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① a) $P = \begin{bmatrix} 0.2 & 0.7 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.4 & 0.4 \end{bmatrix}$

② a) $\pi_\infty^T = \pi_\infty^T P$

$(P^T - I)\pi_\infty = 0$

$P^T = \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.7 & 0.5 & 0.4 \\ 0.1 & 0.3 & 0.4 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (I)

$= \begin{bmatrix} -0.8 & 0.2 & 0.2 \\ 0.7 & -0.5 & 0.4 \\ 0.1 & 0.3 & -0.6 \end{bmatrix} \pi_\infty = 0$

$\Rightarrow$

r_1 $\begin{bmatrix} -0.8 & 0.2 & 0.2 \\ 0.7 & -0.5 & 0.4 \\ 0.1 & 0.3 & -0.6 \end{bmatrix}$

Getting it to RREF

$\begin{bmatrix} -0.8 & 0.2 & 0.2 \\ 0.7 & -0.5 & 0.4 \\ 0.8 & -0.2 & -0.2 \end{bmatrix}$ $r_3 + r_2$

$\begin{bmatrix} 0 & 0 & 0 \\ 0.7 & -0.5 & 0.4 \\ 0.8 & -0.2 & -0.2 \end{bmatrix}$ $r_1 + (r_3 + r_2)$

$\begin{bmatrix} 0 & 0 & 0 \\ 7 & -5 & 4 \\ 8 & -2 & -2 \end{bmatrix}$ $\times 10$

$\begin{bmatrix} 0 & 0 & 0 \\ 7 & -5 & 4 \\ 1 & -1/4 & -1/4 \end{bmatrix}$ $\times 1/8$

$-20/4$ $16/4$

-7 $+7/4$ $+7/4$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -13/4 & 23/4 \\ 1 & -1/4 & -1/4 \end{bmatrix}$ $-7r_3$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -23/13 \\ 1 & -1/4 & -1/4 \end{bmatrix}$

$[23/4] / [13/4]$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -23/13 \\ 1 & -1/4 & -1/4 \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi_1 \\ \pi_2 \end{bmatrix}$

$\pi_1 - 23/13\,\pi_2 = 0$ $\quad \pi_1 = 23/13\,\pi_2$

$\pi_0 - 1/4\,\pi_1 - 1/4\,\pi_2 = 0$

$\pi_0 - 1/4(23/13)\pi_2 - 1/4\,\pi_2 = 0$

$\pi_0 - 23/52\,\pi_2 - 13/52\,\pi_2 = 0$

$\pi_0 - 36/52\,\pi_2 = 0$ $\quad \pi_0 = 9/13\,\pi_2$

constraint $\quad 9/13\,\pi_2 + 23/13\,\pi_2 + \pi_2 = 1$

$45/13\,\pi_2 = 1$ $\quad \pi_2 = 13/45$

$\pi_2 = 13/45$ $\quad \pi_1 = 23/45$ $\quad \pi_0 = 9/45$

$$\boxed{\pi_\infty = [9/45,\ 23/45,\ 13/45]}$$

③ b) $\mu_i = 1 + \sum_{j=1}^{3} p_{ij}\mu_j$ $\quad \mu_i = \mathbb{E}[T_i]$ $\quad T_3 = 0$

$\mu_1 = 1 + p_{1\to1}\mu_1 + p_{1\to2}\mu_2$

$\mu_2 = 1 + p_{2\to2}\mu_2 + p_{2\to1}\mu_1$

$\Rightarrow$ $\mu_1 = 1 + 0.2\mu_1 + 0.7\mu_2$ $\quad \mu_1 = 1 + 0.2\mu_1 + 0.7\mu_2$

$\mu_2 = 1 + 0.5\mu_2 + 0.2\mu_1$ $\quad \mu_2 = 1 + 0.2\mu_1 + 0.5\mu_2$

$\mu_1 - \mu_2 = 0.2\mu_2$ $\quad \mu_1 = 1.2\mu_2$

$1.2\mu_2 = 1 + 0.2(1.2\mu_2) + 0.7\mu_2$

$0.5\mu_2 = 1 + 0.24\mu_2$

$0.26\mu_2 = 1$

$$\boxed{\mu_1 = 4.62 \\ \mu_2 = 3.85}$$