Problem Set # 1

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Problem 1

Part (a). In this assignment, I focus on a statistical model from Jump Regression(Li, Todorov and Tauchen 2017) from Econometrica.

Part (b). Li, Jia, Viktor Todorov, and George Tauchen, "Jump regressions", *Econometrica*, 2017, 85 (1), 173-195.

Part (c). This model is from high-frequency financial econometrics. The authors made great contributions to inference for jump move in asset prices. In this part, I will mainly focus on the basic form of the model. By saying a jump, we mean a price move in an asset that cannot be accounted for(with extremely low probability) based on continuous-time models. These moves are quite usual in the stock market when unexpected information or news is released to the investors. From basic asset pricing theory, we know that an asset comoves with the market index(e.g. S&P 500). The jump regression is basically the regression of individual stock price change on market jumps. The β has great significance in financial literature because it measures the degree to which an individual stock covaries with the systematic risk factor.

In the jump regression setting, we have

$$X = \begin{pmatrix} Z \\ Y \end{pmatrix}$$

where Z is the log of the market index and Y is the log of an asset price. The dynamics are:

$$\begin{aligned} dZ_t &= \sigma_{zt} dW_{1t} + \Delta Z_t \\ dY_t &= \beta_t^c dZ_{1t}^c + \tilde{\sigma}_{yt} dW_{2t} + \beta_t^J \Delta Z_t + \Delta \tilde{Y}_t \end{aligned}$$

where W's are independent Brownian motions and rest of the notations is as follows:

σ_{zt}:	Market diffusive volatility
ΔZ_t:	Jump part of the market return
β_t^c:	Beta on diffusive (continuous) move in Z
dZ_{1t}^c:	The diffusive (continuous) move in Z
$\tilde{\sigma}_{yt}$:	Idiosyncratic diffusive volatility
β_t^J:	Beta on jump moves in Z
$\Delta \tilde{Y}_t$:	Idiosyncratic jumps in the asset return

The key aspect to understanding jump regression is that, by definition, the idiosyncratic jumps in Y are independent of the market jumps, and two independent processes never jump at the same time. Thus, we have:

$$\Delta Z_t \Delta \tilde{Y}_t = 0, \;\; t \in [0, T]$$

In this article, the authors mainly discuss the statistical inference of the jump covariation move: β_t^J.

Part (d). In this statistical model, for exogenous variables: Firstly, we can get data on the price of market portfolio (e.g. S&P 500) as well as the price of individual stocks directly. Additionally, by specifying a threshold, we can get the market jumps and individual stock jumps. (The jumps are defined as the moves in stock prices that can not be accounted for by any continuous-time model) Therefore, we have Z_t, Y_t, ΔZ_t and $\Delta\tilde{Y}_t$ as exogenous variables.

With the model, we can estimate the Betas on diffusive move and jump move in Z. We can also estimate the market diffusive volatility as well as individual stock's idiosyncratic diffusive volatility. Therefore, the endogenous variables are: β_t^c, β_t^J, σ_{zt} and $\tilde{\sigma}_{yt}$.

Part (e). This model is dynamic. It has a linear structure and it is stochastic. High frequency financial econometric model needs the dynamics to capture the time-varying market change. The stochastic process is also one of the fundamental building blocks of modeling the asset prices. In the basic setup of the paper, the authors mainly focus on the linear relationship between the jump move in the market and the jump in individual stocks.

Part (f). One variable that might be missing in the model is the drift term μ of the individual stock and the market. Based on this model, we can only make inference for a relatively short period of time in which the drift term for stock return is negligible. (e.g. a few days) However, for longer-term modeling, we have to consider the drift of stock prices. It can not be seen as purely as a martingale(purely diffusive move). There exists a risk premium in the stock market. In this sense, we have to account for the drift when it comes to longer time horizons.

References

Li, Jia, Viktor Todorov, and George Tauchen, "Jump regressions," *Econometrica*, 2017, *85* (1), 173–195.