Stat 201A Problem set

Matthew Dworkin
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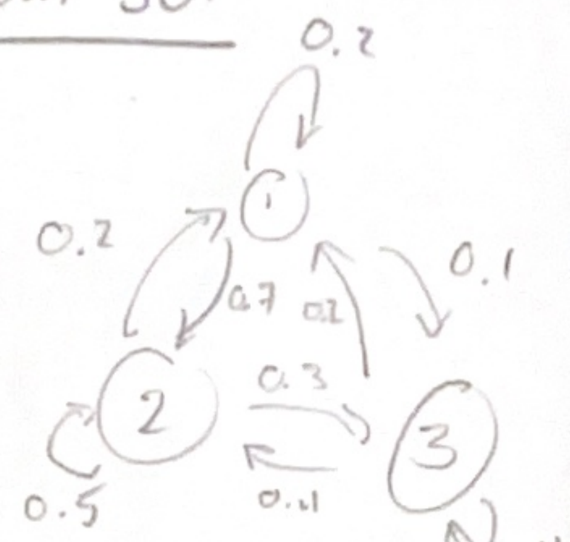
Problem 1



a)

$P =$

$$\begin{bmatrix} 0.2 & 0.7 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.4 & 0.4 \end{bmatrix}$$

b) See code

Problem 2

Let $\vec{\pi} = \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{bmatrix}$

Solve $\vec{\pi}^T P = \vec{\pi}^T$

$$\begin{bmatrix} \pi_1, & \pi_2 & \pi_3 \end{bmatrix} \begin{bmatrix} 0.2 & 0.7 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.4 & 0.4 \end{bmatrix}$$

$$= \begin{bmatrix} \pi_1, & \pi_2 & \pi_3 \end{bmatrix}$$

$$\begin{cases} 0.2\pi_1 + 0.2\pi_2 + 0.2\pi_3 = \pi_1 & (1) \\ 0.7\pi_1 + 0.5\pi_2 + 0.4\pi_3 = \pi_2 & (2) \\ 0.1\pi_1 + 0.3\pi_2 + 0.4\pi_3 = \pi_3 & (3) \end{cases}$$

In the code, I instead solve for the eigenvector of P^T corresponding to $\lambda = 1$.

Problem 3

$$\mathbb{E}[T_1] = 0.1\,\mathbb{E}[1+T_3] + 0.7\,\mathbb{E}[1+T_2] + 0.2\,\mathbb{E}[1+T_1]$$

$$= 1 + 0.7\,\mathbb{E}[T_2] + 0.2\,\mathbb{E}[T_1]$$

$$0.8\,\mathbb{E}[T_1] = 1 + 0.7\,\mathbb{E}[T_2] \quad (4)$$

$$\mathbb{E}[T_2] = 0.3\,\mathbb{E}[1+T_3] + 0.5\,\mathbb{E}[1+T_2] + 0.2\,\mathbb{E}[1+T_1]$$

$$= 1 + 0.5\,\mathbb{E}[T_2] + 0.2\,\mathbb{E}[T_1]$$

$$0.2\,\mathbb{E}[T_1] = 0.5\,\mathbb{E}[T_2] - 1$$

$$0.8\,\mathbb{E}[T_1] = 2\,\mathbb{E}[T_2] - 4 \quad (5)$$

$(4) = (5) \Rightarrow 1 + 0.7\,\mathbb{E}[T_2] = 2\,\mathbb{E}[T_2] - 4$

$$5 = 1.3\,\mathbb{E}[T_2]$$

$$\mathbb{E}[T_2] = \frac{5}{1.3} \approx \boxed{3.846}$$

$$\mathbb{E}[T_1] = \frac{1}{0.8}\left[1 + 0.7 \cdot \frac{5}{1.3}\right] \approx \boxed{4.615}$$