Problem Set #[1]

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Problem 1 Part (a). This statistical model comes from "Optimal Annuitization with Stochastic Mortality and Correlated Medical Costs" by Felix Reichling and Kent Smetters. This article was published in 2015, on the *American Economic Review*.
Part (b). This article reviewed Yaari's paper in 1965 on the demand for annuities, raising questions on Yaari's claim that "households without a bequest motive should fully annuitize their investments". Reichling and Smetters modified Yaari's framework to include health shocks so that in their new multi-period model, a household's mortality risk is stochastic. They concluded that most households should not hold positive annuities, and many of them should short annuities.
Part (c). Multi-period model:
The health state h follows an M-state Markov process, where M = 3: healthy (h1), impaired (h2), and very sick(h3). This Markov Process is with an age-dependent transition matrix:

$$P_j(m,n) = 1,2,...M \tag{1}$$

$$V_j(A_j,\eta_j,h_j,j) = \max_{c_j,\alpha_j}\{u(c_j) + \beta s_j(h_j)E_j[V_{j+1}(A_{j+1},\eta_{j+1},h_{j+1},j+1)\} \tag{2}$$

subject to

$$A_{j+1} = R(\alpha_j,h_j,h_{j+1})(A_j + X_j - c_j)$$
$$\alpha \le 1$$
$$c \le c_j \le A_j + X_j$$

where

$$E_j[V_{j+1}] = \int_{h_{j+1}}\int_{eta_{j+1}} V_{j+1}A_{j+1},\eta_{j+1},h_{j+1},j+1)Q(\eta_j,d\eta_{j+1})P_j(h_j,dh_{j+1})$$

Where $R(\alpha_j, h_j, h_{j+1}) = \alpha_j\rho_j(h_j, h_{j+1}) + (1 - \alpha_j)r$ is the portfolio return, X_j is the value of income, $Q(\eta_j, d\eta_{j+1})$ is the changes in productivity, $P_j(h_j, dh_{j+1})$ is the changes in health, α_j is the share of investments made into annuities at age j, c_j is the consumption at age j.
Part (d). Exogenous variables: c_j, h_j, j, r, X_j, α_j, η_j, ρ_j,
Endogenous: V_j, A_j, R_j, E_j, P_j, $Q(\eta_j, d\eta_{j+1})$
Part (e). The model is dynamic, nonlinear, and stochastic.
Part (f). Additional variable to consider: Change in mortality credit from time j to time j + 1. As mentioned in Appendix G by the authors, the model assumes asymmetric information, and a reduction in mortality credit from asymmetric information could further reduce positive annuitization.

Problem 2 Part (a), (b), (c). Getting Married = 1 if

$$U_{i,1} - U_{i,0} \geq 0 \tag{3}$$

where

$$U_{i,1} = 0.2Age - 0.3Education + 1.5DisposableIncome + 0.6(7 - LengthofRelationship)^2 \\ +1.2Pregnancy - 0.3ParentsDivorced + \epsilon_{i,1}$$

$$U_{i,0} = -0.2Age + 0.5Education + 1.5DisposableIncome + 0.6(LengthofRelationship - 7)^2 \\ -Pregnancy + 0.2ParentsDivorced + \epsilon_{i,0}$$

Part (d) The key factors are disposable income and pregnancy.

Part (e) I decided on these factors because one decides to get married when the utility of getting married is greater than the utility of not getting married. From an economic perspective, one benefits from marriage if disposable income increases after marriage. Pregnancy and the birth of a child brings additional expenses and the economic burden will be less if shared by two.

Part (f) I could cite data from U.S. Census Beareau on On Marital Status by Mean Income. I could also conduct a digital survey on people's marital status, age, education attainment, income, length of relationship, pregnancy, and marital status of parents. Then I will compare the data with the model. I will also perform maximum likelihood test.