Var	w=Sqrt[1+ε^2-2*ε*Cos[θ]];
	s=Sin[θ]; c=Cos[θ];
	g[ε_,θ_]:=$-1+\frac{1}{w}$;
		Pn2=-2Cot[θ]dpn-n(n+1)pn;
		opt={Direction→"FromAbove",Assumptions→{0<θ≤π}};

ID	*Expression*	*Wolfram code*
1	$\sum_{n=1}^{\infty} \varepsilon^{-1+n}$ P[n, 0, c]	p1 = $\frac{g[\varepsilon,\theta]}{\varepsilon}$
2	$\sum_{n=1}^{\infty} n\, \varepsilon^{-1+n}$ P[n, 0, c]	p2 = $\partial_\varepsilon (\varepsilon\, p1)$
3	$\sum_{n=1}^{\infty} n^2\, \varepsilon^{-1+n}$ P[n, 0, c]	p3 = $\partial_\varepsilon (\varepsilon\, p2)$
4	$\sum_{n=1}^{\infty} n^3\, \varepsilon^{-1+n}$ P[n, 0, c]	p4 = $\partial_\varepsilon (\varepsilon\, p3)$
5	$\sum_{n=1}^{\infty} \frac{\varepsilon^{-1+n} P[n,0,c]}{n}$	p5x = $\int p1\, d\varepsilon$; p5 = $\frac{p5x - \lim_{\varepsilon\to 0} p5x}{\varepsilon}$
6	$\sum_{n=2}^{\infty} \frac{\varepsilon^{-1+n} P[n,0,c]}{-1+n}$	p6x = $\int \frac{p1-lf1}{\varepsilon}\, d\varepsilon$; p6 = p6x − $\lim_{\varepsilon\to 0}$ p6x
7	$\sum_{n=1}^{\infty} \frac{\varepsilon^{-1+n} P[n,0,c]}{1+n}$	p7x = $\int \varepsilon\, p1\, d\varepsilon$; p7 = $\frac{p7x - \lim_{\varepsilon\to 0} p7x}{\varepsilon^2}$
8	$\sum_{n=1}^{\infty} n^2\, \varepsilon^{-1+n}$ P[n, 1, c]	p8 = $-\partial_\theta$ p3
9	$\sum_{n=1}^{\infty} n\, \varepsilon^{-1+n}$ P[n, 1, c]	p9 = $-\partial_\theta$ p2
10	$\sum_{n=1}^{\infty} \varepsilon^{-1+n}$ P[n, 1, c]	p10 = $-\partial_\theta$ p1
11	$\sum_{n=1}^{\infty} \frac{\varepsilon^{-1+n} P[n,1,c]}{n}$	p11 = $-\partial_\theta$ p5
12	$\sum_{n=1}^{\infty} \frac{\varepsilon^{-1+n} P[n,1,c]}{1+n}$	p12 = $-\partial_\theta$ p7
13	$\sum_{n=1}^{\infty} \frac{\varepsilon^{-1+n} P[n,1,c]}{n^2}$	p13x = $\int p11\, d\varepsilon$; p13 = $\frac{p13x - \lim_{\varepsilon\to 0} p13x}{\varepsilon}$
14	$\sum_{n=2}^{\infty} n\, \varepsilon^{-1+n}$ P[n, 2, c]	p14f = Collect[n ε^{n-1} Pn2, {pn, dpn}]; p14 = −(p4 + p3) − 2 Cot[θ] ∂_θ p2
15	$\sum_{n=2}^{\infty} \varepsilon^{-1+n}$ P[n, 2, c]	p15f = Collect[ε^{n-1} Pn2, {pn, dpn}]; p15 = −(p3 + p2) − 2 Cot[θ] ∂_θ p1
16	$\sum_{n=2}^{\infty} \frac{\varepsilon^{-1+n} P[n,2,c]}{n}$	p16f = Collect[$\frac{\varepsilon^{n-1} Pn2}{n}$, {pn, dpn}]; p16 = −(p2 + p1) − 2 Cot[θ] ∂_θ p5
17	$\sum_{n=2}^{\infty} \frac{\varepsilon^{-1+n} P[n,2,c]}{-1+n}$	p17x = $\int \frac{p15}{\varepsilon}\, d\varepsilon$; p17 = p17x − $\lim_{\varepsilon\to 0}$ p17x
18	$\sum_{n=2}^{\infty} \frac{\varepsilon^{-1+n} P[n,2,c]}{1+n}$	p18x = $\int p15\, \varepsilon\, d\varepsilon$; p18 = $\frac{p18x - \lim_{\varepsilon\to 0} p18x}{\varepsilon^2}$
19	$\sum_{n=2}^{\infty} \frac{\varepsilon^{-1+n} P[n,2,c]}{n^2}$	p19x = $\int p16\, d\varepsilon$; p19 = $\frac{p19x - \lim_{\varepsilon\to 0} p19x}{\varepsilon}$