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**Induction heater + temperature sensor mathematical model**

Assume a current-controlled, high-frequency sinusoidal power source in the primary coil.

Primary coil has N turns of wire, over a coil length  p. Current in primary:

The coil is wound around a glass tube of outer diameter D = 6 mm. Magnetic flux through the tube eproduced by the primary:

Inside is a narrow metal ring of outer diameter d = 3mm, with a wall thickness *t*. This is the secondary “coil”. If the ring is at the center of the primary, the flux contained inside (due to the primary) is

By Faraday’s law, the electromotive force induced in the secondary by the primary is:

The only time-varying quantity here is Ip.

The secondary coil has a resistance Rs, which is a function of temperature and also frequency. This voltage is equal to the sum of resistive and inductive voltages in the secondary:

What’s the inductance of the secondary coil? Can we determine it? I think so.

If a current spontaneously appears in the secondary, it will create a flux:

That flux will induce a voltage in the loop that opposes it. That induced voltage will be

Treating it as an inductor, the voltage that would appear is

So the inductance of the secondary ought to be

Dimensional analysis sanity check: It does have units of henries...

Okay, so, back to where we were:

The current in the secondary has to be at the same frequency as the driving coil, so it’s of the form:

So

Match that to the voltage supplied by the primary coil:

Whew! That’s one heck of an equation. But it probably has what we need.

This equality has to be true at all times, which should let us find the phase shift *ф*. I don’t know if that will be helpful, but we may as well solve for it. Plug in t = 0….

That’s a transcendental equation, so I think that it will need to be solved numerically. It’s probably not so important. Although, according to Mathematica, the exact solution is:

Where

*The phase, if you trust Mathematica.*

*Heating power in the secondary coil*

Continuing on, the most important thing we’re interested in is the current in the secondary, Is. Can we solve for *that?* Sure…

Nice!

The power transformed into heat in the secondary will be

This is the instantaneous power at a given time, t, but we’d rather know the power over an entire cycle. Integrating this expression over one period, and dividing by the period, gives:

That’s a bit of a mouthful, but it boils down to:

*Heating power in the secondary coil*

Next step: How does this load appear to the primary coil?

The primary coil is assumed to be current-controlled, so the voltage that appears across it should be pure Faraday’s law: the sum of the terms from the primary’s self-inductance, and from the flux generated by the current in the secondary. Oh, okay, also from the coil resistance; why not be precise.

K is the amount of flux from the secondary that ends up going through the primary. (Why didn’t this factor appear before? It did but I hid it. That was the factor , the area ratio. If the primary coil is an ideal solenoid, then the magnetic field inside it is perfectly uniform, and so the fraction that passes through the secondary is just the ratio of the areas. But is the same true going the other way? What is ?

It turns out there’s a reciprocity theorem that says that the mutual inductance between coils 1 and 2 is the same as between coils 2 and 1. That is:

So,

And from before:

So

The voltage on the primary should therefore be:

The inductance of the primary is

So

And, since

We can solve for the primary voltage:

Now the problem becomes that Rp and Rs are both functions of temperature. How do we know which temperature we’re measuring? Well, we can change our input current by adding a DC offset:

This won’t affect anything in the AC end of the calculation, since it doesn’t affect dIp/dt. But it does let us measure the resistance of the primary coil!

Then splitting the measured coil voltage into AC and DC frequencies:

*Sensed AC and DC Voltage across primary coil*

Next up: We have to include a model of Rs as a function of temperature, frequency, geometry, and material properties.

For a thin-walled cylinder secondary, the resistance would be expected to just be:

But actually this is only true if the current flow is uniform through the material. In fact, that’s not the case because the current on the outside layer of the material cancels out some of the flux inside that layer. So there’s less current deeper in the material compared with on the surface. The resistance seen by the current should be the average of the resistance over the area in which the current is flowing.

We probably ought to derive an exact expression for the resistance of a thin-walled cylinder. But for a thick cylinder, the resistance can be approximated by

Where the skin depth is

And for all materials we’re interested in, it’s likely that

So substituting:

This assumes that the skin depth , and also . This second one is the assumption least likely to be valid, so we ought to come back and check this later, and replace our expression with a more exact one for a thin-walled cylinder. (This is where the question of “magnetic transparency” comes in).

The resistivity of metal varies as a function of temperature, which can be fit to a polynomial:

Usually T0 = 0°C or 20°C, and the nonlinear terms are small enough to be neglected.

|  |  |  |
| --- | --- | --- |
| **Substance** | **[Ω·m]** **at 20°C** | **[°C-1] at 20°C** |
| Aluminum (Al) | 2.82x10-8 | 0.00429 |
| Copper (Cu) | 1.68x10-8 | 0.0043 |
| Gold (Au) | 2.44x10-8 | 0.004 |
| Iron (Fe) | 1.0x10-7 | 0.00651 |
| Lead (Pb) | 2.2x10-7 | 0.0042 |
| Nickel (Ni) | 6.99x10-8 | 0.0067 |
| Nichrome | 1.10x10-6 | 0.00017 |
| Platinum (Pt) | 1.06x10-7 | 0.003927 |
| Tungsten (W) | 5.60x10-8 | 0.0048 |

Substituting into the resistivity relationship:

For copper, aluminum, gold, or lead at a temperature of 260°C,

So we ought to expect resistivity to double, which means the resistance should increase by about 43%, taking into account the skin effect.

We also should determine the sensitivity of the signal about small changes in the temperature. A further increase of 1°C will lead to

The resistance increase of the primary for one degree C about 260°C is therefore

In other words, we should aim for our electronics to be sensitive enough to detect a 0.1% change in the resistance of the secondary. We can plug this into the voltage expression found earlier to determine what change in voltage that will result in across the primary, but that might be best done numerically (the equation is getting big enough as it is).

Thin-walled tube:

Magnetic field inside the tube: