Markov Lab Problems

1 A

Let P_{ij} be the probability of transitioning from state i to state j. For a 3×3 matrix we have...

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} = \begin{bmatrix} 0.2 & 0.7 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.4 & 0.4 \end{bmatrix}$$

where $P_{ij} \geq 0$ & $\sum_{j=1}^{3} P_{ij} = 1$ by definition

3b $P_{absorbing} = \begin{bmatrix} 0.2 & 0.7 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0 & 0 & 1 \end{bmatrix}$

using $\mu_i = 1 + \sum_{j=1}^{3} P_{ij}\mu_j = E[T_i]$ & $T_3 = 0$

$$\Rightarrow \begin{aligned} \mu_1 &= 1 + 0.2\mu_1 + 0.7\mu_2 + 0.1\mu_3 \\ \mu_2 &= 1 + 0.2\mu_1 + 0.5\mu_2 + 0.3\mu_3 \\ &\boxed{\mu_3 = 0} \end{aligned}$$

$$\Rightarrow \left.\begin{aligned} \mu_1 &= 1 + 0.2\mu_1 + 0.7\mu_2 \\ \mu_2 &= 1 + 0.2\mu_1 + 0.5\mu_2 \end{aligned}\right\} \begin{aligned} 0.8\mu_1 &= 1 + 0.7\mu_2 \\ 0.5\mu_2 &= 1 + 0.2\mu_1 \end{aligned}$$

$$\Rightarrow \left.\begin{aligned} 0.8\mu_1 &= 1 + 0.7\mu_2 \\ \mu_2 &= 2 + 0.4\mu_1 \end{aligned}\right\} \Rightarrow 0.8\mu_1 = 1 + (0.7)(2 + 0.4\mu_1)$$

$$\Rightarrow 0.8\mu_1 = 1 + 1.4 + 0.28\mu_1 \Rightarrow 0.52\mu_1 = 2.4$$

$$\Rightarrow \boxed{\mu_1 = \frac{2.4}{.52} = 4.61538\ldots} \Rightarrow \boxed{\begin{aligned} \mu_2 &= 2 + 0.4\left(\frac{2.4}{.52}\right) \\ \mu_2 &= 3.84615\ldots \end{aligned}}$$

This very closely aligns with the simulated values & validates my results from part A.