70 Marcellán Fest

International Conference on Orthogonal Polynomials

Cádiz, April 21-23 2022

Book of Abstracts

uc3m | Universidad Carlos III de Madrid

UCAdatalab

Book of Abstracts

Invited Talks

Alexander Aptekarev - Keldysh Institute Moscow

Asymptotic analysis of matrix orthogonal polynomials

We consider a class of the selfadjoint discrete Schr odinger operators defined on an infinite homogeneous rooted graph-tree. The potential of this operator consists of the coefficients of the Nearest Neighbor Re-currence Relations (NNRRs) for the Multiple Orthogonal Polynomials (MOPs).
For the general class of potentials, generated by Angelesco MOPs we prove that the essential spectrum of these operators is a union of the supports of the components of the vector orthogonality measure $\sim \mu := (1, ..., d)$ for the Angelesco MOPs. It is a joint work with Sergey Denisov (Madison University) and Maxim Yattselev (IUPUI).

María Ángeles García-Ferrero - BCAM

The last explored territories of exceptional orthogonal polynomials

A new age in orthogonal polynomials began in 2009 with the discovery of exceptional families. Since then, systems of exceptional Hermite, Laguerre and Jacobi polynomials have been found and described. Nevertheless, there are still some territories on this map to be charted.
In this talk, we will introduce exceptional orthogonal polynomials with an arbitrary number of continuous parameters. We will see the methods to construct them from the classical polynomials. We will also talk about expeditions to be made in this direction.
This is based on joint works with D. Gómez-Ullate, R. Milson and J. Munday.

Walter van Assche - KU Leuven

Chebyshev polynomials in the 16-17th century

We give a few examples of Chebyshev polynomials that appeared in mathematical problems from the 16th and 17th century. The main example is the famous equation of Adrianus Romanus (Adriaan van Roomen) containing a polynomial of degree 45.

Alfredo Deaño - Universidad Carlos III de Madrid

Asymptotic analysis of matrix orthogonal polynomials

In this talk we consider matrix orthogonal polynomials (MVOPs) on a finite interval of the real line. We are particularly interested in the asymptotic behavior as the degree of the polynomials tends to infinity, in different regions of the complex plane, obtained with the method of steepest descent applied to the corresponding Riemann-Hilbert problem. We include examples with Jacobi and Gegenbauer weights, motivated by group theory. This is joint work with Arno Kuijlaars (KU Leuven, Belgium) and Pablo Román (Universidad de Córdoba, Argentina).

Ana Loureiro - University of Kent

On symmetric multiple orthogonal polynomials

At the centre of this talk are polynomials on a single variable satisfying higher order recurrence relations with all recurrence coefficients, except the last one, equal to zero. The polynomials at issue are orthogonal with respect to a vector of measures, are rotational invariant and all the zeros lie on a star in the complex plane. The focus will be on asymptotically periodic recurrence coefficients. The discussion will include ratio asymptotics as well as the zero limit distribution. The discussion will include the analysis of some examples that motivated and also illustrate the results.

Yuan Xu - University of Oregon

Orthogonal structure on conic domains

Spherical harmonics are OPs on the unit sphere and they enjoy two characteristic properties: (1) They are eigenfunctions of a second order PDE (the Laplace-Beltrami operator), (2) Their reproducing kernels satisfy a closed-form formula (the addition formula). Analogs of these properties also hold for classical OPs on the unit ball. The two properties play essential roles for analysis on the sphere and on the ball. In this talk, we explain our recent work on OPs on conic surfaces and solid cones, where several families of OPs are shown to possess these characteristic properties and they have been used to extend a substantial portion of analysis on spherical domains to conic domains.

Book of Abstracts

Contributed Talks

Amílcar Branquinho - Universidade de Coimbra

Random Walks interpretation via multiple orthogonal polynomials

Given a non-negative Hessenber matrix describing higher order recurrence relations, a general strategy for constructing a pair of stochastic matrices, dual to each other, is provided. The connection between the Markov chains describing by these two stochastic matrices is discussed at the light of the Poincaré's theorem on ratio asymptotics for homogeneous linear recurrence relations and the Christoffel-Darboux formula within the sequence of multiple orthogonal polynomials and type I linear forms. The Jacobi-Piñeiro multiple orthogonal polynomials are taken as a case study of the described results.

Jorge Arvesú - Universidad Carlos III de Madrid

On second order holonomic difference equations and rational approximants to $Z(3)$

Some second order holonomic difference equations are derived from a set of Hermite-Padé approximation problems near infinity along with some extra conditions. The involved vector polynomial solutions constitute the numerator and denominator sequences of rational approximants to $Z(3)$. The new constructed rational approximants reprove the irrationality of $Z(3)$. A comparison of these rational approximants and Apéry's approximants is given.

Ana Foulquié Moreno - Universidade de Aveiro

Oscillatory banded Hessenberg matrices, multiple orthogonal polynomials and random walks

The large knowledge on the spectral and factorization properties of oscillatory and totally nonnegative matrices leads to a spectral Favard theorem for the class of the so called regular oscillatory banded Hessenberg matrices, so that bidiagonal positive factorization holds, in terms of sequences of multiple orthogonal polynomials of types II and I with respect to a set of positive Lebesgue-Stieltjes measures.

The spectral Favard theorem is then applied to Markov chains with tetradiagonal transition matrices, i.e. beyond birth and death. In the finite case, the Karlin-McGregor spectral representation is given, it is shown that the random walks are recurrent and explicit expressions in terms of the orthogonal polynomials for the stationary distributions are given. Similar results are obtained for the countable infinite Markov chain. This is a joint work with Amílcar Branquinho, University of Coimbra and Manuel Mañas, University Complutense of Madrid.

Robert Milson - Dalhousie University

Ladder operators and rational extensions

We present the classification of ladder operators corresponding to the class of rational extensions of the harmonic oscillator. We show that it is natural to endow the class of rational extensions and the corresponding intertwining operators with the structure of a category REXT. The combinatorial data for this interpretation is realized as a functor MD -> REXT, where MD refers to the set of Maya diagrams appropriately endowed with categorical structure. Our formalism allows us to easily reproduce and extend earlier results on ladder operators.

Antonio J. Durán - Universidad de Sevilla

Exceptional Hahn and Jacobi polynomials with an arbitrary number of continuous parameters

We construct new examples of exceptional Hahn and Jacobi polynomials. Exceptional polynomials are orthogonal polynomials with respect to a measure which are also eigenfunctions of a second order difference or differential operator. In mathematical physics, they allow the explicit computation of bound states of rational extensions of classical quantum-mechanical potentials. The most apparent difference between classical or classical discrete orthogonal polynomials and their exceptional counterparts is that the exceptional families have gaps in their degrees, in the sense that not all degrees are present in the sequence of polynomials. The new examples have the novelty that they depend on an arbitrary number of continuous parameters. These families are constructed by using new families of Krall dual Hahn polynomials depending on an arbitrary number of continuous parameters. Krall polynomials are orthogonal polynomials which are eigenfunctions of a higher order differential or difference operator. The new Krall dual Hahn families provide further examples for the problem explicitly posed by Richard Askey in 1991.

Helder Lima - University of Kent

Multiple orthogonal polynomials and branched continued fractions

The central theme of this talk is the recently found connection between two completely different corners of mathematics: multiple orthogonal polynomials and branched continued fractions introduced to solve total-positivity problems arising from combinatorics. Firstly, we give an overview of the connection between these two topics. Then, we give further evidence of this connection via the case study of the link between branched-continued-fraction representations for ratios of contiguous hypergeometric series and multiple orthogonal polynomials with respect to measures (or linear functionals) whose moments are ratios of products of Pochhammer symbols. Specialisations of these multiple orthogonal polynomials include the classical Laguerre, Jacobi, and Bessel orthogonal polynomials, multiple orthogonal polynomials with respect to two measures involving the Macdonald function, confluent hypergeometric functions, or Gauss' hypergeometric function $_2F_1$, and multiple orthogonal polynomials with respect to Meijer G-functions used to investigate the singular values of products of Ginibre random matrices. This is joint work with Alan Sokal.

Misael Enrique Marriaga Castillo - Universidad Rey Juan Carlos

A class of Bernstein-type operators on the unit ball

The purpose of this work is to study extensions of the Bernstein polynomials to the bivariate case. To this end, we analyse the bivariate Bernstein-Stancu operators, we study Bernstein-type operators by means of continuously differentiable transformations of the function, and we introduce Bernstein-type operators on disk quadrants by means of domain transformations. We state convergence results for continuous functions. Numerical examples are given, comparing approximations using the Bernstein-Stancu and the Bernstein-type operator on the unit disk.

Book of Abstracts

Henrik Laurberg Pedersen - University of Copenhagen

Generalized Bernstein functions

We introduce and investigate the class of generalized Bernstein functions. The fundamental properties of this class are given including its relation to generalized Stieltjes functions. The subclass of generalized Thorin-Bernstein functions is characterized in different ways. Examples include incomplete gamma functions, Lerch's trancendent and some hypergeometric functions.
This is based on joint work with Stamatis Koumandos.

Ester Pérez Sinusía - Universidad de Zaragoza

A general theory of uniform convergent expansions of special functions in terms of elementary functions

Series expansions of special functions with respect to different systems of functions are interesting representations from an analytical and numerical point of view. Usually, existing expansions for these functions are not simultaneously valid for small and large values of the variables. In this work, we face the problem of designing a general theory of uniform convergent expansions of special functions in terms of elementary functions valid in a large region of the complex plane that includes small and large values of the variables. Error bounds and numerical experiments showing the accuracy of the approximations are given and its application to important special functions.

Grzegorz Swiderski - KU Leuven

Christoffel functions for multiple orthogonal polynomials

We study weak convergence of the Christoffel-Darboux kernel on the diagonal corresponding to a vector of positive measures. In the classical case of one compactly supported measure it is well-known that the weak limit of the Christoffel-Darboux kernel on the diagonal and of the normalized zero counting measure are the same. Under some mild conditions we shall prove an extension of this statement to the general vector case. In the proof we study the Hessenberg matrix corresponding to multiple orthogonal polynomials along any ray of multi-indices tending to infinity.
This is a joint work with Walter Van Assche (KU Leuven).

Ignacio Zurrian - Universidad de Sevilla

Double bispectrality and special functions

In this talk, we will discuss some generalities of the time- and band-limiting problem (in a possibly matrix-valued setup) as well as particular cases of special functions taken from representation theory related to symmetric spaces.

Arno Kuijlaars - KU Leuven

Matrix orthogonality versus scalar orthogonality on a Riemann surface

Matrix valued orthogonal polynomials play a role in random tiling models with periodic weightings [1]. The main goal of the talk will be to show how the matrix orthogonality can be transformed into orthogonality for meromorphic functions on a Riemann surface. The higher genus cases are of particular interest since these are believed to correspond to random tiling models with three different phases in the large size limit.
[1] M. Duits and A.B.J. Kuijlaars, The two-periodic Aztec diamond and matrix valued orthogonal polynomials, J. Eur. Math. Soc. 23 (2021), 1075–1131.

Pablo Palacios Herrero - Universidad Pública de Navarra

A systematic version of the uniform asymptotic method "saddle point near an end point

The main technical difficulties in the application of the uniform asymptotic method for integrals denominated "saddle point near an end point" are originated by a change of variables. In this work, we present a variant of the method that avoids that change of variables and simplifies the computations. On the one hand, as in the standard method, the asymptotic sequence is given in terms of parabolic cylinder functions. On the other hand, the calculation of the coefficients is simpler and systematic. A new asymptotic expansion of the confluent hypergeometric function is given as an illustration.

Miguel A. Piñar González - Universidad de Granada

Generalized classical orthogonal polynomials

Generalized classical orthogonal polynomials are multivariable polynomials that are orthogonal with respect to the weight functions $\mathrm{B}_\gamma(\mathrm{x}) = \prod_{i=1}^d \omega(x_i) \prod_{i<j} |x_i - x_j|^{2\gamma+1}$, whith $\omega(t)$ being one of the classical weight functions (Hermite, Laguerre, or Jacobi) on the real line. They occur as the polynomial part of the eigenfunctions of certain Schr¨odinger operators for Calogero-Sutherland-type quantum systems. Applying the change of variables x_i, $i = 1, 2, \ldots, d$, into u_r, $r = 1, 2, \ldots, d$, where u_r is the r-th elementary symmetric function, we show that, in terms of the variables u_r, the respective Hermite, Laguerre and Jacobi generalized multivariate orthogonal polynomials are the eigenfunctions of a second order partial differential operator with polynomial coefficients.

Maria das Neves Rebocho - Universidade da Beira Interior

On symmetric semi-classical orthogonal polynomials and some of their extensions

This talk concerns the so-called direct problem for orthogonal polynomials. We take the sequences of orthogonal polynomials whose Stieltjes function, S, satisfies a Riccati type differential equation with polynomial coefficients, AS'=BS$^2 + CS + D$, $\quad A \neq 0$, and we seek formulae for the three-term recurrence relation coefficients of the orthogonal polynomials, given A, B, C, D. The symmetric case under the restriction $\max\{\deg(C) - 1, \max\{\deg(A), \deg(B)\} - 2\} = 2$ will be described in detail: we deduce non-linear difference equations for the recurrence relation coefficients, some of them identified as discrete Painlevé equations.

Book of Abstracts

Posters

Carmen Escribano - Universidad Politécnica de Madrid

Some applications of generalized eigenvalues to orthogonal polynomials

In this work we use the notion of generalized eigenvalues for infinite hermitian matrices, and in particular, for moment matrices associated to measures with support in the complex plane to obtain some properties of such measures and the orthogonal polynomials asociated with them. With this matrix tools we study some applications related the support of a measure in comparison with another. With these techniques , we obtain some results concerning the convex support of a measure. Moreover, we deal with some problems of polynomial approximation. Finally, we give some applications of localization of the zero set of Sobolev orthogonal polynomials.

Javier Jiménez de la Jara - Universidad de Cádiz

Document Classification and Segmentation combining image and text processing models

In recent years, all documents normally stored in folders and drawers are being digitalised. These must be processed and studied automatically in order to take advantage of current technological tools. In this project, a tool capable of classifying and segmenting each page of a scanned document will be developed. Every single page of 1000 PDF files has been annotated and classified into 60 categories. Our system extracts the text and image from each of them, which will serve as input for a model that combines CNNs and Transformers, to finally separate each document into different categories.

David Lara Veslaco - Universidad de Granada

On Bernstein-type operators preserving derivatives

As it is well known, Bernstein polynomials were introduced by S. Bernstein in 1912 to provide a constructive proof of the Weierstrass approximation theorem, establishing that every continuous function defined on [0, 1] can be uniformly approximated by Bernstein polynomials.

In this work we study Bernstein-type operators that preserves the derivatives in the sense that the operator applied to the derivative of a function can be expressed as the derivative of the operator applied to the original function.

Almudena del Pilar Márquez - Universidad de Cádiz

Conservation laws and Painlevé analysis for a generalized seventh-order KdV equation

In this work, we study a generalized seventh-order KdV equation depending on seven arbitrary nonzero parameters. A complete classification of the low-order local conservation laws is given by using the multiplier method. We also apply the Lie symmetry method to transform the original partial differential equation into an ordinary differential equation. Next, we analyse if the equation satisfies the Painlevé property. Finally, we give some conclusions about the results obtained.

Juan Enrique Fernández Díaz - Universidade de Aveiro

Multiple Orthogonal Polynomials and Random Walks

The study of birth and death processes is linked to orthogonal polynomials theory since the research of Karlin and McGregor. This relation arises from the connection between the stochastic matrices describing such processes and tridiagonal Jacobi matrices describing the recurrence relations of orthogonal polynomials.

The aim here is to generalize this relation to random walks beyond birth and death chains, that is, with probability of transition beyond first neighbours. This can be done using multiple orthogonal polynomials, which are a generalization of the usual ones.

Chelo Ferreira - Universidad de Zaragoza

A convergent version of Watson Lemma

Watson's Lemma provides an asymptotic expansion of Laplace transforms for large values of the transformation parameter z. It is a useful tool in the asymptotic approximation of special functions that have an integral representation in the form of the Laplace transform of a certain function $f(t)$. But in most of the important examples of special functions, the asymptotic expansion derived by means of Watson's Lemma is not convergent. We investigate a modification of Watson's Lemma that transforms the unbounded integration interval $[0,\infty)$ of the Laplace transform into the bounded interval $(0,1]$. Then, we derive an asymptotic expansion of the transformed integral for large z that it is convergent under a mild condition over the function $f(t)$. Moreover, using the same technique we obtain asymptotic expansions of the two-dimensional Laplace transforms of $f(t,s)$ for large values of the transformation parameters x and y that are also convergent. The expansions are accompanied by error bounds. Some examples of special functions are given as illustration, deriving new convergent and asymptotic expansions of these functions.

Juan F. Mañas Mañas - Universidad de Almería

Symbolic computation of Mehler–Heine formulae for Sobolev–Type orthogonal polynomials

The theory about nonstandard polynomials has been developed along the last 40 years. In this context, we consider Sobolev–Type orthogonal polynomials, which are orthogonal with respect to an inner product involving derivatives. Local asymptotics of these polynomials can be described by the Mehler–Heine formulae, which connect the polynomials with the Bessel functions of the first kind. Recently, the formulae have been computed for Sobolev–Type orthogonal polynomials in several particular cases. We improve various known results by unifying them and, using the software Mathematica, we present a computer program that automatically obtains the corresponding Mehler–Heine formulae. This is a joint work with Juan J. Moreno–Balcázar.