1. a.

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} = \begin{bmatrix} 0.2 & 0.7 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.4 & 0.4 \end{bmatrix}$$

b. $X_0 = 1$

$$f_1^1 = [1 \quad 0 \quad 0] \begin{bmatrix} 0.2 & 0.7 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.4 & 0.4 \end{bmatrix} = [0.2 \quad 0.7 \quad 0.1]$$

sample state using these probs

2. $(P^T - I)\,\pi_\infty = 0$

$$P^T = \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.7 & 0.5 & 0.4 \\ 0.1 & 0.3 & 0.4 \end{bmatrix}$$

$$\begin{bmatrix} -0.8 & 0.2 & 0.2 \\ 0.7 & -0.5 & 0.4 \\ 0.1 & 0.3 & -0.6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} = \begin{bmatrix} 0.2 & 0.7 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.4 & 0.4 \end{bmatrix}$$

3) $u_1 = 1 + \sum_{j=1}^{3} p_{ij} u_j$

$$\begin{cases} u_1 = 1 + p_{11}u_1 + p_{12}u_2 + p_{13}u_3 \\ u_2 = 1 + p_{21}u_1 + p_{22}u_2 + p_{23}u_3 \\ u_3 = 0 \end{cases}$$

$$\begin{cases} (1 - p_{11})u_1 - p_{12}u_2 - p_{13}u_3 = 1 \\ -p_{21}u_1 + (1 - p_{22})u_2 - p_{23}u_3 = 1 \\ u_3 = 0 \end{cases}$$

$$\begin{cases} 0.8u_1 - 0.7u_2 = 1 \\ -0.2u_1 + 0.5u_2 = 1 \end{cases}$$

$$0.8u_1 - 0.7u_2 = 1$$
$$-0.8u_1 + 2u_2 = 4$$
$$1.3u_2 = 5$$

$$u_2 = \frac{5}{1.3} = \boxed{3.846}$$

$$0.8u_1 - 0.7\left(\frac{5}{1.3}\right) = 1$$

$$u_1 = \boxed{4.615}$$