

3. **Absorbing state.** Consider now that node 3 is an absorbing state and we want to estimate the waiting time until the process arrives at  $X_i = 3$  from any other node.

- b) Compute theoretically the mean arrival time to the absorbing state and compare it with part a. To do so, notice that if  $T_i$  denotes the random variable associated to the arrival time starting from  $X_0 = i$ , then

$$\mu_i = 1 + \sum_{j=1}^3 p_{ij} \mu_j, \quad (1)$$

with  $\mu_i = \mathbb{E}[T_i]$ . This is a linear system of equations that you can solve. Notice  $T_3 = 0$ .

$$\mu_1 = 1 + 0.2\mu_1 + 0.7\mu_2$$

$$\mu_2 = 1 + 0.2\mu_1 + 0.5\mu_2$$

$$\Downarrow$$

$$0.5\mu_2 = 1 + 0.2\mu_1 \Rightarrow \mu_2 = 2 + 0.4\mu_1$$

$$\Downarrow$$

$$\mu_1 = 1 + 0.2\mu_1 + 0.7(2 + 0.4\mu_1)$$

$$= 1 + 0.2\mu_1 + 1.4 + 0.28\mu_1$$

$$\Rightarrow 0.52\mu_1 = 2.4 \Rightarrow \mu_1 \approx 4.615$$

$$\Rightarrow \mu_2 \approx 2 + 0.4(4.615)$$

$$\Rightarrow \mu_2 \approx 3.846$$