

HW 3

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1a

$$P = \begin{bmatrix} 0.2 & 0.7 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.4 & 0.4 \end{bmatrix}$$

$$\pi_1 + \frac{23}{9}\pi_1 + \frac{13}{9}\pi_1 = 1$$

$$\frac{9+23+13}{9}\pi_1 = 1$$

$$\pi_1 = \frac{1}{5}$$

$$\frac{45}{9}\pi_1 = 1$$

2a.

$$\pi \pi^T P = \pi^T \pi$$

$$\Rightarrow [\pi_{0,1} \quad \pi_{0,2} \quad \pi_{0,3}] \begin{bmatrix} 0.2 & 0.7 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.4 & 0.4 \end{bmatrix} = [\pi_{0,1} \quad \pi_{0,2} \quad \pi_{0,3}]$$

$$0.2\pi_1 + 0.2\pi_2 + 0.2\pi_3 = \pi_1 \quad (3) \quad -0.8\pi_1 + \frac{46}{90}\pi_1 + 0.2\left(\frac{1}{6}\pi_1 + \frac{1}{2} \cdot \frac{23}{9}\pi_1\right)$$

$$0.7\pi_1 + 0.5\pi_2 + 0.4\pi_3 = \pi_2 \quad = -0.8\pi_1 + \frac{46}{90}\pi_1 + \frac{1}{30}\pi_1$$

$$0.1\pi_1 + 0.3\pi_2 + 0.4\pi_3 = \pi_3 \quad + \frac{2}{10} \cdot \frac{1}{2} \cdot \frac{23}{9}\pi_1$$

$$-0.8\pi_1 + 0.2\pi_2 + 0.2\pi_3 = 0$$

$$0.7\pi_1 - 0.5\pi_2 + 0.4\pi_3 = 0$$

$$0.1\pi_1 + 0.3\pi_2 - 0.6\pi_3 = 0$$

$$\frac{-8.9\pi_1 + 46\pi_1 + 3\pi_1 + 23\pi_1}{90} = 0 \checkmark$$

$$0.6\pi_3 = 0.1\pi_1 + 0.3\pi_2$$

$$\Rightarrow \pi_0 = \left[\pi_1 \quad \frac{23}{9}\pi_1 \quad \frac{26}{18}\pi_1 \right]$$

$$\Rightarrow \pi_0 = \left[\frac{1}{5} \quad \frac{23}{45} \quad \frac{13}{45} \right]$$

$$(1) \quad \pi_3 = \frac{1}{6}\pi_1 + \frac{1}{2}\pi_2 = \frac{1}{6}\pi_1 + \frac{1}{2} \cdot \frac{23}{9}\pi_1$$

$$\frac{3\pi_1 + 23\pi_1}{18}$$

$$(2) \quad 0.7\pi_1 - 0.5\pi_2 + 0.4\left(\frac{1}{6}\pi_1 + \frac{1}{2}\pi_2\right) = 0$$

$$\frac{26}{18}\pi_1$$

$$0.7\pi_1 - 0.5\pi_2 + \frac{1}{15}\pi_1 + \frac{1}{5}\pi_2 = 0$$

$$\frac{23}{30}\pi_1 - 0.3\pi_2 = 0$$

$$\pi_2 = \frac{10}{3} \cdot \frac{23}{30}\pi_1 = \frac{23}{9}\pi_1$$

2a cont.

We notice that:

$$P^T \vec{v} = \lambda \vec{v} \quad \text{where } \lambda \in \mathbb{R}, \vec{v} \in \mathbb{R}^3$$

\vec{v} is eigenvector of P^T .

if we let $\lambda = 1$, $\vec{v} = \pi_{\infty}$

$$P^T \vec{v} = 1 \cdot \vec{v} \Rightarrow P^T \pi_{\infty} = \pi_{\infty}$$

subtract π_{∞} from both sides:

$$P^T \pi_{\infty} - \pi_{\infty} = 0$$

$$\Rightarrow (P^T - I_{(3 \times 3)}) \pi_{\infty} = 0 \quad \square$$

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$$P = \begin{bmatrix} 0.2 & 0.7 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E[T_1] = 1 + 0.2 E[T_1] + 0.7 E[T_2] + 0.1 E[T_3]$$

$$E[T_2] = 1 + 0.2 E[T_1] + 0.5 E[T_2] + 0.3 E[T_3]$$

$$E[T_3] = 0$$

$$\Rightarrow \mu_1 = 1 + 0.2 \mu_1 + 0.7 \mu_2 + 0$$

$$\mu_2 = 1 + 0.2 \mu_1 + 0.5 \mu_2 + 0$$

$$\mu_3 = 0$$

$$\Rightarrow \frac{1}{2} \mu_2 = \frac{1}{5} \mu_1 + 1 \quad \mu_1 = 1 + \frac{1}{5} \mu_1 + \frac{7}{10} \left(\frac{2}{3} \mu_1 + 2 \right)$$

$$\mu_2 = \frac{2}{5} \mu_1 + 2$$

$$\Rightarrow \mu_1 = 1 + \frac{1}{5} \mu_1 + \frac{7}{25} \mu_1 + \frac{7}{5}$$

$$\frac{25-5-7}{25} \mu_1 = \frac{12}{5}$$

$$\frac{13}{25} \mu_1 = \frac{12}{5} \cdot \frac{25}{13} = \frac{60}{13}$$

$$\Rightarrow \mu_2 = \frac{24}{13} + 2 = \frac{50}{13}$$