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Thm

Stat 201a Hw 3 Written

1a) $P = \begin{bmatrix} 0.2 & 0.7 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.4 & 0.4 \end{bmatrix}$

2a) Let $\pi_\infty^T = [P_1 \quad P_2 \quad P_3]$

Want π_∞ s.t. $\pi_\infty^T \cdot P = \pi_\infty^T$

$$\Rightarrow \begin{aligned} 0.2P_1 + 0.2P_2 + 0.2P_3 &= P_1 \\ 0.7P_1 + 0.5P_2 + 0.4P_3 &= P_2 \\ 0.1P_1 + 0.3P_2 + 0.4P_3 &= P_3 \end{aligned} \quad \text{and } P_1+P_2+P_3 = 1$$

$$\Rightarrow 0.7\left(0.2P_1 + 0.2P_2 + 0.2P_3\right) + 0.5P_2 + 0.4P_3 = P_2$$

$$0.1\left(0.2P_1 + 0.2P_2 + 0.2P_3\right) + 0.3P_2 + 0.4P_3 = P_3$$

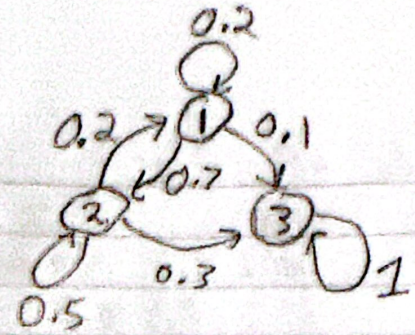
$$\Rightarrow 0.2(P_1+P_2+P_3) = P_1 \Rightarrow P_1 = 0.2$$

$$0.14 = 0.5P_2 - 0.4P_3 \Rightarrow P_2 = 0.5\overline{11}$$
$$0.02 = -0.3P_2 + 0.6P_3$$

$$\Rightarrow P_3 = 1 - 0.2 - 0.5\overline{11} = 0.2\overline{88}$$

Thus, our stationary dist. $\pi_\infty^T = [0.2 \quad 0.5\overline{11} \quad 0.2\overline{88}]$

3b) Now our MC is



$\mu_i = E[T_i]$

Trivially, $\mu_3 = E[T_3] = E[0] = 0$

$$\mu_1 = E[T_1] = 1 + P_{11}\cdot\mu_1 + P_{12}\cdot\mu_2 + P_{13}\cdot\mu_3$$
$$= 1 + 0.2\,\mu_1 + 0.7\,\mu_2$$

$$\mu_2 = E[T_2] = 1 + P_{21}\cdot\mu_1 + P_{22}\cdot\mu_2 + P_{23}\cdot\mu_3$$
$$= 1 + 0.2\cdot\mu_1 + 0.5\cdot\mu_2$$

$$\Rightarrow \quad 0 = 1 - 0.8\mu_1 + 0.7\mu_2$$
$$0 = 1 + 0.2\mu_1 - 0.5\mu_2$$

$$\Rightarrow \boxed{\begin{aligned} \mu_1 &\simeq 4.62 \\ \mu_2 &\simeq 3.85 \\ \mu_3 &= 0 \end{aligned}}$$