## ② b) Find stationary distribution

$\pi_\infty^T = \pi_\infty^T P$ ★ Default: vectors are columns

$0 = \pi_\infty^T (P - I)$

$0^T = (P - I)^T \pi_\infty$

$0 = (P^T - I)\pi_\infty$

$$P^T - I = \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.7 & 0.5 & 0.4 \\ 0.1 & 0.3 & 0.4 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -0.8 & 0.2 & 0.2 \\ 0.7 & -0.5 & 0.4 \\ 0.1 & 0.3 & -0.6 \end{bmatrix}$$

Solve sys. of linear equations (next page)

## ② b) Find stationary distribution

$$\begin{bmatrix} -0.8 & 0.2 & 0.2 \\ 0.7 & -0.5 & 0.4 \\ 0.1 & 0.3 & -0.6 \end{bmatrix} \longrightarrow \begin{bmatrix} -8 & 2 & 2 \\ 7 & -5 & 4 \\ 1 & 3 & -6 \end{bmatrix}$$

$R_2 + R_3 \to R_3'$

$$\begin{bmatrix} -8 & 2 & 2 \\ 7 & -5 & 4 \\ 8 & -2 & -2 \end{bmatrix}$$

$R_3 - R_1 \to R_1'$

$$\begin{bmatrix} 0 & 0 & 0 \\ 7 & -5 & 4 \\ 8 & -2 & -2 \end{bmatrix}$$

$R_3 \div 8 \to R_3'$

$R_3 - 8 \times R_2' \to R_3'$

$$\begin{bmatrix} 0 & 0 & 0 \\ 7 & -5 & 4 \\ 1 & -1/4 & -1/4 \end{bmatrix}$$

$R_2 - 7 \times R_3 \to R_2'$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -13/4 & 23/4 \\ 1 & -1/4 & -1/4 \end{bmatrix}$$

$-\frac{4}{13} R_2 \to R_2$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -23/13 \\ 1 & -1/4 & -1/4 \end{bmatrix}$$

$x_2 - \frac{23}{13} x_3 = 0 \longrightarrow x_2 = \frac{23}{13} x_3$

$x_1 - \frac{1}{4}\left(\frac{23}{13} x_3\right) - \frac{1}{4} x_3 = 0$

$x_1 = \frac{1}{4} x_3 \left(1 + \frac{23}{13}\right) \longrightarrow x_1 = \frac{9}{13} x_3$

$\frac{9}{13} x_3 - \frac{1}{4}\left(\frac{23}{13} x_3\right) - \frac{1}{4} x_3 = 0 \longrightarrow x_3 = \frac{13}{45}$

$x_2 = \frac{23}{45}$, $x_1 = \frac{9}{45}$

$$\pi_\infty^T = \left[\frac{9}{45}, \frac{23}{45}, \frac{13}{45}\right]$$

③ b) $\mu_i = E[T_i]$

$$\mu_1 = 1 + P_{11}\mu_1 + P_{12}\mu_2 + \underbrace{P_{13}\mu_3}_{=0}$$

$$\mu_2 = 1 + P_{21}\mu_1 + P_{22}\mu_2 + P_{23}\mu_3$$

$$\mu_3 = 0$$

---

$$\mu_1 = 1 + 0.2\mu_1 + 0.7\mu_2$$

$$\mu_2 = 1 + 0.2\mu_1 + 0.5\mu_2$$

$$0.8\mu_1 = 1 + 0.7\mu_2$$

$$\mu_1 = 1.25 + 0.875\mu_2$$

$$0.5\mu_2 = 1 + 0.2\mu_1$$

$$\mu_2 = 2 + 0.4\mu_1$$

$$\mu_2 = 2 + 0.4(1.25 + 0.875\mu_2)$$

$$\boxed{\begin{aligned}\mu_1 &= 4.6 \\ \mu_2 &= 3.85\end{aligned}}$$