Homework 3: Lab Nov 20

2) Stationary Distribution

a) $\pi_\infty^T P = \pi_\infty^T$

$P^T \pi_\infty = \pi_\infty$

$$P = \begin{bmatrix} 0.2 & 0.7 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.4 & 0.4 \end{bmatrix}$$

$(P^T - I)\pi_\infty = 0$

$$\begin{bmatrix} -0.8 & 0.2 & 0.2 \\ 0.7 & -0.5 & 0.4 \\ 0.1 & 0.3 & -0.6 \end{bmatrix} \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-0.8\pi_1 + 0.2\pi_2 + 0.2\pi_3 = 0$ — (1)

$0.7\pi_1 - 0.5\pi_2 + 0.4\pi_3 = 0$ — (2)

$0.1\pi_1 + 0.3\pi_2 - 0.6\pi_3 = 0$ — (3)

$\pi_1 + \pi_2 + \pi_3 = 1$ — (4)

(1) & (4) $\quad -0.8 + 0.8\pi_2 + 0.8\pi_3 + 0.2\pi_2 + 0.2\pi_3 = 0$

$\pi_2 + \pi_3 = 0.8$ — (5)

(3) & (4) $\quad 0.1 - 0.1\pi_2 - 0.1\pi_3 + 0.3\pi_2 - 0.6\pi_3 = 0$

$0.2\pi_2 - 0.7\pi_3 = -0.1$ — (6)

(5) $\times 0.7 +$ (6) $\quad 0.9\pi_2 = 0.56 - 0.1$

$\pi_2 = \frac{46}{90} = \frac{23}{45}$

(5): $\quad \pi_3 = 0.8 - \frac{23}{45} = \frac{36-23}{45} = \frac{13}{45}$

(4): $\quad \pi_1 = 1 - \frac{36}{45} = \frac{9}{45} = \frac{1}{5}$

$\therefore \pi_\infty^T = \left[\frac{1}{5} \quad \frac{23}{45} \quad \frac{13}{45} \right]$

3) Absorbing State

b) $\mu_3 = \mathbb{E}[T_3] = 0$

$$\mu_1 = 1 + (p_{11}\mu_1 + p_{12}\mu_2 + p_{13}\mu_3)$$
$$\mu_1 = 1 + (0.2\mu_1 + 0.7\mu_2)$$
$$0.8\mu_1 - 0.7\mu_2 = 1$$
$$8\mu_1 - 7\mu_2 = 10 \quad — ①$$

$$\mu_2 = 1 + (p_{21}\mu_1 + p_{22}\mu_2 + p_{23}\mu_3)$$
$$\mu_2 = 1 + (0.2\mu_1 + 0.5\mu_2)$$
$$0.5\mu_2 - 0.2\mu_1 = 1$$
$$5\mu_2 - 2\mu_1 = 10 \quad — ②$$

① + 4 × ② $\quad 20\mu_2 - 7\mu_2 = 50$
$$\mu_2 = \frac{50}{13} = \underline{\underline{3.8462}}$$

from ①: $\quad 8\mu_1 = \dfrac{350 + 130}{13}$
$$8\mu_1 = \frac{480}{13}$$
$$\mu_1 = \frac{60}{13} = \underline{\underline{4.6154}}$$

μ_1 & μ_2 are similar to the numerically calculated values in part a).