# Demand and supply of differentiated products

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# References

- Reviews:
  - Ackerberg et al. (2007) section 1 (these slides use their notation)
  - Aguirregabiria (2019) chapter 2
  - Reiss and Wolak (2007) sections 1-7, especially 7
- Classic papers:
  - Berry (1994)
  - Berry, Levinsohn, and Pakes (1995)

# Section 1

# Introduction

# Introduction

- Typical market for consumer goods has many differentiated, but similar products, e.g.
  - Cars
  - Cereal
- Differentiated products are a source of market power
- Having many products can result in many parameters creating estimation difficulties and requiring departures from textbook demand and supply models

# Motivation

- Counterfactuals that do not change production technology
  - Mergers
  - Tax changes
- Effects of new goods
- Cost-of-living indices
- Product differentiation and market power
  - Cross-price elasticities

# Section 2

# Demand in product space

# Demand in product space 1

- $J$ products, each treated as separate good
- Classical demand,

$$
\begin{aligned}
q_1 =& D_1(p_1, \ldots, p_J, z_1, \eta_1; \beta_1) \\
\vdots =& \vdots \\
q_J =& D_J(p_1, \ldots, p_J, z_J, \eta_J; \beta_J),
\end{aligned}
$$

and supply (firms' first-order conditions for prices):

$$
\begin{aligned}
p_1 =& g_1(q_1, \ldots, q_J, w_1, \nu_1; \theta_1) \\
\vdots =& \vdots \\
p_J^d =& g_J(q_1, \ldots, q_J, w_J, \nu_J; \theta_J),
\end{aligned}
$$

where

# Demand in product space 2

- $p_j$ = price
- $q_j$ = quantity
- $z_j$ = observed demand shifter
- $\eta_j$ = unobserved demand shock
- $\beta_j$ = demand parameters
- $w_j$ = observed supply shifter
- $\nu_j$ = unobserved supply shock
- $\theta_j$ = supply parameters

- $D_j$ typically parametrically specified, e.g.

$$\ln q_j = \beta_{j0} + \beta_{j1} p_1 + \cdots + \beta_{jJ} p_J + \beta_{jy} \ln y + Z_1 \gamma + \nu_j$$

# Demand in product space

- Use reduced form to find instruments

$$
\begin{aligned}
q_1 =& \Pi_1^q(Z, W, \nu, \eta; \beta, \theta) \\
\vdots =& \vdots \\
q_J =& \Pi_J^q(Z, W, \nu, \eta; \beta, \theta) \\
p_1 =& \Pi_1^p(Z, W, \nu, \eta; \beta, \theta) \\
\vdots =& \vdots \\
p_J =& \Pi_J^p(Z, W, \nu, \eta; \beta, \theta)
\end{aligned}
$$

- Cost shifters of product $j$ excluded from demand and supply of product $k$, but in reduced form
  - Cost data often not available
  - If available, unlikely to be product specific
- Attributes of other products
  - Hausman (1996) uses prices of other products
  - Hard to justify, especially with prices

# Demand in product space 1

- Advantages of product space:
  - Flexible substitution patterns
  - Does not require detailed product attribute data
- Problems with product space:
  1. Representative agent and aggregation issues
     - With heterogeneous preferences, aggregate market demand need not meet restrictions on individual demand derived from economic theory
     - Cannot use restrictions easily to improve estimates
     - Can use simulation to aggregate (Pakes, 1986)
  2. Too many parameters, $O(J^2)$
     - Can limit by restricting cross-price elasticities, e.g. Pinkse, Slade, and Brett (2002)
  3. Too many instruments needed, $J$
  4. Cannot analyze new goods

# Section 3

# Demand in characteristic space

# Demand in characteristic space

- Motivation:
  - Why do firms differentiate products?
  - Because consumers have heterogeneous tastes for product characteristics
    - E.g. cars: tastes for size, safety, fuel efficiency, etc
- Main idea: model consumer preferences for characteristics and treat products as bundles of characteristics
- Early work: Lancaster (1971), McFadden (1973)
- Key extension to early work: Berry, Levinsohn, and Pakes (1995)

# Early work in characteristic space

- Consumer chooses one or none of $J$ products
- Utility of consumer $i$ from product $j$

$$u_{ij} = x_j \beta + \epsilon_{ij}$$

  with $\epsilon_{ij}$ iid across $i$ and $j$ (usually Type I extreme value)
- Implies aggregate demand (for logit)

$$q_j = \frac{\exp(x_j \beta)}{1 + \sum_{k=1}^{J} \exp(x_k \beta)}$$

- Problem: restrictive substitution "independence of irrelevant alternatives"
  - Two goods with the same shares have the same cross price elasticities with any third good (think about a luxury and bargain good with equal shares)
  - Goods with same shares should have same markups
- Solution: add heterogeneity in $\beta$ and/or allow correlation across $j$ in $\epsilon_{ij}$

# Model 1

- Consumers $i$, goods $j$, markets $t$
- Utility: (include good $0 =$ buy nothing)

$$u_{ijt} = U(\underbrace{\tilde{x}_{jt}}_{\text{observed}}, \overbrace{\xi_{jt}}^{\text{unobserved}}, \underbrace{z_{it}}_{\text{observed}}, \overbrace{\nu_{it}}^{\text{unobserved}}, \underbrace{p_{jt}}_{\text{observed}}; \theta)$$

  - $x_{jt} = (\tilde{x}_{jt}, p_{jt})) \in \mathbb{R}^K$, $z_{it} \in \mathbb{R}^R$, $\nu_{it} \in \mathbb{R}^L$
- Choose $j$ if $u_{ijt} > u_{ikt} \; \forall k \neq j$

# Model 1

- Usually $U(\cdot)$ linear:

$$u_{ijt} = \overbrace{x_{jt}}^{1\times K} \overbrace{\underbrace{\theta_{it}}_{=\bar{\theta}+\theta^o z_{it}+\theta^u \nu_{it}}}^{K\times 1} + \overbrace{\xi_{jt}}^{1\times 1} + \epsilon_{ijt}$$

for $j = 1 \ldots J$ and normalize $u_{i0t} = 0$

- Assume $\epsilon_{ijt}$ i.i.d. double exponential
- Assume $\nu_{it} \sim f_\nu(\cdot;\theta)$, e.g. independent normal
- Write as product specific + observed interactions + unobserved interactions

$$u_{ijt} = \underbrace{\delta_j}_{=x_{jt}\bar{\theta}+\xi_{jt}} + x_{jt}\underbrace{\theta^o}_{K\times R} z_{it} + x_{jt}\underbrace{\theta^u}_{K\times L} \nu_{it} + \epsilon_{ijt}$$

# Endogeneity

- Usually assume $\mathrm{E}[\nu_{it}|x_{jt}, z_{it}] = 0$ and $\mathrm{E}[\epsilon_{ijt}|x_{jt}, z_{it}] = 0$
  - Not interested in counterfactuals with respect to changes in $z_{it}$, so can treat as residual, i.e.

$$\nu_{it} = \theta_{it} - \mathrm{E}[\theta_{it}|z_{it}]$$

  - Market average $\nu_{it}$ or $\epsilon_{ijt}$ plausibly correlated with $p_{jt}$ or other product characteristics, but this correlation absorbed into $\xi_{jt}$ and/or market fixed effects

# Endogeneity

- Problem is $\xi_{jt}$
  - Prices and other flexible product characteristics must be correlated with $\xi_{jt}$
  - If $\xi_{jt}$ serially correlated, then likely also correlated with inflexible product characteristics
  - Need instrument, $w_{jt}$ such that $\mathrm{E}[\xi_{jt}|w_{jt}] = 0$
    - Cost shifters
    - Characteristics of other products

# Estimation and identification

- Depends on data:
  - Aggregate product market shares and characteristics
  - Individual characteristics and choices
- Additional assumptions:
  - Use supply and equilibrium assumptions to get a pricing equation

# Aggregate data 1

- Often only have data on product characteristics and market shares
- Maybe also distribution of some individual characteristics for each market (e.g. income and education from CPS or census)
- Instrument $w$ such that $\mathrm{E}[\xi_j|w] = 0$
- Distribution of $\nu \sim f_\nu(\cdot; \theta_\nu)$
  - Combination of estimated market level distribution of observed individual characteristics and parametric distributions of unobserved individual characteristics
  - e.g. $\nu_{it} = (educ_{it}, income_{it}, e_{it})$

$$F_{\nu,t}(s, y, e; \theta_\nu) = \underbrace{\hat{F}_t(s, y)}_{\text{empirical distribution}} \Phi\left(\frac{e - \theta_\nu^\mu}{\theta_\nu^\sigma}\right)$$

$\hat{F}_t(s, y)$ estimated from CPS or other similar data set

# Aggregate data 2

- Assume $\epsilon_{ijt} \sim$ double exponential (aka Gumbel or type I extreme value) as in logit
  - Computationally convenient, but other distributions feasible too

# Estimation outline

- Estimate $\theta$ from moment condition

$$\mathrm{E}[\xi(\cdot;\theta)|w] = 0$$

- Where $\xi(\cdot;\theta)$ is such that model predicted market shares = observed market shares[1]
  1. Compute shares given $\theta$, $\sigma(\cdot;\theta,\delta)$
  2. Find $\delta(\cdot;\theta) = x_{jt}\bar{\theta} + \xi(\cdot;\theta)$ such that observed shares, $s_{jt}$ = model shares, $\sigma(\cdot;\theta,\delta)$, then

$$\xi(\cdot;\theta) = \delta(\cdot;\theta) - x_{jt}\bar{\theta}$$

[1] In this slide $\cdot$ means the data. I will leave the $\cdot$ out of the notation in subsequent slides. I will also leave out $t$ subscripts.

# Computing model shares

- Integrate over $\nu$

$$\sigma_j(\theta,\delta) = \int \frac{\exp(\delta_j + x_j \theta^u \nu)}{1 + \sum_{k=1}^{j} \exp(\delta_k + x_k \theta^u \nu)} dF_\nu(\nu)$$

- Integral typically has no closed form, so compute numerically, usually by Monte Carlo integration

$$\sigma_j(\theta,\delta) = \sum_{r=1}^{N_s} \frac{\exp(\delta_j + x_j \theta^u \nu_r)}{1 + \sum_{k=1}^{j} \exp(\delta_k + x_k \theta^u \nu_r)}$$

where $\nu_r$ are $N_s$ random draws from $f_\nu$

  - Issues about how best to compute integral – simulation vs quadrature, type of simulation (Skrainka and Judd, 2011)
  - Simulation (more generally approximation) of integral affects distribution of estimator

# Solving for $\delta$ and $\xi$

- Want $\delta$ s.t. $\sigma_j(\theta, \delta) = \hat{s}_j$
- Berry, Levinsohn, and Pakes (1995) show

$$T(\delta) = \delta + \log(\hat{s}_j) - \log(\sigma_j(\theta, \delta))$$

  is a contraction
  - Unique fixed point $\delta$ such that $\delta = \delta + \log(\hat{s}_j) - \log(\sigma_j(\theta, \delta))$, i.e. $\hat{s}_j = \sigma_j(\theta, \delta)$
  - Can compute $\delta(\theta)$ by repeatedly applying contraction (in theory and practice often faster to use other method)
- $\xi_j(\theta) = \delta_j(\theta) - x_j\bar{\theta}$
- Important identifying assumption: only $\theta$ s.t. $\xi_j(\theta) = \xi_j^0$ is true $\theta_0$

# Estimating $\theta$

- Conditional moment restriction $\mathrm{E}[\xi_j(\theta)|w] = 0$
- Empirical unconditional moments:

$$G_{J,T,N,N_s} = \frac{1}{JT}\sum_{j=1}^{J}\sum_{t=1}^{T}\xi_{jt}(\theta)f(w_t)$$

  where
  - $f(w)$ = vector of function of $w$
  - $J$ = number of products
  - $T$ = number of markets
  - $N$ = number of observations in each market underlying $\hat{s}_j$
  - $N_s$ = number of simulations
- Asymptotic properties (consistency, distribution), depend on which of $J, T, N$, and $N_s$ are $\rightarrow\infty$, see Berry, Linton, and Pakes (2004)
- Reynaert and Verboven (2014): using optimal instruments greatly improves efficiency and stability

# Pricing equation 1

- More moments give more precise estimates
- Assumption about form of equilibrium allows use of firm first order condition (pricing equation) as additional moment
- Nash equilibrium in prices
- Log linear marginal cost

$$\log mc_j = r_j \theta^k + \omega_j$$

  - $r_j$ = observed product characteristics, input prices, maybe quantity, etc
  - $\omega_j$ = unobserved productivity, possibly endogenous
- Firm $f$ producing set of product $\mathcal{J}_f$,

$$\max_{p_j : j \in \mathcal{J}_f} \sum_{j \in \mathcal{J}_f} \left( p_j - C_j(\cdot) \right) M s_j(\cdot, p)$$

# Pricing equation 2

- First order condition:

$$\sigma_j(\cdot) + \sum_{l \in \mathcal{J}_f} (p_l - mc_l) \frac{\partial \sigma_l(\cdot)}{\partial p_j} = 0$$

- Collect as

$$s + (p - mc)\Delta = 0$$

- Rearrange and use log linear marginal cost

$$\log(p - \Delta^{-1}\sigma) - r\theta^c = \omega(\theta)$$

- Conditional moment restriction $\mathrm{E}[\omega(\theta)|w] = 0$
- Add empirical moments to $G$, $\frac{1}{JT}\sum_{jt} \omega_{jt}(\theta) f(w_t)$

# Micro data

- Berry, Levinsohn, and Pakes (2004)
- Data on individual choices and characteristics

$$u_{ijt} = \underbrace{\delta_j}_{=x_{jt}\bar{\theta}+\xi_{jt}} + x_{jt} \underbrace{\theta^o}_{K\times R} z_{it} + x_{jt} \underbrace{\theta^u}_{K\times L} \nu_{it} + \epsilon_{ijt}$$

- Random coefficients discrete choice model, so can identify and estimate $\delta$, $\theta^o$, and $\theta^u$ without assumptions about $\xi$ and $x$
    - Ichimura and Thompson (1998) give conditions for nonparametric identification of random coefficients binary choice models
    - Estimate by MLE or (usually) GMM
- Still need $\bar{\theta}$ for price elasticities, etc

$$\delta_j = x_{jt}\bar{\theta} + \xi_{jt}$$

    - Use IV
    - Use IV with a pricing equation

# Section 4

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