Dynamic Oligopoly

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References

- Reviews:
  - Aguirregabiria (2019) chapters
  - Ackerberg, Caves, and Frazer (2015) section 3
  - Aguirregabiria and Mira (2010)
  - Doraszelski and Pakes (2007)
  - My notes from 628
- Key papers:
  - Ericson and Pakes (1995), Aguirregabiria and Mira (2007), Bajari, Benkard, and Levin (2007)

Section 1

Introduction

Introduction

Section 2

Model

Model primitives 1

- N players indexed by i
- Discrete time index by t
- Player i chooses action $a_{it} \in A$; actions of all players $a_t = (a_{1t}, ..., a_{Nt})$
- State $x_t = (x_{1t}, ..., x_{Nt}) \in X$ observed by econometrician and all players at time t
- Private shock $\epsilon_{it} \in \mathcal{E}$
- Payoff of player i is $U_i(a_t, x_t, \epsilon_{it})$
- Discount factor β

Assumptions 1

1. A is a finite set
2. Payoffs additively separable in ϵ_{it},
$$U_i(a_t, x_t, \epsilon_{it}) = u(a_t, x_t) + \epsilon_{it}(a_{it})$$
3. x_t follows a controlled Markov process
$$F(x_{t+1} | \underbrace{\mathcal{I}_t}_{\text{all information at time } t}) = F(x_{t+1} | a_t, x_t)$$
4. The observed data is generated by a single Markov Perfect equilibrium
5. β is known
6. ϵ_{it} i.i.d. with CDF G, which is known up to a finite dimensional parameter

Assumptions 2

Each of these assumptions could be (and in some papers has been) relaxed; relaxing 6 is probably most important empirically

Value function 1

- Strategies $\alpha : (X \times \mathcal{E})^N \to A^N$
    - α_i is the strategy of player i
    - α_{-i} is the strategy of other players
- "Value" functions
    - Value function given strategies: $V_i^\alpha(x_t, \epsilon_{it})$
    - Integrated (over ϵ) value function

$$\begin{aligned}\bar{V}^\alpha(x) = & \int V_i^\alpha(x_t, \epsilon_{it}) dG(\epsilon_{it}) \\ = & \int \left(\max_{a_{it} \in A} v_i^\alpha(x_t, a_{it}) + \epsilon_{it}(a_{it}) \right) dG(\epsilon_{it})\end{aligned}$$

- Choice specific value function

$$v_i^\alpha(a_{it}, x_t) = \mathrm{E}_{\epsilon_{-i}} \left[\begin{array}{c} u(a_{it}, \alpha_{-i}(x_t, \epsilon_{-it}), x_t) + \\ + \beta \mathrm{E}_x[\bar{V}_i^\alpha(x_{t+1}) | a_{it}, \alpha_{-i}(x_t, \epsilon_{-it}), x_t] \end{array} \right]$$

Equilibrium

- Markov perfect equilibrium: given α_{-i}, α_i maximizes v_i

$$\alpha_i(x_t, \epsilon_{it}) \in \arg\max_{a_i} \mathrm{E}_{\epsilon_{-i}} \begin{bmatrix} u\left(a_i, \alpha_{-i}(x_t, \epsilon_{-it}), x_t\right) + \epsilon_{it}(a_i) + \\ +\beta \mathrm{E}_x \left[\bar{V}_i^{\alpha}(x_{t+1}) | a_{it}, \alpha_{-i}(x_t, \epsilon_{-it}), x_t\right] \end{bmatrix}$$

Equilibrium in conditional choice probabilities 1

- Conditional choice probabilities

$$
\begin{aligned}
\mathrm{P}_i^\alpha(a_i|x) =& \mathrm{P}\left(a_i = \arg\max_{j\in A} v_i^\alpha(j,x) + \epsilon_{it}(j)|x\right) \\
=& \int 1\left\{a_i = \arg\max_{j\in A} v_i^\alpha(j,x) + \epsilon_{it}(j)\right\} dG(\epsilon_{it}).
\end{aligned}
$$

- Choice specific value function with $\mathrm{E}_{\epsilon_{-i}}$ replaced with $\mathrm{E}_{a_{-i}}$

$$
v_i^P(a_{it},x_t) = \sum_{a_{-i}\in A^{N-1}} \mathrm{P}_{-i}(a_{-i}|x_t)\begin{pmatrix} u(a_{it},a_{-i},x_t)+ \\ +\beta \mathrm{E}_x[\bar{V}_i^\alpha(x_{t+1})|a_{it},a_{-i},x_t] \end{pmatrix}
$$

Equilibrium in conditional choice probabilities 2

where

$$\mathrm{P}_{-i}(a_{-i}|x) = \prod_{j \neq i}^{N} \mathrm{P}(a_j|x).$$

Equilibrium in conditional choice probabilities

- Let

$$\Lambda(a|v_i^P(\cdot, x_t)) = \int 1\left\{a_i = \arg\max_{j \in A} v_i^P(j, x) + \epsilon_{it}(j)\right\} dG(\epsilon_{it}).$$

  Then the equilibrium condition is that

$$P_i(a|x) = \Lambda(a|v_i^P(\cdot, x))$$

  or in vector form $\mathbf{P} = \mathbf{\Lambda}(v^{\mathbf{P}})$
- Fixed point equation in $\mathbf{P}$
- Generally not a contraction mapping, so existence and computation more difficult than in single agent models
- Equilibrium existence:
  - If $\mathbf{\Lambda} : [0,1]^{N|X|} \to [0,1]^{N|X|}$ is continuous, then by Brouwer's fixed point theorem, there exists at least one equilibrium
  - $\mathbf{\Lambda}$ need not be continuous, see Gowrisankaran (1999) and Doraszelski and Satterthwaite (2010)

Section 3

Identification

Identification – expected payoff 1

- As in single-agent dynamic decision problems given G, β, and $\mathrm{E}_\epsilon[u(0, \alpha_{-i}(x, \epsilon_{-i}), x_t)] = 0$, we can identify the expectation over other player's actions of the payoff function,

$$\mathrm{E}_\epsilon[u(a_i, \alpha_{-i}(x, \epsilon_{-i}), x)] = \sum_{a_{-i}} \mathrm{P}(a_{-i}|x) u(a_i, a_{-i}, x)$$

- See Bajari et al. (2009), which builds on Hotz and Miller (1993) and Magnac and Thesmar (2002)

Identification – expected payoff (details) 1

- Hotz and Miller (1993) inversion shows

$$v_i^{\alpha^*}(a,x) - v_i^{\alpha^*}(0,x) = q(a, \mathrm{P}(\cdot|x); G)$$

for some known function q

- Use normalization and Bellman equation to recover $v_i^{\alpha^*}$

$$\begin{aligned} v_i^{\alpha^*}(0,x) = & \underbrace{\mathrm{E}[u(0, \alpha^*_{-i}(x, \epsilon_{-i}), x)]}_{=0} + \\ & + \beta \mathrm{E}[\max_{a' \in A} v_i^{\alpha^*}(a', x') + \epsilon(a')|a, x] \\ = & \underbrace{\beta \mathrm{E}[\max_{a' \in A} v_i^{\alpha^*}(a', x') - v_i^{\alpha^*}(0, x') + \epsilon(a')|0, x]}_{\equiv q(x, \mathrm{P}(\cdot|x), G)} + \\ & + \beta \mathrm{E}[v_i^{\alpha^*}(0, x')|0, x] \end{aligned}$$

Identification – expected payoff (details) 2

q is known; can solve this equation for $v_i^{\alpha^*}(0,x)$, then

$$v_i^{\alpha^*}(a,x) = v_i^{\alpha^*}(0,x) + q(a, \mathrm{P}(\cdot|x); G)$$

- Recover $\mathrm{E}[u(a_i, \alpha^*_{-i}(x, \epsilon_{-i}), x)]$ from $v_i^{\alpha^*}$ using Bellman equation

$$\mathrm{E}\left[u(a_i, \alpha^*_{-i}(x, \epsilon_{-i}), x)\right] = v_i^{\alpha^*}(a_i, x) - \\ - \beta \mathrm{E}\left[\max_{a' \in A} v_i^{\alpha^*}(a', x') + \epsilon(a') | a, x\right]$$

Identification of $u(a, x)$

- Separating $u(a, x)$ from $\mathrm{E}_\epsilon[u(a_i, \alpha_{-i}(x, \epsilon_{-i}), x)]$ is new step compared to single-agent model
- Need exclusion to identify $u(a, x)$
- Without exclusion order condition fails

$$\mathrm{E}_\epsilon[u(a_i, \alpha_{-i}(x, \epsilon_{-i}), x)] = \sum_{a_{-i}} \mathrm{P}(a_{-i}|x) u(a_i, a_{-i}, x)$$

  Left side takes on $|A||X|$ identified values, but $u(a, x)$ has $|A|^N|X|$ possible values
- Assume $u(a, x) = u(a, x_i)$ where x_i is some sub-vector of x. u identified if

$$\mathrm{E}_\epsilon[u(a_i, \alpha_{-i}(x, \epsilon_{-i}), x)] = \sum_{a_{-i}} \mathrm{P}(a_{-i}|x) u(a_i, a_{-i}, x_i)$$

  has a unique solution for u

Section 4

Estimation

Estimation 1

- Can use similar methods as in single agent dynamic models
- Maximum likelihood

$$\max_{\theta \in \Theta, \mathbf{P} \in [0,1]^N} \sum_{m=1}^{M} \sum_{t=1}^{T_m} \sum_{i=1}^{N} \log \Lambda \left(a_{imt} | v_i^{\mathbf{P}}(\cdot, x_{mt}; \theta) \right)$$
$$\text{s.t.} \mathbf{P} = \mathbf{\Lambda}(v^{\mathbf{P}}(\theta))$$

- Nested fixed point: substitute constraint into objective and maximize only over θ
    - For each θ must solve for equilibrium – computationally challenging
    - $\mathbf{\Lambda}$ not a contraction
    - What to do when equilibrium not unique?

Estimation approaches

- MPEC (Su and Judd, 2012): use high quality optimization software to solve constrained optimization problem

Estimation approaches

- 2-step estimators: estimate $\hat{\mathrm{P}}(a|x)$ from observed actions and then

$$\max_{\theta\in\Theta} \sum_{m=1}^{M} \sum_{t=1}^{T_m} \sum_{i=1}^{N} \log \Lambda(a_{imt}|v_i^{\hat{\mathbf{P}}}(\cdot, x_{mt}; \theta))$$

  - Can replace pseudo-likelihood with GMM (Bajari, Benkard, and Levin, 2007) or least squares (Pesendorfer and Schmidt-Dengler, 2008) objective
  - Unlike single agent case, efficient 2-step estimators do not have same asymptotic distribution as MLE[1]

[1] In single agent models efficient 2-step and ML estimators have the same asymptotic distribution but different finite sample properties.

Estimation approaches

- Nested pseudo likelihood (Aguirregabiria and Mira, 2007): after 2-step estimator update $\hat{\mathbf{P}}^{(k)} = \mathbf{\Lambda}(v^{\hat{\mathbf{P}}^{(k-1)}}(\hat{\theta}^{(k-1)}))$, re-maximize pseudo likelihood to get $\hat{\theta}^{(k)}$
  - Asymptotic distribution depends on number of iterations; if iterate to convergence, then equal to MLE

Incorporating static parameters

- Often some portion of payoffs can be estimated without estimating the full dynamic model
  - E.g. Holmes (2011) estimates demand and revenue from sales data, costs from local wages, and only uses dynamic model to estimate fixed costs and sales
- Bajari, Benkard, and Levin (2007) and Pakes, Ostrovsky, and Berry (2007) incorporate a similar ideas

Section 5

Examples

Dunne et al. (2013) "Entry, Exit and the Determinants of Market Structure" 1

- Market structure = number and relative size of firms
- Classic question in IO: how does market structure affect competition?
- Here: how is market structure determined? Entry and exit
  - Sunk entry costs
  - Fixed operating costs
  - Expectations of profits (nature of competition)
    - Like Bresnahan and Reiss (1991) summarize with profits as a function of number of firms, $\pi(n)$
- Estimate dynamic model of entry and exit to determine relative importance of factors affecting market structure
- Context: dentists and chiropractors

Model 1

- Similar to Pakes, Ostrovsky, and Berry (2007)
- State variables $s = (n, z)$
  - n = number of firms, z = exogenous profit shifters
  - Follow a finite state Markov process
- Parameters θ
- Profit $\pi(s; \theta)$ (leave θ implicit henceforth)
- Fixed cost $\lambda_i \sim G^\lambda = 1 - e^{-\lambda_i/\sigma}$
- Discount factor δ
- Value function

$$V(s; \lambda_i) = \pi(s) + \max\{\delta VC(s) - \delta\lambda_i, 0\}$$

where VC is expected next period's value function

$$VC(s) = \mathrm{E}^c_{s'}\left[\pi(s') + \mathrm{E}_{\lambda'}\left[\max\{\delta VC(s') - \delta\lambda', 0\}|s\right]|s\right]$$

Model 2

- Probability of exit:

$$p^x(s) = \mathrm{P}(\lambda_i > VC(s)) = 1 - G^\lambda(VC(s)).$$

- Assume λ exponential, $G^\lambda = 1 - e^{-(1/\sigma)\lambda}$, then

$$VC(s) = \mathrm{E}^c_{s'}\left[\pi(s') + \delta VC(s') - \delta\sigma\left(1 - p^x(s')\right)|s\right]$$

- Let $\mathbf{M}_c$ be the transition matrix, then

$$\begin{aligned}\mathbf{VC} =& \mathbf{M}_c\left[\boldsymbol{\pi} + \delta\mathbf{VC} - \delta\sigma(1 - \mathbf{p}^x)\right] \\ \mathbf{VC} =& (I - \delta\mathbf{M}_c)^{-1}\mathbf{M}_c\left[\boldsymbol{\pi} - \delta\sigma(1 - \mathbf{p}^x)\right] \qquad (1)\end{aligned}$$

  - Pakes, Ostrovsky, and Berry (2007): use non parametric estimates of $\mathbf{M}_c$ and $\mathbf{p}^x$ in (1) to form **VC**

Model 3

- Here: use non parametric estimate of $\mathbf{M}_c$ and form $\mathbf{VC}$ by solving

$$\mathbf{VC} = \mathbf{M}_c \left[\pi + \delta \mathbf{VC} - \delta \sigma G^{\lambda}(\mathbf{VC}) \right]$$

- Potential entrants:
  - Expected value after entering

$$VE(s) = \mathrm{E}^{e}_{s'} \left[\pi(s') + \delta VC(s') - \delta \sigma \left(1 - p^x(s') \right) | s \right]$$

  - Cost of entry $\kappa_i \sim G^{\kappa}$
  - Entry probability

$$p^e(s) = \mathrm{P}\left(\kappa_i < \delta VE(s) \right) = G^{\kappa}\left(\delta VE(s) \right)$$

  - As before can use Bellman equation in matrix form to solve for VE

Empirical specification 1

- Data: U.S. Census of Service Industries and Longitudinal Business Database
  - 5 periods – 5 year intervals from 1982-2002
  - 639 geographic markets for dentists; 410 for chiropractors
  - Observed average market-level profits π_{mt}
  - Number of firms n_{mt}, entrants, e_{mt}, exits x_{mt}, potential entrants p_{mt}
  - Market characteristics $z_{mt} = (pop_{mt}, w_{mt}, inc_{mt})$

Empirical specification 1

- Profit function

$$\pi_{mt} = \theta_0 + \sum_{k=1}^{5} \theta_k 1\{n_{mt} = k\} + \theta_6 n_{mt} + \theta_7 n_{mt}^2 + \\ + \text{quadratic polynimal in } z_{mt} + \\ + f_m + \epsilon_{mt}$$

  Key assumption: ϵ_{mt} independent over time
- Transition matrix $\mathbf{M}_c$
  - Define $\hat{z}_{mt}$ = estimate value polynomial in z_{mt} in profit function
  - Discretize $\hat{z}_{mt}$ into 10 categories and use sample averages to estimate transition probabilities
- Fixed (G^{λ}) and entry costs (G^{κ})
  - $\widehat{VC}(\sigma)$ and $\widehat{VE}(\sigma)$ as described above

Empirical specification 2

- Log-likelihood

$$L(\sigma,\alpha)=\sum_{m,t}\begin{array}{l}(n_{mt}-x_{mt})\log\left(G^{\lambda}\left(\widehat{VC}_{mt}(\sigma);\sigma\right)\right)+\\ +x_{mt}\log\left(1-G^{\lambda}\left(\widehat{VC}_{mt}(\sigma);\sigma\right)\right)+\\ +e_{mt}\log\left(G^{\kappa}\left(\widehat{VE}_{mt}(\sigma);\alpha\right)\right)+\\ +(p_{mn}-e_{mt})\log\left(1-G^{\kappa}\left(\widehat{VE}_{mt}(\sigma);\alpha\right)\right)\end{array}$$

Results

Profit function:

- Decreasing with n increasing in w, inc, pop
- Compare fixed effects and OLS estimates
    -
    - More relevant concern is assumption of ϵ_{mt} independent over time – this is empirically testable, but they do not do anything about it

$\hat{\pi}(n)$

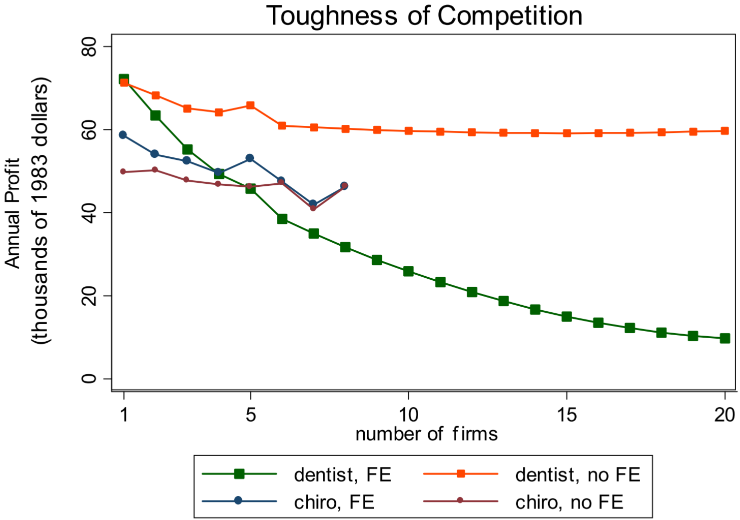

Toughness of Competition
Annual Profit
(thousands of 1983 dollars)
0
20
40
60
80
1
5
10
15
20
number of firms
dentist, FE
dentist, no FE
chiro, FE
chiro, no FE

TABLE 4 Fixed Cost and Entry Cost Parameter Estimates (standard errors in parentheses)

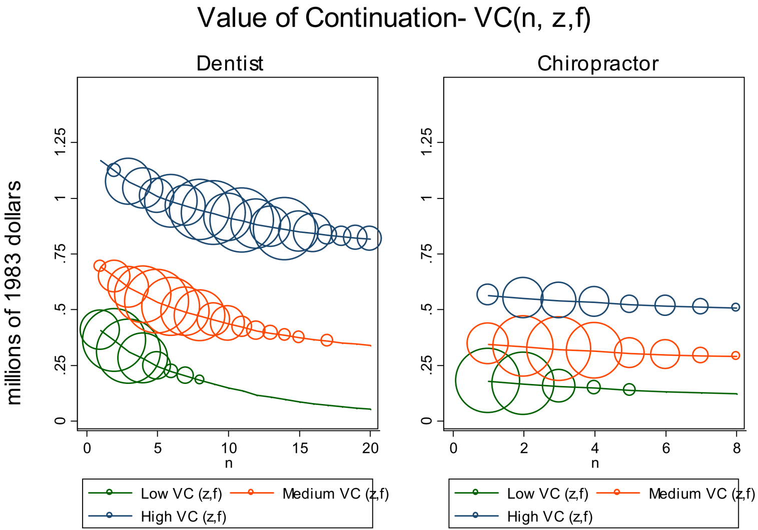
	Maximum Likelihood Estimator			GMM Estimator		
Panel A. Dentist (all markets)						
Entry Pool	σ	α		σ	α	
Internal	0.373 (0.006)	2.003 (0.013)		0.362 (0.004)	2.073 (0.031)	
External	0.375 (0.006)	3.299 (0.039)		0.362 (0.004)	2.644 (0.067)	
Panel B. Dentist (HPSA versus non-HPSA markets)						
Entry Pool	σ	α (HPSA)	α (non-HPSA)	σ	α (HPSA)	α (non-HPSA)
Internal	0.366 (0.009)	1.797 (0.069)	2.019 (0.041)	0.351 (0.005)	1.877 (0.076)	2.098 (0.032)
External	0.368 (0.008)	3.083 (0.169)	3.376 (0.079)	0.351 (0.005)	1.943 (0.213)	2.695 (0.092)
Panel C. Chiropractor						
Entry Pool	σ	α		σ	α	
Internal	0.275 (0.005)	1.367 (0.015)		0.254 (0.004)	1.337 (0.023)	
External	0.274 (0.005)	1.302 (0.022)		0.254 (0.004)	1.302 (0.028)	

TABLE 7 Distribution of the Number of Dental Establishments

	non-HPSA Markets		HPSA Markets	
Number of Establishments	Data	Model	Data	Model
$n = 1$	.018	.043	.034	.059
$n = (2,3)$	.166	.162	.314	.268
$n = (4,5)$	.223	.209	.275	.251
$n = (6,7,8,9,10)$	.376	.382	.305	.340
$n > 10$	.217	.204	.072	.081

TABLE 6 Predicted Probabilities of Exit and Entry (evaluated at different values of the state variables)

	Probability of Exit — Dentist			Probability of Entry — Dentist		
	Low(z, f)	Mid(z, f)	High(z, f)	Low(z, f)	Mid(z, f)	High(z, f)
$n = 1$	0.313	0.129	0.032	0.141	0.216	0.382
$n = 2$	0.358	0.148	0.036	0.126	0.204	0.371
$n = 3$	0.412	0.170	0.042	0.110	0.191	0.360
$n = 4$	0.451	0.186	0.046	0.100	0.182	0.352
$n = 5$	0.497	0.205	0.050	0.088	0.173	0.344
$n = 6$	0.531	0.219	0.054	0.080	0.166	0.338
$n = 8$	0.593	0.244	0.060	0.067	0.155	0.328
$n = 12$	0.713	0.294	0.072	0.044	0.136	0.312
$n = 16$	0.787	0.324	0.080	0.032	0.124	0.303
$n = 20$	0.836	0.345	0.085	0.024	0.117	0.297
	Probability of Exit — Chiro			Probability of Entry — Chiro		
$n = 1$	0.524	0.286	0.129	0.133	0.245	0.371
$n = 2$	0.547	0.299	0.135	0.127	0.239	0.367
$n = 3$	0.569	0.311	0.141	0.119	0.233	0.362
$n = 4$	0.585	0.319	0.144	0.114	0.228	0.358
$n = 5$	0.606	0.331	0.150	0.107	0.222	0.352
$n = 6$	0.620	0.339	0.153	0.103	0.219	0.350
$n = 7$	0.629	0.344	0.155	0.101	0.217	0.348
$n = 8$	0.639	0.349	0.158	0.098	0.215	0.346


Value of Continuation- VC(n, z,f)
Dentist
Chiropractor
millions of 1983 dollars
0
.25
.5
.75
1
1.25
0
5
10
15
20
n
0
2
4
6
8
n
Low VC (z,f)
Medium VC (z,f)
High VC (z,f)

Subsidies to entry and fixed costs

- Health Professional Shortage Areas (HPSA) have entry subsidies
- Entry cost subsidy = change distribution of entry costs for all markets to the distribution estimated for HPSA markets
- Fixed cost subsidy = reduce mean of fixed cost by 8% (chosen to generate similar number of firms as HPSA subsidy)

TABLE 11 Cost-Benefit Comparison of Alternative Policies

Impact on Market Structure	Benchmark non-HPSA costs	Entry Cost Reduction	Fixed Cost Reduction	Expand Program
$\Pr(n = 1)$	0.062	0.055	0.056	0.034
$\Pr(n \leq 3)$	0.338	0.313	0.319	0.246
$\Pr(n \leq 5)$	0.592	0.562	0.571	0.475
Average number of entrants/market	1.396	1.657	1.423	2.563
Average number of exits/market	1.029	1.131	0.950	1.477
Net change in establishments/market	0.367	0.526	0.473	1.086
Cost/additional entrant (millions 1983 $)		0.103		0.075
Cost/additional establishment (millions 1983 $)		0.170	0.503	0.140

Lin (2015)

Quality choice and market structure: a dynamic analysis of nursing home oligopolies

- Poor quality common in nursing homes
  - 30% of nursing homes violated federal regulations in 2006
- Policies designed to inform consumers about nursing home quality
  - Nursing Home Quality Initiative began in 2002 in US
  - NPR: Rule Change Could Push Hospitals To Tell Patients About Nursing Home Quality
  - Performance of 1,000 Canadian long-term care facilities now publicly available
  - Ontario nursing homes feed seniors on $8.33 a day

Lin (2015)

- Dynamic model of quality choice
- Effect of eliminating low quality nursing homes
    - Raises quality, but reduces supply and alters competition
- Effect of competition

Data

- 1996-2005 Online Survey Certification and Reporting System (OSCAR)
- Not his paper, but if you wanted similar, more recent data see Provider of Services (POS) files from CMS
  - Annual (possibly quarterly) 2006-2016
  - Very detailed staff and service information
- Market = county
- Limit sample to counties with 6 or fewer nursing homes
- Quality = nurses/beds above or below median

Table 1
FACILITY ATTRIBUTES FOR LOW- AND HIGH-QUALITY NURSING HOMES

	Low Quality		High Quality	
	Mean	Std. Dev.	Mean	Std. Dev.
Number of beds	96.76	41.86	90.86	50.40
For-profit ownership	0.73	0.45	0.54	0.50
Occupacy rate	0.83	0.16	0.84	0.18
Proportion of non-Medicaid patients	0.28	0.16	0.37	0.20
Total observations	24,413		24,733	

Table 2
ENTRY, EXIT, AND QUALITY ADJUSTMENT

Count	Entry	Exit	Continue	Transition
Low quality	822	763	18,552	4,171
High quality	599	499	19,464	4,276
Total	1,421	1,262	38,016	8,447

Model 1

- Common knowledge state
  $$x_t = (\underbrace{M_t}_{marketsize}, \underbrace{I_t}_{marketincome}, \underbrace{\tau}_{markettype}, \underbrace{\mathbf{s}_t}_{firmstates})$$
  - All variables are market (county) specific, but suppressed from notation
- $$s_{it} = \begin{cases} 0 & \text{if out of market} \\ 1 & \text{if low quality} \\ 2 & \text{if high quality} \end{cases}$$
- Private info of firm i, ϵ_{it}
- Action $a_{it} = s_{it+1}$
- Assumptions (same as general setup):
  1. Additive separability: $\pi_{it}(x_t, a_t, \epsilon_t) = \pi_{it}(x_t, a_t) + \epsilon_{it}(a_{it})$
  2. Conditional independence:
     $F(x_{t+1}, \epsilon_{t+1} | x_t, \epsilon_t, a_t) = F_t(x_{t+1} | x_t, a_t) F_\epsilon(\epsilon_{t+1})$

Market type

- Market type used to capture unobserved market heterogeneity
- Market type estimation:
  - Fixed effects regressions

$$\begin{aligned} N_{highquality,mt} =& \theta_{m,H} + \beta_{1,H} M_{mt} + \beta_{2,H} I_{mt} + u_{mt} \\ N_{lowquality,mt} =& \theta_{m,L} + \beta_{1,L} M_{mt} + \beta_{2,L} I_{mt} + u_{mt} \end{aligned}$$

  - Market m, type H_L if $\hat{\theta} m, H$ below its median
  - Similarly define H_H, L_L, L_H, to get 4 types
- Ad-hoc? similar to Collard-Wexler (2013)
  - Method of Bonhomme and Manresa (2015) could be better way to capture similar idea

Table 3

ESTIMATE OF THE MULTINOMIAL LOGIT MODEL

Variables	I Low	II High	III Low	IV High
State low	7.63***	6.54***	7.37***	6.50***
	(0.052)	(0.058)	(0.052)	(0.060)
State high	6.72***	8.34***	6.73***	8.18***
	(0.061)	(0.062)	(0.063)	(0.062)
Log elderly population	0.66***	0.66***	0.92***	0.40***
	(0.030)	(0.031)	(0.033)	(0.034)
Log per-capita income	−0.08	0.91***	0.05	0.53***
	(0.115)	(0.116)	(0.119)	(0.120)
First low competitor	−0.30***	−0.65***	−0.82***	−0.71***
	(0.050)	(0.051)	(0.054)	(0.055)
Second low competitor	0.12**	−0.15**	−0.38***	−0.27***
	(0.060)	(0.063)	(0.063)	(0.066)
No. of additional low competitors	0.19***	0.01	0.01	−0.04
	(0.054)	(0.058)	(0.052)	(0.057)
First high competitor	−0.72***	−0.36***	−0.86***	−0.93***
	(0.051)	(0.053)	(0.058)	(0.060)
Second high competitor	−0.17***	0.08	−0.33***	−0.03
	(0.065)	(0.065)	(0.066)	(0.065)
No. of additional high competitors	−0.19***	−0.05	−0.21***	0.03
	(0.055)	(0.053)	(0.055)	(0.052)
Market type II (L, H)			0.36***	1.46***
			(0.090)	(0.090)
Market type III (H, L)			1.58***	0.15*
			(0.080)	(0.084)
Market type IV (H, H)			1.96***	1.79***
			(0.092)	(0.095)
Constant	−8.44***	−18.56***	−12.29***	−13.34***
	(1.129)	(1.151)	(1.193)	(1.207)

Notes: This table reports results from a multinomial logit model of choosing quality levels with (columns III and IV) and without (columns I and II) the inclusion of market-specific dummies. Each group type is characterized by the profitability for being low- and high-quality firms. The omitted market type (type I) refers to low profitability for both low- and high-quality firms. Standard errors are in parentheses. ***$p < 0.01$, **$p < 0.05$.

Payoff function

$$
\begin{aligned}
\pi_{it}(x_t, a_t|\theta) = & I(a_{it}=1)\cdot[\theta_L^1 + \theta_L^2 M_t + \theta_L^3 I_t + g_L(a_{1t}, a_{2t}, ..., a_{Nt})\cdot\theta_L] \\
& + I(a_{it}=2)\cdot[\theta_H^1 + \theta_H^2 M_t + \theta_H^3 I_t + g_H(a_{1t}, a_{2t}, ..., a_{Nt})\cdot\theta_H] \\
& + I(s_{it}=0, a_{it}=1)\theta_{0L} + I(s_{it}=0, a_{it}=2)\theta_{0H} \\
& + I(s_{it}=1, a_{it}=2)\theta_{LH} + I(s_{it}=2, a_{it}=1)\theta_{HL}.
\end{aligned}
$$

with

$$
\begin{aligned}
g_L\cdot\theta_L = & \theta_L^{L1} \times \text{(presence of the 1st low competitor)} \\
& + \theta_L^{L2} \times \text{(presence of the 2nd low competitor)} \\
& + \theta_L^{LA} \times \text{(no. of additional low competitors)} \\
& + \theta_L^{H1} \times \text{(presence of the first high competitor|with low competitors)} \\
& + \theta_L^{HA} \times \text{(no. of additional high competitors |with low competitors)} \\
& + \theta_L^{0H1} \times \text{(presence of the first high competitor |without low competitors)} \\
& + \theta_L^{0HA} \times \text{(no. of additional high competitors |without low competitors)}.
\end{aligned}
$$

and similar for g_H

Estimation

- Estimate $\tilde{P}(a|x)$ by multinomial logit
- Form value function

$$\hat{V}(x,a;\theta,\tilde{P}) = \pi(x,a;\theta) + (I - \beta F^{\tilde{P}})^{-1} \left(\sum_a \tilde{P}(a|x)\pi(x,a;\theta) \right) + \\ + (I - \beta F^{\tilde{P}})^{-1} \left(\sum_a \tilde{P}(a|x)\mathrm{E}[\epsilon|a,x] \right)$$

π linear in θ, so

$$\hat{V}(x,a;\theta,\tilde{P}) = Z(a)\theta + \hat{\epsilon}(a|\tilde{P})$$

- Model predicted probabilities: $\hat{P}(a|x;\theta,\tilde{P}) = \frac{e^{Z(a)\theta + \hat{\epsilon}(a|\tilde{P})}}{\sum_{a'} e^{Z(a')\theta + \hat{\epsilon}(a'|\tilde{P})}}$
- Moments:

$$\mathrm{E}\left[\left(\hat{P}(a|x;\theta,\tilde{P}) - P^0(a|x) \right) X \right] = 0$$

- Estimate θ by GMM

TABLE 4
ESTIMATES OF THE MAIN MODEL

Variables	Entry, Exit, and Quality Adjustment		
Log elderly population	Low quality	0.18***	(0.006)
	High quality	0.11***	(0.007)
Log per-capita income	Low quality	0.05***	(0.020)
	High quality	0.11***	(0.028)
	First low competitor	−0.35***	(0.029)
	Second low competitor	−0.22***	(0.019)
Competition effect on low	No. of additional low competitors	−0.07***	(0.007)
	First high | low competitor	−0.15**	(0.065)
	No. of additional high | low competitor	−0.03	(0.038)
	First high | no low competitor	−0.28***	(0.037)
	No. of additional high | no low competitor	−0.03	(0.039)
	First high competitor	−0.66***	(0.034)
	Second high competitor	−0.17***	(0.041)
Competition effect on high	No. of additional high competitors	−0.03	(0.041)
	First low | high competitor	−0.04	(0.053)
	No. of additional low | high competitor	−0.02	(0.017)
	First low | no high competitor	−0.53***	(0.037)
	No. of additional low | no high competitor	−0.28***	(0.012)
Markets type I	Low	−1.98***	(0.198)
	High	−2.03***	(0.284)
Markets type II	Low	−2.04***	(0.199)
	High	−1.62***	(0.286)
Markets type III	Low	−1.56***	(0.197)
	High	−2.08***	(0.282)
Markets type IV	Low	−1.56***	(0.194)
	High	−1.46***	(0.281)
Quality adjustment	Low to high	−1.42***	(0.083)
	High to low	−0.76***	(0.083)
Sunken entry cost	Low	−7.06***	(0.109)
	High	−8.17***	(0.160)
Number of observations		132,138	

NOTES: Standard errors are in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$.

TABLE 5

MONOPOLY PROFITS FOR LOW- AND HIGH-QUALITY NURSING HOMES

	Type I (L_L, H_L)	Type II (L_L, H_H)	Type III (L_H, H_L)	Type IV (L_H, H_H)
Low	0.14	0.08	0.56	0.56
	(0.048)	(0.053)	(0.052)	(0.058)
High	0.26	0.67	0.21	0.82
	(0.064)	(0.065)	(0.072)	(0.073)

TABLE 6

MODEL FIT

	Data	Simulated Data
% of Low Quality	49.39%	50.50%
% of entry and exit	5.60%	6.44%
% of Low to High	8.71%	8.95%
% of High to Low	8.93%	8.92%
% of Low Quality		
Markets Type I	49.39%	50.76%
Markets Type II	15.44%	15.91%
Markets Type III	88.41%	88.33%
Markets Type IV	53.47%	56.15%
% of Markets with Number of Homes		
Zero	7.80%	9.59%
One	32.38%	33.56%
Two	24.13%	24.81%
Three	16.45%	15.27%
More	19.24%	16.76%

Counterfactuals

- Simulate beginning in 2000 for markets with 4 or fewer firms (2195 markets)

I. Baseline

II. Elderly populations grows 3% faster years 6-15

III. Low quality forbidden

IV. Lower entry cost

TABLE 8
SUMMARY OF COUNTERFACTUALS

	0		I				II			
	Year 0	Year 5	Year 1	Year 5	Year 15	Year 25	Year 1	Year 5	Year 15	Year 25
Total	4,227	4,185	4,275	4,342	4,342	4,352	4,449	4,945	5,454	5,480
Low quality	1,991	2,209	2,112	2,191	2,214	2,242	2,306	2,834	3,242	3,285
High quality	2,236	1,976	2,163	2,151	2,128	2,110	2,143	2,111	2,212	2,195
% of low quality										
Overall	47.10%	52.78%	49.40%	50.46%	50.99%	51.52%	51.83%	57.31%	59.44%	59.95%
Markets type I	45.82%	49.05%	47.53%	51.13%	53.38%	48.33%	47.58%	48.13%	42.75%	46.20%
Markets type II	11.68%	18.97%	16.02%	15.51%	15.69%	17.16%	17.14%	22.46%	24.43%	24.18%
Markets type III	86.81%	89.65%	88.09%	88.83%	86.82%	88.58%	87.94%	88.75%	88.55%	90.12%
Markets type IV	48.98%	52.78%	49.68%	53.39%	52.44%	54.05%	55.97%	63.58%	67.80%	66.02%
% of markets with number of homes										
Zero	7.84%	8.25%	8.25%	8.38%	9.61%	9.02%	5.42%	1.46%	0.27%	0.27%
One	34.67%	35.31%	34.21%	34.17%	33.12%	33.94%	34.35%	32.39%	26.47%	26.83%
Two	26.74%	26.92%	25.97%	26.29%	27.47%	26.65%	27.24%	28.97%	31.34%	30.98%
Three	18.59%	18.00%	18.82%	17.13%	15.13%	15.54%	19.73%	21.64%	22.64%	22.55%
More	12.16%	11.53%	12.76%	14.03%	14.67%	14.85%	13.26%	15.54%	19.27%	19.36%

	III				IV			
	Year 1	Year 5	Year 15	Year 25	Year 1	Year 5	Year 15	Year 25
Total	3,479	3,228	3,121	3,124	5,028	5,763	5,911	5,865
Low quality					2,846	3,632	3,756	3,753
High quality					2,182	2,131	2,155	2,112
% of low quality								
Overall					56.60%	63.02%	63.54%	63.99%
Markets type I					60.16%	71.39%	73.25%	69.65%
Markets type II					24.08%	30.20%	27.92%	30.29%
Markets type III					86.65%	88.07%	88.78%	88.81%
Markets type IV					54.78%	60.64%	61.16%	62.22%
% of markets with number of homes								
Zero	15.63%	20.23%	25.56%	27.70%	7.15%	4.87%	3.83%	4.56%
One	41.37%	41.46%	38.50%	37.72%	23.55%	16.67%	16.86%	17.72%
Two	20.36%	18.54%	17.86%	16.95%	27.65%	29.02%	27.70%	25.88%
Three	14.40%	12.48%	9.70%	7.38%	22.32%	23.78%	24.37%	25.10%
More	8.25%	7.29%	8.38%	10.25%	19.32%	25.65%	27.24%	26.74%

NOTES: This table summarizes industry structure for various scenarios: 0 for raw data; I for simulation based on equilibrium policy function; II for a 10-year positive growth of the elderly population starting in year 6; III for low-quality firms being prohibited; and IV for a 20% reduction in entry costs.

Section 6

Generalizations and extensions

Generalizations and extensions

- Unobserved state variables
- Multiple equilibria
- Continuous time

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