

$$1. (a) \quad P = \begin{bmatrix} 0.2 & 0.7 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.4 & 0.4 \end{bmatrix}$$

$$2. (a) \quad P^T = \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.7 & 0.5 & 0.4 \\ 0.1 & 0.3 & 0.4 \end{bmatrix}$$

$$(P^T - I) \lambda_{\infty} = 0$$

$$\begin{bmatrix} 0.2-1 & 0.2 & 0.2 \\ 0.7 & 0.5-1 & 0.4 \\ 0.1 & 0.3 & 0.4-1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -0.8 & 0.2 & 0.2 \\ 0.7 & -0.5 & 0.4 \\ 0.1 & 0.3 & -0.6 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -0.8\lambda_1 + 0.2\lambda_2 + 0.2\lambda_3 = 0 \\ 0.7\lambda_1 - 0.5\lambda_2 + 0.4\lambda_3 = 0 \\ 0.1\lambda_1 + 0.3\lambda_2 - 0.6\lambda_3 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \lambda_1 = \frac{1}{5} \\ \lambda_2 = \frac{23}{45} \\ \lambda_3 = \frac{13}{45} \end{cases}$$

3.(b)

$$\mu_i = 1 + \sum_{j=1}^3 p_{ij} \mu_j$$

$$\begin{cases} \mu_1 = 1 + 0.2\mu_1 + 0.7\mu_2 + 0.1\mu_3 \\ \mu_2 = 1 + 0.2\mu_1 + 0.5\mu_2 + 0.3\mu_3 \\ \mu_3 = E[T_3] = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \mu_1 = \frac{60}{13} \\ \mu_2 = \frac{50}{13} \\ \mu_3 = 0 \end{cases}$$