201A-Lab 11/20 Calculations

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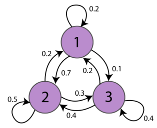
1. **Simulation of Markov Process.**



a) Write the Markov process in matrix representation, that is, define the matrix $P \in \mathbb{R}^{3\times 3}$ such that P_{ij} is the probability of transitioning from the node i to j.

In transition matrix, the 1st row represents the probabilities of transitioning from state 1 to 1,2,3. The 2nd row from state 2 to 1,2,3. And the 3rd row from state 3 to 1,2,3.

$$P = \begin{bmatrix} P_{1\to1} & P_{1\to2} & P_{1\to3} \\ P_{2\to1} & P_{2\to2} & P_{2\to3} \\ P_{3\to1} & P_{3\to2} & P_{3\to3} \end{bmatrix} = \begin{bmatrix} 0.2 & 0.7 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.4 & 0.4 \end{bmatrix}$$

b) Compute theoretically the mean arrival time to the absorbing state and compare it with part a. To do so, notice that if T_i denotes the random variable associated to the arrival time starting from $X_0 = i$, then

$$\mu_i = 1 + \sum_{j=1}^{3} p_{ij}\mu_j, \tag{1}$$

with $\mu_i = \mathbb{E}[T_i]$. This is a linear system of equations that you can solve. Notice $T_3 = 0$.

Given our transition matrix P we will set up and solve a system of equations to find μ_1 and μ_2. Given $\mu_i = 1 + \sum_{j=1}^{3} p_{ij}\mu_j$. Note that $\mu_3 = 0$ as state 3 is the absorbing state.

For μ_1:
$$\mu_1 = 1 + (p_{11}\mu_1) + (p_{12}\mu_2) + (p_{13}\mu_3)$$
$$= 1 + (0.2\cdot\mu_1) + (0.7\cdot\mu_2) + (0.1\cdot 0)$$

For μ_2:
$$\mu_2 = 1 + (p_{21}\mu_1) + (p_{22}\mu_2) + (p_{23}\mu_3)$$
$$= 1 + (0.2\mu_1) + (0.5\mu_2) + (0.3\cdot 0)$$

So we get a system of equations:
$$\mu_1 = 1 + 0.2\mu_1 + 0.7\mu_2$$
$$\mu_2 = 1 + 0.2\mu_1 + 0.5\mu_2$$

Combining like terms and rearranging we get,
$$1 \quad 0.8\mu_1 - 0.7\mu_2 = 1$$
$$2 \quad -0.2\mu_1 + 0.5\mu_2 = 1$$

We will first solve 2 for μ_2
$$0.5\mu_2 = 1 + 0.2\mu_1$$
$$\mu_2 = \frac{1 + 0.2\mu_1}{0.5}$$

Then plug μ_2 into 1: $0.8\mu_1 - 0.7\left(\frac{1+0.2\mu_1}{0.5}\right) = 1$
$$0.8\mu_1 - 1.4(1 + 0.2\mu_1) = 1$$
$$0.8\mu_1 - 1.4 - 0.28\mu_1 = 1$$
$$0.52\mu_1 = 2.4$$
$$\mu_1 \approx 4.615$$

Now plug μ_1 into μ_2, to solve for μ_2: $\mu_2 = \frac{1 + 0.2(4.615)}{0.5} \approx 3.846$

And so we get $\mu_1 \approx 4.615$ and $\mu_2 \approx 3.846$. We observe that the theoretical mean arrival times are consistent with the simulation results we obtained in part a. Thus, confirming the accuracy of our simulation.