Carbon cycle, Math 101

Seckin Demirbas, Raphael Kelly, Sven Bachmann, and Peter Harrington

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The CO_2 concentration in the atmosphere at a particular point on earth follows a cyclic pattern, due to seasonal changes, in particular in the relative proportions of photosynthesis and respiration in plants at different months of the year. In order to explore this cycle, we can decompose the net flux of CO_2 as

$$\frac{dc}{dt} = i(t) - o(t),$$

where $i(t)$ is the rate of carbon influx into the atmosphere due to respiration and emissions and $o(t)$ is the rate of carbon outflux out of the atmosphere due to photosynthesis. Both are given in PgC/year (Petagrams of carbon per year, where 1 petagram $= 10^{15}$ grams).

1. Let $a \geq 0$ and $b > a$ represent two different points in time.

   (a) **Describe concisely** the physical quantity that is represented by $\int_a^b i(t)dt$.

   (b) **Describe concisely** the physical quantity that is represented by $\int_a^b (i(t) - o(t))\, dt$.

2. Suppose we let the start of the year (January 1st) be time $t = 0$ and start measuring the change in CO_2 at this time. Assume that $i(t) = 25 + 5\cos(2\pi t)$ and that $o(t) = 25(1 + \cos(2\pi t - \pi))$.

   (a) **Compute the earliest time** $T > 0$ at which the amount of CO_2 in the atmosphere starts decreasing

   (b) Does the amount of CO_2 in the atmosphere return to its original value? If so, determine the earliest time $t' > 0$ at which this occurs.

   On climatological time scales, the annual oscillations of CO_2 discussed above are small compared to the overall increase in carbon. One possible way to determine the overall increase in carbon is by using averages.

   For a function $f(t)$, its average value from $t = a$ to $t = b > a$ is given by

$$\frac{1}{b-a}\int_a^b f(t)\mathrm{d}t.$$

   Below is a plot of the CO_2 concentration at Mauna Loa Observatory since 1960. By observation a simple model of the concentration of atmospheric CO_2 since 1960 is

$$C(t) = 310 + 4\cos(2\pi t) + 1.65t$$

   where t is the time in years after 1960 and here $C(t)$ is measured in ppm (parts per million). The long-term trend of rising carbon dioxide levels is driven by human activities.

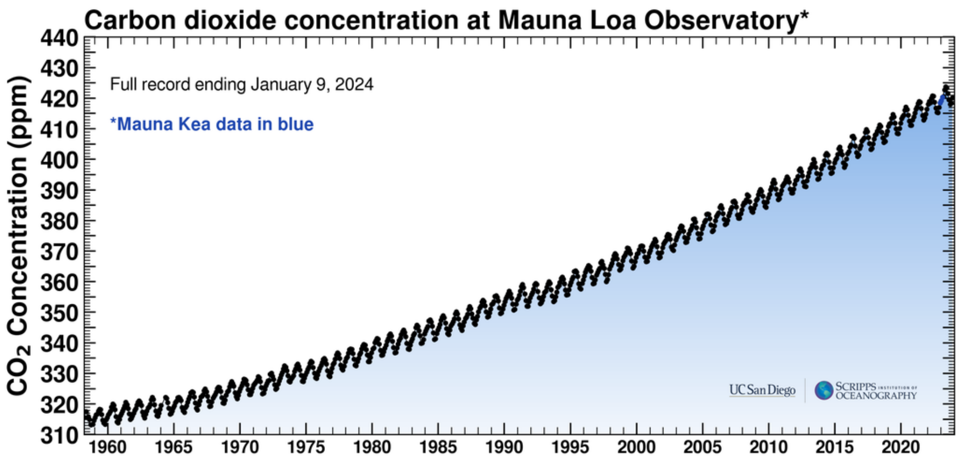

Figure 1. Graph showing the concentration of atmospheric CO_2 since 1960. See `https://keelingcurve.ucsd.edu/` for more details

Let $A(t)$ be the yearly average of the concentration, namely

$$A(t) = \int_t^{t+1} C(s)ds.$$

3. For $t \in [0, 1]$, find the time at which the yearly average is maximal.
4. Calculate the rate of increase of $A(t)$. That is, how much is the yearly average concentration of atmospheric CO_2 increasing per year?