Urban Computing

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What did we talk about last sessions?

- **Session 1:** Urban computing
  - Urban applications
  - Urban data
    - Old data: Questionnaires, census surveys
    - New data: citizens as sensors, accidental data, open data
- **Session 2:** Time-series data
  - Methods of representing time-series data (time domain, frequency domain)
    - Auto-correlation
    - Periodogram
  - Methods of processing time-series
    - Time-series forecasting using auto-regressive models
    - Time-series classification

Third Session: Urban Computing - Processing Spatial Data

Agenda for this session

- **Part 1:** Preliminaries
  - What is spatial data?
  - How do we represent it?
- **Part 2:** Methods for processing spatial data
  - Spatial auto-correlation
  - Neighborhoods and weight matrices
  - Spatial regression and auto-regressive models

Part 1: Preliminaries

What is spatial data?

- What is spatial data?
- Spatial datasets?
- Spatial statistics versus classical statistics?

What is spatial data?

- Data that associates locations to each data instance
- Examples:
  - Temperature values for different cities
  - GDP values for countries
  - Number of crimes happening across a city
  - Pixel values in a grayscale image
  - Frequency band values of remote sensing images
  - ...
- Spatial versus geo-spatial $\rightarrow$ Any image versus geo-spacial images

Spatial databases

- **A spatial database**: is a database optimized for storing and querying objects defined in a geometric space.
  - Geometric objects:
    - Points
    - Lines
    - Polygons

Geometric feature

Vector data structures that represent specific features on the Earths surface, and assign attributes to those features.

Figure: Point data

Figure: line data

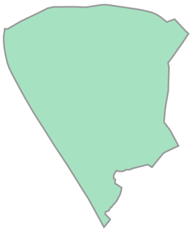


Figure: polygon data

Spatial statistics versus classical statistics

- **Case:** You have the data on the amount of rainfall in different locations in the Netherlands and you want to predict the value of temperature in Leiden
  - **Data you have:** $\rightarrow$ temperature, wind power, rainfall
- How can you define a regression task to solve this? (dependent value, independent value)

Spatial statistics versus classical statistics

Key difference:

- **The assumption in classical statistics:** Data samples are Independent and identically distributed (i.i.d. or iid or IID)
  - Each random variable has the same probability distribution as the others and all are mutually independent

iid versus spatial correlation

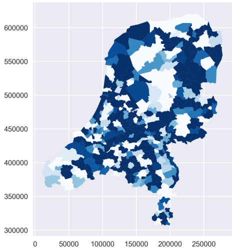


Figure: Independent and identically distributed data

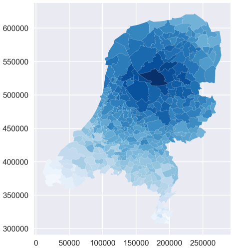


Figure: Data distributed with correlation over space

Spatial data

First law of geography:
All things are related, but nearby things are more related than distant things. [Tobler70]

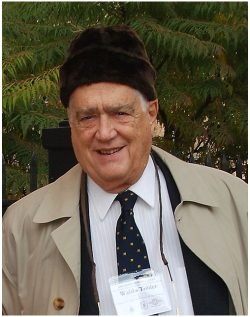


Figure: Waldo Tobler [1]

[1] https://en.wikipedia.org/wiki/WaldoR.Tobler

Spatial statistics versus classical statistics

Classical statistics:

- Data samples are IID
  - Simplified mathematical ground (Example: Linear Regression)

Spatial statistics:

- Data samples are non-IID distributed.
- Methods should be able to capture spatial affects:
  - **Spatial correlation:** What happens north, south east, and west of here depends is very likely to be dependent on what is happening here.
  - **Spatial heterogeneity:** Different concentration of events, etc over space. Similarity of values decay with distance.

Temporal statistics:

- Data are non-IID
  - **Temporal correlation:** What happens now determines what happens next (one directional flow from past to present)
  - **Temporal heterogeneity**: Non-stationarity over time

How do we represent spatial data to algorithms?

- How do you represent each of these examples (space domain):
  - Crime events and coordinates
  - Rainfall and coordinates
  - Population and coordinates

How do we represent spatial data to algorithms?

What points should you consider:

- What is a variable's nature?
  - Discrete, continuous
- What is the location data nature?
  - Discrete, continuous
    - To answer this question we need to know about the nature of the underlying process

How to represent data over space?

In general there are three classic approaches for dealing with spatial data. This depends on the underlying process: [CW15]

- Geostatistical process
- Lattice process
- Point process

Geo-statistical process

- **Fixed continuous location:** observations with a continuously varying quantity; a spatial process that varies continuously being observed only at few points
- Examples: rainfall, wind speed, temperature
- Statistical methods based on geo-spatial data:
  - **Gaussian process regression (Kriging)**: spatial interpolation

Kriging [CW15]

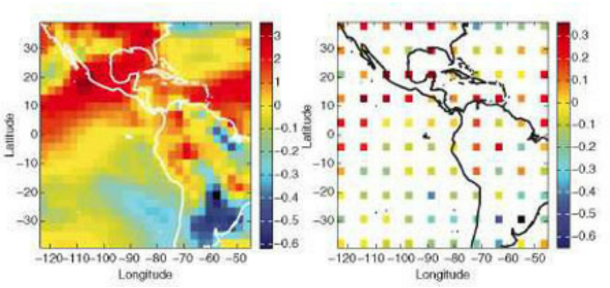


Figure: simple geo-statistical data and recovering through simple kriging predictor

Lattice process

- **Fixed discrete location**: Counts or spatial averages of a quantity over regions of space; aggregated unit level data.
- Examples: aggregate data of census, income, number of residents
- Data is represented in discrete spatial units (grid cells, regions, pixels, areas)
- Statistical methods designed based on lattice processes:
  - **Spatial auto-correlation:** Is there a correlation between neighboring units?

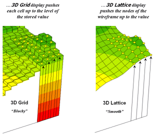


Figure: 3D Grid and Lattice [2]

[2] https://blogs.ubc.ca/advancedgis/schedule/slides/spatial-analysis-2/lattices-vs-grids/

Lattice process

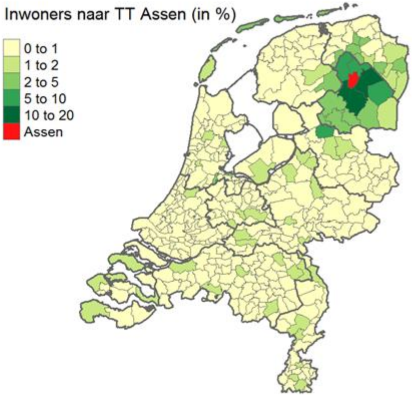


Figure: People who went to TT Assen from other cities

Point process

- **Random continuous location:** the spatial process is observed at a set of locations; the locations are interesting as well
- Examples: location of wildfires, earthquakes, accidents, burglaries
- Data is represented by arrangement of points on a region
- Methods designed based on point processes:
  - **K-function**: considers the distance between points in a set

Point process

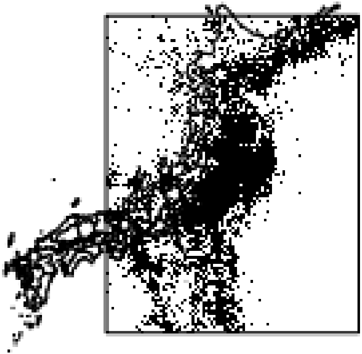


Figure: The Japan Earthquake data contained earthquake locations and magnitudes from 2002 to 2011[3]

[3]http://www.stat.purdue.edu/ huang251/pointlattice1.pdf

Various statistical indicators and methods for different representation

- **Geo-statistical process:** kriging, variogram, etc.
- **Lattice process:** cluster and clustering detection, spatial autocorrelation, etc.
- **Point processes:** point pattern analysis, marked point patterns, K-functions, etc.

We can't take a look at all of them but we will look at some

Other ways to represent data

- Space domain (point, geo-spatial, lattice)
- Alternative domains (out of the scope of this session):
  - Applying Fourier, Wavelet transform on the Lattice representation
  - Inspired from the image processing literature
  - Convolutional neural networks: (convolutions are multiplication of signals in frequency domain)

Part 2: Methods for processing spatial data

Spatial auto-correlation

Spatial auto-correlation, does spatial correlations exist?

Problem: Are the data instances IID or non-IID? Does spatial correlation exist?

- Exploration

Spatial auto-correlation

What does +1, 0, -1 spatial auto-correlation value mean when observed in data?

- Positive
  - Typical in Urban data
  - Similar values happen in neighboring locations. (High, High), (Low, Low)
  - Closer values are more similar to each other than further ones
- Zero
  - IID
  - Randomly arranged data over space
  - No spatial pattern
- Negative
  - Dissimilar values happen in neighboring locations (High, Low), (Low, High), Checker board pattern
  - Closer values are more dissimilar to each other than further ones
  - Typically a sign of spatial competition

How spatial auto-correlation function is designed:

We learned about the temporal auto-correlation. How should be implement spatial auto-correlation?

- We need to capture
  - Attribute similarity
  - Neighborhood similarity

The different between temporal and spatial auto-correlation

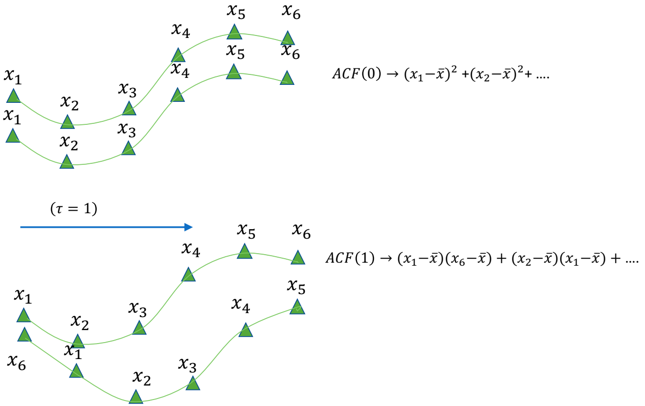
What do you remember about temporal auto-correlation?

- **Temporal:** Self-similarity of data over time, Previous data instances determine future data instances
- $ACF_\tau = \frac{1}{T}\sum_{t=1}^{t=T-\tau(orT)}$ [4] $(x_t - \overline{x})(x_{t+\tau} - \overline{x}), \tau = 0, 1, 2, ..., T$ [5]
- **Spatial:** Self-similarity over space, Neighboring data instances determine each other
- ?

[4] T is used in circular autocorrelation

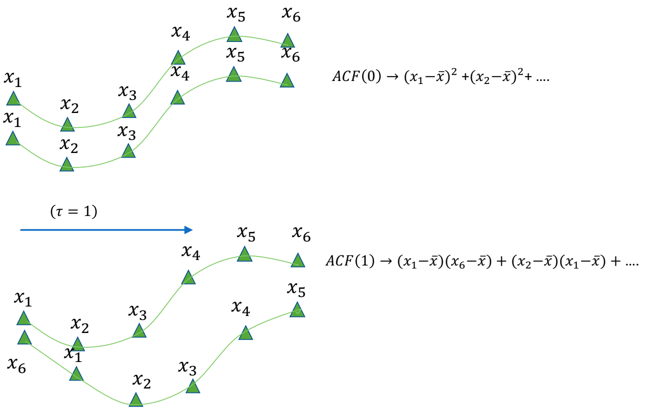
[5] max value of τ can be smaller

Temporal auto-correlation



How did we capture attribute and neighborhood (in time) similarity?

Temporal auto-correlation



How did we capture attribute and neighborhood (in time) similarity? Attribute: multiplication, neighborhood in time: shift, lags

Spatial auto-correlation

What is the equivalent of temporal lag in space? $\rightarrow$ Distance

- Moran's I
- $I(d) = \frac{N}{|W|} \frac{\sum_i \sum_j w_{ij}(x_i - \overline{x})(x_j - \overline{x})}{\sum_i (x_i - \overline{x})^2}$
  - $I(d)$= Moran's I correlation coefficient as a function of distance d, $d \in \{1, 2, ...\}$
  - x_i is the value of a variable at location i
  - W is a matrix of weighted values. Each w_{ij} in W represents the weight representing the effect of element x_i on element x_j
  - $|W|$ is sum of the values of w_{ij}
  - N is the sample size

How to show spatial dependence over neighborhoods?

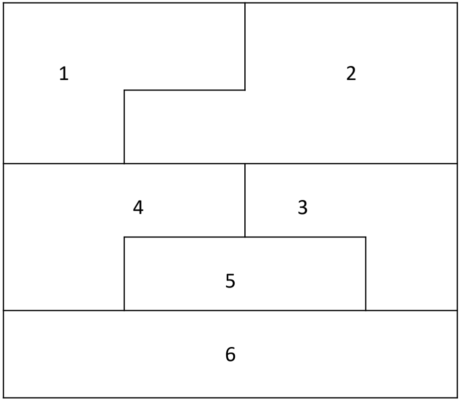
- We need some representation of dependence and interactions over space
- The most common way people are considering these effects is by using Spatial Weights Matrices W
  - N$\times$ N positive matrix containing the strength of interactions between spatial point i and j
- Many algorithms designed for spatial data make use of weight matrices
  - Spatial auto-correlation
  - Spatial regression
  - Spatial clustering

How to assign weights to neighbors

- N variables and N^2 comparisons to make to consider all neighbors $\rightarrow$ for the sake of efficiency some can be ignored (the interaction can be set to zero)
- Ignored neighbors: $w_{ij} = 0$
- Important neighbors:
  - $w_{ij} = 1$
  - $w_{ij} = 0 < wij < 1$
- Non-binary weights can be a function of:
  - Distance
  - Strength of interaction (e.g. commuting flows, trade, etc.)
  - ...

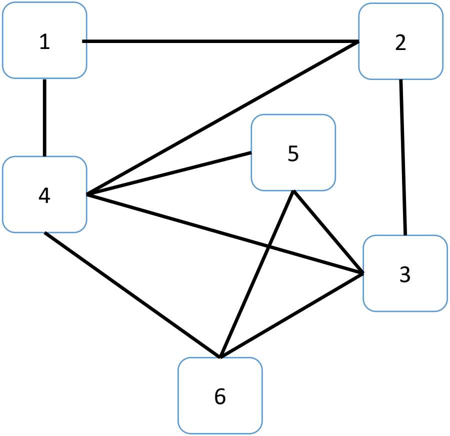
Weights matrix

How do we represent interactions from raster and polygon data in a matrix?



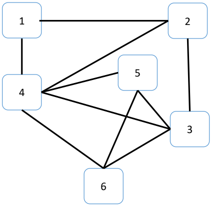
Weights matrix

Create a graph representation showing neighboring cells based on having a common border



Graph representation and adjacency matrix

Use the adjacency matrix of the graph to create the weight matrix:



$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

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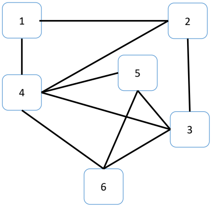


$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Is there a solution?

This way we can only show neighbors that have common edges. What if we cared about the physical distance? or two-hop away neighbors?

Neighbors

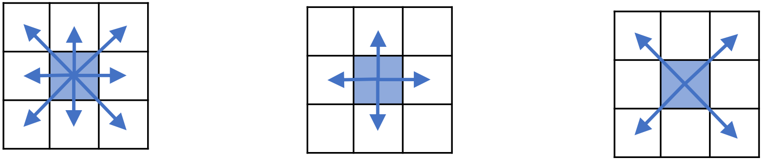
How do we define neighborhood? What neighbors do we care about? (i.e. select non-zero elements of W):

- **Contiguity-based**: Having a common border
- **Distance-based**: Being in the vicinity
- **Block-based**: Being in the same place based on an official agreement
  - Provinces
  - Cities and countries
  - ..
- ...

Contiguity-based weights

Figure: How can you move to a neighboring cell?

Contiguity-based weights



Queen’s case Rook’s case Bishop’s case

Figure: neighborhood cases

Queen's case

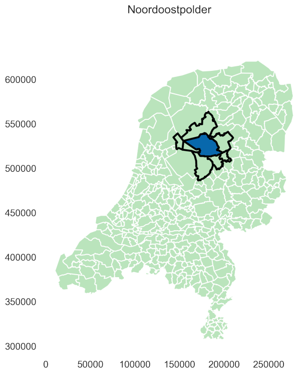


Figure: Queen's case

Rook's case

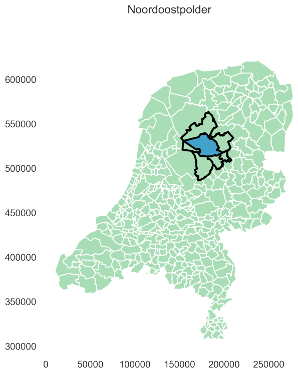


Figure: Rook's case

Bishop's case

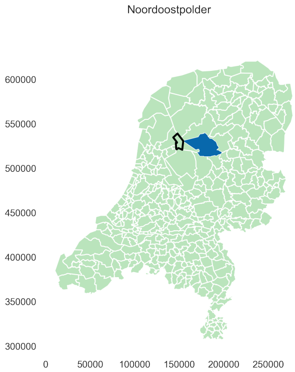


Figure: Bishop's case

Distance-based

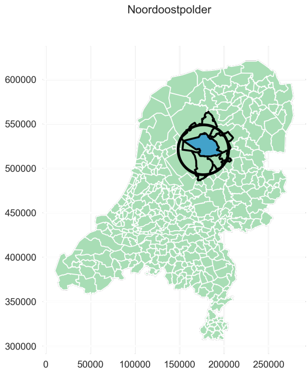


Figure: distance-based neighborhoods

Block neighborhood

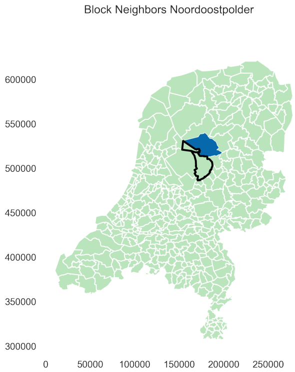


Figure: Block neighborhood based on province (Flevoland)

What neighborhood to choose from

Neighborhood should reflect how interaction happens for the question at hand.

What neighborhood to choose from

Neighborhood should reflect how interaction happens for the question at hand.

- **Contiguity weights:** Processes that propagate geographically from borders
- **Distance weights:** Accessibility
- **Block weights:** Effects of provincial laws

[AB17]

Spatial auto-regressive models

Regressive models over space

Problem: A regression model for predicting the value of a dependent variables (represented in a vector Y_n)

- **Regression model** (no temporal and spatial effect)
- **Auto-regressive models** (temporal effect)
- **Auto-regressive models** (spatial effect)
- Key factors to consider:
  - How the phenomenon diffuses in space? (spatial lag model)
  - Local and Global effect

Autoregressive models

$\rightarrow X_n$ and Y_n are vectors of independent and dependent variables of size n. ϕ, λ, ρ are model parameters. c is a constant. E represents the noise term. W_n is the spatial weights matrix

- **Regression**
  - $Y_n = c + \phi X_n + \epsilon_n$
- **Spatial Autoregressive model (SAR)**
  - $Y_n = c + \lambda W_n Y_n + \epsilon_n,$
  - $W_n Y_n$ is referred to as the spatial lag term in the models
  - How we use W_n determines global and local effect
- **Regression model with SAR disturbance**
  - $Y_n = c + \phi X_n + U_n,\ U_n = \rho W_n U_n + \epsilon_n,$
  - U_n Captures the effect of variables that we do not have in our data
- **Mixed regressive, spatial autoregressive model (MRSAR)**
  - $Y_n = c + \lambda W_n Y_n + \phi X_n + \epsilon_n,$

Lessons learned

- Spatial statistics versus classical statistics
  - Spatial correlation effect $\rightarrow$ many statistical indicators designed for non-spatial data are not valid for spatial data
- Ways to represent data (points versus polygons, continuous versus discrete)
  - **Geo-statistics**: locations are fixed and continuous, numbers are random values
  - **Point Processes**: location and numbers are both random
  - **Lattice Data**: locations are fixed and discrete, numbers are random aggregate values
- Spatial auto-correlation
- Neighborhoods and spatial weights for capturing the effects
  - Contiguity
  - Distance
  - Block

Lessons learned continued...

- Spatial auto-regressive models
  - **SAR:** value of neighboring points as predictive value
  - **SAR disturbance:** Noise on the neighboring values as predictive values
  - **MRSAR:** combination of independent predictive values and neighboring values as predictive values

End of theory!

References I

Dani Arribas-Bel, *Geographic data science'16*, 2017.

Noel Cressie and Christopher K Wikle, *Statistics for spatio-temporal data*, John Wiley & Sons, 2015.