RQ4

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1 When all predictors are considered together, which variables contribute the most to predicting affordability?

1.1 Approach

RQ4 asks which variables contribute most when predictors are considered jointly. We evaluate two feature sets:

- **Full:** `median_income + cost_yr + {region_name, geotype, race_eth_name}`
- **NoCost:** `median_income + {region_name, geotype, race_eth_name}`

We fit several model families using a fixed train/test split and select the best-performing model within each feature set. To quantify each variable's contribution, we use **permutation importance** on the held-out test set.

Note: If `cost_yr` is mechanically related to how `affordability_ratio` is constructed, it may dominate importance and inflate performance in the Full setting. The NoCost setting helps isolate how much predictive signal remains in income + contextual variables alone.

```
import numpy as np
import pandas as pd
from pathlib import Path
import matplotlib.pyplot as plt

from sklearn.model_selection import train_test_split
from sklearn.pipeline import Pipeline
from sklearn.dummy import DummyRegressor
from sklearn.linear_model import LinearRegression
from sklearn.ensemble import RandomForestRegressor, HistGradientBoostingRegressor
```

```
from sklearn.metrics import mean_absolute_error, r2_score
from sklearn.inspection import permutation_importance
from utils.model_utils import rmse, make_ohe, split_cols, make_preprocessor, wrap_log1p, eva

RANDOM_STATE = 159
np.random.seed(RANDOM_STATE)

DATA_PATH = Path("./data/food_affordability.csv")
OUT_DIR = Path("./outputs"); OUT_DIR.mkdir(exist_ok=True)
FIG_DIR = Path("./figures"); FIG_DIR.mkdir(exist_ok=True)

TARGET = "affordability_ratio"
```

```
df = pd.read_csv(DATA_PATH)

# Define feature sets
CONTEXT_FEATURES = ["region_name", "geotype", "race_eth_name"]
FULL_FEATURES    = ["median_income", "cost_yr"] + CONTEXT_FEATURES
NOCOST_FEATURES  = ["median_income"] + CONTEXT_FEATURES

ALL_USED = sorted(set([TARGET] + FULL_FEATURES + NOCOST_FEATURES))
df_m = df.dropna(subset=ALL_USED).copy()

print("Rows kept:", df_m.shape[0])
df_m[ALL_USED].head()
```

```
Rows kept: 3473
```

	affordability_ratio	cost_yr	geotype	median_income	race_eth_name	region_name
0	0.315779	7508.289655	CA	23777.0	AIAN	California
1	0.194980	7508.289655	CA	38508.0	Asian	California
2	0.286664	7508.289655	CA	26192.0	AfricanAm	California
3	0.328475	7508.289655	CA	22858.0	Latino	California
4	0.204379	7508.289655	CA	36737.0	NHOPI	California

```
X_all = df_m[sorted(set(FULL_FEATURES + NOCOST_FEATURES))].copy()
y = df_m[TARGET].copy()

idx = np.arange(len(df_m))
idx_train, idx_test = train_test_split(idx, test_size=0.2, random_state=RANDOM_STATE)

X_train_all, X_test_all = X_all.iloc[idx_train], X_all.iloc[idx_test]
y_train, y_test = y.iloc[idx_train], y.iloc[idx_test]

X_train_all.shape, X_test_all.shape
```

```
((2778, 5), (695, 5))

feature_sets = {
    "RQ4 -Full (income+cost+context)": FULL_FEATURES,
    "RQ4 -NoCost (income+context)": NOCOST_FEATURES
}

rows = []
best_models = {}
all_preds = {}

for fs_name, feats in feature_sets.items():
    Xtr = X_train_all[feats]
    Xte = X_test_all[feats]

    # Baseline
    dummy = DummyRegressor(strategy="mean")
    dummy.fit(Xtr, y_train)
    pred_dummy = dummy.predict(Xte)
    rows.append({
        "feature_set": fs_name,
        "model": "Dummy(mean)",
        "RMSE": rmse(y_test, pred_dummy),
        "MAE": float(mean_absolute_error(y_test, pred_dummy)),
        "R2":  float(r2_score(y_test, pred_dummy))
    })
    all_preds[(fs_name, "Dummy(mean)")] = pred_dummy

    # Linear (good baseline)
    pre_lin = make_preprocessor(feats, X_train_all, dense=False, scale_num_for_linear=True)
    lin = wrap_log1p(Pipeline([("pre", pre_lin), ("lin", LinearRegression())]))
    m_lin, pred_lin = eval_model_rq4(lin, Xtr, Xte, y_train, y_test)
    rows.append({"feature_set": fs_name, "model": "Linear(log1p y)", **m_lin})
    all_preds[(fs_name, "Linear(log1p y)")] = pred_lin

    # Random Forest
    pre_tree = make_preprocessor(feats, X_train_all, dense=False, scale_num_for_linear=False
    rf = RandomForestRegressor(
        n_estimators=300,
        random_state=RANDOM_STATE,
        n_jobs= -1,
        min_samples_leaf=2
    )
    rf_m = wrap_log1p(Pipeline([("pre", pre_tree), ("rf", rf)]))
    m_rf, pred_rf = eval_model_rq4(rf_m, Xtr, Xte, y_train, y_test)
    rows.append({"feature_set": fs_name, "model": "RandomForest(log1p y)", **m_rf})
```

```
    all_preds[(fs_name, "RandomForest(log1p y)")] = pred_rf

    pre_hgb = make_preprocessor(feats, X_train_all, dense=True, scale_num_for_linear=False)
    hgb = HistGradientBoostingRegressor(
        random_state=RANDOM_STATE,
        max_depth=6,
        learning_rate=0.05,
        max_iter=400
    )
    hgb_m = wrap_log1p(Pipeline([("pre", pre_hgb), ("hgb", hgb)]))
    m_hgb, pred_hgb = eval_model_rq4(hgb_m, Xtr, Xte, y_train, y_test)
    rows.append({"feature_set": fs_name, "model": "HistGB(log1p y)", **m_hgb})
    all_preds[(fs_name, "HistGB(log1p y)")] = pred_hgb

metrics_rq4 = pd.DataFrame(rows).sort_values(["feature_set", "RMSE"])
metrics_rq4
```

	feature_set	model	RMSE	MAE	R2
2	RQ4-Full (income+cost+context)	RandomForest(log1p y)	0.023730	0.006445	0.996411
3	RQ4-Full (income+cost+context)	HistGB(log1p y)	0.027553	0.009115	0.995162
1	RQ4-Full (income+cost+context)	Linear(log1p y)	0.340012	0.145797	0.263215
0	RQ4-Full (income+cost+context)	Dummy(mean)	0.396309	0.219434	-0.000969
7	RQ4-NoCost (income+context)	HistGB(log1p y)	0.084735	0.037181	0.954241
6	RQ4-NoCost (income+context)	RandomForest(log1p y)	0.094195	0.039957	0.943453
5	RQ4-NoCost (income+context)	Linear(log1p y)	0.337358	0.145718	0.274672
4	RQ4-NoCost (income+context)	Dummy(mean)	0.396309	0.219434	-0.000969

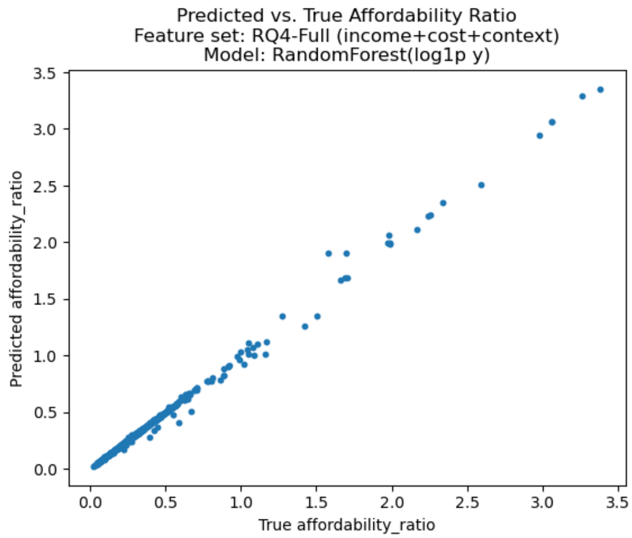
Tree-based models strongly outperform the linear baseline. With the Full feature set, Random Forest achieves near-perfect performance ($R^2 \approx 0.996$; RMSE ≈ 0.024), while without `cost_yr`, the best model (HistGradientBoosting) still performs very well ($R^2 \approx 0.954$; RMSE ≈ 0.085). The linear model remains around $R^2 \approx 0.26$–0.27, indicating that nonlinear models capture structure that linear regression does not.

Note: Because `cost_yr` may be mechanically related to `affordability_ratio` (depending on how the affordability ratio was constructed), the extremely high R^2 in the Full setting should be interpreted cautiously as potentially reflecting a definitional/structural relationship rather than purely "discovering" new predictive patterns.

```
# Save metrics
metrics_rq4.to_csv(OUT_DIR / "rq4_model_metrics.csv", index=False)

def pick_best(df_metrics, fs_name):
```

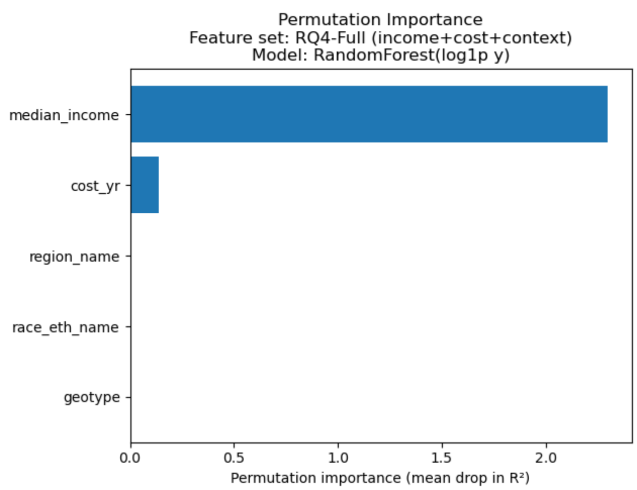
```
    sub = df_metrics[df_metrics["feature_set"] == fs_name].copy()
    sub = sub.sort_values("RMSE")
    return sub.iloc[0]["model"]

for fs_name, feats in feature_sets.items():
    best_model_name = pick_best(metrics_rq4, fs_name)
    preds = all_preds[(fs_name, best_model_name)]

    # Rebuild and refit the best model (so we can run permutation importance cleanly)
    Xtr = X_train_all[feats]
    Xte = X_test_all[feats]

    if best_model_name.startswith("Linear"):
        pre = make_preprocessor(feats, X_train_all, dense=False, scale_num_for_linear=True)
        best = wrap_log1p(Pipeline([("pre", pre), ("lin", LinearRegression())]))
    elif best_model_name.startswith("RandomForest"):
        pre = make_preprocessor(feats, X_train_all, dense=False, scale_num_for_linear=False)
        best = wrap_log1p(Pipeline([("pre", pre), ("rf", RandomForestRegressor(
            n_estimators=300, random_state=RANDOM_STATE, n_jobs= -1, min_samples_leaf=2
        ))]))
    elif best_model_name.startswith("HistGB"):
        pre = make_preprocessor(feats, X_train_all, dense=True, scale_num_for_linear=False)
        best = wrap_log1p(Pipeline([("pre", pre), ("hgb", HistGradientBoostingRegressor(
            random_state=RANDOM_STATE, max_depth=6, learning_rate=0.05, max_iter=400
        ))]))
    else:
        continue

    best.fit(Xtr, y_train)

    # Pred vs True
    plt.figure()
    plt.scatter(y_test, preds, s=10)
    plt.xlabel("True affordability_ratio")
    plt.ylabel("Predicted affordability_ratio")
    plt.title(
    f"Predicted vs. True Affordability Ratio\n"
    f"Feature set: {fs_name}\n"
    f"Model: {best_model_name}"
)

    fig_path = FIG_DIR / f"rq4_pred_vs_true__{fs_name.split()[0].lower()}__{best_model_name.
    plt.savefig(fig_path, dpi=150, bbox_inches="tight")
    plt.show()

    # Permutation importance on RAW features (region/geotype/race/income/cost)
```

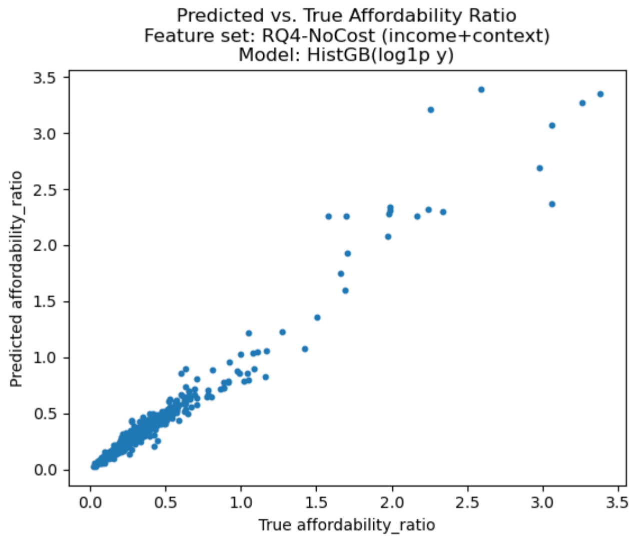
```
imp = permutation_importance(
    best,
    Xte,
    y_test,
    n_repeats=20,
    random_state=RANDOM_STATE,
    n_jobs= -1,
    scoring="r2"
)

imp_df = pd.DataFrame({
    "feature": Xte.columns,
    "importance_mean": imp.importances_mean,
    "importance_std": imp.importances_std,
}).sort_values("importance_mean", ascending=False)

print("\n=== Permutation importance:", fs_name, "|", best_model_name, "===\n")
display(imp_df)

# Save + plot
out_csv = OUT_DIR / f"rq4_perm_importance__{fs_name.split()[0].lower()}__{best_model_nam
imp_df.to_csv(out_csv, index=False)

plt.figure()
plt.barh(imp_df["feature"][:: -1], imp_df["importance_mean"][:: -1])
plt.xlabel("Permutation importance (mean drop in R²)")
plt.title(
f"Permutation Importance\n"
f"Feature set: {fs_name}\n"
f"Model: {best_model_name}"
)
out_fig = FIG_DIR / f"rq4_perm_importance__{fs_name.split()[0].lower()}__{best_model_nam
plt.savefig(out_fig, dpi=150, bbox_inches="tight")
plt.show()
```




```
=== Permutation importance: RQ4 -Full (income+cost+context) | RandomForest(log1p y) ===
```

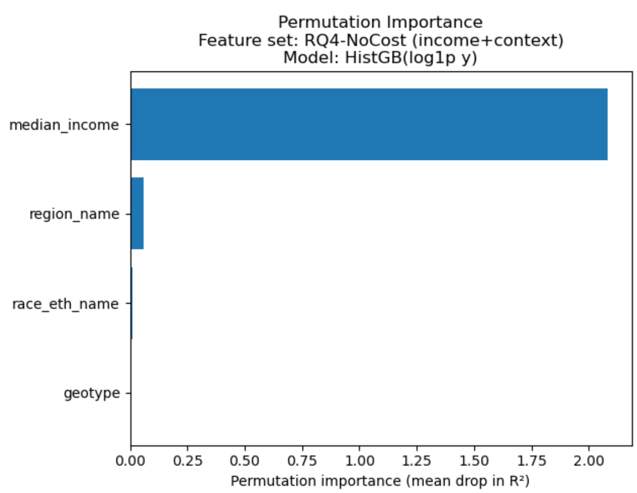
	feature	importance_mean	importance_std
0	median_income	2.297895	0.130425
1	cost_yr	0.135856	0.024027
2	region_name	0.000696	0.000430
4	race_eth_name	0.000067	0.000039
3	geotype	0.000043	0.000042






```
=== Permutation importance: RQ4 -NoCost (income+context) | HistGB(log1p y) ===
```

	feature	importance_mean	importance_std
0	median_income	2.081926	0.093943
1	region_name	0.060948	0.019441
3	race_eth_name	0.008749	0.006657
2	geotype	0.000395	0.000102



Permutation importance (measured as mean drop in test R^2 when a feature is permuted) shows that **median_income dominates** in both settings.

- In the **Full** model, `median_income` is by far the largest driver, with `cost_yr` as a distant second; `region_name`, `race_eth_name`, and `geotype` contribute negligibly.
- In the **NoCost** model, `median_income` remains dominant; `region_name` becomes the next most informative feature, while `race_eth_name` is small and `geotype` is near zero.

Overall, when all predictors are considered together, **income is the primary driver**, and **cost (when included) adds additional predictive power**, while geographic/race context variables contribute comparatively little in this dataset/modeling setup.