Perspectives PSet 2 Part 1

January 15, 2020

1 Macs 30150 PSet 2

1.1 Part 1

[23]:

```
import sympy as sy
import numpy as np
import math
from matplotlib import pyplot as plt
import warnings
warnings.filterwarnings("ignore")
```

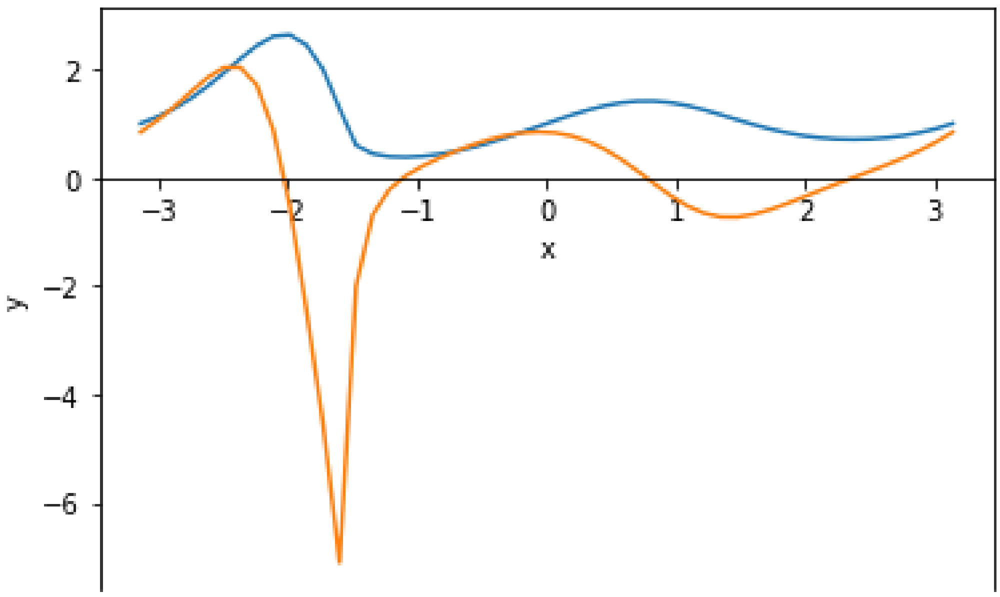
1.1.1 Problem 1

[24]:

```
x = sy.symbols('x')
ex = (sy.sin(x) + 1)**(sy.sin(sy.cos(x)))
f = sy.lambdify(x, ex, 'numpy')
f_dif = sy.lambdify(x, sy.diff(ex), 'numpy')
ax=plt.gca()
ax.spines['bottom'].set_position('zero')
a = np.linspace(-np.pi, np.pi)
b, c = f(a), f_dif(a)
plt.plot(a,b)
plt.plot(a,c)
plt.xlabel('x')
plt.ylabel('y')
```

[24]:

```
Text(0, 0.5, 'y')
```

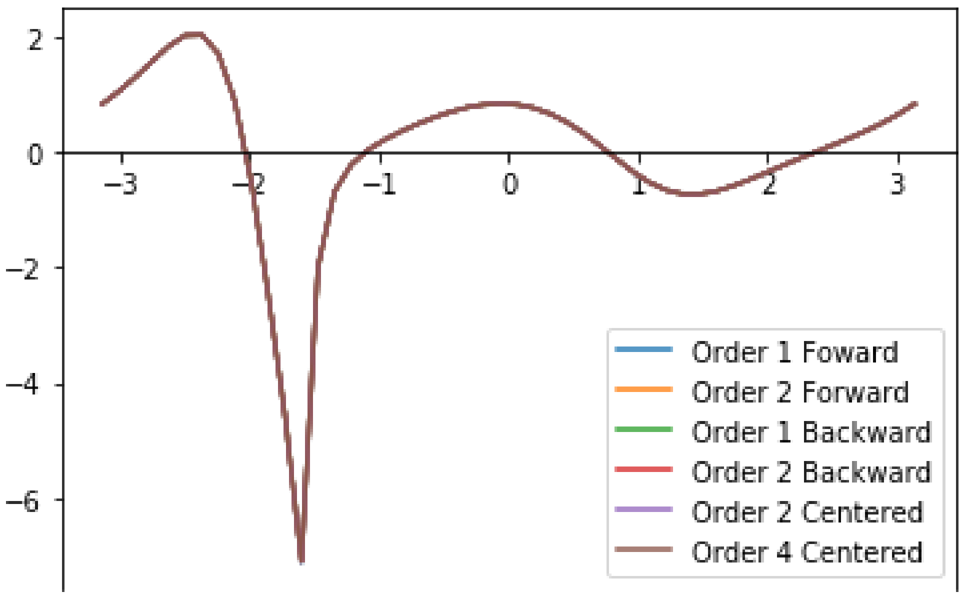


1.1.2 Problem 2

[25]:
```
def ffw_1(f, x, h):
    return (f(x+h) - f(x))/h
def ffw_2(f, x, h):
    return (-3*f(x) + 4*f(x+h) - f(x+2*h))/(2*h)
def fbw_1(f, x, h):
    return (f(x) - f(x-h))/h
def fbw_2(f, x, h):
    return (3*f(x) - 4*f(x-h) + f(x-2*h))/(2*h)
def fc_2(f, x, h):
    return (f(x+h) - f(x-h))/(2*h)
def fc_4 (f, x, h):
    return (f(x-2*h) - 8*f(x-h) + 8*f(x+h) - f(x+2*h))/(12*h)
```

[26]:
```
plt.plot(a,ffw_1(f,a,0.001))
plt.plot(a,ffw_2(f,a,0.001))
plt.plot(a,fbw_1(f,a,0.001))
plt.plot(a,fbw_2(f,a,0.001))
plt.plot(a,fc_2(f,a,0.001))
plt.plot(a,fc_4(f,a,0.001))
ax = plt.gca()
ax.spines["bottom"].set_position("zero")
plt.legend(['Order 1 Foward', "Order 2 Forward", "Order 1 Backward", \
            "Order 2 Backward", "Order 2 Centered", "Order 4 Centered"], loc =␣
 ↪'lower right')
```

[26]:
```
<matplotlib.legend.Legend at 0x1df130e6ef0>
```



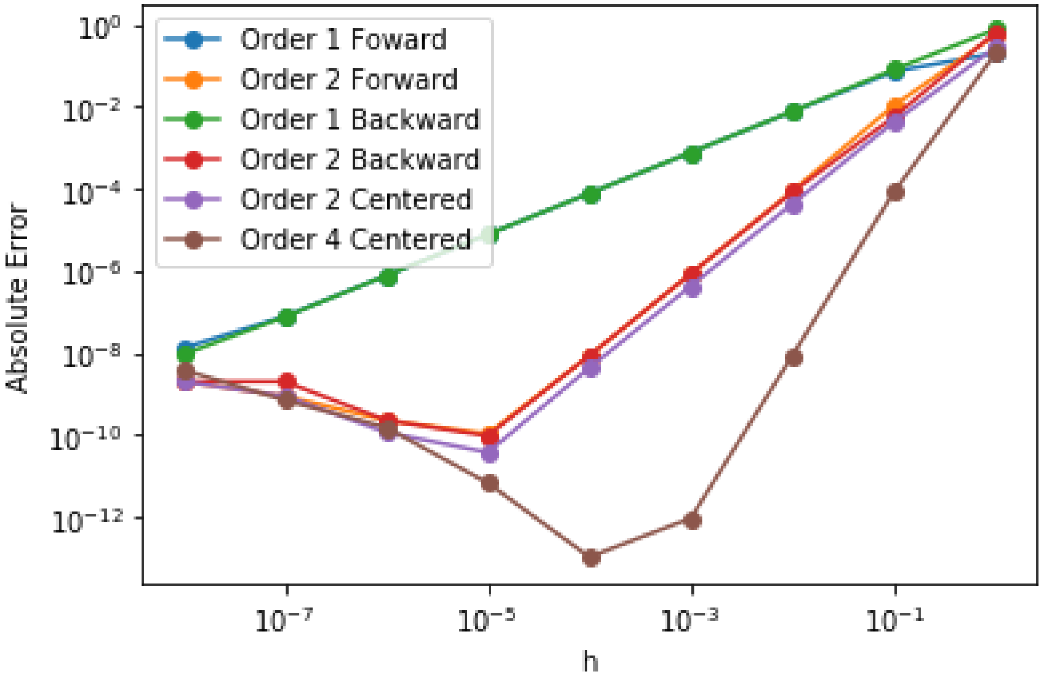
1.1.3 Problem 3

[27]:

```
h = np.logspace(-8,0,9)
x = 1
h1 = abs(f_dif(x)-ffw_1(f,x,h))
h2 = abs(f_dif(x)-ffw_2(f,x,h))
h3 = abs(f_dif(x)-fbw_1(f,x,h))
h4 = abs(f_dif(x)-fbw_2(f,x,h))
h5 = abs(f_dif(x)-fc_2(f,x,h))
h6 = abs(f_dif(x)-fc_4(f,x,h))
plt.loglog(h,h1,marker = 'o')
plt.loglog(h,h2,marker = 'o')
plt.loglog(h,h3,marker = 'o')
plt.loglog(h,h4,marker = 'o')
plt.loglog(h,h5,marker = 'o')
plt.loglog(h,h6,marker = 'o')
plt.legend(['Order 1 Foward', "Order 2 Forward", "Order 1 Backward", \
            "Order 2 Backward", "Order 2 Centered", "Order 4 Centered"], loc =␣
 ↪'upper left')
plt.xlabel('h')
plt.ylabel('Absolute Error')
```

[27]:

```
Text(0, 0.5, 'Absolute Error')
```



1.1.4 Problem 4

[28]:

```
import pandas as pd
df = pd.DataFrame(np.load('plane.npy'), columns = ['time', 'a', 'b'])
df['a'] = np.deg2rad(df['a'])
df['b'] = np.deg2rad(df['b'])
def x_cord(a,b):
    return 500*(np.tan(b))/(np.tan(b)-np.tan(a))
def y_cord(a,b):
    return 500*(np.tan(b)*np.tan(a))/(np.tan(b)-np.tan(a))
df['x'] = x_cord(df['a'],df['b'])
df['y'] = y_cord(df['a'],df['b'])
#time: 8 - 13
df['x_prime'] = (df.x.shift(-1)-df.x.shift(1))/2
df['y_prime'] = (df.y.shift(-1)-df.y.shift(1))/2
#time: 7
df['x_prime'][0] = df['x'][1]- df['x'][0]
df['y_prime'][0] = df['y'][1]- df['y'][0]
#time: 14
df['x_prime'][7] = df['x'][7]- df['x'][6]
df['y_prime'][7] = df['y'][7]- df['y'][6]
#calculate speed
df['speed'] = np.sqrt(df['x_prime']**2 + df['y_prime']**2)
df
```

[28]:

```
   time         a         b            x            y    x_prime    y_prime  \
0   7.0  0.981748  1.178795  1311.271337  1962.456239  44.665140  12.658266
1   8.0  0.969181  1.161866  1355.936476  1975.114505  45.323531  12.444889
2   9.0  0.956440  1.144761  1401.918398  1987.346016  47.280265  12.863104
3  10.0  0.943525  1.127308  1450.497006  2000.840713  48.360976  13.083197
4  11.0  0.930959  1.110378  1498.640350  2013.512411  46.650974  12.475760
5  12.0  0.919614  1.095020  1543.798955  2025.792234  49.700516  13.739086
6  13.0  0.906524  1.077217  1598.041382  2040.990583  51.898569  14.636669
7  14.0  0.895005  1.061509  1647.596093  2055.065571  49.554711  14.074988

       speed
0  46.424201
1  47.001039
2  48.998805
3  50.099442
4  48.290351
5  51.564559
6  53.923034
7  51.514801
```

1.1.5 Question 5

[29]:

```python
def Jac(f, p, h):
    n = len(f)
    m = len(p)
    J = np.zeros((n, m))
    e = np.eye(m)
    for i, f_i in enumerate(f):
        for j, p_j in enumerate(p):
            fcn = sy.lambdify((x, y), f_i, 'numpy')
            x_1 = p_j - h * e[:,j]
            x_2 = p_j + h * e[:,j]
            J[i, j] = (fcn(x_2[0], x_2[1]) - fcn(x_1[0], x_1[1])) / (2 * h)
    return J
```

[30]:

```python
# Test
x = sy.Symbol('x')
y = sy.Symbol('y')
Jac([x ** 2, x ** 3-y], [1,1], 0.01)
```

[30]:

```
array([[ 2.    ,  0.    ],
       [ 3.0001, -1.    ]])
```

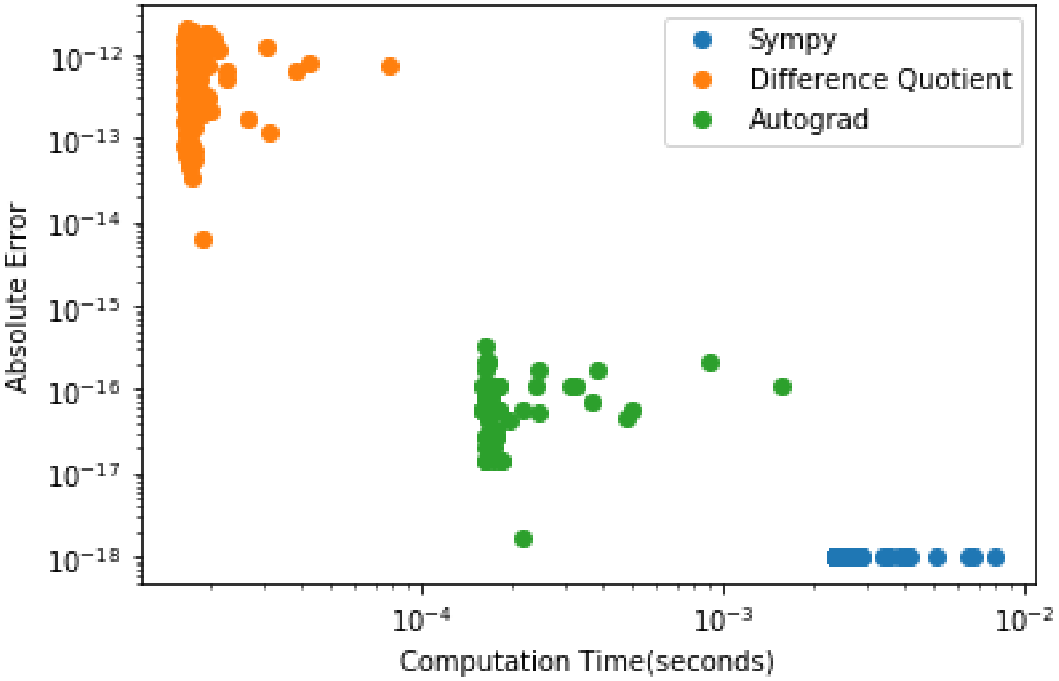
1.1.6 Question 7

[31]:
```python
import time
import random
from autograd import grad
from autograd import numpy as npy

g = lambda x: (npy.sin(x) + 1)**(npy.sin(npy.cos(x)))
g_grad = grad(g)
x = sy.symbols('x')
fx = (sy.sin(x) + 1)**(sy.sin(sy.cos(x)))
```

[32]:
```python
def TIME(N):
    err1 = np.ones(N,dtype='float') * 1e-18
    err2 = np.zeros(N,dtype='float')
    err3 = np.zeros(N,dtype='float')
    time1 = np.zeros(N,dtype='float')
    time2 = np.zeros(N,dtype='float')
    time3 = np.zeros(N,dtype='float')
    for i in range(N):
        # Step 1
        x0 = np.random.uniform()
        # Step 2
        start1 = time.clock()
        dfx = sy.diff(fx, x)
        f_sy = sy.lambdify(x, dfx)
        num1 = f_sy(x0)
        end1 = time.clock()
        time1[i] = end1-start1
        # Step 3
        start2 = time.clock()
        num2 = fc_4(f, x0, h=0.0001)
        end2 = time.clock()
        time2[i] = end2-start2
        err2[i] = abs(num1 - num2)
        # Step 4
        start3 = time.clock()
        num3 = g_grad(x0)
        end3 = time.clock()
        time3[i] = end3-start3
        err3[i] = abs(num1 - num3)

    ax = plt.gca()
    ax.loglog(time1, err1,'o', label='Sympy')
    ax.loglog(time2, err2, 'o', label='Difference Quotient')
    ax.loglog(time3, err3, 'o', label='Autograd')
    plt.legend()
    plt.xlabel('Computation Time(seconds)')
    plt.ylabel('Absolute Error')
    return
```

[33]:

```
TIME(200)
```



[ ]: