Question 2.1:- 2D Transformations

a) Translation Matrix ($R_{translation}$) :- $T_x = 10$ and $T_y = 20$

Translation to origin is given by:-

$$R_T = \begin{bmatrix} 1 & 0 & -T_x \\ 0 & 1 & -T_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad R_T = \begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -20 \\ 0 & 0 & 1 \end{bmatrix} \#$$

$\Rightarrow$ We are having negative T_x & T_y because we are shifting to the origin.

b) Rotation Matrix ($R_{rotation}$):

Given,

rotation angle (θ) = $30°$

Rotation is given by: $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

substituting $\theta = 30°$, in the above rotation matrix.

$$R_{rotation} = \begin{bmatrix} \cos 30° & -\sin 30° & 0 \\ \sin 30° & \cos 30° & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \#$$

c) Combined Transformation matrix (R).

$$R = R_{rotation} \times R_{translation}$$

$$= \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -20 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{3}/2 & -1/2 & -(\sqrt{3}/2 \times 10 + (-1/2) \times 20) \\ 1/2 & \sqrt{3}/2 & -(1/2 \times 10 + \sqrt{3}/2 \times 20) \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{3}/2 & -1/2 & -(5\sqrt{3} - 10) \\ 1/2 & \sqrt{3}/2 & -(5 + 10\sqrt{3}) \\ 0 & 0 & 1 \end{bmatrix} \#$$

Question 3:2 ⇒ 3D Rotation

a) The transformations are :-

R_1 : A rotation of π around X-axis.

R_2 : A rotation of $\pi/2$ around Z-axis.

R : rotation matrix obtained by applying R_1 followed by R_2. ($R = R_2 \times R_1$)

we have,

⇒ rotation matrix for rotating a point around X-axis by an angle θ as:,

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

For $\theta = \pi$;

$$R_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

⇒ rotation matrix for rotating a point around Z-axis by an angle θ as:-

$$R_y = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

For $\theta = \pi/2$.

$$R_2 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

⇒ Finally,

$$R = \underset{(R_2)}{\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \times \underset{(R_1)}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \#$$

b) Given the point $A=[10, 0, 20]$, we need to apply rotation Matrix (R) to this point.

$$B = R \times A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} 10 \\ 0 \\ 20 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times10 + 1\times0 + 0\times20 \\ 1\times10 + 0\times0 + 0\times20 \\ 0\times10 + 0\times0 + (-1)\times20 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 20 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ -20 \end{bmatrix} \#$$

c) The inverse rotation matrix (R') that maps from B to A is the inverse of matrix (R). As matrix (R) is orthogonal matrix, its inverse is the transpose.

$$R' = R^T = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}^T = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \#$$

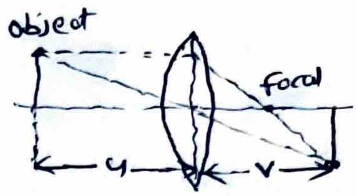
d) Given the point (B) $= [10, 0, 20]$, we need to apply inverse rotation matrix (R') to this point.

$$A = R' \times B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} 10 \\ 0 \\ 20 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times10 + 1\times0 + 0\times20 \\ 1\times10 + 0\times0 + 0\times20 \\ 0\times10 + 0\times0 + 1\times20 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 20 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ -20 \end{bmatrix} \#$$

Question 3.3: Focal length

Given,



distance from object to lens $(u) = 6$

distance between image plane and lens $(v) = 3$

we have,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$= \frac{1}{6} + \frac{1}{3} = \frac{1+2}{6} = \frac{3}{6} = \frac{1}{2}$$

Hence, the focal length (f) for the camera is 2.