

LAB 11/20

STOCHASTIC PROCESS $\overbrace{X_0, X_1, X_2, \dots}^{RV}$

$$X_i = \{1, 2, 3\}$$

EACH NODE IS $P(X_{i+1} = a | X_i = b)$
 $\downarrow \quad \quad \quad \checkmark$
 $\{1, 2, 3\}$

$$X_0 = 1 \leftarrow \pi_0^T P$$

$$X_2 = 2 \quad \pi_0^T P^2 = \pi_1^T P$$

$$X_3 = 2$$

$$X_4 = 1$$

$$\pi_n^T = \pi_0^T P^n$$

π_0 = Prob. dist. at time 0

$$\pi_0 = (1, 0, 0)$$

TRANSPOSE

$$\pi_{i+1}^T = \pi_i^T P$$

$$[p_1, p_2, p_3] \begin{bmatrix} p_{1 \rightarrow 1} & p_{1 \rightarrow 2} & p_{1 \rightarrow 3} \\ p_{2 \rightarrow 1} & p_{2 \rightarrow 2} & p_{2 \rightarrow 3} \\ p_{3 \rightarrow 1} & p_{3 \rightarrow 2} & p_{3 \rightarrow 3} \end{bmatrix}$$

$$= [p_1 p_{1 \rightarrow 1} + p_2 p_{2 \rightarrow 1} + p_3 p_{3 \rightarrow 1}, \dots]$$

\downarrow
 Prob of being in 1 after 1 move

②

$$P = \begin{bmatrix} p_{1 \rightarrow 1} & p_{1 \rightarrow 2} & p_{1 \rightarrow 3} \\ p_{2 \rightarrow 1} & p_{2 \rightarrow 2} & p_{2 \rightarrow 3} \\ p_{3 \rightarrow 1} & p_{3 \rightarrow 2} & p_{3 \rightarrow 3} \end{bmatrix} = \begin{bmatrix} 0.2 & 0.7 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.4 & 0.4 \end{bmatrix}$$

$$\pi_{i+1}^T = \pi_i^T P$$

2

$$\lim_{n \rightarrow \infty} \pi_n = \pi_\infty \rightarrow \boxed{\pi_\infty^T = \pi_\infty^T P} \text{ save}$$

$$\pi_\infty^T (P - I) = 0$$

3

$$\pi_\infty^T = \pi_\infty^T P \Rightarrow (\pi_\infty^T \cdot I) - \pi_\infty^T P = 0$$

$$\begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.3 & -0.5 & 0.4 \\ 0.1 & 0.3 & 0.4 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \pi_\infty = 0$$

$$\begin{array}{ccc|c} -0.8 & 0.2 & 0.2 & 0 \\ 0.3 & -0.5 & 0.4 & 0 \\ 0.1 & 0.3 & -0.6 & 0 \end{array} \xrightarrow{L1+8L3} \begin{array}{ccc|c} -0.8 & 0.2 & 0.2 & 0 \\ 0.3 & -0.5 & 0.4 & 0 \\ 0 & 2.6 & -4.6 & 0 \end{array}$$

$$\xrightarrow{L1+8L3} \begin{array}{ccc|c} -0.8 & 0.2 & 0.2 & 0 \\ 0 & -2.6 & 4.6 & 0 \\ 0 & 2.6 & -4.6 & 0 \end{array} \rightarrow \begin{array}{ccc|c} -0.8 & 0.2 & 0.2 & 0 \\ 0 & -2.6 & 4.6 & 0 \\ 0 & 0 & 0 & 0 \end{array}$$

$$-2.6X_2 + 4.6X_3 = 0 \Rightarrow 2.6X_2 = 4.6X_3$$

$$X_2 = 1.77X_3$$

$$-0.8X_1 + 0.2(1.77X_3) + 0.2X_3 = 0$$

$$0.55X_3 = 0.8X_1 \Rightarrow X_1 = 0.69X_3$$

$$\pi_\infty = \begin{pmatrix} 0.69 \\ 1.77 \\ 1 \end{pmatrix}$$

$$\text{Normalizing by } 3.46 \Rightarrow \pi_\infty = \begin{pmatrix} 0.69/3.46 \\ 1.77/3.46 \\ 1/3.46 \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.51 \\ 0.28 \end{pmatrix}$$

③

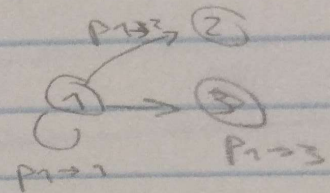
④

$$\frac{T_1}{PV} \quad T_3 < 0$$

$$\mu_1 = E[T_1]$$

$$= 1 + p_{11}/\mu_1 + p_{12}/\mu_2 + p_{13}/\mu_3$$

"0"
↓
T=C



$$\mu_1 = 1 + p_{11}/\mu_1 + p_{12}/\mu_2$$

$$(1 - p_{11})/\mu_1 = 1 + p_{12}/\mu_2 \Rightarrow \mu_1 = \frac{1 + p_{12}/\mu_2}{1 - p_{11}}$$

$$\mu_2 = E[T_2] = 1 + p_{21}/\mu_1 + p_{22}/\mu_2 + p_{23}/\mu_3$$

"0"
↓
T=C

$$(1 - p_{22})/\mu_2 = 1 + p_{21}/\mu_1 \Rightarrow (1 - p_{22})/\mu_2 = 1 + p_{21} \left(\frac{1 + p_{12}/\mu_2}{1 - p_{11}} \right)$$

$$(1 - p_{22})/\mu_2 = 1 + \frac{p_{21} + p_{12}p_{21}/\mu_2}{1 - p_{11}}$$

$$(1 - p_{22})/\mu_2 - \frac{p_{12}p_{21}}{1 - p_{11}} \mu_2 = 1 + \frac{p_{21}}{1 - p_{11}}$$

$$\mu_2 \left(\frac{1 - p_{22} - \frac{p_{12}p_{21}}{1 - p_{11}}}{1 - p_{22} - p_{11} + p_{11}p_{22} - p_{12}p_{21}} \right) = 1 + \frac{p_{21}}{1 - p_{11}}$$

$$\mu_2 = \frac{1 - p_{11} + p_{21}}{(1 - p_{11})(1 - p_{22}) - p_{12}p_{21}} = \frac{1 - 0.2 + 0.2}{(1 - 0.2)(1 - 0.5) - 0.3(0.2)} = 3.89$$

$$\mu_1 = \frac{1}{1 - p_{11}} + \frac{p_{12}}{1 - p_{11}} \left(\frac{1 - p_{11} + p_{21}}{(1 - p_{11})(1 - p_{22}) - p_{12}p_{21}} \right)$$

$$= \frac{1}{1 - p_{11}} + \frac{p_{12}(1 - p_{11} + p_{21})}{(1 - p_{11})^2(1 - p_{22}) - (1 - p_{11})(p_{12}p_{21})}$$

$$= \frac{1}{1 - 0.2} + \frac{0.3(1 - 0.2 + 0.2)}{(1 - 0.2)^2(1 - 0.5) - (1 - 0.2)(0.3)(0.2)}$$

$$= 1.25 + \frac{0.3}{0.208} = 4.59$$