

# STAT 201A Lab (11/20)

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## Problem 1a

$$P = \begin{bmatrix} 0.2 & 0.7 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.4 & 0.4 \end{bmatrix}$$

## Problem 1b

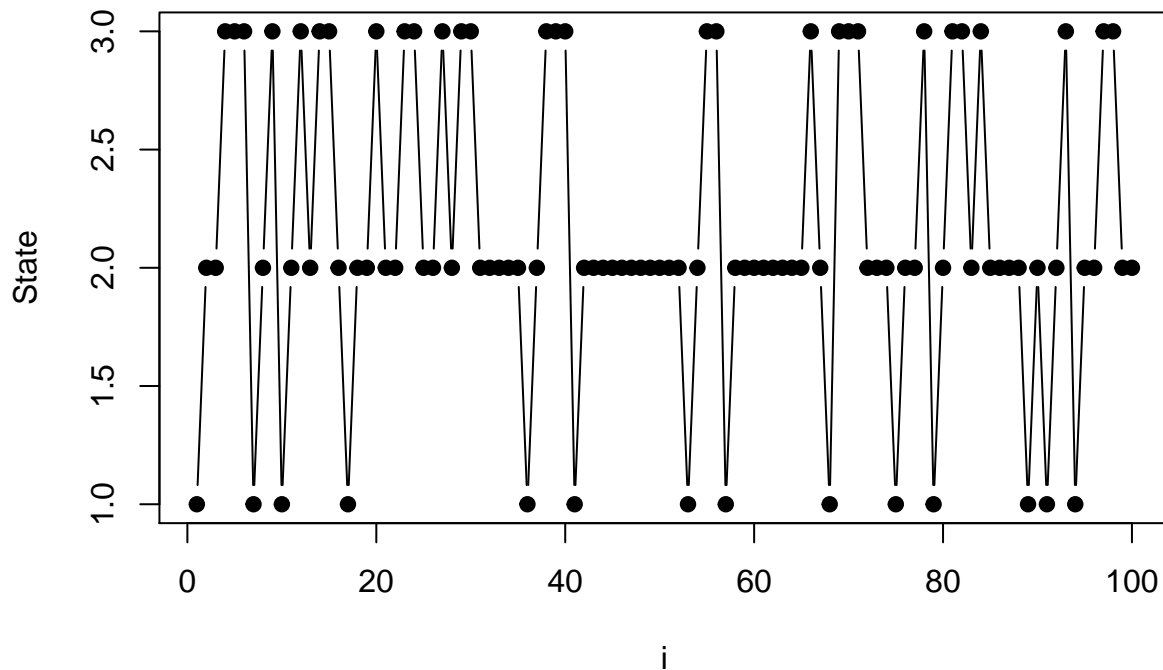
```
set.seed(24)
P <- matrix(c(0.2, 0.7, 0.1, 0.2, 0.5, 0.3, 0.2, 0.4, 0.4), nrow = 3, byrow = T)
P

##      [,1] [,2] [,3]
## [1,]  0.2  0.7  0.1
## [2,]  0.2  0.5  0.3
## [3,]  0.2  0.4  0.4

simulate_mc <- function(n, P, X0) {
  # Generates sequences of length n (including X0)
  # P: transition probability matrix
  # X0: initial state
  cur_X <- X0
  states <- 1:3
  MC_seq <- numeric(n)
  MC_seq[1] <- X0
  for (i in 2:n) {
    transition_probs <- P[cur_X, ]
    next_X <- sample(states, size = 1, prob = transition_probs)
    MC_seq[i] <- next_X
    cur_X <- next_X
  }
  MC_seq
}

plot(1:100, simulate_mc(100, P, 1),
     xlab = "i", ylab = "State",
     type = "b", pch = 19,
     main = "Realization of Markov Chain (100 Steps, X0 = 1)")
```

## Realization of Markov Chain (100 Steps, $X_0 = 1$ )



### Problem 2a

```
eigs <- eigen(t(P))
stationary_state <- eigs$vectors[, 1]
stationary_state_norm <- stationary_state / sum(stationary_state)
eigs
```

```
## eigen() decomposition
## $values
## [1] 1.000000e+00 1.000000e-01 1.363926e-16
##
## $vectors
##      [,1]      [,2]      [,3]
## [1,] 0.3224585 1.073314e-16 0.2672612
## [2,] 0.8240605 -7.071068e-01 -0.8017837
## [3,] 0.4657733 7.071068e-01 0.5345225
```

Solving numerically, we see that 0.3224585, 0.8240605, 0.4657733 is the solution to  $(P^T - I)\pi_\infty = 0$ . Normalized, we have  $\pi_\infty = 0.2, 0.5111111, 0.2888889$ .

### Problem 2b

We plot the convergence for an initial  $\pi_0$  close to  $\pi_\infty$ :

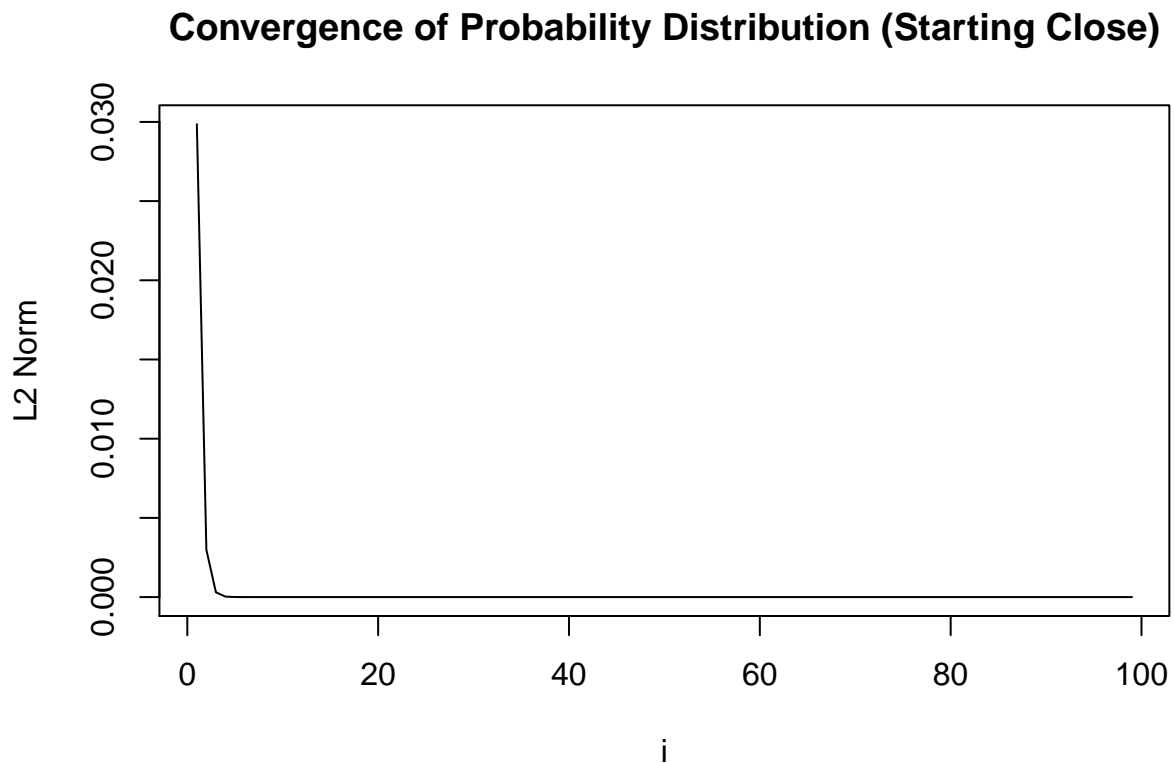
```
# initial distn close to pi_infinity
close_initial <- c(0.1, 0.6, 0.3)
pis <- list()
```

```

pis[[1]] <- close_initial
n <- 100
# calculates L2 norm
l2norm <- function(u, v) {
  sqrt(sum((u - v)^2))
}
l2_norms <- numeric(n)
for (i in 2:n) {
  pis[[i]] <- pis[[i - 1]] %*% P
  l2_norms[i] <- l2norm(pis[[i]], stationary_state_norm)
}

plot(1:(n - 1), l2_norms[2:n], type = "l",
     xlab = "i", ylab = "L2 Norm",
     main = "Convergence of Probability Distribution (Starting Close)")

```



We plot the convergence for an initial  $\pi_0$  far from  $\pi_\infty$ :

```

# initial distn far from pi_infinity
far_initial <- c(0.7, 0.2, 0.1)
pis <- list()
pis[[1]] <- far_initial
n <- 100
# calculates L2 norm
l2norm <- function(u, v) {
  sqrt(sum((u - v)^2))
}

```

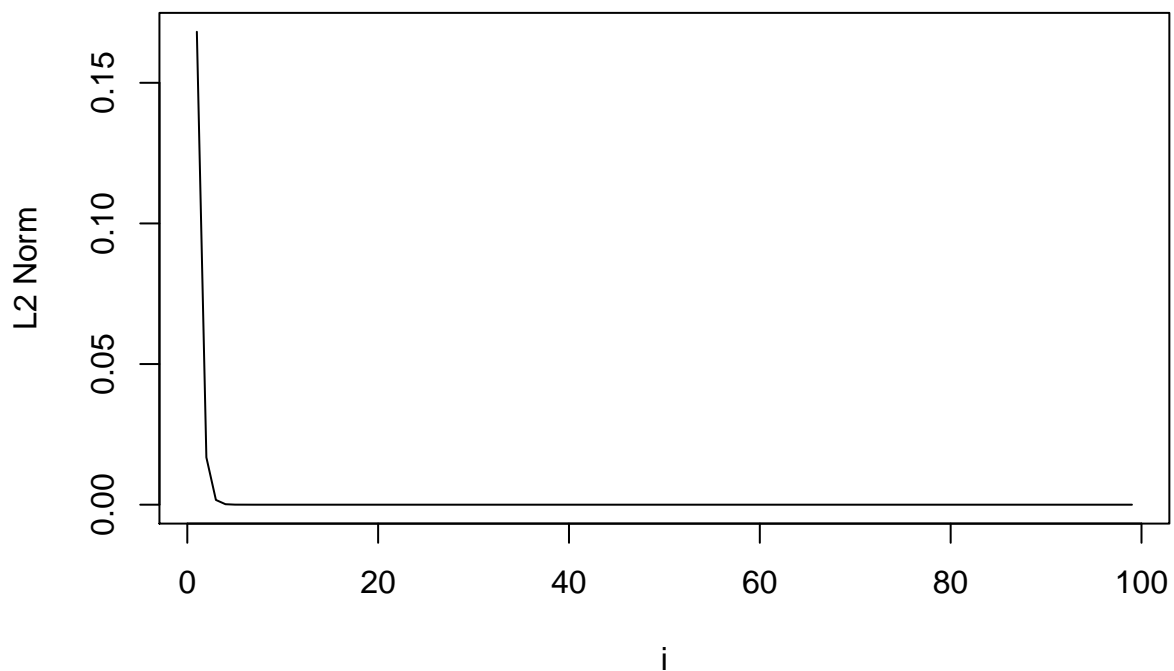
```

}
l2_norms <- numeric(n)
for (i in 2:n) {
  pis[[i]] <- pis[[i - 1]] %*% P
  l2_norms[i] <- l2norm(pis[[i]], stationary_state_norm)
}

plot(1:(n - 1), l2_norms[2:n], type = "l",
     xlab = "i", ylab = "L2 Norm",
     main = "Convergence of Probability Distribution (Starting Far)")

```

## Convergence of Probability Distribution (Starting Far)



We see  $\pi_i$  converges to  $\pi_\infty$  ( $\pi_i \rightarrow \pi_\infty$ ) quite quickly regardless of the specific  $\pi_0$  value.

### Problem 3a

```

# Function that returns arrival time
until_X3 <- function(X0) {
  t <- 0
  cur_X <- X0
  while (cur_X != 3) {
    cur_X <- simulate_mc(2, P, cur_X)[2]
    t <- t + 1
  }
  t
}

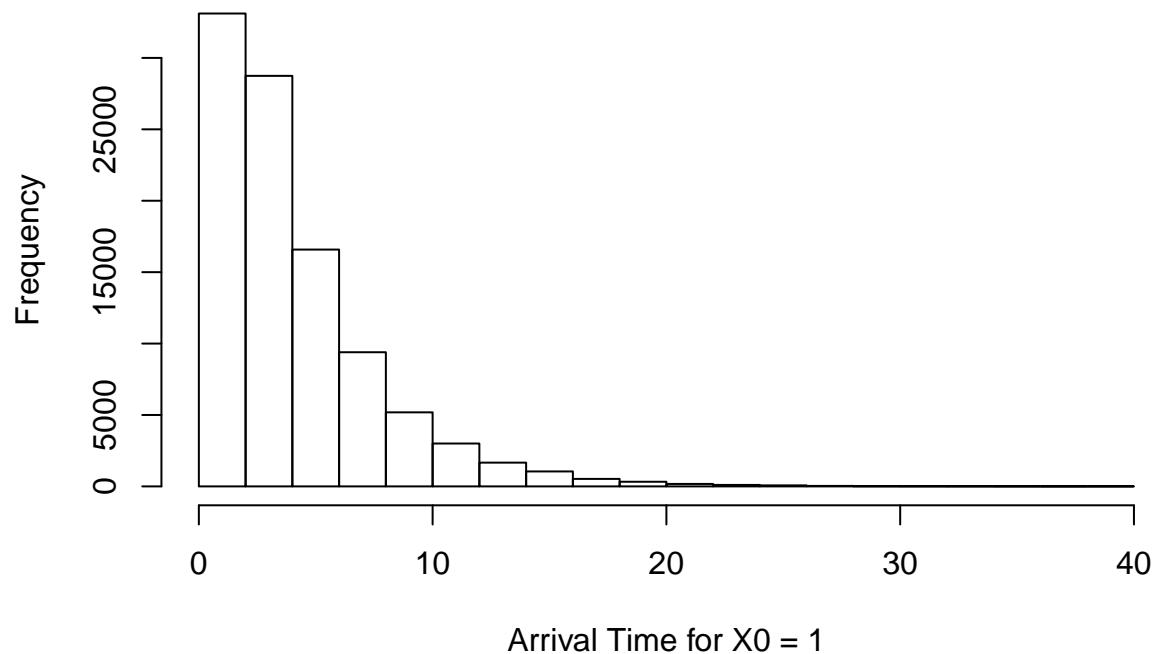
```

```

# Simulate for X0 = 1
N <- 1e5
arrivals_1 <- numeric(N)
for (i in 1:N) {
  arrivals_1[i] <- until_X3(1)
}
hist(arrivals_1, xlab = "Arrival Time for X0 = 1",
     main = "Histogram of Arrival Times for X0 = 1")

```

**Histogram of Arrival Times for X0 = 1**

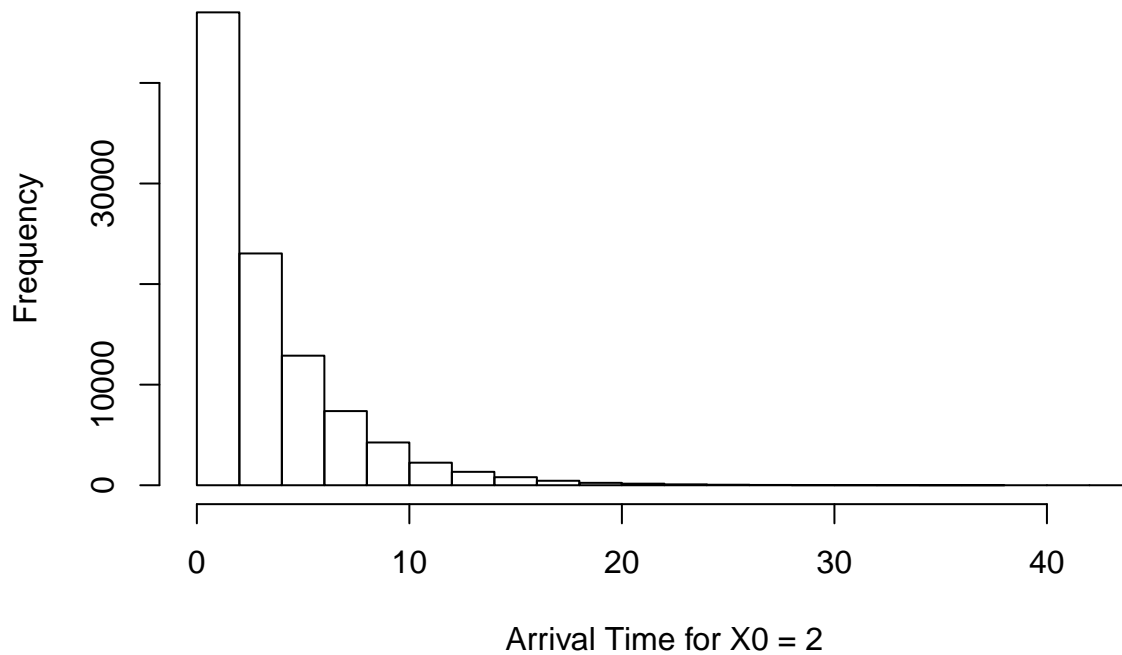


```

# Simulate for X0 = 2
arrivals_2 <- numeric(N)
for (i in 1:N) {
  arrivals_2[i] <- until_X3(2)
}
hist(arrivals_2, xlab = "Arrival Time for X0 = 2",
     main = "Histogram of Arrival Times for X0 = 2")

```

## Histogram of Arrival Times for $X_0 = 2$



```
# mean arrival times
mean_arrival_time1 <- mean(arrivals_1)
mean_arrival_time2 <- mean(arrivals_2)
mean_arrival_time1
```

```
## [1] 4.61894
```

```
mean_arrival_time2
```

```
## [1] 3.84996
```

We see that starting from  $X_0 = 1$ , we have a mean arrival time of 4.61894 and starting from  $X_0 = 2$ , we have a mean arrival time of 3.84996.