

# Homework 3

1. See Jupyter Notebook

2. a) The linear system that must be solved:

$$(P^T - I)\pi_a = 0$$

$$P^T - I = \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.7 & 0.5 & 0.4 \\ 0.1 & 0.3 & 0.4 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -0.8 & 0.2 & 0.2 \\ 0.7 & -0.5 & 0.4 \\ 0.1 & 0.3 & -0.6 \end{bmatrix}$$

$$\begin{bmatrix} -0.8 & 0.2 & 0.2 \\ 0.7 & -0.5 & 0.4 \\ 0.1 & 0.3 & -0.6 \end{bmatrix} \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{bmatrix} = 0 \quad \left. \vphantom{\begin{bmatrix} -0.8 & 0.2 & 0.2 \\ 0.7 & -0.5 & 0.4 \\ 0.1 & 0.3 & -0.6 \end{bmatrix} \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{bmatrix} = 0} \right\} \text{solve this, see jupyter notebook}$$

2b) See Jupyter notebook

3a) see Jupyter Notebook

$$3b) \mu_1 = 1 + 0.2\mu_1 + 0.7\mu_2$$

$$0.8\mu_1 = 1 + 0.7\mu_2$$

$$0.8\mu_1 = 1 + 0.7(2 + 0.4\mu_1)$$

$$0.8\mu_1 = 1 + 1.4 + 0.28\mu_1$$

$$\boxed{\mu_1 = 4.615}$$

$$\mu_2 = 0.5\mu_2 + 0.2\mu_1 + 1$$

$$0.5\mu_2 = 1 + 0.2\mu_1$$

$$\xleftarrow{\text{substitute in}} \mu_2 = 2 + 0.4\mu_1$$

$$\mu_2 = 2 + 0.4(4.615)$$

$$\boxed{\mu_2 = 3.846}$$

These results are very similar to what we observed in part a.