Lab1120

November 27, 2023

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats as stats
from scipy.stats import expon
import matplotlib as mpl
import pandas as pd
```

[2]:
```
import scipy as sp
```

0.1 Exercise 1b

[18]:
```
array = np.array([1,2,3])
transition_matrix = (np.array([[0.2, 0.7, 0.1], [0.2, 0.5, 0.3], [0.2, 0.4, 0.
 ↪4]]))

x = [1]
x_iteration = 1

for i in range(30):
    if x_iteration==1:
        x_iteration=np.random.choice(array, p = transition_matrix[0])
        x.append(x_iteration)
    if x_iteration==2:
        x_iteration=np.random.choice(array, p = transition_matrix[1])
        x.append(x_iteration)
    if x_iteration==3:
        x_iteration=np.random.choice(array, p = transition_matrix[2])
        x.append(x_iteration)
```

[9]:
```
#np.transpose(pi)@transition_matrix
```

[9]:
```
matrix([[0.2, 0.7, 0.1]])
```

[19]:
```
x
```

[19]:
```
[1,
 2,
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3,
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3,
2,
1]
```

0.2 Exercise 2b

[3]:
```
array = np.array([1,2,3])
transition_matrix = (np.array([[0.2, 0.7, 0.1], [0.2, 0.5, 0.3], [0.2, 0.4, 0.
 ↪4]]))

pi = [0,0,1]
result_2b = np.transpose(pi)@np.linalg.matrix_power(transition_matrix, 100)
result_2b
```

[3]:
```
array([0.2       , 0.51111111, 0.28888889])
```

We can see we got the same results as in the handwritten part

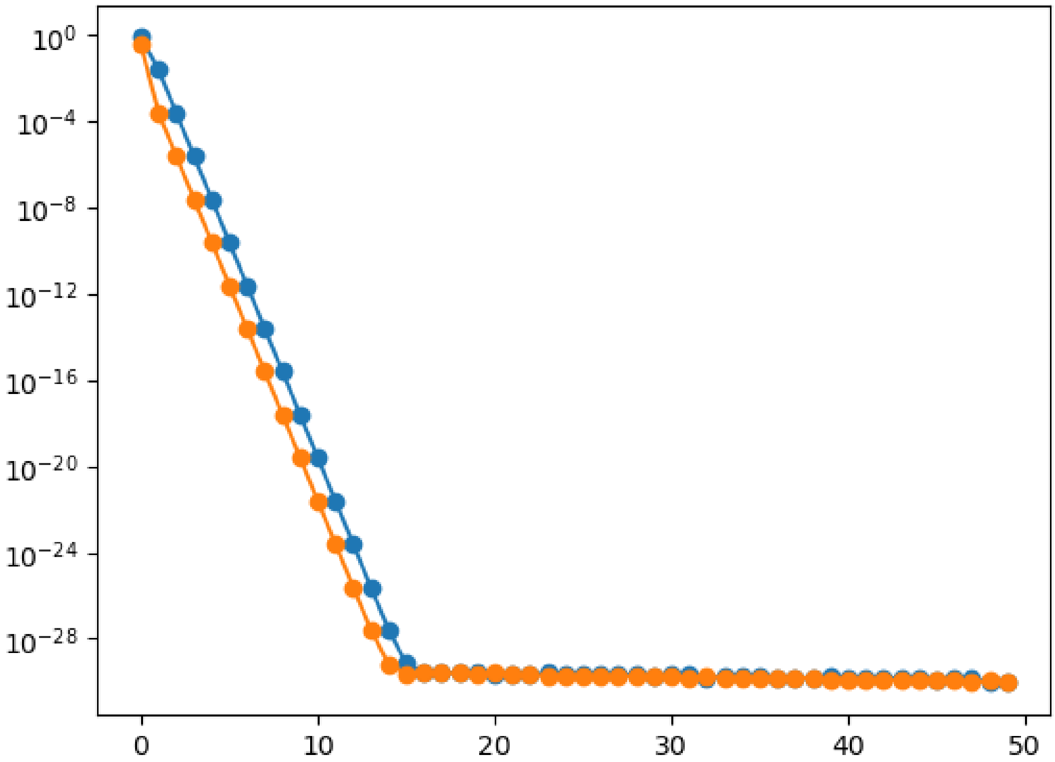
[15]:
```
x_0 = [0,0,1]
list_x0 = []
x_1 = [0,1,0]
list_x1 = []

def norm2(x):
    return(np.sum(x**2) ** 0.5)

for i in range(50):
    #Source consulted: https://stackoverflow.com/questions/35213592/
 ↪numpy-calculate-square-of-norm-2-of-vector
    iterate_0 = (norm2((np.transpose(x_0)@np.linalg.
 ↪matrix_power(transition_matrix, i))- (np.transpose(x_0)@np.linalg.
 ↪matrix_power(transition_matrix, 100)))**2)
    list_x0.append([i,iterate_0])
    iterate_1 = (norm2((np.transpose(x_1)@np.linalg.
 ↪matrix_power(transition_matrix, i))- (np.transpose(x_1)@np.linalg.
 ↪matrix_power(transition_matrix, 100)))**2)
    list_x1.append([i,iterate_1])

df_x0 = pd.DataFrame(list_x0)
df_x0=df_x0.rename(columns={0: "i", 1: "(|| _i -  _∞||)^2"})
df_x1 = pd.DataFrame(list_x1)
df_x1=df_x1.rename(columns={0: "i", 1: "(|| _i -  _∞||)^2"})
```

[16]:
```
plt.plot(df_x0['i'], df_x0["(|| _i -  _∞||)^2"],'o-');
plt.plot(df_x1['i'], df_x1["(|| _i -  _∞||)^2"],'o-');
plt.yscale('log')
```



We can see that after around 14 iterations both cases converge, and that before the norm square of [0,0,1] is greater than the norm square of [0,1,0]

0.3 Exercise 3 a

We create a function from the code of question 1a changed

```
[27]: def get_markov_chain(input_array, input_transition_matrix, x_start):
          time_saved = []
          for i in range(10000):
              x_iteration = x_start
              time = 0
              while x_iteration!=3:
                  if x_iteration==1:
                      x_new=np.random.choice(input_array, p =␣
       ↪input_transition_matrix[0])
                  elif x_iteration==2:
                      x_new=np.random.choice(input_array, p =␣
       ↪input_transition_matrix[1])
                  elif x_iteration==3:
                      x_new=np.random.choice(input_array, p =␣
       ↪input_transition_matrix[2])
```

```
            time+=1
            x_iteration=x_new
        time_saved.append(time)
    return time_saved
```

```
[28]: array = np.array([1,2,3])
      transition_matrix = (np.array([[0.2, 0.7, 0.1], [0.2, 0.5, 0.3], [0.2, 0.4, 0.
        ↪4]]))
```

```
[29]: x_1 = get_markov_chain(array, transition_matrix,1)
      x_2 = get_markov_chain(array, transition_matrix,2)
```

```
[26]: np.mean(x_1)
```

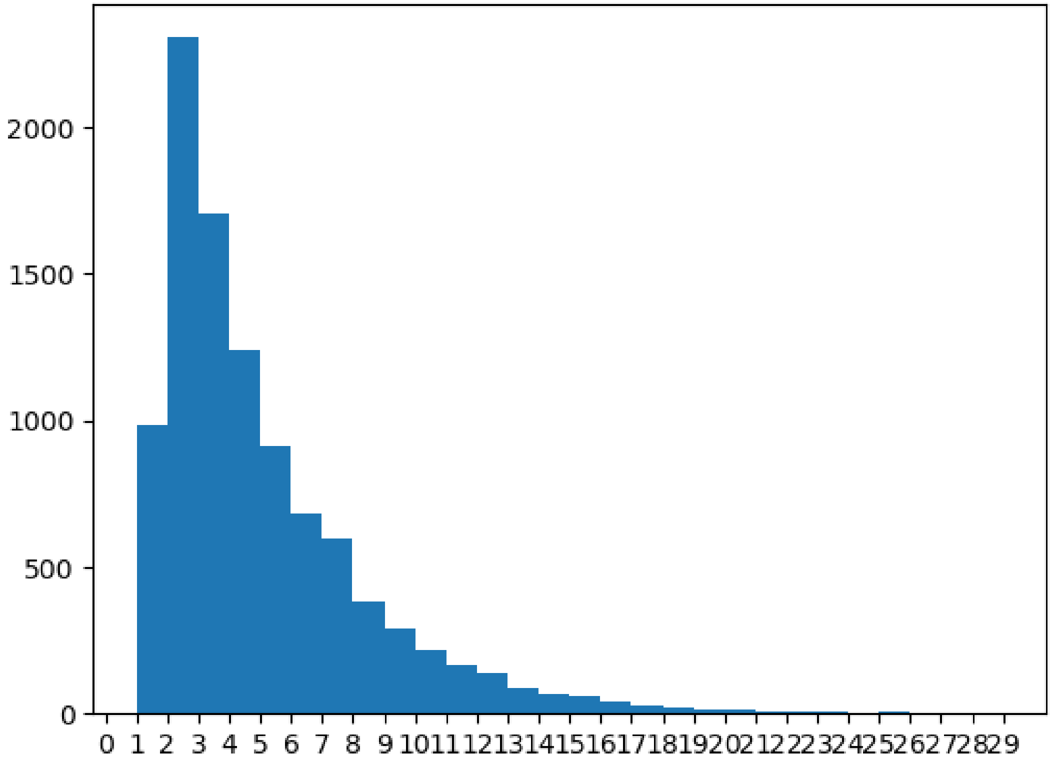
```
[26]: 4.6017
```

```
[25]: np.mean(x_2)
```

```
[25]: 3.8665
```

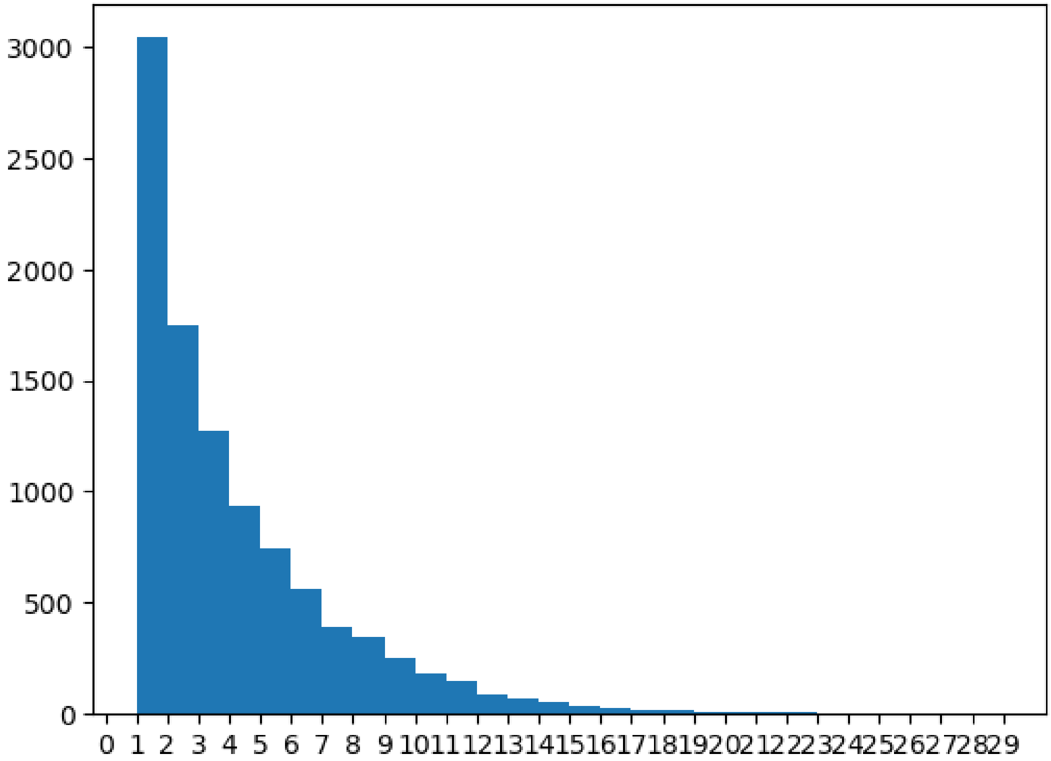
We can see we got the same results as in the handwritten part

```
[36]: plt.hist(x_1, bins=np.arange(1,30,1)); #, bins=100, density=True
      plt.xticks(range(30));
```



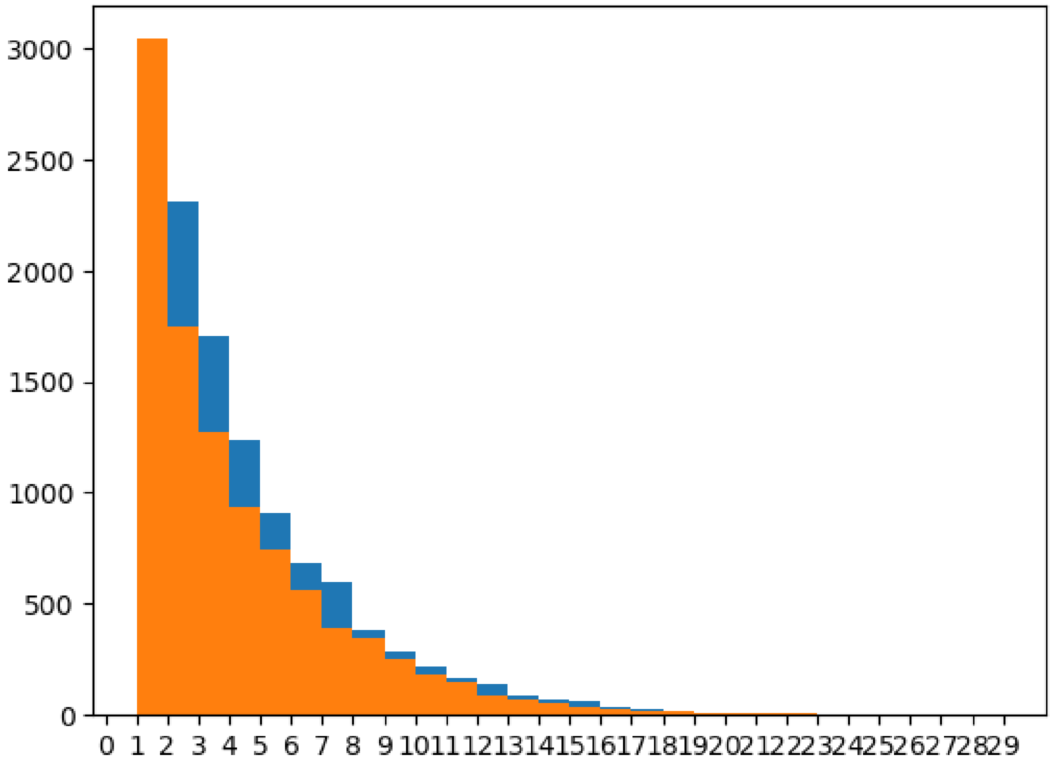
[38]:

```
plt.hist(x_2, bins=np.arange(1,30,1)); #, bins=100, density=True
plt.xticks(range(30));
```



[40]:

```
plt.hist(x_1, bins=np.arange(1,30,1)); #, bins=100, density=True
plt.hist(x_2, bins=np.arange(1,30,1)); #, bins=100, density=True
plt.xticks(range(30));
```


3000
2500
2000
1500
1000
500
0
0
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