Problem Set #1

MACS 30000, Dr. Evans
Ye Sun

Problem 1 Classify a model from a journal.
Part (a). I picked Davis and Dingel (2019) for this assignment.
Part (b). Please see the reference list attached to the end of this file for detailed citation.
Part (c). This paper considers a general equilibrium of a continuum of individuals of mass L choosing their locations, occupations, and time allocations optimally. There are two sectors of production in every city: tradables (t) and non-tradables (n). First, individual preferences in city c are characterized by the indirect utility function

$$V(p_{n,c}, p_{h,c}, y) = y - p_{n,c}\bar{n} - p_{h,c}$$

where $\bar{n}$ is the non-tradable necessities, and $p_{h,c}$ is the congestion cost and takes the form of $p_{h,c} = \theta L_c^{\gamma}$. Consumers spend the rest of their income on tradables which in the model is seen as the numéraire. The productivity of one unit of labor in sector σ is

$$y = \begin{cases} p_{n,c} & \sigma = n \\ \tilde{z}(z, Z_c) & \sigma = t \end{cases}$$

The productivity of the non-tradables is normalized to 1. Heterogeneity are allowed in the productivity of tradables and is governed by the individual ability parameter z and learning opportunities available through local interactions Z_c. $\tilde{z}(z, Z_c)$ is also regarded as the trade-off result between time spent on learning from others and time spent on producing output directly. Namely,

$$\tilde{z}(z, Z_c) := \max_{\beta_{z,c}\in[0,1]} B(1-\beta_{z,c}, z, Z_c)$$

$$Z_c := Z(\{1-\beta_{z,c}\}, \{L\cdot\mu(z,c)\})$$

Moreover, by assuming (i) $B(1-\beta_{z,c}, z, Z_c)$ is concave in $1-\beta_{z,c}$, strictly increasing in z and Z_c, (ii) supermodularity in $\tilde{z}(\cdot,\cdot)$, (iii) continuity and boundedness of function $Z(\cdot,\cdot)$, (iv) non-tradable market clear, and (v) labor market clear, we can attain the equilibrium.
Part (d). The exogenous variables are θ, L, γ and $\bar{n}$. Also, other exogenous parts are functional forms of $B(1-\beta_{z,c}, z, Z_c)$, $Z(\{1-\beta_{z,c}\}, \{L\cdot\mu(z,c)\})$, and distribution of talents $\mu(z)$. The output of the model are the price of non-tradables $p_{n,c}$, the sizes of cities $\mu(c)$, and talent distributions across cities $\mu(z,c)$.
Part (e). This model is static, non-linear and deterministic
Part (f). The model assumes no cost of shifting sectors and moving to a different city, or the individuals are perfectly mobile among cities and sectors. If these variables (fixed cost of moving) are added in the model we might foresee multiple equilibria.

Problem 2 Make your own model.

Part (a)-(c). In the book *The Economics of the Family* (Browning, Chiappori and Weiss 2014, Chapter 2, The Gains from Marriage), the author introduced a unitary model. The rationale is simple: the wife a and the husband b have their own utility function, U_a and U_b respectively, over two commodities Q_i and q_i[1]. These two people tend to optimize their utility through adjusting the amounts consumed of two commodities Q_i and q_i whereas the commodity Q is shared by two:

$$\max_{Q_a,q_a} U_a(Q_a + Q_b, q_a)$$
$$\max_{Q_b,q_b} U_b(Q_a + Q_b, q_b)$$

They are facing the budget constraint that the expenditure cannot be larger than the individual income plus/minus the transfer ρ from/to the spouse. That is:

$$\max_{Q_a,q_a} U_a(Q_a + Q_b, q_a) \quad \text{subject to} \quad p \cdot q_a + P \cdot Q_a = I_a + \rho$$

for a, and

$$\max_{Q_b,q_b} U_b(Q_a + Q_b, q_b) \quad \text{subject to} \quad p \cdot q_b + P \cdot Q_b = I_b - \rho$$

for b. The first-order condition would be

$$\begin{aligned} \frac{(U_a)_Q}{(U_a)_q}(Q_a + Q_b, q_a) &= \frac{P}{p} \\ \frac{(U_b)_Q}{(U_b)_q}(Q_a + Q_b, q_b) &= \frac{P}{p} \end{aligned}$$

respectively. In the above we can tell that how one family member acts directly influences the utility of the other one. In this case we can reduce the two optimization into one and eliminate the parameter ρ. The two member make their decisions always end up in the Nash Equilibrium $(\hat{Q}^s, \hat{q}^s)$.

$$\frac{(U_a)_Q}{(U_a)_q}(\hat{Q}, \hat{q}_a) = \frac{P}{p} \tag{1}$$
$$\frac{(U_b)_Q}{(U_b)_q}(\hat{Q}, \hat{q}_b) = \frac{P}{p} \tag{2}$$
$$p \cdot (\hat{q}_a + \hat{q}_b) + P \cdot \hat{Q} = I_a + I_b \tag{3}$$

Solve the equations regarding $\hat{Q}, \hat{q}_a, \hat{q}_b$, and we can deduce the one and only Nash Equilibrium.

If person a chooses not to marry b, its optimal utility level under its own budget constraint, which is

$$\max_{Q_a,q_a} U_a(Q_a, q_a) \quad \text{subject to} \quad p \cdot q_a + P \cdot Q_a = I_a$$

[1] $i = a, b$ and *sic passim.*

should be larger than that of getting married.

In a world where the shared goods Q and the private goods q have the same price which we use as the numéraire, let the utility function takes the Cobb-Douglas form with the parameter α, $U := Q^{\alpha}q^{1-\alpha}$ The result of the aforementioned framework is presented below (Figure 1, 2, and 3). The y-axis denotes the income (or other endowments) share r of a person in the total household income. The x-axis denotes the decided share β of a person's contribution in the shared goods. The curve starting from $(0, 0)$ is person a's indifference curve of marriage. The lower contour is the area where this person decide to marry. The curve that ends at $(1, 1)$ is the other person b's indifference curve of marriage. The upper contour is the area this person decide to marry. Only when the person's income fraction r and public goods fraction β lies in the union of the contours, these two people may consider marry each other. When we are tuning the value of α, we can observe the process of the union of these two contours shrinks and disappears. The intuition is clear: when the household public goods entering personal utility function with a bigger weight, the balance of income or other endowment is more important. The more you share with your family members, the matching of income is more strict. Also, fixing α such that the union of contours are not empty, both upper bound $\bar{r}$ and lower bound $\underline{r}$ of the income fraction shows a positive correlation with the share of public goods β. Put it another way, the interval $[\bar{r}, \underline{r}]$ is moving upwards with β shifting to bigger.

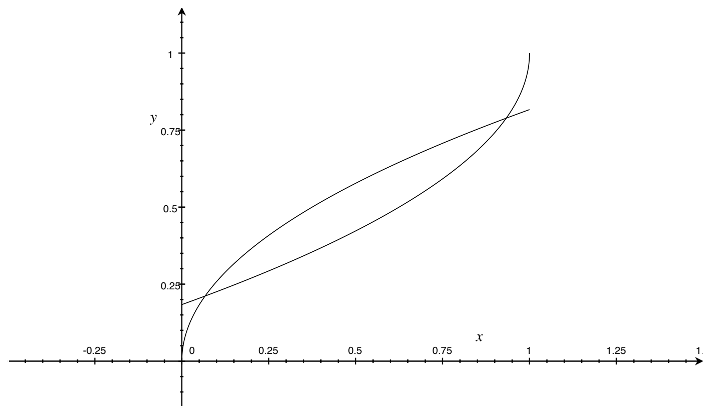

Figure 1: α=.5

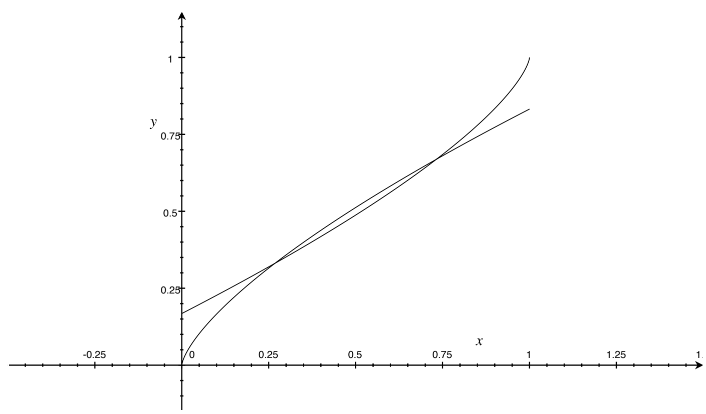

Figure 2: α=.7

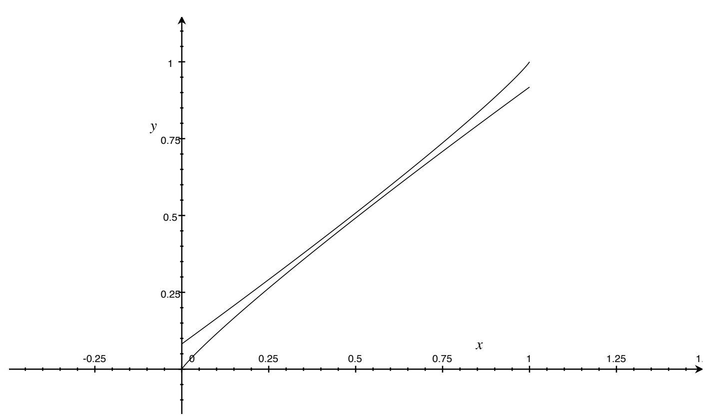

Figure 3: α=.9

Part (d). The key factors that influences this outcome is the income fraction r and the contribution to public goods β.

Part (e). Of course, other factors also contributes to one's marital status. This model is not exhaustive. The decision of getting married is a race of multiple factors including financial condition, appearance, personality compatibility, among other things. Our model depicts how the difference of personal financial status influences marriage decision controlling other factors, and personal income is one of the most inclusive proxies for personal financial factors.

Part (f). We can test the model by surveying married couples how they allocate their income. We can ask the proportion of one's income that he/she spend solely for him/herself to infer the value of $1-\alpha$ and the proportion of one's income out of the family income r, and see if there's a positive correlation between α and $Var(r|\alpha)$. Also, very intuitively, we can run regression over the income share and public good expenditure share and see if 1 is in the coefficient CI.

References

Browning, Martin, Pierre Chiappori, and Yoram Weiss, *Economics of the Family*, Cambridge University Press, 2014.

Davis, Donald R. and Jonathan I. Dingel, "A Spatial Knowledge Economy," *American Economic Review*, January 2019, *109* (1), 153–70.