Symbol	Name	Definition	Formula
d_υ	Volume diameter, equivalent diameter or nominal diameter of particle	Diameter of a sphere having the same volume (υ_p) as the particle	$\upsilon_p = \frac{\pi}{6} d_\upsilon^3$
d_S	Surface diameter	Diameter of a sphere having the same surface as the particle	$S_p = \pi d_S^2$
$d_{S\upsilon}$	Surface-volume diameter	Diameter of a sphere having the same external surface to volume ratio as a sphere	$d_{s\upsilon} = \frac{d_\upsilon^3}{d_S^2}$
d_d	Drag diameter	Diameter of a sphere having the same resistance to motion as the particle in a fluid of the same viscosity and at the same velocity (d_υ approximates d_s when *Re* is small)	$F_D = C_D A \rho_f v^2/2$ Where $C_D A = f(d_d)$ $F_D = 3\pi d_d \eta v$ $(Re<0.2)$
d_f	Free-falling diameter	Diameter of a sphere having the same density and the same free-falling speed as the particle in a fluid of the same density and viscosity	
d_{stk}	Stoke's diameter	The free-falling diameter of a particle in the laminar flow region (Re<0.2)	$d_{stk}^2 = \frac{d_\upsilon^3}{d_d}$
d_α	Projected area diameter	Diameter of a circle having the same area as the projected area of the particle in random orientation	$A_p = \frac{\pi}{4} d_\alpha^2$
d_{AR}	Projected area diameter	Diameter of a circle having the same area as the projected area of the particle in random orientation	
d_C	Perimeter diameter	Diameter of a circle having the same perimeter as the projected outline of the particle	
d_A	Sieve diameter	The width of the minimum square aperture through which the particle will pass	
d_F	Feret's diameter	The mean value of the distance between pairs of parallel tangents to the projected outline of the particle	
d_M	Martin's diameter	The mean chord length of the projected outline of the particle	
d_R	Unrolled diameter	The mean chord length through the center of gravity of the particle	$E(d_R) = \frac{1}{\pi}\int_0^{2\pi} d_R d\theta_R$

Particles are rarely perfect spheres. To define their deviation from a sphere, we introduce **the sphericity**. For a **spherical particle** of volume equivalent diameter d_υ, $\psi = 1$; for a **non-spherical particle**

$$\psi \equiv \frac{surface-area-of-sphere-of-same-volume-as-particle}{surface-area-of-particle} = \frac{6\upsilon_p}{d_\upsilon s_p}$$

d_υ is the volume equivalent diameter (sieve analysis),
s_p is the surface area of one particle (adsorption measurements)
υ_p is the volume of one particle

Another measure of the shape is the **volumetric shape factor**, *k*.

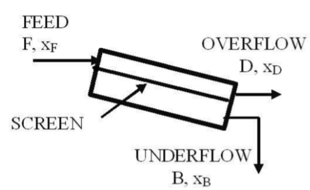
$$k = \upsilon_p / d_p^{\,3}$$

where d_p is a characteristic particle dimension obtained from sieving, frequently related to the projected area diameter d_α. For a sphere $k = \pi/6 = 0.524$

Some approximate values for *k* for typical particles:
Sand 0.26; coal 0.23; limestone 0.16; gypsum 0.13; flake graphite 0.023; mica 0.003.

British fine mesh (B.S.S. 410)(3)			I.M.M.(4)			U.S. Tyler(5)			U.S. A.S.T.M.(5)		
Sieve no.	Nominal aperture in.	Nominal aperture µm	Sieve no.	Nominal aperture in.	Nominal aperture µm	Sieve no.	Nominal aperture in.	Nominal aperture µm	Sieve no.	Nominal aperture in.	Nominal aperture µm
						325	0.0017	43	325	0.0017	44
						270	0.0021	53	270	0.0021	53
300	0.0021	53				250	0.0024	61	230	0.0024	61
240	0.0026	66	200	0.0025	63	200	0.0029	74	200	0.0029	74
200	0.0030	76							170	0.0034	88
170	0.0035	89	150	0.0033	84	170	0.0035	89			
150	0.0041	104				150	0.0041	104	140	0.0041	104
120	0.0049	124	120	0.0042	107	115	0.0049	125	120	0.0049	125
100	0.0060	152	100	0.0050	127	100	0.0058	147	100	0.0059	150
			90	0.0055	139	80	0.0069	175	80	0.0070	177
85	0.0070	178	80	0.0062	157	65	0.0082	208	70	0.0083	210
			70	0.0071	180				60	0.0098	250
72	0.0083	211	60	0.0083	211	60	0.0097	246	50	0.0117	297
60	0.0099	251							45	0.0138	350
52	0.0116	295	50	0.0100	254	48	0.0116	295	40	0.0165	420
			40	0.0125	347	42	0.0133	351	35	0.0197	500
44	0.0139	353				35	0.0164	417	30	0.0232	590
36	0.0166	422	30	0.0166	422	32	0.0195	495			
30	0.0197	500				28	0.0232	589			
25	0.0236	600									
22	0.0275	699	20	0.0250	635	24	0.0276	701	25	0.0280	710
18	0.0336	853	16	0.0312	792	20	0.0328	833	20	0.0331	840
16	0.0395	1003				16	0.0390	991	18	0.0394	1000
14	0.0474	1204	12	0.0416	1056	14	0.0460	1168	16	0.0469	1190
12	0.0553	1405	10	0.0500	1270	12	0.0550	1397			
10	0.0660	1676	8	0.0620	1574	10	0.0650	1651	14	0.0555	1410
8	0.0810	2057				9	0.0780	1981	12	0.0661	1680
7	0.0949	2411				8	0.0930	2362	10	0.0787	2000
6	0.1107	2812	5	0.1000	2540	7	0.1100	2794	8	0.0937	2380
5	0.1320	3353				6	0.1310	3327			
						5	0.1560	3962	7	0.1110	2839
						4	0.1850	4699			
									6	0.1320	3360
									5	0.1570	4000
									4	0.1870	4760

The Figure below is a schematic diagram of a screen



Flow of solid particles represented by F (mass/time) that consists of particles made out of materials "1" (oversize) and "2" (undersize) respectively. **The mass fraction of the component to be separated (assumed component "1") is x_F.** Let D and B be the mass flow rates of the overflow and underflow respectively and **x_D and x_B be the mass fraction of material "1".** The mass fractions of **material "2"** in the feed, overflow and undeflow are $1\text{-}x_F$, $1\text{-}x_D$, and $1\text{-}x_B$.

Overall mass balance gives $\quad F = D + B$

Material balance for "1" $\quad Fx_F = Dx_D + Bx_B$

Elimination of B from the above $\quad \frac{D}{F} = \frac{x_F - x_B}{x_D - x_B}$

While elimination of D gives $\quad \frac{B}{F} = \frac{x_D - x_F}{x_D - x_B}$

Considering **unit mass** of particles consisting of particles of characteristic dimension d_{p1} (from sieving analysis), constituting a mass fraction of x_1, particles of characteristic dimension d_{p2}, constituting a mass fraction of x_2 and so on

Volume-surface mean diameter

$$d_{S\upsilon} \equiv \frac{1}{\sum_{i=1}^{n} (x_i / d_{pi})}$$

Arithmetic mean diameter

$$d_n = \frac{\sum_{i=1}^{n} (n_i d_{pi})}{\sum_{i=1}^{n} n_i} = \frac{\sum_{i=1}^{n} (n_i d_{pi})}{N}$$

Mass mean diameter

$$d_w = \sum_{i=1}^{n} x_i d_{pi}$$

Length average mean diameter

$$d_l = \left[\frac{\sum_{i=1}^{n} (x_i / d_{pi}^2)}{\sum_{i=1}^{n} (x_i / d_{pi}^3)} \right]$$

Dividing the total area of the sample by the number of particles in the mixture gives the average surface of a particle. The diameter of such a particle is the ***surface mean diameter or the Sauter mean diameter (see below)***

$$d_S = \left[\frac{\sum_{i=1}^{n} (x_i / d_{pi})}{\sum_{i=1}^{n} (x_i / d_{pi}^3)} \right]^{1/2}$$

Dividing the total volume of the sample by the number of particles in the mixture gives the average volume of a particle. The diameter of such a particle is the ***volume mean diameter***.

$$d_\upsilon = \left[\frac{1}{\sum_{i=1}^{n} (x_i / d_{pi}^3)} \right]^{1/3}$$

$$\boxed{Screen\ Area = \frac{Flowrate\ of\ underflowin\ short\ tons\ per\ hour}{Basic\ capacity \times Bulk\ density \times Factors(F, E, S, D, O, W)}}$$

Energy Requirements for Comminution

The ratio of the surface energy created by crushing to the energy absorbed by the solid is the **crushing efficiency** (0.0006-0.01).

$$\boxed{\eta_c = \frac{e_s (A_{wb} - A_{wa})}{E_n} = \frac{Surface-energy-created-by-crushing}{Energy-absorbed-by-the-solid}}$$

e_S is the surface energy per unit area,
A_{wb} and A_{wa} are the areas per unit mass of product and feed
E_n is the energy absorbed by a unit mass of the material

We can also define the **mechanical efficiency**, η_m. Note that part of the total energy is used to overcome friction in the bearings and other moving parts, and the rest is available for crushing. Thus,

$$\boxed{\eta_m = \frac{E_n}{E} = \frac{Energy-absorbed-by-the-material}{Total-energy-input}}$$

Combining $E = \dfrac{E_n}{\eta_m} = \dfrac{e_s (A_{wb} - A_{wa})}{\eta_m \eta_c}$

The power required by the machine

$$\boxed{P = E\dot{m} = \frac{\dot{m} e_s (A_{wb} - A_{wa})}{\eta_m \eta_c}}$$

$\dot{m}$ is the feed mass flow rate

Empirical Relations for Comminution Energy

All these three laws can be derived from the basic differential equation

$$\frac{dE}{dL} = -CL^p$$

The energy dE per unit mass required to effect a small change dL in the size of unit mass of material is a simple power function of the size

If p=-2, the Rittinger crushing law is recovered. Simple integration gives

$$\boxed{E = C\left(\frac{1}{L_2} - \frac{1}{L_1}\right)}$$

Writing $C = K_R f_C$ where f_C is the crushing strength of the material, and K_R is the Rittinger's constant,

$$\boxed{E = K_R f_c \left(\frac{1}{L_2} - \frac{1}{L_1}\right)}$$

If p=-1, the Kick crushing law is recovered. Simple integration gives

$$E = C\ln\frac{L_1}{L_2} \quad or \quad E = K_K f_c \ln\frac{L_1}{L_2}$$

K_k is the Kick constant. Neither of K_R or K_K are dimensionless.
Neither of these laws permit an accurate calculation of the energy requirement

Bond has suggested a law intermediate between Rittinger's and Kick's laws, putting p=-3/2. Thus:

$$E = 2C\left(\frac{1}{L_2^{1/2}} - \frac{1}{L_1^{1/2}}\right) = 2C\sqrt{\left(\frac{1}{L_2}\right)\left(1-\frac{1}{q^{1/2}}\right)}$$

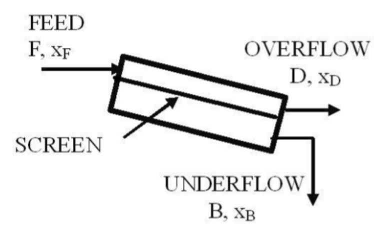
Where $q = L_1 / L_2$ is the reduction ratio. Writing $C = 5E_i$

$$E = E_i\sqrt{\left(\frac{100}{L_2}\right)\left(1-\frac{1}{q^{1/2}}\right)}$$

E_i is the work index and expresses the amount of energy required to reduce unit mass of material from an infinite particle size to a particle size L_2 of 100 μm, that is $q = \infty$. The size of the material is taken as the size of the square hole through which 80% of the material will pass. The L2 in the above equation is in μm

Screening Effectiveness

No screen gives perfect separations. If ideal, all "1" would end up in the overfllow. Thus, the screen effectiveness is defined as a measure of the success of the screen in closely separating "1" from "2"



A common measure of screen effectiveness, E_1 is the ratio of oversize material "1" that is actually in the overflow to the amount of "1" entering with the feed. Thus,

$$E_1 = \frac{Dx_D}{Fx_F}$$

Similarly, an effectiveness E_2, based on the undersize materials is given by,

$$E_2 = \frac{B(1-x_B)}{F(1-x_F)}$$

A combined overall effectiveness can be defined as $E = E_1E_2 = \dfrac{DBx_D(1-x_B)}{F^2x_F(1-x_F)}$

Substituting D/F and B/F

$$E = \frac{(x_F - x_B)(x_D - x_F)x_D(1-x_B)}{(x_D - x_B)^2(1-x_F)x_F}$$

Bond work index values for various materials (E_i is given in kWh/ton)

Material	Specific gravity	Work index E_i
Bauxite	2.20	8.78
Cement clinker	3.15	13.45
Cement raw material	2.67	10.51
Clay	2.51	6.30
Coal	1.4	13.00
Coke	1.31	15.13
Granite	2.66	15.13
Gravel	2.66	16.06
Gypsum rock	2.69	6.73
Iron ore (hematite)	3.53	12.84
Limestone	2.66	12.74
Phosphate rock	2.74	9.92
Quartz	2.65	13.57
Shale	2.63	15.87
Slate	2.57	14.30
Trap rock	2.87	19.32

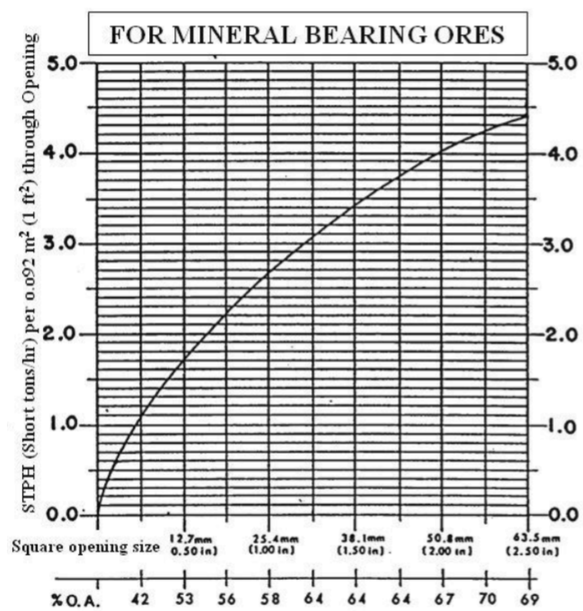


Figure 3.6

The Basic Screen Capacity is obtained from Figure 3.6 where it is plotted as a function of square opening size of the screen. Note that this plot is for mineral bearing ores. Figure 3.6 is also for a material having a density of 1602 kg/m³.
In the formula, calculate **bulk density** by dividing the density of the feed material by 1602.

The factor *F* is a measure of the amount of fines in the feed that is material, which has a size, one half of the screen opening. We can get this from a sieve analysis of the feed. When the fines fraction=0.4 (40%) *F*=1.0. Table 3.2 gives values for F as a function of % fines.

The factor *E* is an efficiency factor. Efficiency is a ratio of the amount of material in the feed that actually passes through the screen to the amount that should pass through. An "ideal" screen would have an efficiency of 95% (no screen is perfect) so its efficiency factor for 95% *E*=1.0. For values for *E* see Table 3.2.

Table 3.2: Fines and Efficiency Factors (*F* and *E*)

%	FACTOR	
	Fines *F*	Efficiency *E*
0	0.44	
10	0.55	
20	0.70	
30	0.80	
40	1.00	
50	1.20	
60	1.40	
70	1.80	2.25
80	2.20	1.75
85	2.50	1.50
90	3.00	1.25
95	3.75	1.00

***S* is a slot factor** and is used if the screen has slots rather than more or less square holes. The rationale is that more particles would pass through a slotted opening because there are fewer interfering cross wires or bars. Values for *S* can be found in Table 3.3.

***D* factor:** It allows for the fact that, except for the top deck of a series of stacked screens, the feed does not arrive at only one end of the deck. Thus, material passing through the top deck, say half way down the screen arrives at the second screen in the stack only half way down, thus the upper half of the second screen is not receiving any feed of this material and so is ineffective. Values for D are in Table 3.4.

Table 3.3: Slotted deck opening factor (*S*)

Typical Deck Preparations	Length/Width ratio	Slotted Opening Factor S
Square and Slight Rectangular Openings	< 2	1.00
Rectangular Openings Ton-Cap	> 2 but < 4	1.15
Slotted Openings Ty-Rod	> 4 but < 25	1.20
Parallel Rod Decks	> 25	SP 1.4* RA 1.3

*SP = Slots Parallel to Flow, RA = Slots Right Angles to Flow

Table 3.4: Deck Factor

Deck	Deck Factor *D*
Top	1.00
2nd	0.90
3rd	0.80

***W* is a wet screening factor**.

It is used if water is sprayed on the screens during screening. The use of water sprays may improve screening if the correct amount of water is used. The recommended range is 14.83-24.81 l/min per m^3 of feed. Values for *W* are provided in Table 3.5.

Table 3.5: Wet screening factor

SIZE OPENING (Square)	*W*
1/32" or less	1.25
1/16"	3.00
1/8" & 3/16"	3.50
5/16"	3.00
3/8"	2.50
1/2"	1.75
3/4"	1.35
1"	1.25
+2"	1.00

$$\text{Circularity} = \frac{\text{Perimeter of projected area equivalent sphere}}{\text{Actual perimeter}}$$

Open area factor, *O*:

Figure 3.6 shows the % open area associated with a square opening screen width for a typical woven wire screen. If the % open area is significantly different than the one shown, the *O* factor is the ratio of the actual % open area to the standard open area shown on Figure 3.6. For example suppose the screen opening is 24 mm and the % open area is 36. From Figure 3.6 if the opening is 25 mm the standard % open area is 58%. Therefore, *O*=36/58=0.62.

The drag force is a combination of skin friction (*wall drag*) and form drag (*pressure effect*) it can be calculated by solution of the Navier-Stokes equations.

For the case of ***creeping flow*** (flow at very small velocities-inertia forces negligible, **Re<0.1**), the drag force F_D on a **spherical particle** was obtained from **Stokes in 1851**.

$$\boxed{F_D = 3\pi\mu\, d_p\, u}$$

i. skin friction: $2\pi\mu\, d_p\, u$

ii. form drag : $\pi\mu\, d_p\, u$

The drag force is characterized using a "**drag coefficient**", C_D

$$\boxed{C_D = \frac{F_D / A_p}{\rho u^2/2} = \frac{2F_D}{\rho u^2 A_p}}$$

***A_p*:** Projected area of particle on a plane vertical to the direction of flow

The flow is characterized by the **Reynolds number** $\boxed{Re = u\, d_p \rho / \mu}$

Drag Coefficients for Spheres

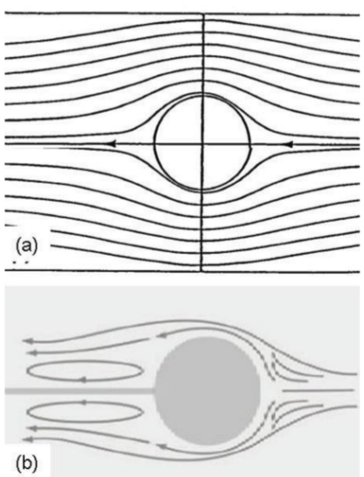
For particle Reynolds numbers less than **(*Re* < 0.1)**, the pressure distribution around the sphere can be solved for and integrated to yield the total drag. The result is familiar to us as **Stoke's law for creeping flow**

$$F_D = 3\pi\mu\, d_p\, u$$

$$\boxed{C_D = \frac{24}{Re} \qquad (Re < 0.1)}$$

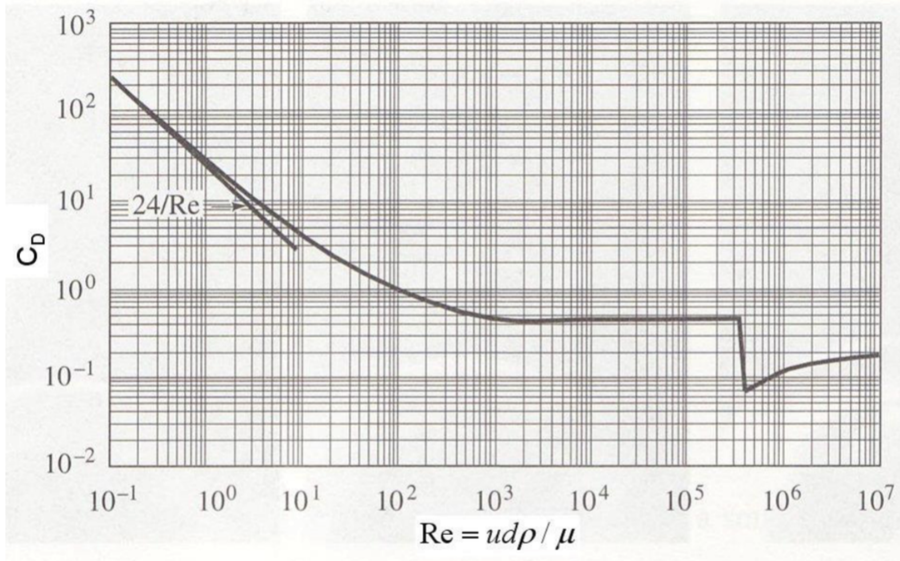
$$Re = u\, d_p \rho / \mu$$

At high Reynolds numbers, the streamlines are not symmetrical about the sphere, inertia becomes important, a recirculating wake begins to form behind the sphere, which grows in size and length with increasing Reynolds number, with the point of "boundary layer separation" gradually shifting forward



$$\boxed{C_D = \frac{18.5}{Re^{3/5}} \qquad (0.1 < Re < 1000)}$$

$$\boxed{C_D = 0.44 \qquad (1000 < Re < 2\times10^5)}$$



Note that the drag coefficient is a function of only the particle Reynolds number.

To calculate the Drag Force: $\boxed{C_D = \frac{F_D / A_p}{\rho u^2/2} = \frac{2F_D}{\rho u^2 A_p}}$ $\quad A_D = \frac{\pi D^2}{4}$

Terminal Velocity Calculations

When **steady-state** (acceleration terms are zero, du/dt=0) is obtained the particles attain their terminal velocities ($u = u_t$)

$$\boxed{u_t = \sqrt{\frac{2g(\rho_P - \rho)m}{A_p \rho_p C_D \rho}}} \qquad \boxed{u_t = \omega\sqrt{\frac{2r(\rho_P - \rho)m}{A_p \rho_p C_D \rho}}}$$

Gravity | **Centrifugal acceleration**

For spherical particles of diameter equal to d_p, $m = \pi d_p^3 \rho_p / 6$ *and* $A_p = \pi d_p^2 / 4$

$$\boxed{u_t = \sqrt{\frac{4}{3}\frac{g(\rho_P - \rho)d_p}{C_D \rho}}}$$

For the alternative case where $\rho_p < \rho$, then drag will act downward

$$\boxed{u_t = \sqrt{\frac{4}{3}\frac{(\rho - \rho_p) g\, d_p}{\rho C_D}}}$$

To avoid iterative procedures for terminal velocity calculations, the problem can be simplified using **dimensional analysis (Buckingham PI Theorem).**

The number of variables characterizing the problem is 6 (ρ, $\Delta\rho$ same units, so 5):

$$u_t,\ d_p,\ \rho,\ \Delta\rho,\ g,\ \mu$$

The number of dimensions is 3: M, L, t (Mass, Length, Time)

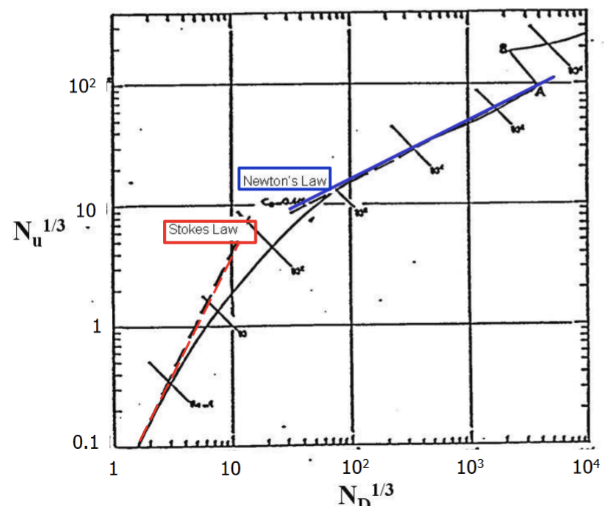
According to the Buckingham PI theorem, any two (5-3) independent dimensionless groups can be used to characterize the problem.

$$C_D = \frac{4}{3}\frac{\Delta\rho\, g\, d_p}{\rho\, u_t^2} \quad \text{and} \quad Re = \frac{d_p u_t \rho}{\mu},$$

However, it is also possible to choose groups, which isolate effects of the important variable u_t and d_p without combining them. The groups, which have been chosen are:

$$\mathrm{N_D} = C_D Re^2 = \frac{4}{3}\frac{g d_p^3 \rho \Delta\rho}{\mu^2} \qquad \mathrm{N_u} = Re/C_D = \frac{\rho^2 u_t^3}{\Delta\rho\, g\, \mu} \qquad \mathrm{N_D} = f(N_u)$$

Plots and Tables have been made of these groups which permit very simple calculations. Solution of problems in the form, "given d_p and other variables find u_t?"



For spheres

$$\mathrm{N_D} = C_D Re^2 = \frac{4}{3}\frac{g d_p^3 \rho \Delta\rho}{\mu^2}$$

$$\mathrm{N_u} = Re/C_D = \frac{\rho^2 u_t^3}{\Delta\rho\, g\, \mu}$$

To identify at which range the motion of the particle lies, the velocity is eliminated from the Reynolds number by substituting the terminal velocity (Stokes' regime) into the *Re* number.

$$\mathrm{Re} = \frac{d_p u_t \rho}{\mu} = \frac{d^3 g\, \rho(\rho_p - \rho)}{18\mu^2} \quad \text{using} \quad u_t = \frac{g(\rho_p - \rho) d_p^2}{18\mu}$$

If Stokes' law applies , *Re* should be less than 1*. Thus, set:

$$K = d\left[\frac{g\rho(\rho_p - \rho)}{\mu^2}\right]^{1/3}$$

Then it turns out that by setting *Re* < 1, to be in the Stokes' regime, *K*<2.6.

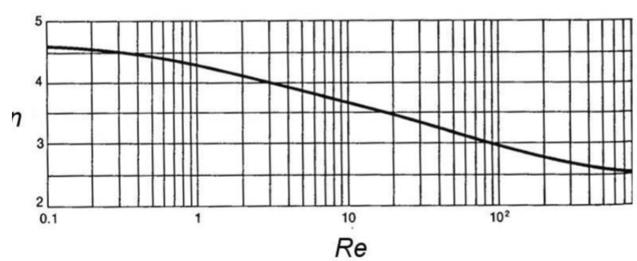
K<2.6	Stokes regime
68.9<*K*<2360	Newton's law applies

*Strictly Stokes' law applies for Re<0.1, however sometimes we relax it to Re<1

In hindered settling, the velocity gradients around each particle are affected by the presence of nearby particles. For such cases, for a uniform suspension, **the settling velocity u_S** can be estimated from the terminal velocity for an isolated particle

$$u_S = u_t(\varepsilon)^n \qquad \text{(Maude and Whitmore Correlation)}$$

where ε is the void fraction of the fine suspension, *n* is the exponent given in Figure as a function of the Reynolds number (for a suspension of 20vol% particles, ε=0.8) .



For very small particles, the ratio u_S/u_t is 0.62 for ε=0.9 and 0.095 for ε =0.6. For large particles the corresponding value is u_S/u_t=0.77 and 0.28.

The viscosity of the suspension is also influenced by the presence of particles.

$$\frac{\mu_S}{\mu} = \frac{1 + 0.5(1-\varepsilon)}{\varepsilon^4}$$

The equation applies for ε>0.6 and it is most accurate when ε>0.9

Heywood's Procedure

(i). Establish the orientation of the particle with respect to the fluid.

(ii). Calculate the projected area equivalent diameter for the particle orientation

(iii.). Define $N_{d_\alpha}^{1/3} = \left[\frac{4}{3}\frac{\rho\,\Delta\rho\, g}{\mu^2}\right]^{1/3} d_\alpha$ where $A_p=\pi d_\alpha^2/4$

(iv). Calculate u_t for the projected area equivalent sphere.

(v.). Begin the correction for a non-spherical particle by calculating the **Heywood volume shape factor, *k*** (Table 4.1. in next page or given or $k=\upsilon_p/d_\alpha^3$).

(vi.). k and $N_{d\alpha}^{1/3}$ have been correlated by Heywood to give **terminal velocity correction factor K_A**, $K_A = \frac{u_t \text{ of non-spherical particle}}{u_t \text{ of sphere with diameter } d_\alpha}$

This correction factor K_A has been measured for many different types of particles (Figure 4.9). Table 4.1 gives values for Heywood's volume shape factor for commonly found regular shapes. The correlation is only valid for $k < 0.524$, the value for a sphere. For $k > 0.524$, naturally, $K_A = 1.0$, which means that u_t(of non-sphere)=u_t(of sphere with $d=d_\alpha$).

TABLE 4.1: Heywood Volumetric Shape Factors

Regular shapes:	
Sphere	0.524
Cube	0.696
Tetrahedron	0.328
Cylinder with $E = 1$:	
viewed along axis	0.785
viewed normal to axis	0.547
Spheroids: $E = 0.5$	0.262
$E = 2$	0.370
Approximate values for isometric irregular shapes, k_c(H2):	
Rounded	0.56
Subangular	0.51
Angular	
tending to prismoidal	0.47
tending to a tetrahedron	0.38
Selected natural particles (D1):	
Sand	0.26
Bituminous coal	0.23
Blast furnace slag	0.19
Limestone	0.16
Talc	0.16
Gypsum	0.13
Flake graphite	0.023
Mica	0.003

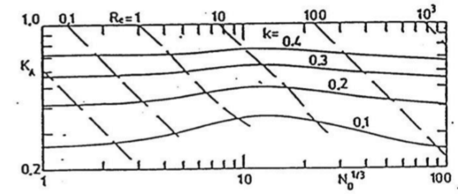


Figure 4.9 :Ratio K_A of terminal velocity of particle of arbitrary shape to that of a sphere having same projected area.

$$K_A = \frac{u_t \text{ of non-spherical particle}}{u_t \text{ of sphere with diameter } d_\alpha}$$

Other Factors Influencing Terminal Velocities

The most practical approach to hindered settling calculations (also discussed above in a graphical form) is to use the **Richardson-Zaki equation**, an empirical expression for the modified terminal velocity found to be satisfactory in many applications This includes the wall effects compared to the method for hindered settling discussed previously.

$$\frac{u_S}{u_{t,\infty}} = \varepsilon^n$$

$$\log u_t = \log u_{t,\infty} + \frac{d_\alpha}{D}$$

ε = void fraction of suspension

1-ε = solids fraction

solid vol./system vol.= v_s/v

$n = 4.65 + 20\frac{d_\alpha}{D}$	$(Re < 0.2)$
$n = \left(4.4 + 18\frac{d_\alpha}{D}\right)Re^{-0.03}$	$(0.2 < Re < 1)$
$n = \left(4.4 + 18\frac{d_\alpha}{D}\right)Re^{-0.1}$	$(1 < Re < 200)$
$n = 4.4\, Re^{-0.1}$	$(200 < Re < 500)$

u_S = particle terminal velocity in presence of other particles

$u_{t,\infty}$ = single particle terminal velocity with column correction

u_t = single particle terminal velocity without column correction

D = column diameter

Calculations using Sphericity

Drag coefficients have been tabulated as a function of sphericity and Reynolds number (based on d_α). The resulting plot (below) is an extension of the standard drag curve developed before. **It can be used in exactly the same way to solve iteratively for u_t** using the drag curve for the appropriate value of sphericity together with Equation for terminal velocity). Sphericities for a number of natural materials are given in Table 4.2 (next page).

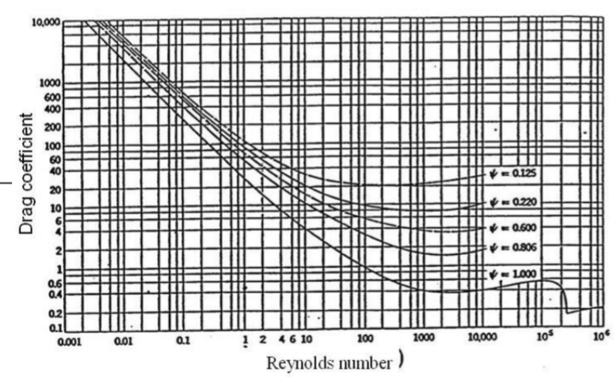


Figure 4.10: Drag coefficient as a function of Re and sphericity

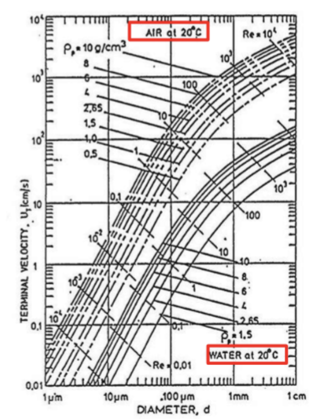


Figure 4.11: Terminal Velocities of spheres in air and water at 20°C