics of the model will be very important and give rise to different types of phenomena. This will be discussed in Section IV.

Relation to the Dahl Model

The model reduces to the Dahl model if $g(v) = F_C/\sigma_0$, and $\sigma_1 = \sigma_2 = 0$. Equations (1) and (3) then give

$$\frac{dF}{dt} = \sigma_0 \frac{dz}{dt} = \sigma_0 v\left(1 - \frac{F}{F_C}\mathrm{sgn}(v)\right). \tag{5}$$

Dahl actually suggested the more general model

$$\frac{dF}{dt} = \sigma_0 v \left|1 - \frac{F}{F_C}\mathrm{sgn}(v)\right|^i \mathrm{sgn}\left(1 - \frac{F}{F_C}\mathrm{sgn}(v)\right)$$

see [11]. Most references to Dahl's work, however, do use the simpler model (5). Dahl's model accounts for Coulomb friction but it does not describe the Stribeck effect.

An Extension of the Dahl Model

An attempt to extend Dahl's model to include the Stribeck effect was made by Bliman and Sorine [5]. They replaced the time variable t by a space variable s through the transformation

$$s = \int_0^t |v(\tau)|d\tau.$$

Equation (5) then becomes

$$\frac{dF}{ds} = -\sigma_0 \frac{F}{F_c} + \sigma_0 \mathrm{sgn}(v) \tag{6}$$

which is a linear first-order system if $\mathrm{sgn}(v)$ is regarded as an input. Bliman and Sorine then replaced (6) by the second-order model

$$\frac{d^2F}{ds^2} + 2\zeta\omega\frac{dF}{ds} + \omega^2 F = \omega^2 F_C \mathrm{sgn}(v)$$

to imitate the Stribeck effect with an overshoot in the response to sign changes in the velocity. This model, however, will only give a spatially transient Stribeck effect after a change of the direction of motion. The Stribeck effect is not present in the steady-state relation between velocity and friction force.

III. Model Properties

The properties of the model given by (1) and (3) will now be explored. To capture the intuitive properties of the bristle model in Fig. 1, the deflection z should be finite. This is indeed the case because we have the following property.

Property 1: Assume that $0 < g(v) \le a$. If $|z(0)| \le a$ then $|z(t)| \le a \ \forall\ t \ge 0$.

Proof: Let $V = z^2/2$, then the time derivative of V evaluated along the solution of (1) is

$$\begin{aligned} \frac{dV}{dt} &= z(v - \frac{|v|}{g(v)}z) \\ &= -|v||z|\left(\frac{|z|}{g(v)} - \mathrm{sgn}(v)\mathrm{sgn}(z)\right) \end{aligned}$$

TABLE I
PARAMETER VALUES USED IN ALL SIMULATIONS

Parameter	Value	Unit
σ_0	10^5	[N/m]
σ_1	$\sqrt{10^5}$	[Ns/m]
σ_2	0.4	[Ns/m]
F_C	1	[N]
F_S	1.5	[N]
v_S	0.001	[m/s]

The derivative $\frac{dV}{dt}$ is negative when $|z| > g(v)$. Since $g(v)$ is strictly positive and bounded by a, we see that the set $\Omega = \{z : |z| \leq a\}$ is an invariant set for the solutions of (1), i.e., all the solutions of $z(t)$ starting in Ω remain there.

Dissipativity

Intuitively we may expect that friction will dissipate energy. Since our model given by (1) and (3) is dynamic, there may be phases where friction stores energy and others where it gives energy back. It can be proven that the map $\varphi : v \mapsto z$ is dissipative for our model. For more details on the concepts and definitions concerning dissipative systems, see [12].

Property 2: The map $\varphi : v \mapsto z$, as defined by (1), is dissipative with respect to the function $V(t) = \frac{1}{2}z^2(t)$, i.e.,

$$\int_0^t z(\tau)v(\tau)\, d\tau \geq V(t) - V(0).$$

Proof: It follows from (1) that

$$\begin{aligned} zv &= z\frac{dz}{dt} + \frac{|v|}{g(v)}z^2 \\ &\geq z\frac{dz}{dt}. \end{aligned}$$

Hence

$$\int_0^t z(\tau)v(\tau)\, d\tau \geq \int_0^t z(\tau)\frac{dz(\tau)}{d\tau}d\tau \geq V(t) - V(0).$$

Linearization in Stiction Regime

To get some insight into the behavior of the model in the stiction regime we will consider a mass m in contact with a fixed horizontal surface. Let x be the coordinate of the mass, i.e., $v = dx/dt$. The equation of motion becomes

$$m\frac{d^2x}{dt^2} = -F = -\sigma_0 z - \sigma_1\frac{dz}{dt} - \sigma_2\frac{dx}{dt} \tag{7}$$

where z is given by (1). Linearizing (1) around $z = 0$ and $v = 0$ we get

$$\frac{dz}{dt} = \frac{dx}{dt}. \tag{8}$$

Inserting (8) into (7) gives

$$m\frac{d^2x}{dt^2} + (\sigma_1 + \sigma_2)\frac{dx}{dt} + \sigma_0 x = 0. \tag{9}$$

This shows that the system behaves like a damped second-order system. Notice that the bristle stiffness, σ_0, is usually very large, and therefore it is essential to have $\sigma_1 \neq 0$ to have a sufficiently damped motion. The viscous friction coefficient, σ_2, is normally not sufficiently large to provide good damping.

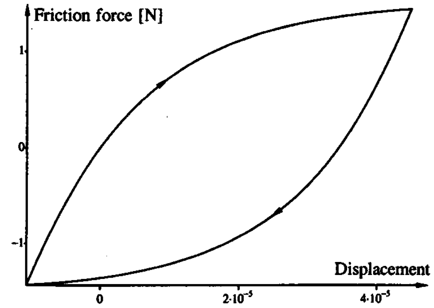

Fig. 2. Presliding displacement as described by the model. The simulation was started with zero initial conditions.

IV. DYNAMICAL MODEL BEHAVIOR

As a preliminary assessment of the model we will investigate its behavior in some typical cases. They correspond to standard experiments that have been performed. In all the simulations the function g has been parameterized according to (4) and the parameter values in Table I have been used. The parameter values have to some extent been based on experimental results [1]. The stiffness σ_0 was chosen to give a presliding displacement of the same magnitude as reported in various experiments. The value of the damping coefficient σ_1 was chosen to give a damping of $\zeta = 0.5$ for the linearized equation (9) with a unit mass. The Coulomb friction level F_C corresponds to a friction coefficient $\mu \approx 0.1$ for a unit mass, and F_S gives a 50% higher friction for very low velocities. The viscous friction σ_2 and the Stribeck velocity v_s are also of the same order of magnitude as given in [1].

The different behaviors shown in the following subsections cannot be attributed to single parameters but rather to the behavior of the nonlinear differential equation (1) and the shape of the function g. The presliding displacement and the varying break-away force are due to the dynamics. This behavior is also present in the Dahl model. The Stribeck shape of g together with the dynamics give rise to the type of hysteresis observed in the subsection on frictional lag.

A. Presliding Displacement

Courtney-Pratt and Eisner have shown that friction behaves like a spring if the applied force is less than the break-away force. If a force is applied to two surfaces in contact there will be a displacement. A simulation was performed to investigate if our model captures this phenomenon. An external force was applied to a unit mass subjected to friction. The applied force was slowly ramped up to 1.425 N which is 95% of F_S. The force was then kept constant for a while and later ramped down to the value –1.425 N, where it was kept constant and then ramped up to 1.425 N again. The results of the simulation are shown in Fig. 2 where the friction force is shown as a function of displacement. The behavior shown in Fig. 2 agrees qualitatively with the experimental results in [13].

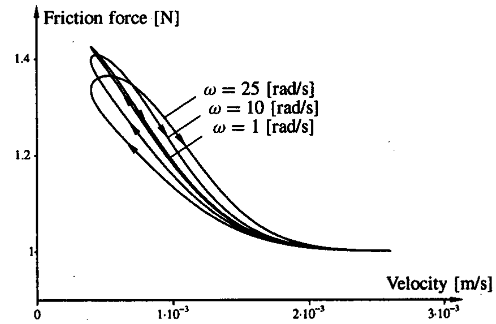

Fig. 3. Hysteresis in friction force with varying velocity. The velocity variation with the highest frequency shows the widest hysteresis loop.

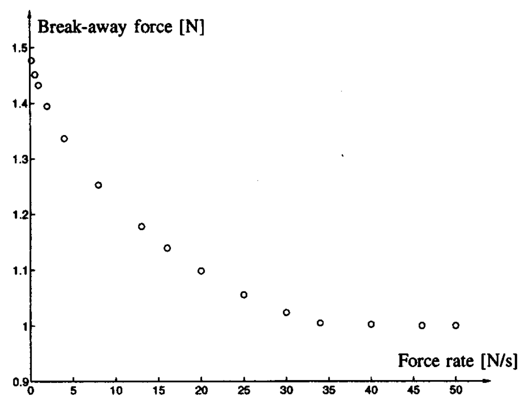

Fig. 4. Relation between break-away force and rate of increase of the applied force.

B. Frictional Lag

Hess and Soom [14] studied the dynamic behavior of friction when velocity is varied during unidirectional motion. They showed that there is hysteresis in the relation between friction and velocity. The friction force is lower for decreasing velocities than for increasing velocities. The hysteresis loop becomes wider at higher rates of the velocity changes. Hess and Soom explained their experimental results by a pure time delay in the relation between velocity and friction force. Fig. 3 shows a simulation of the Hess–Soom experiment using our friction model. The input to the friction model was the velocity which was changed sinusoidally around an equilibrium. The resulting friction force is given as a function of velocity in Fig. 3. Our model clearly exhibits hysteresis. The width of the hysteresis loop also increases with frequency. Our model thus captures the hysteretic behavior of real friction described in [14].

C. Varying Break-Away Force

The break-away force can be investigated through experiments with stick-slip motion. In [15] it is pointed out that in such experiments the dwell-time when sticking and the rate of increase of the applied force are always related and hence the effects of these factors cannot be separated. The experiment was therefore redesigned so that the time in stiction and the rate of increase of the applied force could be varied independently. The results showed that the break-away force

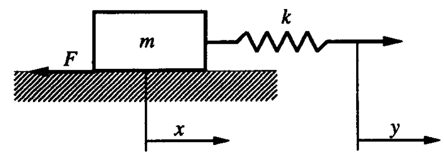

Fig. 5. Experimental setup for stick-slip motion.

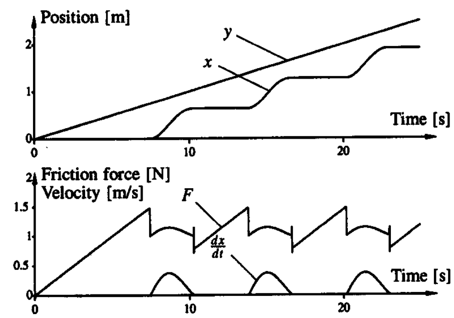

Fig. 6. Simulation of stick-slip motion.

did depend on the rate of increase of the force but not on the dwell-time; see also [16]. Simulations were performed using our model to determine the break-away force for different rates of force application. Since the model is dynamic, a varying break-away force can be expected. A force applied to a unit mass was ramped up at different rates, and the friction force when the mass started to slide was determined. Note that since the model behavior in stiction is essentially that of a spring, there will be microscopic motion, i.e., velocity different from zero, as soon as a force is applied. The break-away force was therefore determined at the time where a sharp increase in the velocity could be observed. Fig. 4 shows the force at break-away as a function of the rate of increase of the applied force. The results agree qualitatively with the experimental results in [15] and [16].

D. Stick-Slip Motion

Stick-slip motion is a typical behavior for systems with friction. It is caused by the fact that friction is larger at rest than during motion. A typical experiment that may give stick-slip motion is shown in Fig. 5. A unit mass is attached to a spring with stiffness $k = 2$ N/m. The end of the spring is pulled with constant velocity, i.e., $dy/dt = 0.1$ m/s. Fig. 6 shows results of a simulation of the system based on the friction model in Section II. The mass is originally at rest and the force from the spring increases linearly. The friction force counteracts the spring force, and there is a small displacement. When the applied force reaches the break-away force, in this case approximately $\sigma_0 g(0)$, the mass starts to slide and the friction decreases rapidly due to the Stribeck effect. The spring contracts, and the spring force decreases. The mass slows down and the friction force increases because of the Stribeck effect and the motion stops. The phenomenon then repeats itself. In Fig. 6 we show the positions of the mass and the spring, the friction force and the velocity. Notice the highly irregular behavior of the friction force around the region where the mass stops.

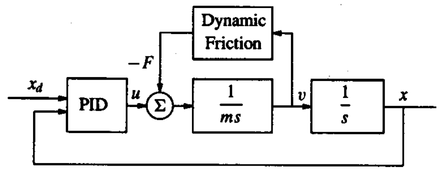

Fig. 7. Block diagram for the servo problem with PID controller

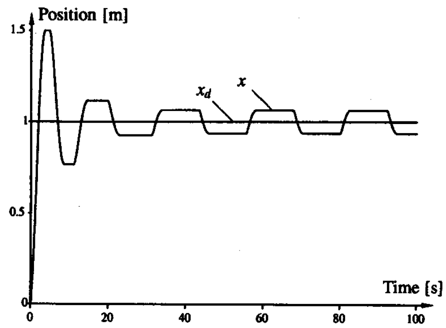

Fig. 8. Simulation of the PID position control problem in Fig. 7.

V. Feedback Control

To further illustrate the properties of our friction model we will investigate its application to some typical servo problems. First we will use it to show that it predicts limit cycle oscillations in servos with PID control. We will then use it to design observer based friction compensators.

A. Limit Cycles Caused by Friction

It has been observed experimentally that friction may give rise to limit cycles in servo drives where the controller has integral action; for references see [2]. This phenomenon is often referred to as hunting.

Consider the linear motion of a mass m at position x. The equation of motion is

$$m\frac{d^2x}{dt^2} = u - F \tag{10}$$

where $dx/dt = v$ is the velocity, F the friction force given by (3) and u the control force which is given by the PID controller

$$u = -K_v v - K_p(x - x_d) - K_i \int (x - x_d). \tag{11}$$

A block diagram of the system is shown in Fig. 7. In Fig. 8 we show the results of a simulation of the system. The friction parameters are given by Table I, $m = 1$ and the controller parameters are $K_v = 6$, $K_p = 3$, and $K_i = 4$. The reference position is chosen as $x_d = 1$. The model clearly predicts limit cycles as have been observed experimentally in systems of this type.

B. Friction Compensation

Only linear feedback from the position was used in the PID control law (11). Knowledge about friction was not used. It is of course more appealing to make a model-based control that uses the model to predict the friction to compensate for it. Since the model is dynamic and has an unmeasurable state, some kind of observer is necessary. This will be discussed next.

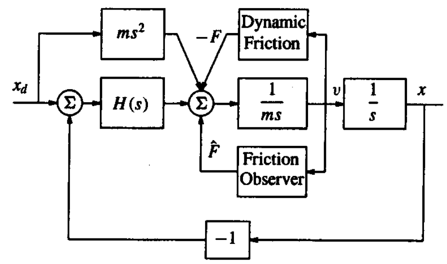

Fig. 9. Block diagram for the position control problem using a friction observer.

Position Control with a Friction Observer: Consider the problem of position tracking for the process (10). Assume that the parameters σ_0, σ_1, and σ_2, and the function g in the friction model are known. The state z is, however, not measurable and hence has to be observed to estimate the friction force. For this we use a nonlinear friction observer given by

$$\frac{d\hat{z}}{dt} = v - \frac{|v|}{g(v)}\hat{z} - ke, \quad k > 0 \tag{12}$$

$$\hat{F} = \sigma_0\hat{z} + \sigma_1\frac{d\hat{z}}{dt} + \sigma_2 v \tag{13}$$

and the following control law

$$u = -H(s)e + \hat{F} + m\frac{d^2x_d}{dt^2} \tag{14}$$

where $e = x - x_d$ is the position error and x_d is the desired reference which is assumed to be twice differentiable. The term ke in the observer is a correction term from the position error. The closed-loop system is represented by the block diagram in Fig. 9. With the observer based friction compensation, we achieve position tracking as shown in the following theorem.

Theorem 1: Consider system (10) together with the friction model (1) and (3), friction observer (12) and (13), and control law (14). If $H(s)$ is chosen such that

$$G(s) = \frac{\sigma_1 s + \sigma_0}{ms^2 + H(s)}$$

is strictly positive real (SPR) then the observer error, $F - \hat{F}$, and the position error, e, will asymptotically go to zero.

Proof: The control law yields the following equations

$$e = \frac{1}{ms^2 + H(s)}(-\tilde{F}) = \frac{\sigma_1 s + \sigma_0}{ms^2 + H(s)}(-\tilde{z}) = -G(s)\tilde{z}$$

$$\frac{d\tilde{z}}{dt} = -\frac{|v|}{g(v)}\tilde{z} + ke$$

where $\tilde{F} = F - \hat{F}$ and $\tilde{z} = z - \hat{z}$. Now introduce

$$V = \xi^T P\xi + \frac{\tilde{z}^2}{k}$$

as a Lyapunov function and

$$\frac{d\xi}{dt} = A\xi + B(-\tilde{z})$$

$$e = C\xi$$

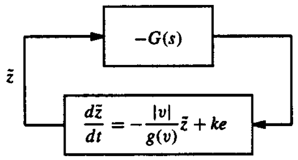

Fig. 10. The block diagram in Fig. 9 redrawn with e and $\tilde{z}$ as outputs of a linear and a nonlinear block, respectively.

which is a state-space representation of $G(s)$. Since $G(s)$ is SPR [17] it follows from the Kalman–Yakubovitch Lemma [17] that there exist matrices $P = P^T > 0$ and $Q = Q^T > 0$ such that

$$\begin{aligned} A^T P + PA &= -Q \\ PB &= C^T. \end{aligned}$$

Now

$$\begin{aligned} \frac{dV}{dt} &= -\xi^T Q\xi - 2\xi^T PB\tilde{z} + \frac{2}{k}\tilde{z}\frac{d\tilde{z}}{dt} \\ &= -\xi^T Q\xi - 2e\tilde{z} + \frac{2}{k}\tilde{z}(-\frac{|v|}{g(v)}\tilde{z} + ke) \\ &= -\xi^T Q\xi - \frac{2}{k}\frac{|v|}{g(v)}\tilde{z}^2 \\ &\leq -\xi^T Q\xi. \end{aligned}$$

The radial unboundedness of V together with the semi-definiteness of dV/dt implies that the states are bounded. We can now apply LaSalle's theorem to see that $\xi \to 0$ and $\tilde{z} \to 0$ which means that both e and $\tilde{F}$ tends to zero and the theorem is proven.

The theorem can also be understood from the following observations. By introducing the observer we get a dissipative map from e to $\tilde{z}$ and by adding the friction estimate to the control signal, the position error will be the output of a linear system operating on $\tilde{z}$. This means that we have an interconnection of a dissipative system and a linear SPR system as seen in Fig. 10. Such a system is known to be asymptotically stable.

Velocity Control: The same type of observer-based control can be used for velocity control. For this control problem the controller is changed to

$$\begin{aligned} u &= -H(s)e + \hat{F} + m\frac{dv_d}{dt} \\ \frac{d\hat{z}}{dt} &= v - \frac{|v|}{g(v)}\hat{z} - k(v - v_d), \quad k > 0 \qquad (15) \\ \hat{F} &= \sigma_0\hat{z} + \sigma_1\frac{d\hat{z}}{dt} + \sigma_2 v \end{aligned}$$

where $v - v_d$ is the velocity error and v_d the desired velocity which is assumed to be differentiable. Velocity tracking is achieved as shown in the following theorem.

Theorem 2: Consider system (10) together with friction model (1) and (3) and observer based control law (15). If $H(s)$ is chosen such that

$$G(s) = \frac{\sigma_1 s + \sigma_0}{ms + H(s)}$$

is strictly positive real, then the observer error, $F - \hat{F}$, and the velocity error will asymptotically go to zero.

Proof: The theorem is proven in the same way as Theorem 1 after observing that the control law yields the following error equations

$$\begin{aligned} v - v_d &= \frac{1}{ms + H(s)}(-\tilde{F}) = \frac{\sigma_1 s + \sigma_0}{ms + H(s)}(-\tilde{z}) = -G(s)\tilde{z} \\ \frac{d\tilde{z}}{dt} &= -\frac{|v|}{g(v)}\tilde{z} + k(v - v_d). \end{aligned}$$

This is again an interconnection of a dissipative system with $\tilde{z}$ as its output and a linear SPR system with $v - v_d$ as its output.

To assume that the friction model and its parameters are known exactly is of course a strong assumption. Investigation of the sensitivity of the results to these assumptions is an interesting problem that is outside the scope of this paper. The accuracy required in the velocity measurement is a similar problem.

VI. Conclusions

A new dynamic model for friction has been presented. The model is simple yet captures most friction phenomena that are of interest for feedback control. The low velocity friction characteristics are particularly important for high performance pointing and tracking. The model can describe arbitrary steady-state friction characteristics. It supports hysteretic behavior due to frictional lag, spring-like behavior in stiction and gives a varying break-away force depending on the rate of change of the applied force. All these phenomena are unified into a first-order nonlinear differential equation. The model can readily be used in simulations of systems with friction.

Some relevant properties of the model have been investigated. The model was used to simulate position control of a servo with a PID controller. The simulations predict hunting as has been observed in applications of position control with integral action. The model has also been used to construct a friction observer and to perform friction compensation for position and velocity tracking. When the parameters are known the observer error and the control error will asymptotically go to zero. Sensitivity studies, parameter estimation and adaptation are natural extensions of this work.

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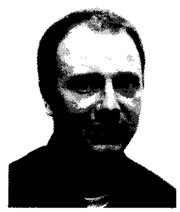

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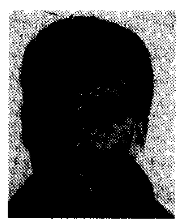

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