Transmission Lines and Electrical Solitons

Advanced Laboratory

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1 Introduction

In recent years, the study of nonlinear phenomena has become popular in physics. One particular aspect of nonlinear science that has attracted a lot of attention is that of the soliton, which is essentially a waveform that propagates through a nonlinear and dispersive medium without its shape being affected.

The concept of the soliton originated in the 1830's due to an engineer named John Scott Russell. His discovery occurred when he noticed that a wave coming from the front of a boat being pulled down a narrow channel was continuing to propagate down the channel without dispersing. He was amazed by this sight and followed the moving wave for a while on his horse. He found that the wave propagated for a very long distance before he finally lost it. Russell's curiousity drove him to perform many experiments to determine the nature of such waves, which he called *solitary waves*. His study of the properties of the solitary wave were useful to mathematicians who later studied the mathematical nature of solutions to wave equations that didn't change shape. This concept and the types of physical phenomena that fit this discription are now referred to collectively as *solitons*.

Today, the study of solitons has been extended to many more areas of physics than water waves. Solitons can be found in a wide range of areas such as optics, electrical transmission, and various other media. On the mathematical end, the study of solitons is now very developed. A mathematical definition of a soliton is now in use and the various wave equations which emit soliton solitions have been generalized.

The purpose of this experiment is to study solitons that occur in electrical transmission lines. These type of solitons are commonly refered to as *electrical solitons*. These are the simplest type of solitons to understand: the theory here is more simple than for optical solitions or water waves. Also, the fact that this experiment uses electronics, it is more experimentally accessible to an undergraduate lab than other types of experiments involving solitons.

In order to understand these solitons, it is necessary to first understand the nature of the medium through which they will be propagating. Thus, it will first be necessary to study the general theory of electrical transmission lines, in the next section. From this development, a physical understanding for the cause and nature of an electronic soliton (and solitons in general) will become possible. Through this development also, some of the basics of the mathematical understanding of solitons will become clear, and the connection between the physical and the mathematical concepts of the soliton can be made. After a basic theoretical understanding is developed, experiments can then be performed using a specially made model of a transmission line that is particularly suited to soliton experiments. With this experimental device, the concepts introduced can be reproduced and the nature of solitons can be explored.

There currently exist many books and papers written about solitons of all types. While many are rather high-level and/or do not apply to the type of solitons that we are studying, there are a few good references that go into much greater depth than this writeup and will be useful for anyone doing this experiment. Some of these references are included at the end of this document. The book by Remoissenet is invaluable for this experiment as it is written at an introductory level and addresses the types of experiments that you will be doing. This book also includes more advanced electronic soliton experiments and discussions of optical and water wave solitons. The book by Drazin and Johnson provides a very mathematical discussion of solitons and will be useful for anyone who wants to understand the math better and finds Remoissenet to be a little lacking in that aspect. Lab copies of both of these books have now been aquired and should be available to anyone doing this experiment.

2 Background

2.1 Transmission lines

In studying transmission lines, there are two concepts that are important for an understanding of solitons: dispersion and nonlinearity. The effects of these two things on waves propagating through a transmission line are different in nature. By starting with the simplest case — a linear nondispersive transmission line — and, one-by-one, including dispersion and nonlinearity, the individual and combined effects of the two can be understood best. When the final case — a nonlinear dispersive transmission line — is understood, the concept of the soliton can be introduced.

A transmission line in general is two wires through which an electronic signal is transmitted. The behavior of an electronic wave or pulse propagating on the transmission line is dependent on the nature of the wires, the interaction of the wires, and the nature of the wave or pulse. The study of these various effects makes up transmission line theory.

2.1.1 Linearity with no dispersion

The simplest case that we will study is the propagation of waves in a linear transmission line that is nondispersive. This situation is represented in Figure 1, where there is a inductance per unit length l of the top wire and a capacitance per unit length c between the two wires. The change in the voltage between the two wires, from x to $x + \mathrm{d}x$ is then approximately

$$V(x) - V(x + \mathrm{d}x) = l\mathrm{d}x\frac{\partial i(x)}{\partial t}. \qquad (1)$$

The change in the current from x to $x + \mathrm{d}x$ is

$$i(x) - i(x + \mathrm{d}x) = c\mathrm{d}x\frac{\partial v(x)}{\partial t}. \qquad (2)$$

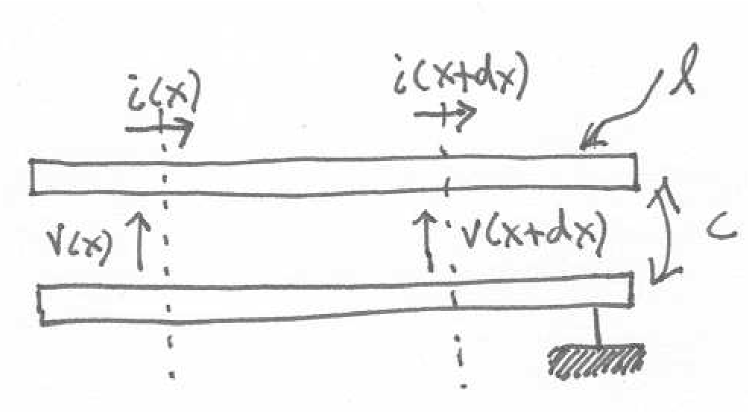

Figure 1: A linear, nondispersive transmission line. This has an inductance per unit length l and a capacitance per unit length c between the wires.

For dx small, these two equations can be written

$$\frac{\partial v}{\partial x} = -l\frac{\partial i}{\partial t}, \tag{3}$$

and

$$\frac{\partial i}{\partial x} = -c\frac{\partial v}{\partial t}, \tag{4}$$

respectively.

By taking the partial derivative of each with respect to x, and then using the new equations to eliminate the partial of i terms, the following equation results,

$$\frac{\partial^2 v}{\partial t^2} - v_0^2\frac{\partial^2 v}{\partial x^2} = 0, \tag{5}$$

with

$$v_0 = \frac{1}{\sqrt{lc}}. \tag{6}$$

Equation 5 is simply a familiar linear wave equation (there is a similar equation in terms of the current). This tells us that a wave will propagate through this particular transmission line linearly, without dispersion, and with a velocity of v_0. Thus, this rather simple case produces wave solutions of the most elementary kind. A wave in this transmission line will propagate through space in the manner shown in Figure 2.

2.1.2 Linearity with dispersion

The next step is to include the effects of dispersion in the transmission line that was just studied. This can be done in various ways (Remoissenet does this by including an inductance per unit length between the two wires, in parallel with the capacitance), but I will discuss here the situation where the capacitance and inductance per unit length in the previous example now becomes discrete elements as shown in Figure 3.

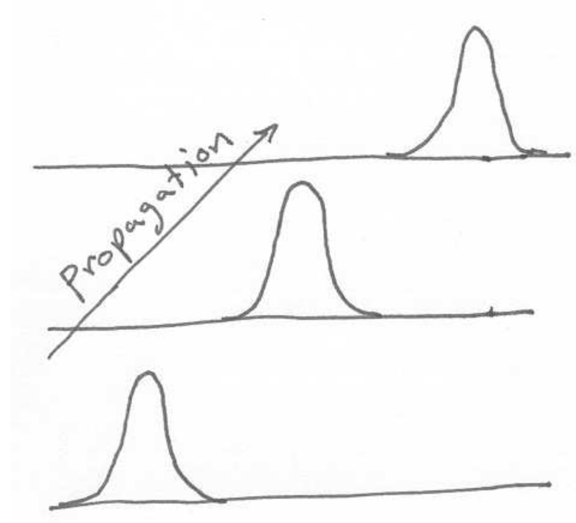

Figure 2: The propagation of a wave in a linear nondispersive transmission line.

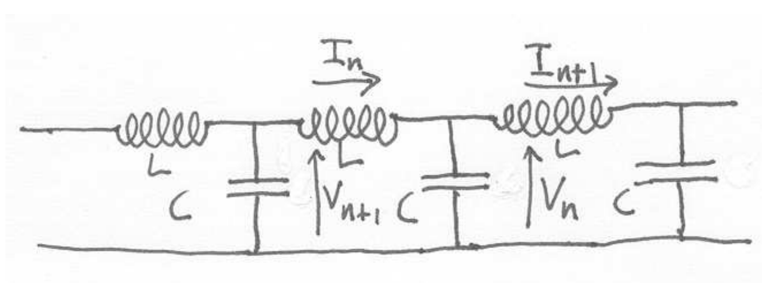

Figure 3: A transmission line similar to the type studied in the previous example, but with the inductance and a capacitance now in discrete elements of L and C.

For this situation, the equations for the change in shunt voltage and shunt current for each element become:

$$V_{n-1} - V_n = L\frac{\mathrm{d}i_n}{\mathrm{d}t} \tag{7}$$

and

$$I_n - I_{n+1} = C\frac{\mathrm{d}V_n}{\mathrm{d}t}. \tag{8}$$

In a manner similar to the previous section, these two equations result in the following wave equation in terms of V,

$$\frac{\mathrm{d}^2V_n}{\mathrm{d}t^2} = \frac{1}{LC}\left(V_{n+1} + V_{n-1} - 2V_n\right), \tag{9}$$

and a similar equation for the current can be found. This equation is not as nice to deal with as the wave equation found previously. Remoissenet inserts a solution of the form

$$V_n(t) = V_0\mathrm{Re}\left[\mathrm{e}^{i(\omega t - \kappa n)}\right], \tag{10}$$

into the wave equation. This results in a dispersion relation of the form

$$\omega^2 = \omega_\mathrm{c}^2 \sin^2\frac{\kappa}{2}, \tag{11}$$

where

$$\omega_\mathrm{c} = \frac{2}{\sqrt{LC}}. \tag{12}$$

This is known as a *dispersion relation*. It indicates that waves of different frequencies will travel with different velocities $v = \omega/\kappa$. This will result in the component frequencies of an arbitrary waveform travelling with different velocities. This causes the waveform to spread out as if propagates, as illustrated in Figure 4.

2.1.3 Nonlinearity with no dispersion

Now, to consider the effects of a nonlinear transmission line, we will begin by working without dispersion again. The nonlinearity is introduced by making the capacitance per unit length no longer simply a constant, but dependent on V,

$$c = c(V). \tag{13}$$

This situation is illustrated in Figure 5.

The equations for this transmission line, found like those in the first example, are

$$\frac{\partial v}{\partial x} = -l\frac{\partial i}{\partial t}, \tag{14}$$

and

$$\frac{\partial i}{\partial x} = -c(V)\frac{\partial v}{\partial t}, \tag{15}$$

the only difference being the change in c. Expanding the capacitance in V,

$$c(V) = C_0\left(1 + a_1V + a_2V^2 + \dots\right), \tag{16}$$

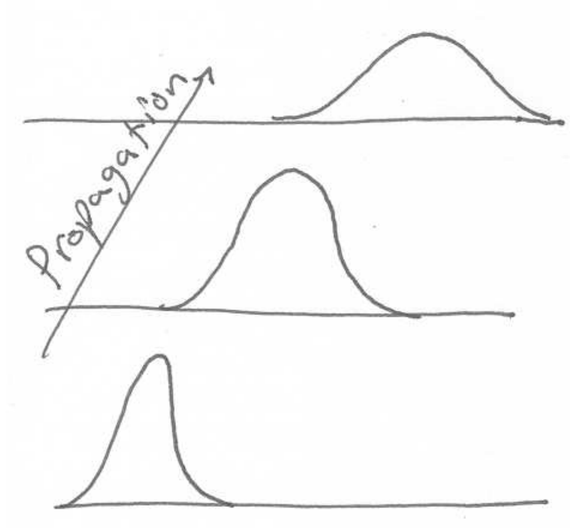

Figure 4: The propagation of a wave in a linear dispersive transmission line. The dispersion of the line results in the spreading out of the waveform.

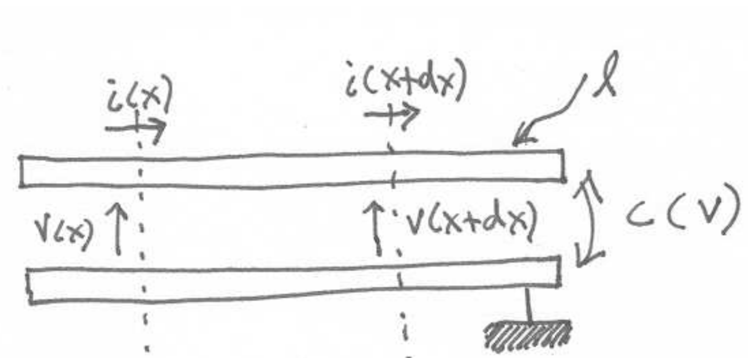

Figure 5: A transmission line similar to the type first studied, but with a capacitance that now depends on V.

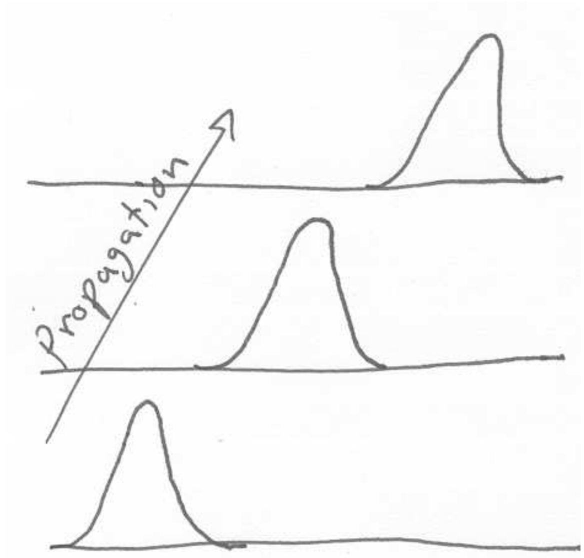

Figure 6: The propagation of a wave in a nonlinear nondispersive transmission line. The nonlinearity introduced causes the higher voltage portions of the waveform to move faster than lower voltage waveforms.

we can simplify things if we consider the case of a small voltage. Then, if only up through the second term is kept, we have

$$c(V) \approx C_0 \left(1 - 2bV\right), \tag{17}$$

where $b = -a_1/2$. By analogy with the case of the linear nondispersive transmission line, the velocity of propagation is

$$v = \frac{1}{\sqrt{lc(V)}} = \frac{1}{\sqrt{lC_0 \left(1 - 2bV\right)}} \approx \frac{1}{\sqrt{lC_0}} \left(1 + bV\right). \tag{18}$$

The result of this is to cause larger amplitude waveforms to travel faster than lower amplitude waveforms. This will cause the type of behavior shown in Figure 6 for propagation of a wave pulse through such a transmission line.

2.1.4 Nonlinearity with dispersion

The final case to consider is the simultaneous effects of both dispersion and nonlinearity in a transmission line. As will discussed later, it is the combination of these two effects that allows for the creation of electrical solitons.

This sitution is described by the diagram shown in Figure 7, where we have a discrete transmission line as in the linear dispersive case, but now with the nonlinear capacitance introduced in the previous section. Setting up the discrete equations, similarly to the linear dispersive case, results in an equation that cannot be solved analytically. In the limit of the continuum, Remoissenet finds a wave equation for this situation that consists of a the terms from a linear nondispersive wave equation (Equation 5, with additional terms: one representing dispersion and the other representing nonlinear effects.

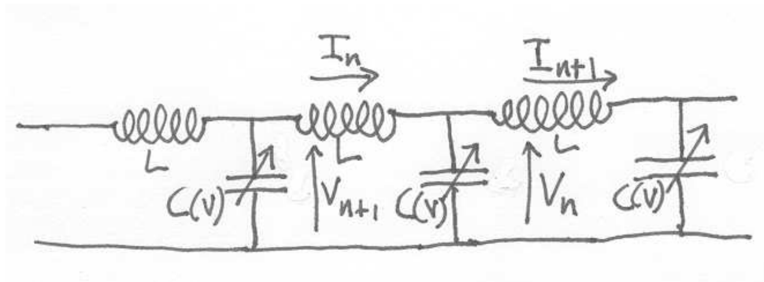

Figure 7: A discrete transmission line (which produces the effect of dispersion), but now with a capacitance that depends on the voltage.

2.2 Electrical solitons

The final case considered in the previous section set the stage for the introduction of electrical solitons. The effects of both nonlinearity and dispersion resulted in the addition of two new terms to the basic wave equation shown in Equation 5. If there is a particular waveform (that is, a solution to the modified wave equation) that causes the effect of the two additional terms to exactly counteract each other, then the waveform will propagate through this nonlinear dispersive transmission line without changing shape. This is precisely what an electrical soliton is: an electronic pulse of a certain shape that allows the effects of the nonlinearity to exactly balance the effects of the dispersion in the transmission line, resulting in the wave propagating through the line unchanged.

This rather physical explanation is very useful for what a physicist wants to understand about solitons. However, mathematicians have developed a theory of solitons that is based on a generalized wave equation called the *Korteweg-de Vries* (KdV) *equation.* The link between the physical understanding of the soliton and the mathematical understanding of the soliton can be made by relating the wave equation found for the nonlinear dispersive case above to the more general KdV equation (see Remoissenet, Appendix 3C). With this understanding, the known solutions to the KdV equation tell us that the form of a soliton for our particular nonlinear dispersive transmission will be that of a squared hyperbolic secant function (this is discussed both in Remoissenet and Drazin & Johnson).

3 Experiments

To experiment with nonlinear transmission lines and solitons, a device made by Dr. Dana Anderson for this purpose can be used. This device, called a *soliton propagator* is the same as the diagram in Figure 7, consisting of 100 discrete elements, each with an inductor and a nonlinear capacitor. By conducting various experiments with the soliton propagator, the concepts of transmission lines and solitons can be explored. Below are some recommended experiments. **Important: only use the soliton propagator with positive input voltages of less thatn 15 volts.**

3.1 Continuous wave experiments

The first type of experiments do not deal with solitons, but rather with the essentials of transmission lines. By simply inputting AC voltages of various frequencies and amplitudes, the response of the soliton propagator to these waves can be determined.

First, measure the propagation velocity of sine waves as a function of frequency. Use the function generator to generate the sine waves, and vary their frequency. A good range of frequencies is from 0 to 10 or 20 MHz. To measure the velocity, use the probe to find the phase shift of the waves at the location of various elements in the soliton propagator. Then, measure the distance between the elements and use that for the distance along the transmission line. This will allow a measurement of the velocity vs. frequency.

Try this for various amplitudes of the input waves. For small enough input amplitudes (< 0.1 V p-p) the behavior of the soliton propagator will be sufficiently linear and the effects of dispersion alone can be explored. When in the low-amplitude regime, sine waves of a large enough amplitude will be longer than the distance between elements in the propagator, and the dispersion relation in this region should be linear. As the wavelength is made shorter and becomes of the order of the element spacing in the propagator, the dispersion effects discussed in the linear dispersive case above should take effect, making the dispersion relation no longer linear.

Measure the velocity of input sine waves as a function of an applied bias voltage. To do this, use the function generator biased with a voltage from the power supply. Measure the phase shift with the probe as described previously. The bias voltage will allow the effects of nonlinearity to become evident. For low bias voltages the propagator should behave linearly, but past a certain point nonlinear effects should take effect. For low frequency sine waves (in the nondispersive regime), record the dependence of the velocity on the DC bias. Be careful not to exceed the 15 V limit of the propagator.

3.2 Pulsed wave experiments

To experiment with solitons, the two HP pulse generators that accompany this experiment can be used. These generators can make triangular-shaped pulses that, althought not perfectly solitons, are close enough to explore soliton behavior with.

To begin with, create a triangular pulse of about 100 ns in length. Then, by varying the pulse height, measure the velocity of these soliton-like waveforms as a function of the pulse height. (Remember to limit the height to less than 15 V.) More qualitatively, explore the behavior of the shape of the soliton as it propagates. Try to find the regimes where you can reproduce the dispersive only behavior (Figure 4), the nonlinearity only behavior (Figure 6), and then include both and try to create a solition for that situation by tuning the shape and size of the pulse so that it doesn't change shape when propagating.

Also, the effect of applying a DC bias to the triangular pulses on the velocity of propagation of the pulses can be explored. E.g. start with a 5V pulse and vary an applied DC bias, measuring the propagation delay as before.

A very interesting effect can be observed by using two soliton-like pulses. By using both HP pulse generators, create a short pulse (about 5V high) and a tall pulse (about 15 V high) that are close together. This can be done by using one HP pulse generator to gate the other.

Then, using the probe, observe the two pulses at various points along the soliton propagator. The higher pulse should move faster, colliding with and then bypassing the smaller pulse. In early work on solitons, the scientists Zabusky and Kruskal showed by numerical simulations that solitons should collide with one another without any interaction. You should be able to qualitatively verify this to some degree.

3.3 Other possible experiments

Supposedly there is a pulse generator in the lab somewhere that allows one to create highly custom pulse shapes by varying the amplitude at various points. With this, the exact waveform of a soliton for the soliton propagator could be much better approximated. By finding the best shape for unaltered propagation, this could then be compared to the theoretical soliton shape. An experiment of this type is discussed in Remoissenet, section 3.4.2.

If you would like to experiment with different type of soliton propagators, Remoissenet provides descriptions of different nonlinear transmission lines such as one in the microwave range and the electrical Toda network.

References

- P. G. Drazin and R. S. Johnson, *Solitons: an introduction* (Cambridge University Press, Cambridge MA, 1989).
- M. Remoissenet, *Waves Called Solitons*, (Springer, New York, 1999).