

New version of Example 3.3 on pages 69-70.

Start with
$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & -7 & -29 \\ 0 & 3 & 0 & -3 & -6 \\ 0 & 0 & 1 & 2 & 11 \\ 0 & 0 & 0 & 2 & 8 \end{array} \right] \sim$$

after the first pass of Gaussian Elimination done in Ex 3.2. We now want to put the system into reduced row echelon form, which requires all leading entries to be 1 and all entries above leading entries to be zero.

To make the later arithmetic easier, we can make all leading entries 1 now, by multiplying row 2 by $\frac{1}{3}$ and row 4 by $\frac{1}{2}$.

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & -2 & \textcircled{-7} & -29 \\ 0 & 1 & 0 & \textcircled{-1} & -2 \\ 0 & 0 & 1 & \textcircled{2} & 11 \\ 0 & 0 & 0 & 1 & 4 \end{array} \right] \sim \left[\begin{array}{cccc|c} 1 & 2 & \textcircled{-2} & 0 & -1 \\ 0 & 1 & \textcircled{0} & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{array} \right]$$

We can now eliminate the circled entries on left above using the bottom row, i.e. replacing row 3 by row 3 - 2x row 4, row 2 by row 2 + row 4, row 1 by row 1 + 7x row 4.

We now eliminate the circled entries on the right of the previous equation using row 3, i.e. replace row 1 by row 1 + 2x row 3

(nothing is needed in row 2 here since that circled entry is already 0).

$$\left[\begin{array}{cccc|c} 1 & \textcircled{2} & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{array} \right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{array} \right]$$

Finally we eliminate the circled entry on the left above using the second row, i.e. row 1 is replaced by row 1 - 2x row 2. The final form on the right above is the reduced row echelon form of the matrix.

Note: It is important in this second stage of elimination to work from right to left, eliminating entries above the diagonal. The presentation currently in the notes works in the wrong order, and involves many more arithmetic operations because of this.