Continuous Time Dynamic Models

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References

- Doraszelski and Judd (2012): less computation in continuous than discrete time
- Estimation and identification: Arcidiacono et al. (2012), Blevins (forthcoming)
- Applications:
  - Schiraldi, Smith, and Takahashi (2013)
  - Cosman (2014)

Why continuous time reduces computation

- Discrete time simultaneous move game suffers from "curse of dimensionality" in computing expectations
    - E.g. entry/exit game with N firms has at least 2^N possible states next period
- If only one player could move each instant then number of possible future states is much lower
- Continuous time: assume move opportunities arrive at stochastically, then $\mathbb{P}(\text{two move at same time}) = 0$

Comparing continuous and discrete time models

- See discussion in Doraszelski and Judd (2012)
- Move order matters – e.g. Cournot vs Stackelberg competition
- Discrete time model limits how often and how much state variables can change
- Embedding problem: sometimes there does not exist a continuous time Markov chain that induces the same probability distribution over states at discrete times as a discrete time Markov chain
- Often no compelling reason to prefer a discrete or continuous time model, but important to remember that they do have slightly different assumptions and implications

Section 1

Model

Model 1

- Notation of Arcidiacono et al. (2012)
- N players indexed by i
- Finite state space $\mathcal{X}$ with K elements, indexed by k
- J actions in $\mathcal{A} = \{0, \dots, J-1\}$.
- Flow payoff u_{ik} from being in state k
- Instantaneous payoff $\psi_{ijk} + \epsilon_{ij}$ from choosing j in state k
- Choice probabilities σ_{ijk}
- Discount rate ρ

Model 2

- States follow an exogenous Markov jump process with intensity matrix:

$$Q = \begin{bmatrix} q_{11} & \cdots & q_{1K} \\ \vdots & \ddots & \vdots \\ q_{K1} & \cdots & q_{KK} \end{bmatrix}$$

  where

$$q_{kl} = \lim_{h \to 0} \frac{\mathrm{P}(X_{t+h} = l | X_t = k)}{h}$$

  is the rate of arrival of moves to state l given state k.
- States also change from actions: $l(m, j, k) =$ state after player m chooses j in state k
- Moves arrive at rate λ
- Beliefs of player $\zeta_{imjk} = \mathrm{P}($ player m chooses j in state $k)$

Model 3

- Value function:

$$V_{ik}(\zeta_i) = \frac{u_{ik} + \sum_{l \neq k} V_{il}(\zeta_i) + \sum_{m \neq i} \lambda \sum_j \zeta_{imjk} V_{i,l(m,j,k)}(\zeta(i)) + \lambda}{\rho + \sum_{l \neq k} q_{kl} + N\lambda}$$

- Best response choice probabilities

$$\sigma_{ijk} = \mathrm{P}(\psi_{ijk} + V_{i,l(i,j,k)}(\zeta_i) + \epsilon_{ij} \geq \psi_{ij'k} + V_{i,l(i,j',k)}(\zeta_i) + \epsilon_{ij'} \forall j')$$

- Equilibrium $\sigma_{-i} = \zeta_i$ for all i

Identification

- Argument is mostly similar to discrete time
- Q and choice probabilities are identified from observed distribution of states
    - Extra argument needed if observed data is at discrete intervals – see Arcidiacono et al. (2012) for details
- Given Q and knowing distribution of ϵ, differences in value functions are given by a known function of choice probabilities
- Expected (over other players actions) payoffs recovered from Bellman equation
- Exclusion identifies payoffs

Section 2

Estimation

Estimation

- Describe 2-step estimator, but could imagine a single step or nested pseudo-likelihood style iteration

Step 1 : estimate hazards and choice probabilities

$$\hat{h} = \arg\max_h \sum_{m=1}^{M} \sum_{n=1}^{T} \underbrace{\log g(\tau_{mn}, k_{mn}; h)}_{\text{likelihood of waiting } \tau_{nm} \text{ to next event given state } k_{mn}} + \underbrace{\sum_{l \neq k_{mn}} I_{mn}(0,l) \log q_{k_{mn} l}}_{\text{next move exogenous state variable}} + \underbrace{\sum_{i} \sum_{j = \neq 0} I_{mn}(i,j) \log(\lambda \sigma_{ijk_{mn}})}_{\text{next move by a player}}$$

Estimation

Step 2 : given $\hat{h}$ compute best response choice probabilites, represent implied hazards as $\Lambda(\theta, \hat{h})$

$$\hat{\theta} = \arg\max_{\theta} \sum_{m=1}^{M} \sum_{n=1}^{T} g(\tau_{mn}, k_{mn}; \Lambda(\theta, \hat{h}) + \\ + \sum_{i} \sum_{j=\neq 0} I_{mn}(i,j) \log(\lambda \Lambda_{ijk_{mn}}(\theta, \hat{h}))$$

Section 3

Applications

Entertainment districts and the value of variety in nightlife: evidence from Chicago

- Competition between businesses in a set of closely related industries
- Structural model: infer consumer preferences, firm's problem from observing entry and exit
- Strong consumer preference from variety — entrant can raise incumbent profits
- High barriers to entry matter for nightlife supply

Related economic literature

- Consumption amenities and valuation of cities
  - Glaeser (2001), Rappaport (2008), Lee (2010), Albouy (2013)
- Measuring consumers' value of access to variety
  - Broda & Weinstein (2006), Consumer goods: Li (2012), Broda & Weinstein (2010), Handbury & Weinstein (2011), Couture (2014)
- Explaining colocation of similar businesses
  - Theoretical: Wolinsky (1983), Fischer & Harrington (1996), Konishi (2005)
  - Empirical: Davis (2006), Jia (2008), Dunne *et al.* (2013), Datta & Sudhir (2013), Yang (2014)
- Profit functions from entry/exit decisions
  - Bresnahan & Reiss (1991), Pesendorfer & Schmidt-Dengler (2003), Aguirregabiria and Mira (2007), Ryan (2012), Collard-Wexler (2013), Dunne *et al.* (2013)

Structural modelling approach

- Data on venue entry and exit — find parameters to rationalize as equilibrium

- Build model in stages:
  1. Static model: consumers choose to go out, venues choose price
  2. Dynamic model: venues choose whether to enter and exit
  3. Estimation: match parameters to observed entry and exit

- Static and dynamic counterfactuals

Static model

Consumer's problem

- Nested CES utility — substitution within, between venue types
- Reservation utility shock: stay in or go out?
- More utility to going out means more consumers choose to do so

Firm's problem

- Firms adjust prices to maximize profits taking into account consumer preferences, each others' behaviour
- Unique equilibrium prices for given number of competitors

Necessary assumption: interact only within neighbourhood

Dynamic model and continuous-time estimation

Dynamic model of entry and exit

- Entrants, incumbents receive opportunities via Poisson process
- Entrants can enter with given type, neighbourhood
- Best-respond to consistent beliefs — Markov-Nash equilibrium

Continuous-time structural estimation

- Arcidiacono, Bayer, Blevins, Ellickson
- Intuition: choose structural parameters so observed entry, exit rates rationalized as equilibrium
- Advantages: feasibility, data usage, flexibility

Data sources

Venues and regulation from City of Chicago Data Portal (2006–2014)

- Divide venues into categories based on licensing:
  1. Amusement only (e.g. Los Globos Ballroom)
  2. Drinks only (e.g. Casual Tap)
  3. Drinks and amusement (e.g. Tabu)
  4. Drinks and music (e.g. New Celebrity Lounge)

- Two types of within-city regulation:
  1. Dry areas: no bars at all
  2. Moratoria: no new bars

- Divide city into neighbourhoods based on community areas

Demographic data from Census, American Community Survey

Estimated preference for variety

Elasticity	Symbol	Estimate
Between sectors	η	2.04 (0.002)
Amusement only	ρ_1	4.90 (0.013)
Drinks only	ρ_2	2.15 (0.001)
Drinks and amusement	ρ_3	3.56 (0.224)
Drinks and music	ρ_4	7.96 (0.290)

- Amusement only, Drinks and amusement $\approx 5^{th} - 25^{th}$ percentile of consumer goods (Broda and Weinstein (2010))
- Drinks and music $\approx$ restaurants (Couture (2014))

Results: entry sunk cost and exit payoff

	Value (thousands of dollars)	
Entry cost	Amusement only baseline	862 [839, 886]
	Drinks only baseline	943 [871, 1023]
	Drinks and amusement baseline	892 [797, 995]
	Drinks and music baseline	670 [83, 7588]
Exit payoff	Amusement only	38.4 [36.6, 3383.7]
	Drinks only	38.3 [37.5, 39.8]
	Drinks and amusement	42.9 [36.8, 201.4]
	Drinks and music	40.5 [38.5, 44.3]

Barriers to entry

Is \$700k-\$900k to open a bar reasonable?

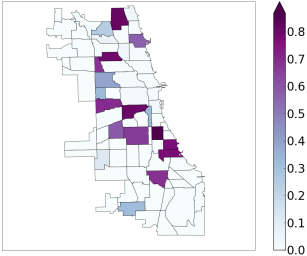
- Small business literature:
  - PowerHomeBiz: \$239k-\$837k depending on jurisdiction
  - Houston Chronicle: up to \$1 million depending on licensing requirements
  - IBISWorld Industry Reports: \$200k-\$1 million
- Regulatory expenses: fees, time uncertainty, renovations to comply
- Marketing, hiring, cash on hand for payment systems

One more venue: impacts on profits

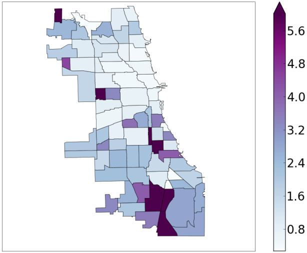
Percentage of observations where counterfactual new venue would *increase* incumbent profit

	Amusement only	Drinks only	Drinks and amusement	Drinks and music
Amusement only	36.3 [0.0,36.3]	13.2 [0.0,13.6]	6.7 [6.4,19.1]	14.1 [0.0,14.1]
Drinks only	13.3 [12.7,13.6]	13.2 [0,14.5]	17.8 [9.5,18.5]	8.4 [0.0,8.6]
Drinks and amusement	0.0 [0.0,0.3]	1.1 [0.0,1.2]	32.2 [0.0,86.8]	12.4 [0.0,12.4]
Drinks and music	0.0 [0.0,0.0]	1.1 [0.0,1.1]	13.3 [0.0,13.3]	25.3 [0.0,26.3]

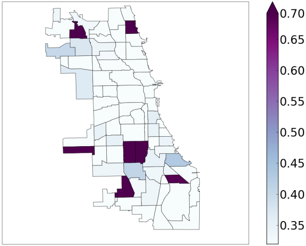
Dynamic counterfactual: lower barriers to entry



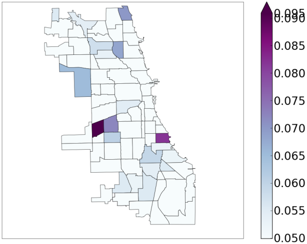
Amusement only



Drinks only



Drinks and amusement



Drinks and music

Discussion and further research

- Dynamic structural model for competition of related businesses
- Strong preferences for variety, high barriers to entry
- Further research: non-pecuniary benefits and goodness of fit

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