We might not be able to get away with the quasistatic approximation, because our frequency is so high. In that case, the magnetic field inside (and *outside*) a solenoid is *not* constant, but actually takes the form of radio waves. So we will have to calculate this again, using Maxwell’s equations.

# Regions

Here’s the setup: Each of these regions represents an area covered by a different differential equation, which we’ll have to stitch together with matching boundary conditions.

EM waves in free space:

EM waves in conductor:

EM waves in free space:

Sinusoidally varying current:

EM waves in free space:

# Equations:

## In free space

From Griffiths chapter 9,

We’re expecting the magnetic fields to be only pointing along z, and the electric fields should all be pointing along only. They should both vary sinusoidally with time. Also, they should be constant in z and . That should make our work considerably easier!

In cylindrical coordinates:

Magnetic:

The speed of light is , so we can define as the wave number, . Then:

Electric:

The solutions to these are, unfortunately, Bessel functions (aka the “cylindrical harmonics”). These functions always tend to appear in cylindrical coordinate problems. The solutions are:

is a Bessel function “of the first kind” of order 0, and is a Bessel function “of the second kind” of order 0. C1,2,3,4 are just constants that we can use to stitch these equations together at the boundaries.

Also, B and E here represent the amplitudes of the electric fields, since the time-dependence cancelled out of the equation. Of course, they are time-varying.

The electric and magnetic fields are not independent though; they are related by

And since

so

## In the conductor

From Griffiths, chapter 9, the electromagnetic field in a conductor is:

In cylindrical coordinates:

And then requiring them to be functions of s and t only,

So now there’s a bit of a complication. Unlike before, we have terms that are proportional to and . You might recognize these as damping terms. When we plug in the time-dependence, it doesn’t cancel out the way it did before. Using separation of variables, let

For the time half of this equation, we have the constant U that we have to solve for (with units of per-metres).

The characteristic polynomial of this differential equation is:

So the general solution for this is:

Where I’ve set the frequency to be

Whew… so it’s an exponentially decaying solution? That’s not what we wanted at all! We need something that’s just sinusoidal, but perhaps exponentially decaying in z. Clearly, I chose the wrong value of U. What value will cancel out the exponential decrease? I need

Plugging this in,

And now, since the solution is supposed to be just oscillating in time, we can say =0.

And it is worth noting also that

Now, we just need to solve the spatial side:

This is just the same as in free space, except that the wave number is more complicated.

Now the magnetic field should be doing something like this:

The electric field is found from Maxwell’s equations:

## In region with current flow

The electric field is continuous, but the magnetic field is not.

# Matching up the boundary conditions

1. EM waves in center area free space:

Condition: Must be finite as s->0

1. EM waves in conductor:

Conditions: Let’s call , and for notational simplicity.

Both E and B must be continuous, so

Let’s try to solve c5 and c6 in terms of c1.

Substitute back to solve for c5.

Simplify for the sake of sanity:

Continuing on...

1. EM waves between conductor and solenoid:

Both E and B must be continuous, so

Let’s try to solve c7 and c8 in terms of c5 and c6.

Solve for c8:

Substitute back to solve for c7. It’s a bit messy.

Simplify for the sake of sanity:

1. EM waves outside of the solenoid:

Sinusoidally varying current:

Let’s call for simplicity.

Also, the solution for s>R will be travelling waves, which happens to require a particular combination of J and Y so that, as we get the solution for a plane wave. That is:

At R, E must be continuous, and B must be discontinuous (it gets boosted inside by the current), so

- **not satisfied yet!**

**Start with E this time**

**Simplifying:**

**So two equations here, and two unknowns: c9 and c1**

**Simplify:**

The problem is now solved, essentially.

Now we need to work backwards, now that the electric field is known everywhere.

All quantities here are known! Also, in conductive materials,

The total current is:

RMS: Square the current, and take the average over a whole period:

The power dissipated by the ring should be

Averaged over a whole period:

The resistance should be:

Dimensionally, you might also be concerned about the presence of in the numerator of the resistance expression. That should cancel out when the integral in the numerator is evaluated, so let’s see if we can remove it.

Change of variables:

Unfortunately, the integral in the numerator is a physicist’s nightmare, and probably best done numerically.

Also, I don’t think that this is frequency-independent! The frequency is just hidden inside the constants c5 and c6. But since c5 and c6 are in both the numerator and denominator, some things will have to cancel out. Let’s look at these:

This probably doesn’t look much simpler, but it actually is, because A56 and A65 don’t depend on extraneous factors like the current I0.

((o\*((c^2 + d^2)\*k^2\*Sqrt[Pi]\*o^2\*HypergeometricPFQ[ {3/2, 3/2}, {2, 5/2, 3}, -(k^2\*o^2)] + 12\*d\*(d\*MeijerG[{{1/2}, {-1/2, 1/2}}, {{-1, 0, 1}, {-1/2, -1/2}}, k\*o, 1/2] + c\*MeijerG[{{1/2, 1/2}, {-1/2}}, {{0, 1}, {-1, -1/2, -1/2}}, k\*o, 1/2])))/ (12\*Sqrt[Pi])) -((n\*((c^2 + d^2)\*k^2\*Sqrt[Pi]\*n^2\*HypergeometricPFQ[ {3/2, 3/2}, {2, 5/2, 3}, -(k^2\*n^2)] + 12\*d\*(d\*MeijerG[{{1/2}, {-1/2, 1/2}}, {{-1, 0, 1}, {-1/2, -1/2}}, k\*n, 1/2] + c\*MeijerG[{{1/2, 1/2}, {-1/2}}, {{0, 1}, {-1, -1/2, -1/2}}, k\*n, 1/2])))/ (12\*Sqrt[Pi]))