

# Scientific Machine Learning Workshop

## Lab 2: PIGPs for Forward and Inverse Problems

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# Forward PIGP Assignment

- 1 Run the notebook `SciML_Advection_GP.ipynb`. (We will run it together.)
- 2 It is well-known that the linear advection equation with a periodic boundary condition becomes “stiff” as the velocity increases. Indeed, PINNs are known to fail as the velocity increase beyond  $v = 40$ . Compute the GP solution for  $v = 20, 30, 40, 50, 60$ . Record the trained values of  $\sigma_k$ ,  $\ell_x$ ,  $\ell_t$ , log-likelihood, and relative L2 error in each case. What happens to the accuracy of the solution as  $v$  increases? Is there a relationship between  $\ell_x$  and  $\ell_t$ ?

# Inverse PIGP Assignment

- 1 Run the notebook `SciML_Advection_GP_Inverse.ipynb`. (We will run it together.)
- 2 Plot the evolution of the velocity estimate and the log-likelihood value, over 10 independent runs, for  $N_x = 16, 32, 64$  uniformly-distributed spatial sensors, and the same 8 temporal snapshots as in the original code. Explain in detail what the effect of the number of sensors and the true velocity is on the accuracy of the results. In particular, compare the results to the forward problem results in the previous assignment for large values of the velocity.