Supplementary Materials for "Impedance Control of Co-robot on Biased Sliding Surface with Jerk Adaptation"

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I. LINEAR-IN-PARAMETER DYNAMICS MODEL

The linear-in-parameter (LIP) dynamics model for co-robot under the experimental setting, is given as follows,

$$\boldsymbol{u}_{\text{ff}} = \boldsymbol{Y}\hat{\boldsymbol{a}}, \quad \boldsymbol{Y} \in \mathbb{R}^{3\times 12}, \quad \hat{\boldsymbol{a}} \in \mathbb{R}^{12} \tag{S1}$$

where the entries of $\boldsymbol{Y}$ are given as

$$Y_{11} = \ddot{q}_2, \quad Y_{12} = \ddot{q}_4, \quad Y_{13} = \ddot{q}_6, \quad Y_{16} = \sin(q_2)$$

$$\begin{aligned} Y_{14} = &-(\dot{q}_4^2 + \dot{q}_6^2 + 2\dot{q}_2\dot{q}_4 + 2\dot{q}_2\dot{q}_6 + \dot{q}_4\dot{q}_6)\sin(q_4+q_6)/2 \\ &-(\ell_4+\ell_6)(\dot{q}_6^2 + 2\dot{q}_2\dot{q}_6 + 2\dot{q}_4\dot{q}_6)\sin(q_6)/(2\ell_2) \\ &+(\ell_4+\ell_6)(2\ddot{q}_2 + 2\ddot{q}_4 + \ddot{q}_6)\cos(q_6)/2\ell_2 \\ &+(2\ddot{q}_2 + \ddot{q}_4 + \ddot{q}_6)\cos(q_4+q_6)/2 \\ &-9.81\sin(q_2+q_4+q_6)/(2\ell_2) \end{aligned}$$

$$\begin{aligned} Y_{15} = &(2\ddot{q}_2 + \ddot{q}_4)\cos(q_4)/2 - (\dot{q}_4^2 + 2\dot{q}_2\dot{q}_4)\sin(q_4)/2 \\ &-9.81\sin(q_2+q_4)/(2\ell_2) \end{aligned}$$

$$Y_{1,9} = Y_{1,10} = Y_{1,11} = Y_{1,12} = 0, \ \ Y_{1,7} = \text{sgn}(\dot{q}_2), \ \ Y_{1,8} = \dot{q}_2$$

$$Y_{2,1} = Y_{2,6} = Y_{2,7} = Y_{2,8} = Y_{2,11} = Y_{2,12} = 0,$$

$$Y_{2,2} = \ddot{q}_2 + \ddot{q}_4, \quad Y_{2,3} = \ddot{q}_6, \quad Y_{2,9} = \text{sgn}(\dot{q}_4), \quad Y_{2,10} = \dot{q}_4$$

$$\begin{aligned} Y_{2,4} = &-(\ell_4+\ell_6)(2\dot{q}_2\dot{q}_6 + 2\dot{q}_4\dot{q}_6 + \dot{q}_6^2)\sin(q_6)/(2\ell_2) \\ &+(\dot{q}_2^2\sin(q_4+q_6))/2 + (\ddot{q}_2\cos(q_4+q_6))/2 \\ &+(\ell_4+\ell_6)(2\ddot{q}_2 + 2\ddot{q}_4 + \ddot{q}_6)\cos(q_6)/(2\ell_2) \\ &-9.81\sin(q_2+q_4+q_6)/(2\ell_2) \end{aligned}$$

$$Y_{2,5} = q_2^2\sin(q_4)/2 + \ddot{q}_2\cos(q_4)/2 - 9.81\sin(q_2+q_4)/(2\ell_2)$$

$$Y_{3,1} = Y_{3,2} = Y_{3,5} = Y_{3,6} = Y_{3,7} = Y_{3,8} = Y_{3,9} = Y_{3,10} = 0$$

$$Y_{3,3} = \ddot{q}_2 + \ddot{q}_4 + \ddot{q}_6, \quad Y_{3,11} = \text{sgn}(\dot{q}_6), \quad Y_{3,12} = \dot{q}_6$$

$$\begin{aligned} Y_{3,4} = &(\ddot{q}_2\cos(q_4+q_6))/2 + (\dot{q}_2^2\sin(q_4+q_6))/2 \\ &+(\ell_4+\ell_6)(\dot{q}_2^2 + \dot{q}_4^2 + 2\dot{q}_2\dot{q}_4)\sin(q_6)/(2\ell_2) \\ &+(\ell_4+\ell_6)(\ddot{q}_2 + \ddot{q}_4)\cos(q_6)/(2\ell_2) \\ &-9.81\sin(q_2+q_2+q_6)/(2\ell_2) \end{aligned}$$

TABLE S1
PARAMETERS OF LIP DYNAMIC MODEL (UNITS IN SI)

$\ell_2 = 0.35$	$\ell_4 = 0.35$	$\ell_6 = 0.28$	$\hat{a}_1 = 0.27$	$\hat{a}_2 = 0.33$
$\hat{a}_3 = -0.65$	$\hat{a}_4 = 27$	$\hat{a}_5 = -1.58$	$\hat{a}_6 = -3.75$	$\hat{a}_7 = 8.08$
$\hat{a}_8 = 2.66$	$\hat{a}_9 = 4.24$	$\hat{a}_{10} = 1.43$	$\hat{a}_{11} = -2.46$	$\hat{a}_{12} = -0.98$

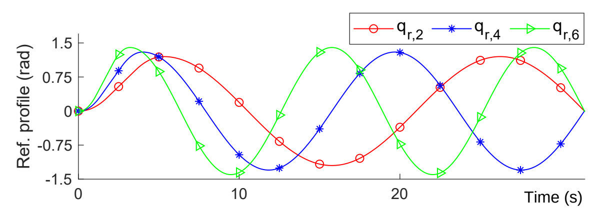

Fig. S1. Reference profile of computing torque by LIP model.

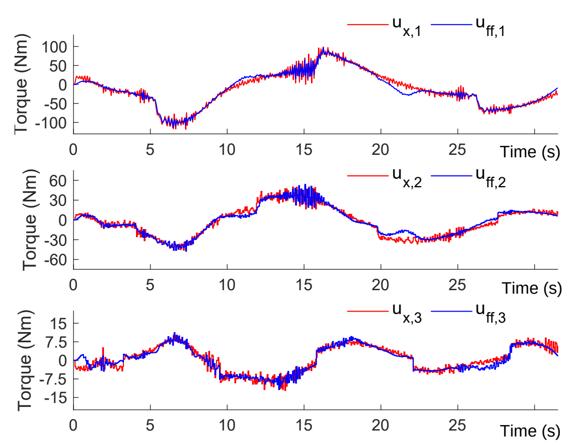

Fig. S2. Total control torque and computing torque by LIP dynamics model.

To achieve computing torque for feedforward control, the estimated parameters of LIP model are given in Table S1. To further show the accuracy of the computing torque by the LIP model, the reference profiles to compute the LIP model are given in Fig. S1, while the comparison between total control torque and computing torque by the LIP model are displayed in Fig. S2 which shows high accuracy of the LIP model.