

# Spatial Visualization: Grids

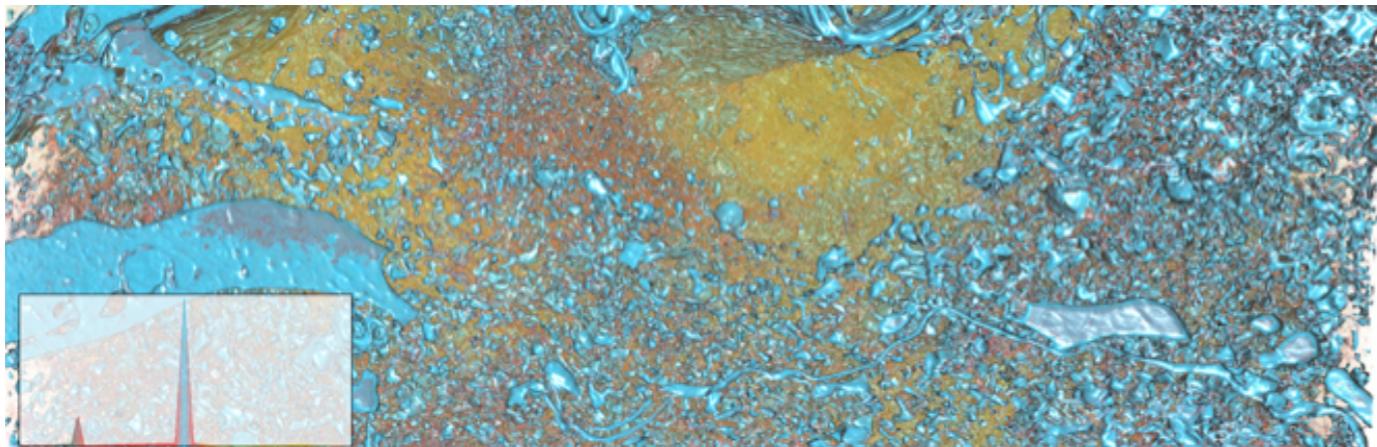
CS 5630/6630, Fall 2016  
Alexander Lex  
Guest Lecturer: Aaron Knoll

# The traditional “branches” of visualization (IEEE Visweek)

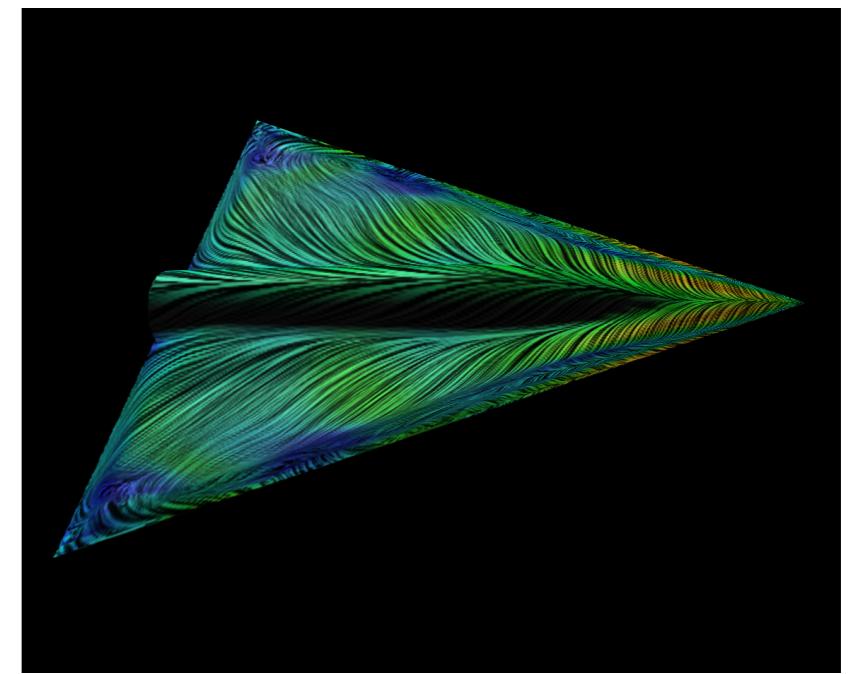
- Scientific (spatial) Visualization
- Information Visualization
- Visual Analytics

# Scientific visualization

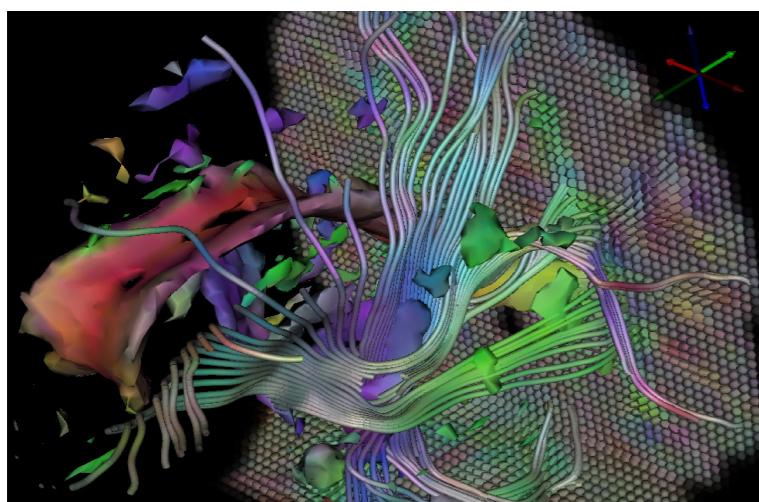
- Data have **spatial** context (usually from simulation or scan)
- Map spatial quantities to colors or geometry,  
 $f(\text{space}, \text{time}) \rightarrow \text{rgba}$
- **2D or 3D graphics for visualization.**



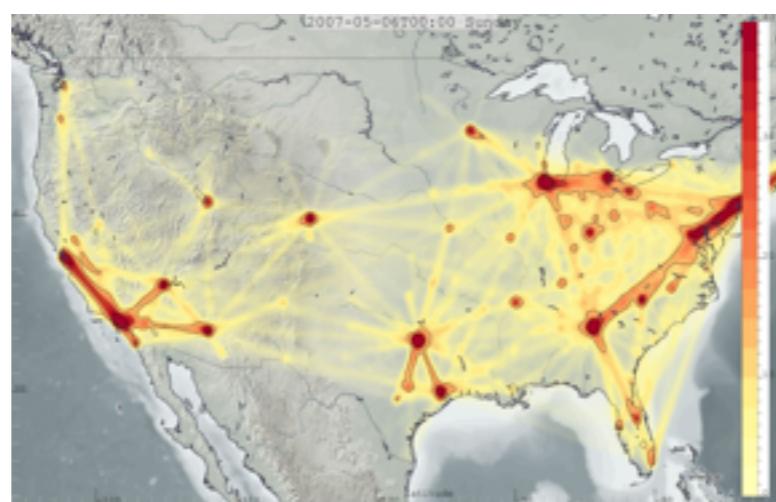
Volume rendering



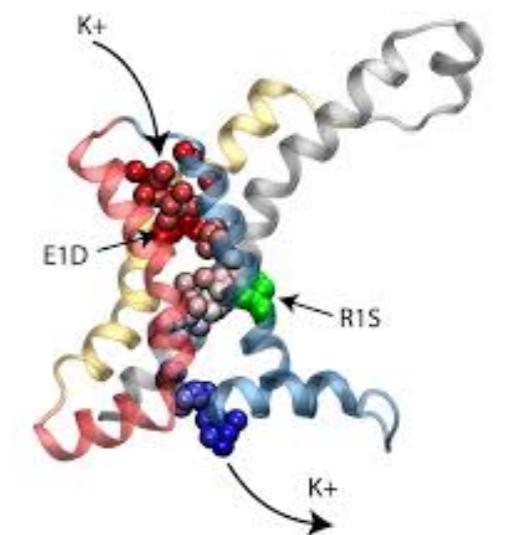
Flow visualization



Tensor field visualization



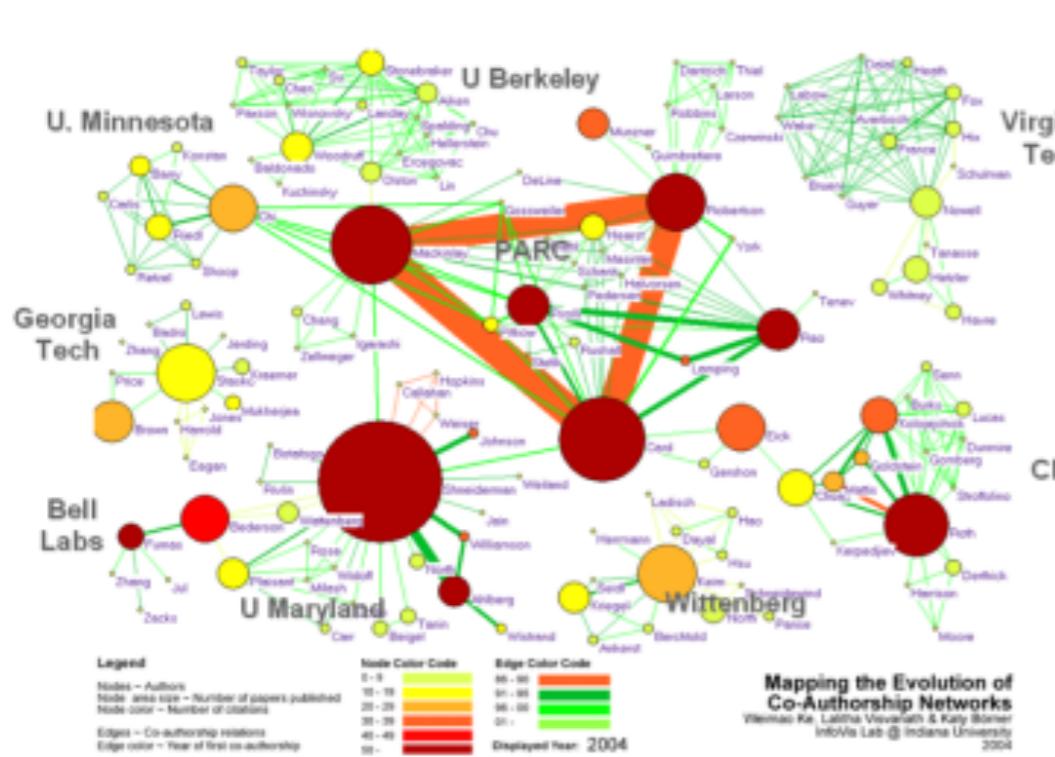
Map and GIS visualization



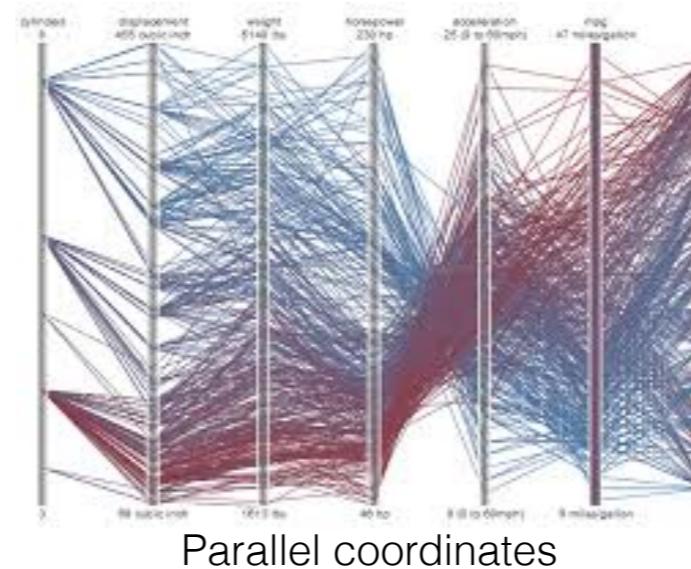
Molecular visualization

# Information visualization

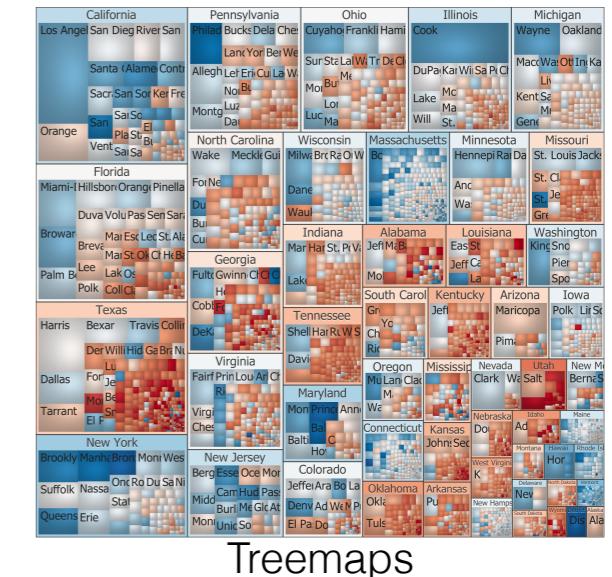
- Spatial position is secondary or non-existent.
- Illustrate relationships between abstract attributes.
- **Plots, charts, graphs, diagrams.**



Graph and network visualization



Parallel coordinates



Treemaps



Scatterplots



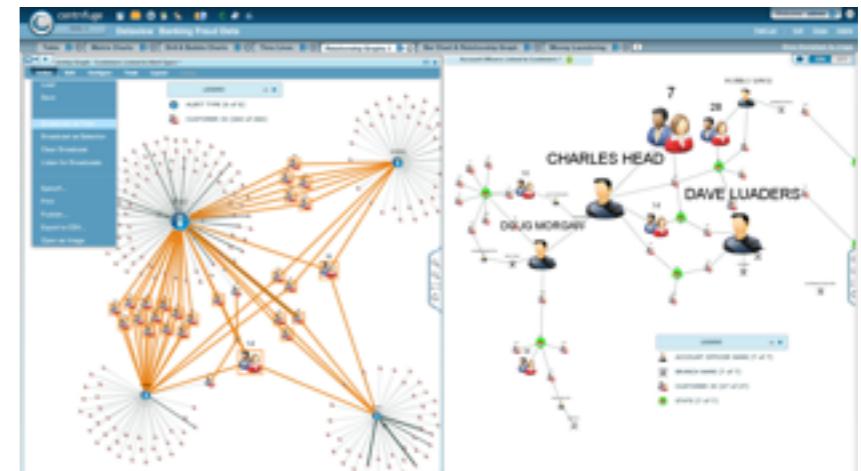
Charts

# Visual Analytics

- More about **interactive user interfaces** for data analysis.
- Uses techniques from both scientific visualization and information visualization, as well as statistics, perception, cognition.
- D3+Javascript, R, Matlab
- “Putting it all together”



Management Information Systems (SAS)



Security visualization (Centrifuge)



Genomics (Meyer et al. "Mizbee")



# Scientific Visualization

- “Sci-vis” is about interpreting and rendering spatial data.
- Today:
  - where do spatial data come from? (science domains)
  - how are they represented (*grids*)
  - what can we do with them (*direct* and *indirect* vis)
  - interpolation
- Tues, Nov 17: Volumes  
Tues, Nov 29: Isosurfaces  
Thurs, Dec 1: Advanced topics: Vector and Tensor Fields

# Scientific Visualization

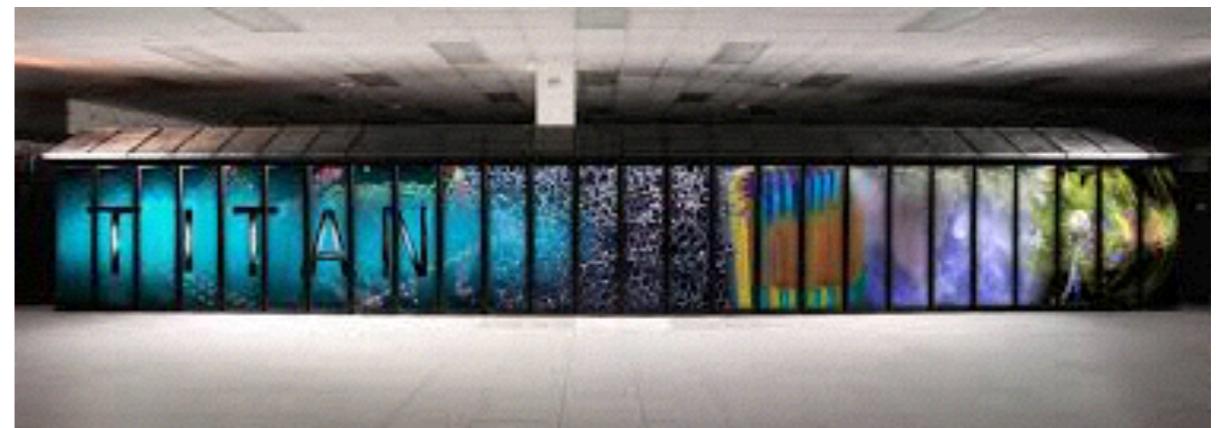
- Data sources
- Data representation
  - fields
  - grids
- Data interpretation
  - The scientific visualization pipeline
  - Interpolation

# Data sources

# Computational Data

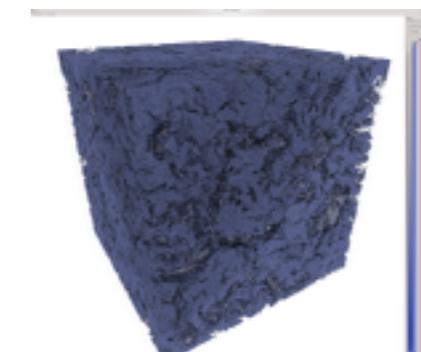
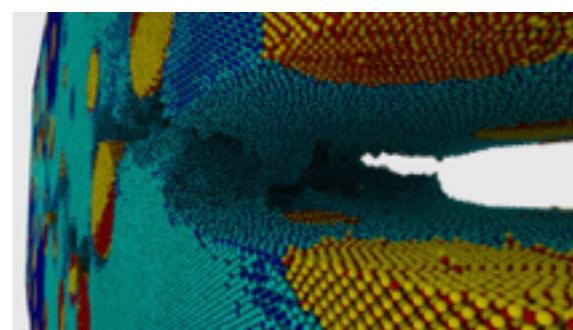
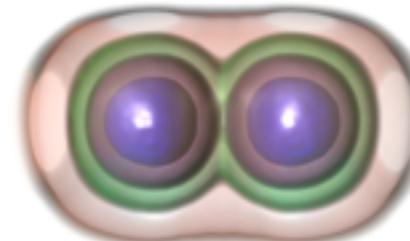
## The output of scientific computing:

physics, chemistry, blood flow,  
neurophysiology, meteorology,  
climatology, astronomy...

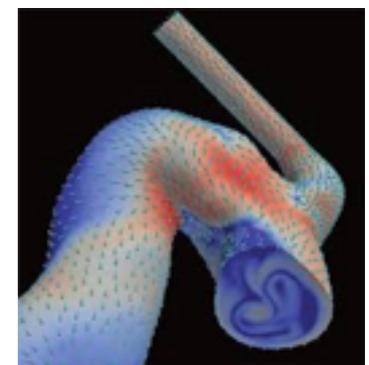


- Nuclear physics
- Quantum chemistry
- Molecular dynamics
- Computational fluid dynamics
- Rigid-body and structural mechanics
- Coarse-grained dynamics, agents simulations
- Meteorology
- Astrophysics
- Cosmology

kilobytes



petabytes

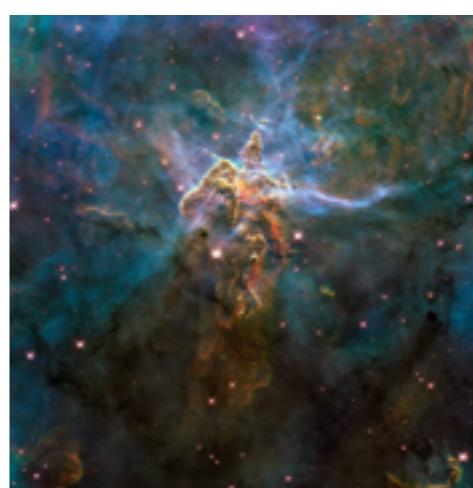


# Scanned data

**The output of instruments** in medical imaging, microscopy, telescropy, GIS

- X-ray crystallography
- Synchrotron / radiation light sources
- Transmission electron microscopy
- Confocal microscopy
- Camera imagery
- Ultrasound
- Magnetic resonance imaging
- X-ray tomography
- Satellite
- Telescope

Angstroms



Megaparsecs



# Data representation (grids)

# Fields

- Mathematically, a *field* is a set of elements with addition, multiplication operators that satisfy the field axioms

name	addition	multiplication
associativity	$(a + b) + c = a + (b + c)$	$(a b) c = a (b c)$
commutativity	$a + b = b + a$	$a b = b a$
distributivity	$a (b + c) = a b + a c$	$(a + b) c = a c + b c$
identity	$a + 0 = a = 0 + a$	$a \cdot 1 = a = 1 \cdot a$
inverses	$a + (-a) = 0 = (-a) + a$	$a a^{-1} = 1 = a^{-1} a \text{ if } a \neq 0$

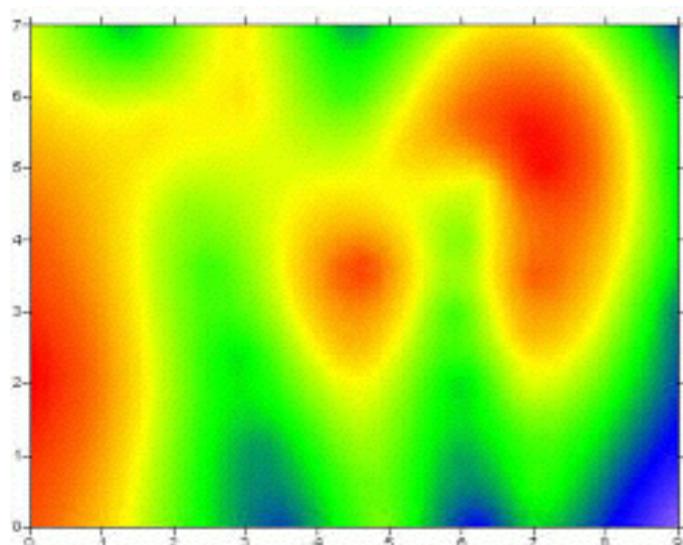
[wolfram.com](http://wolfram.com)

- Intuitively, a field is a varying quantity defined continuously over space.

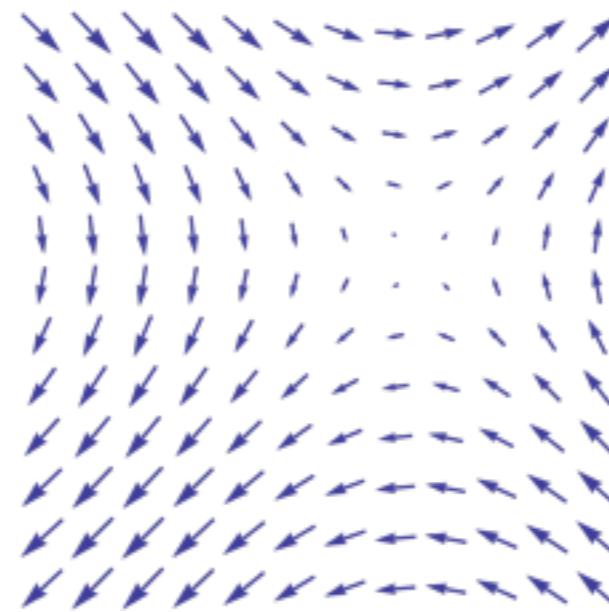
# Fields

with a 2D domain

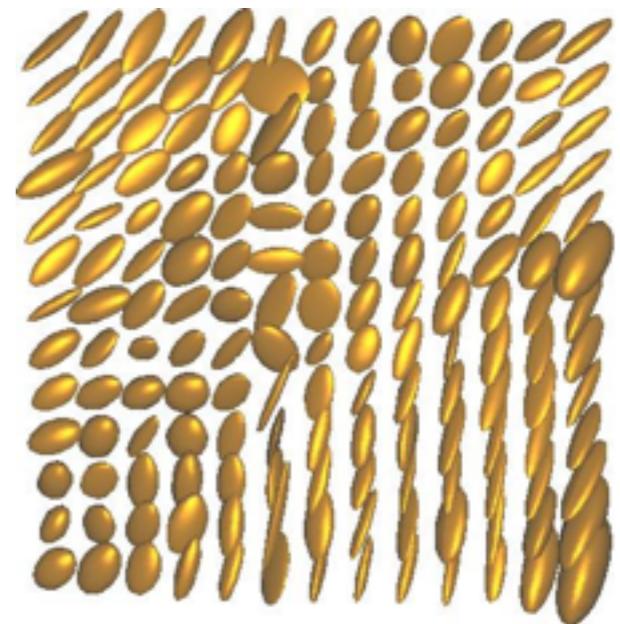
scalar field



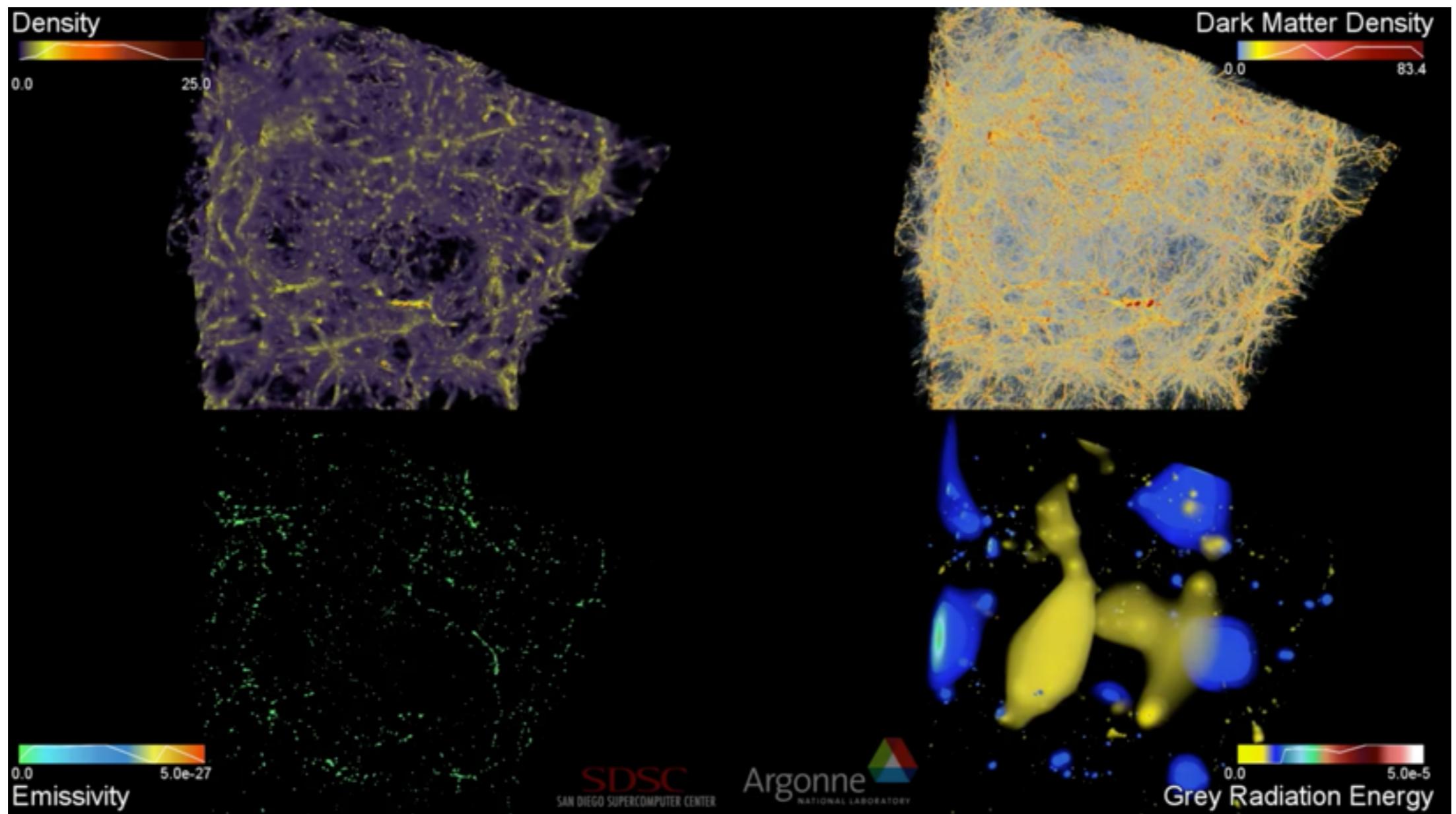
vector field



tensor field

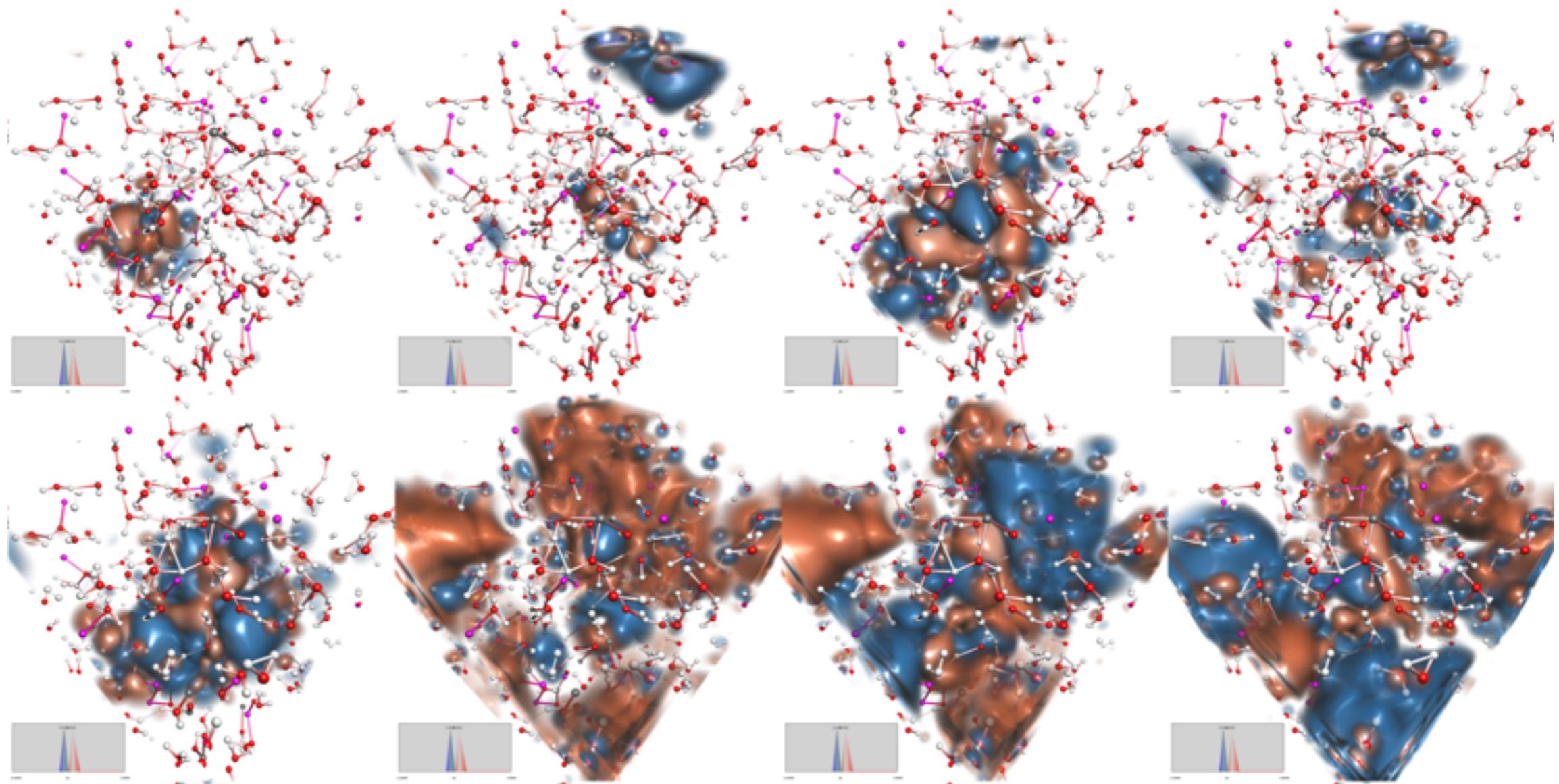


# Multifields



Radiation hydrodynamics in Enzo: Joe Insley (ANL), Rick Wagner (SDSC)  
<https://vimeo.com/17771397>

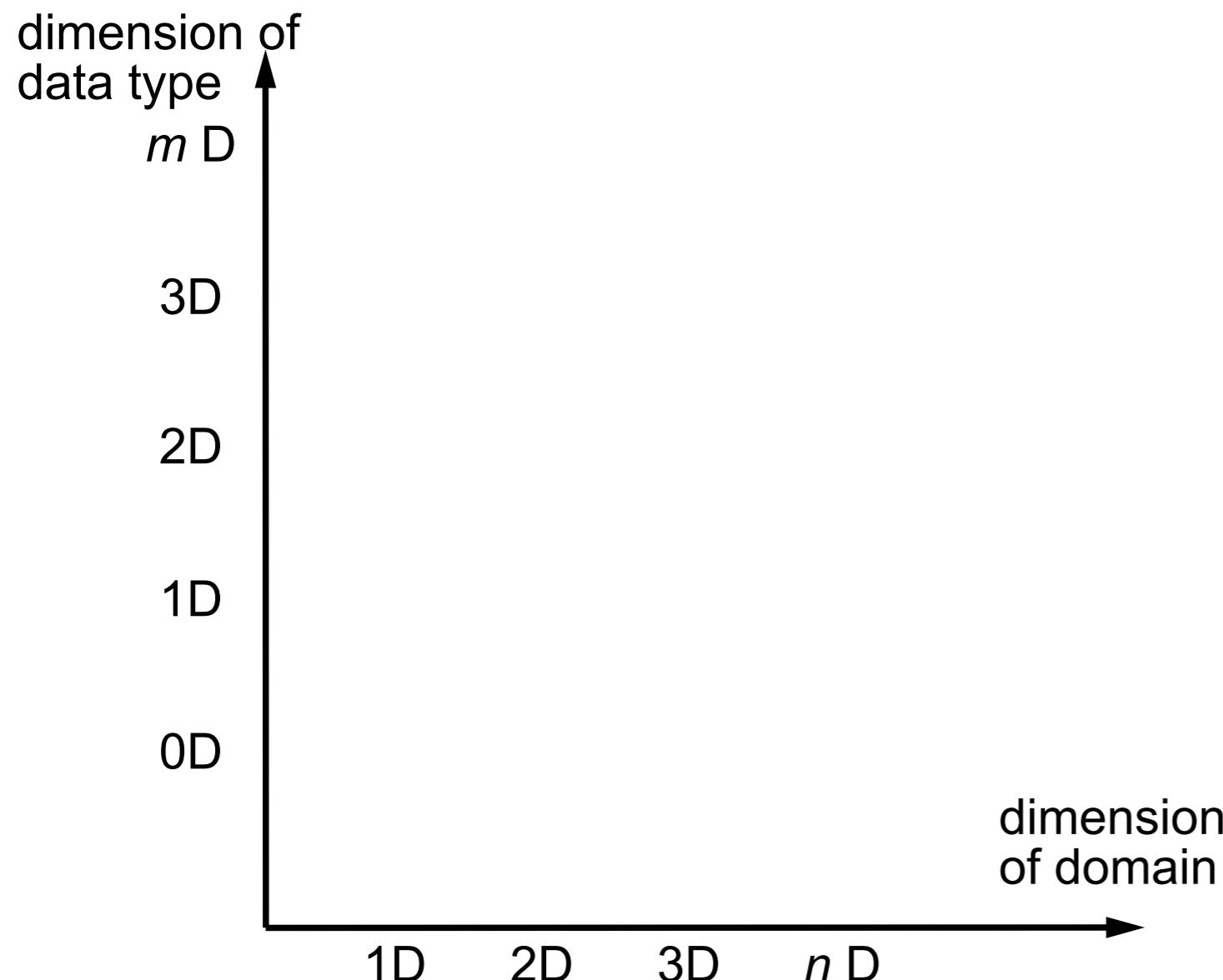
# Multifields



8 molecular orbitals of a LiAlH<sub>2</sub>O DFT simulation, courtesy Aiichiro Nakano, University of Southern California

# Types and Classification of Field Data

- dimension of domain (the field)
- dimension of the data to visualize (the geometry)

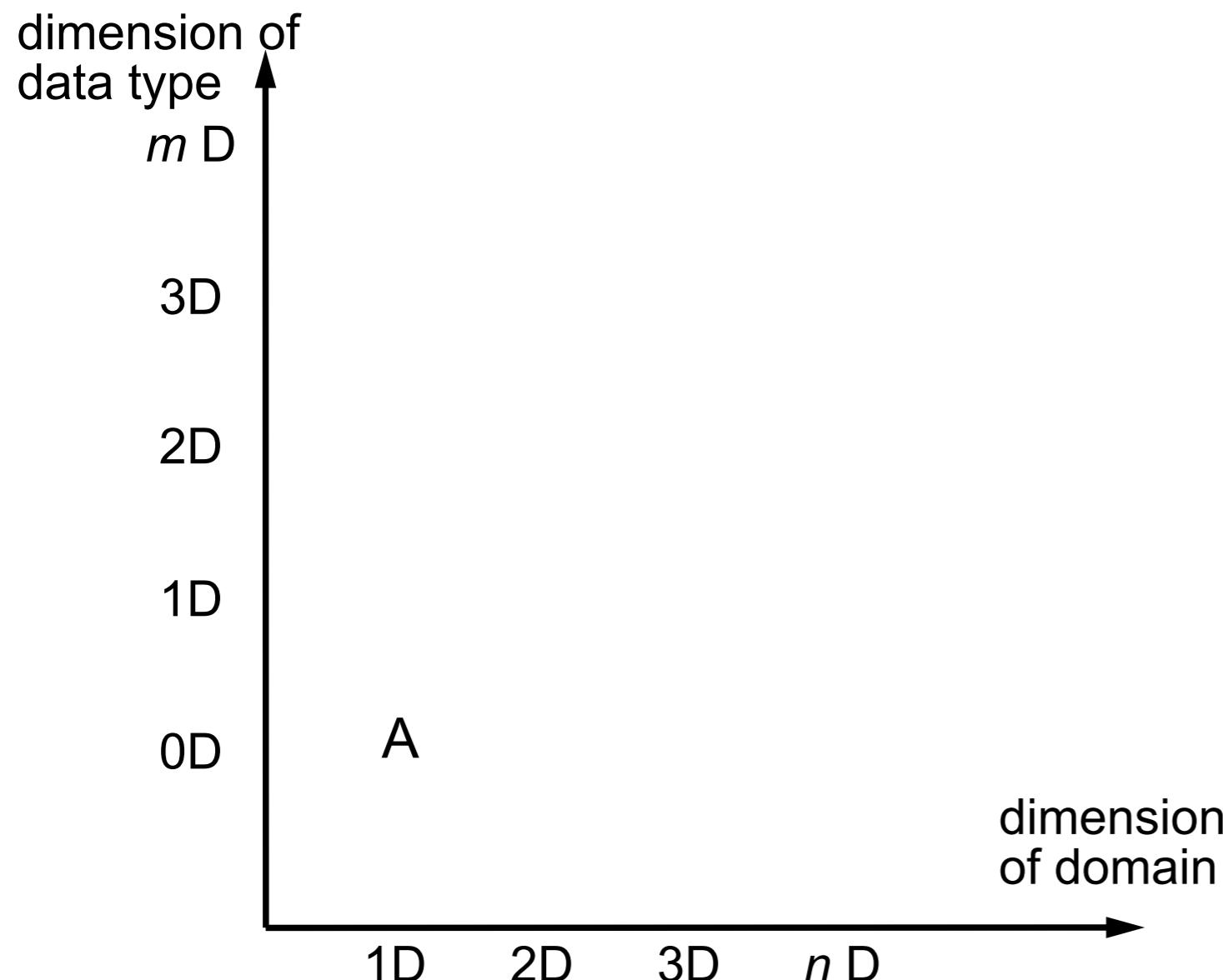


Examples:

- A: gas station along a road
- B: map of cholera in London
- C: temperature along a rod
- D: height field of a continent
- E: 2D air flow
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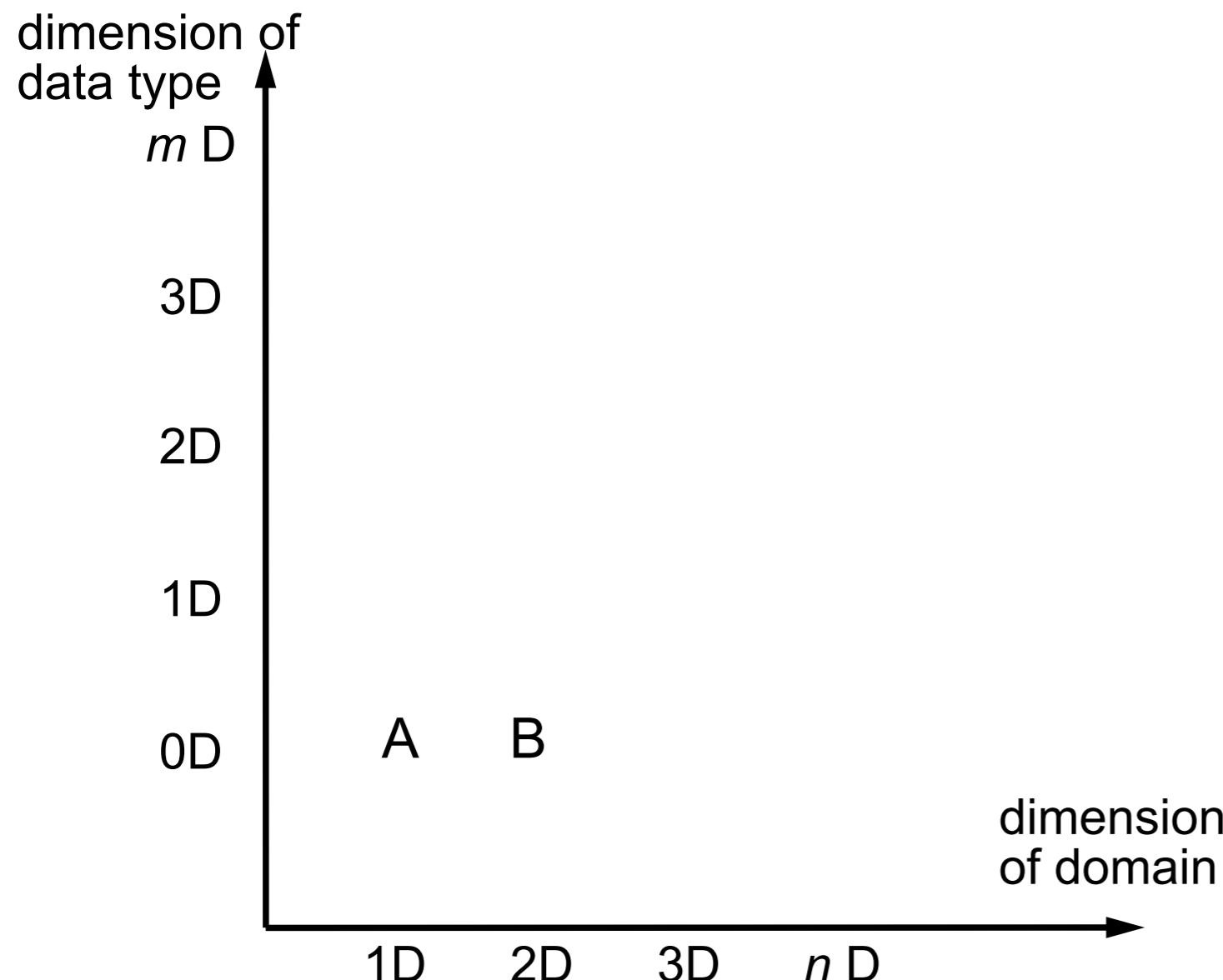


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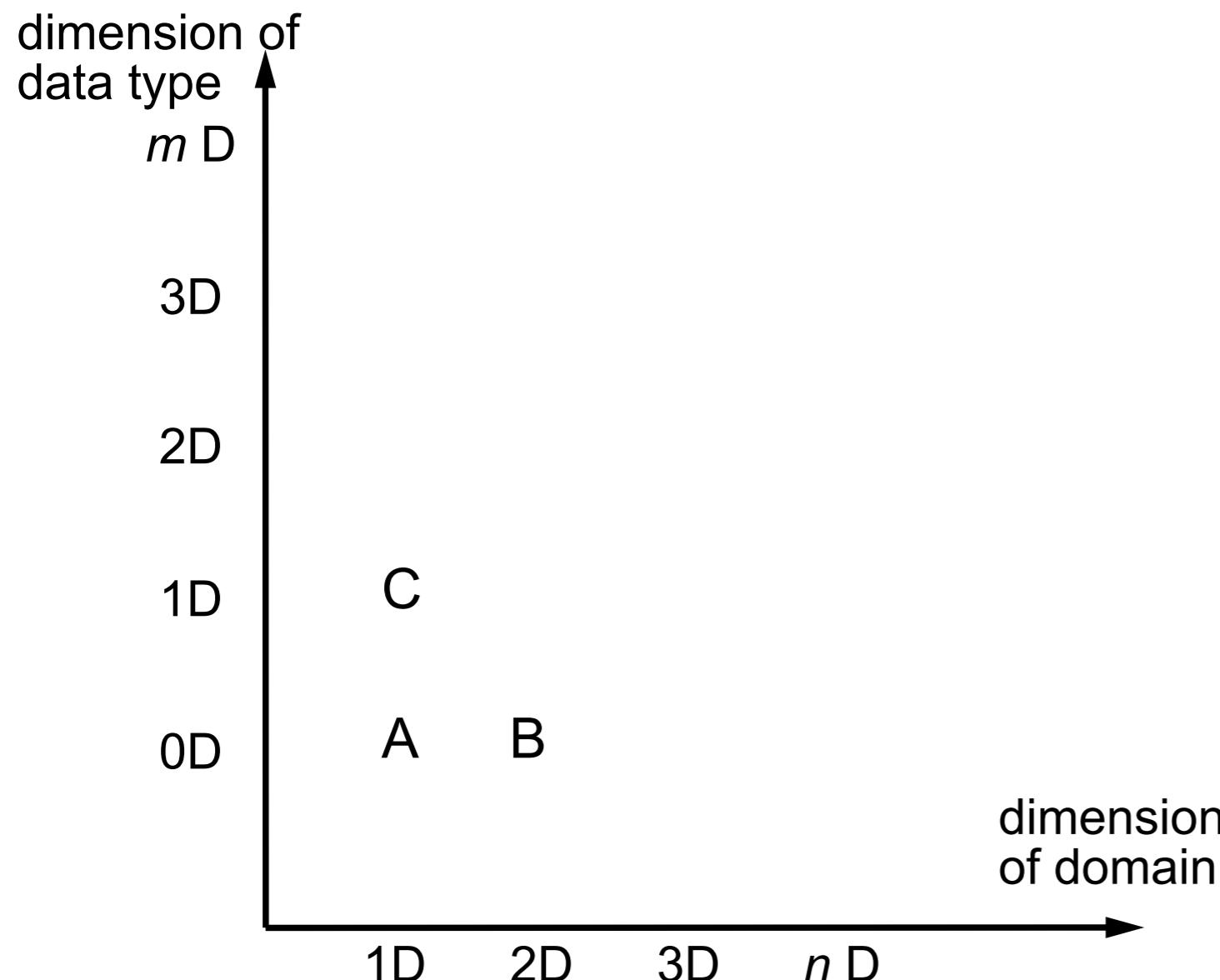


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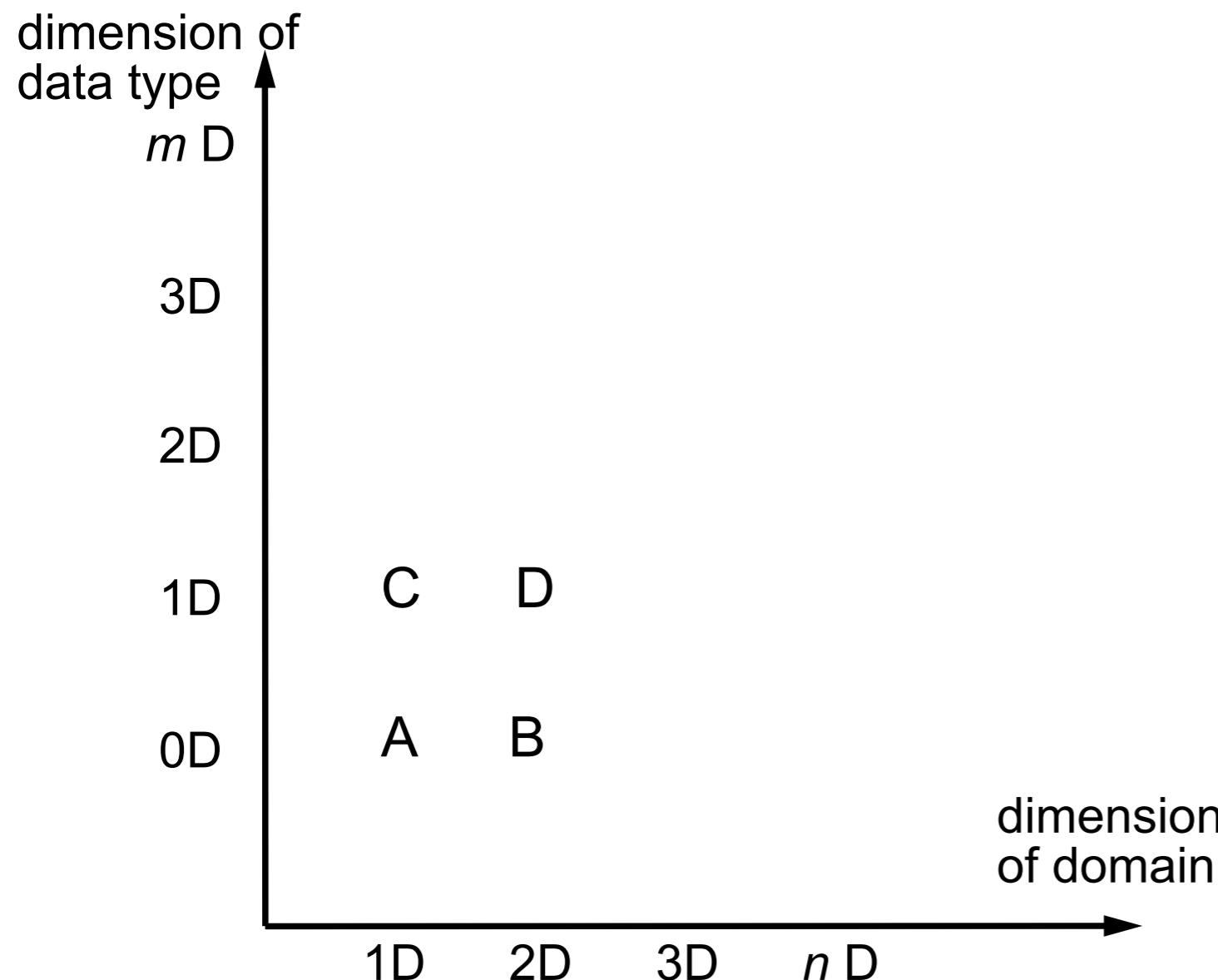


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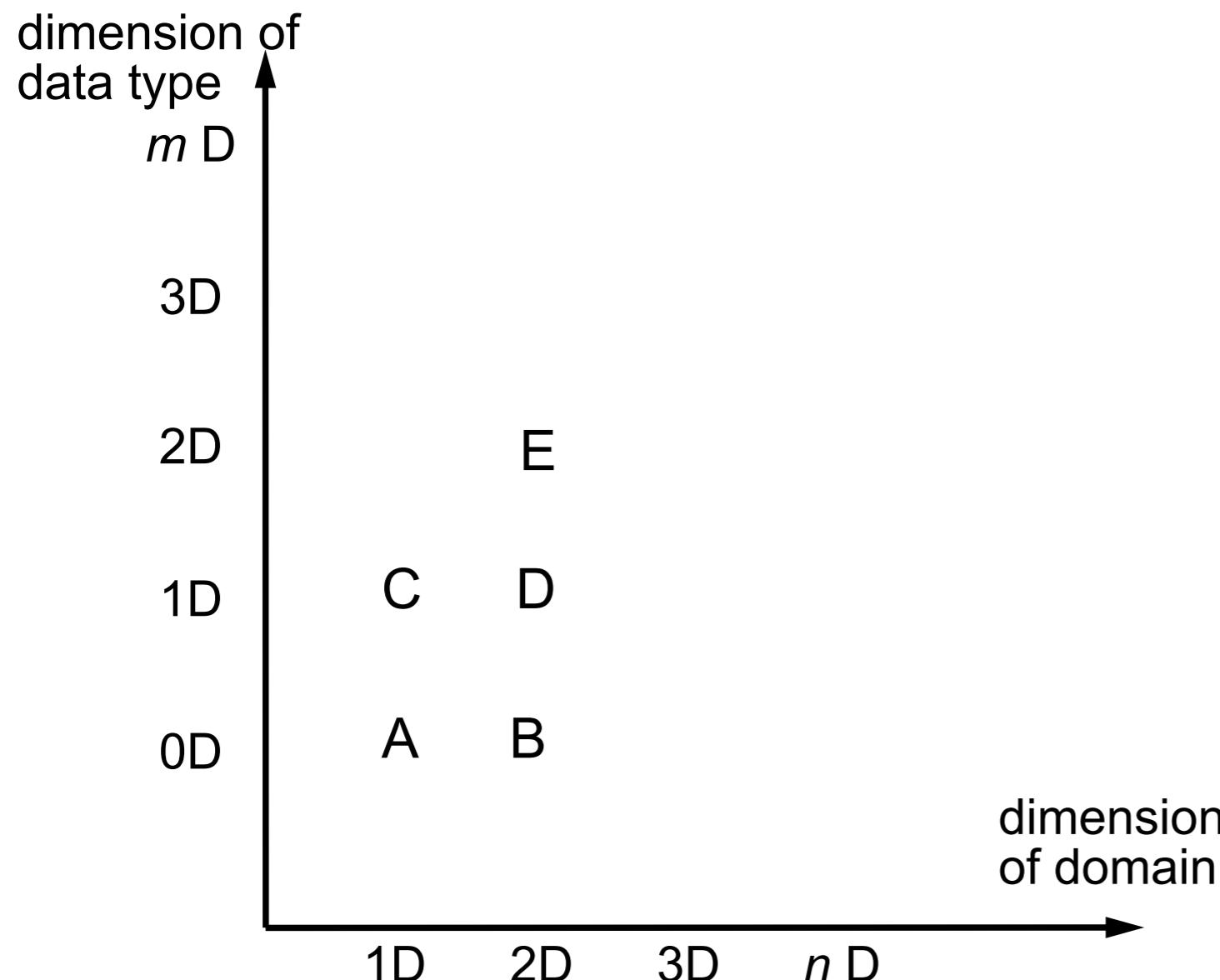


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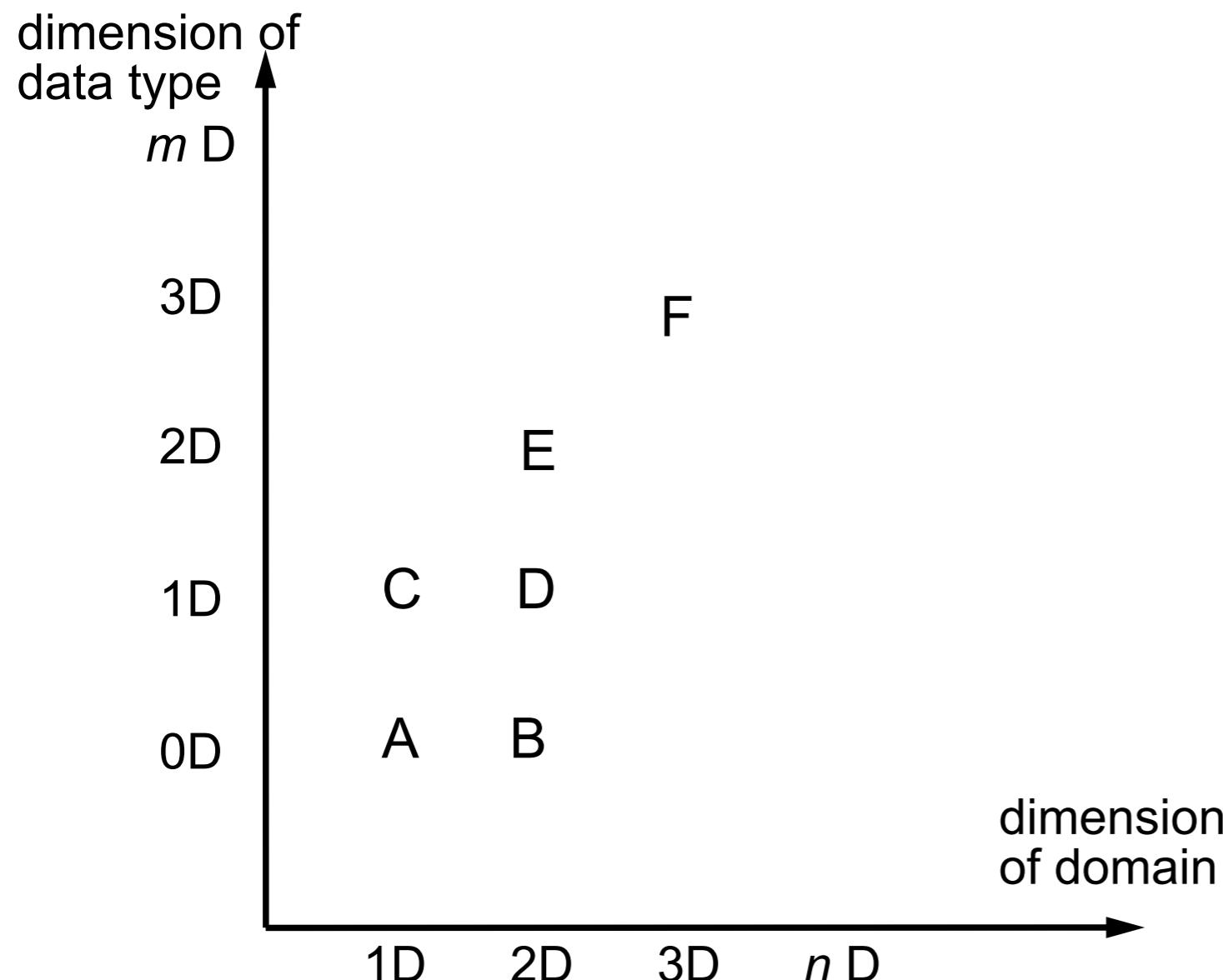


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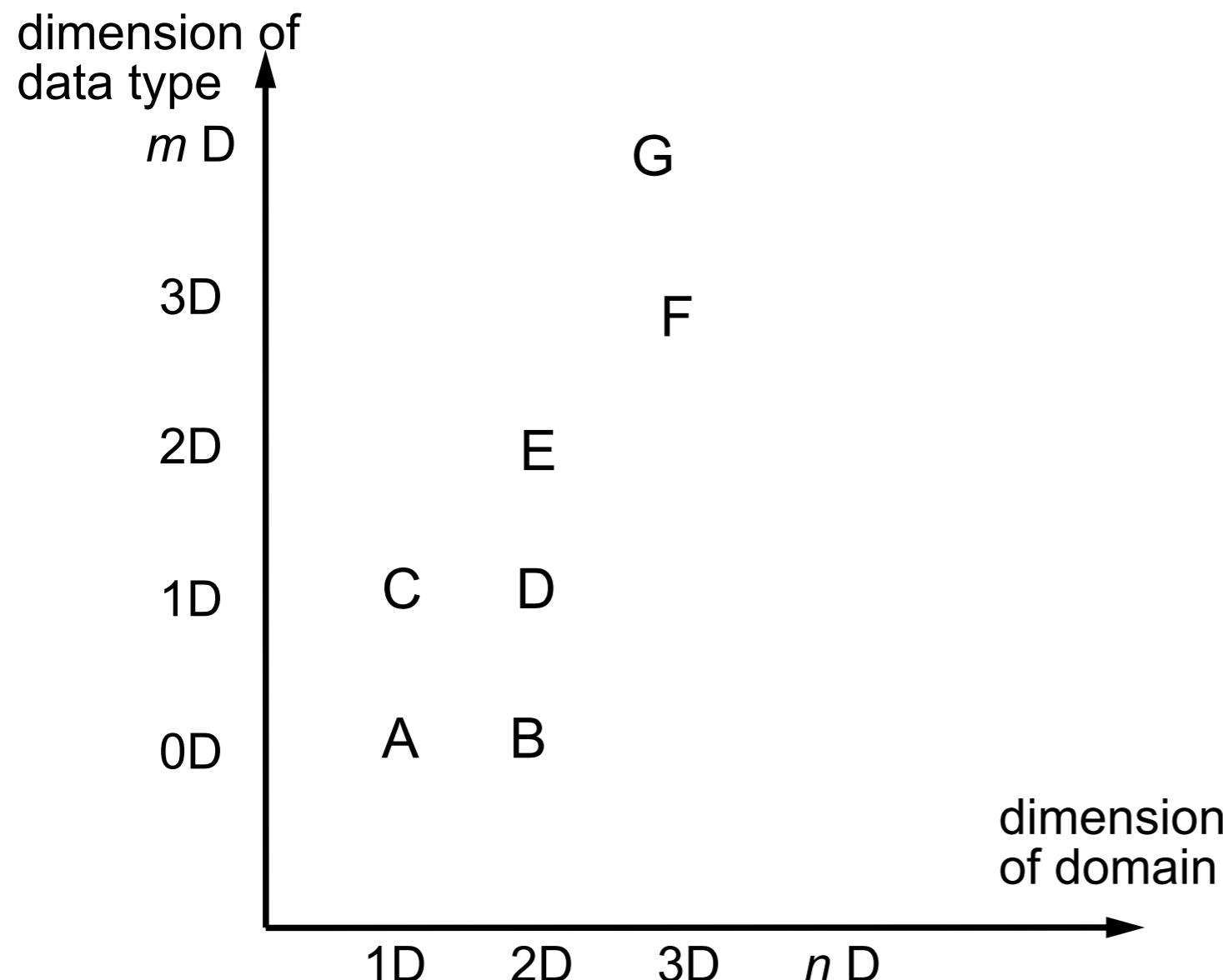


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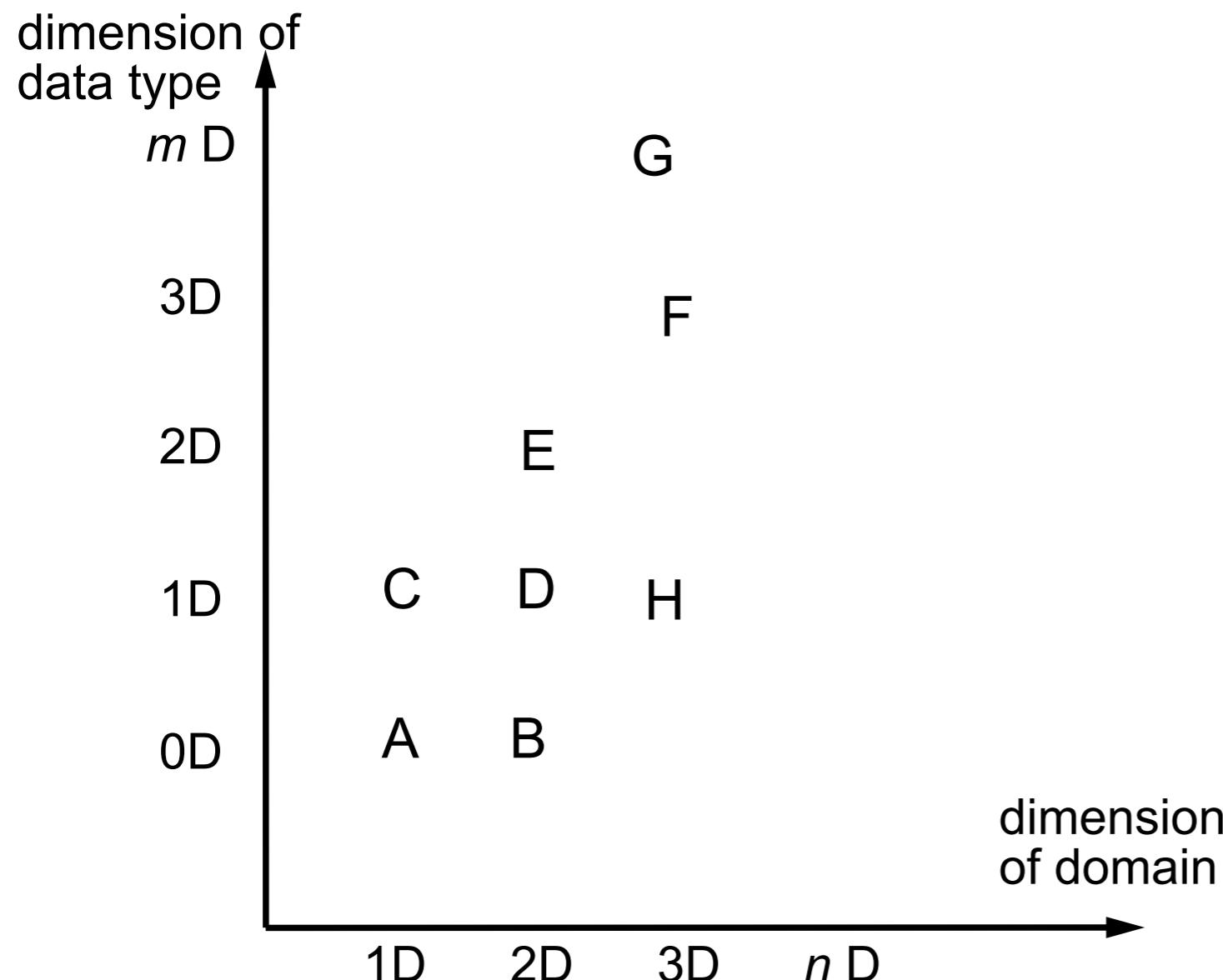


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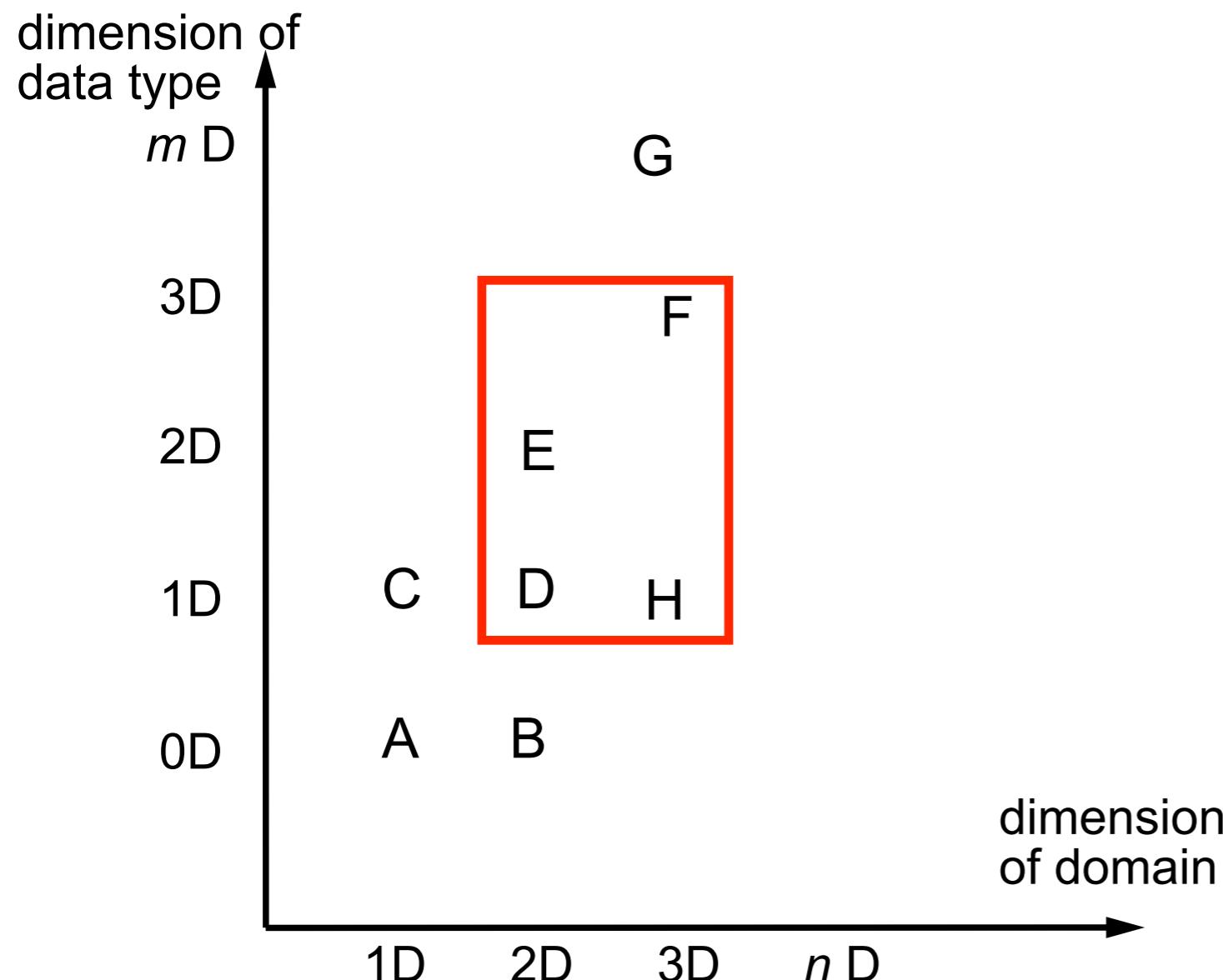


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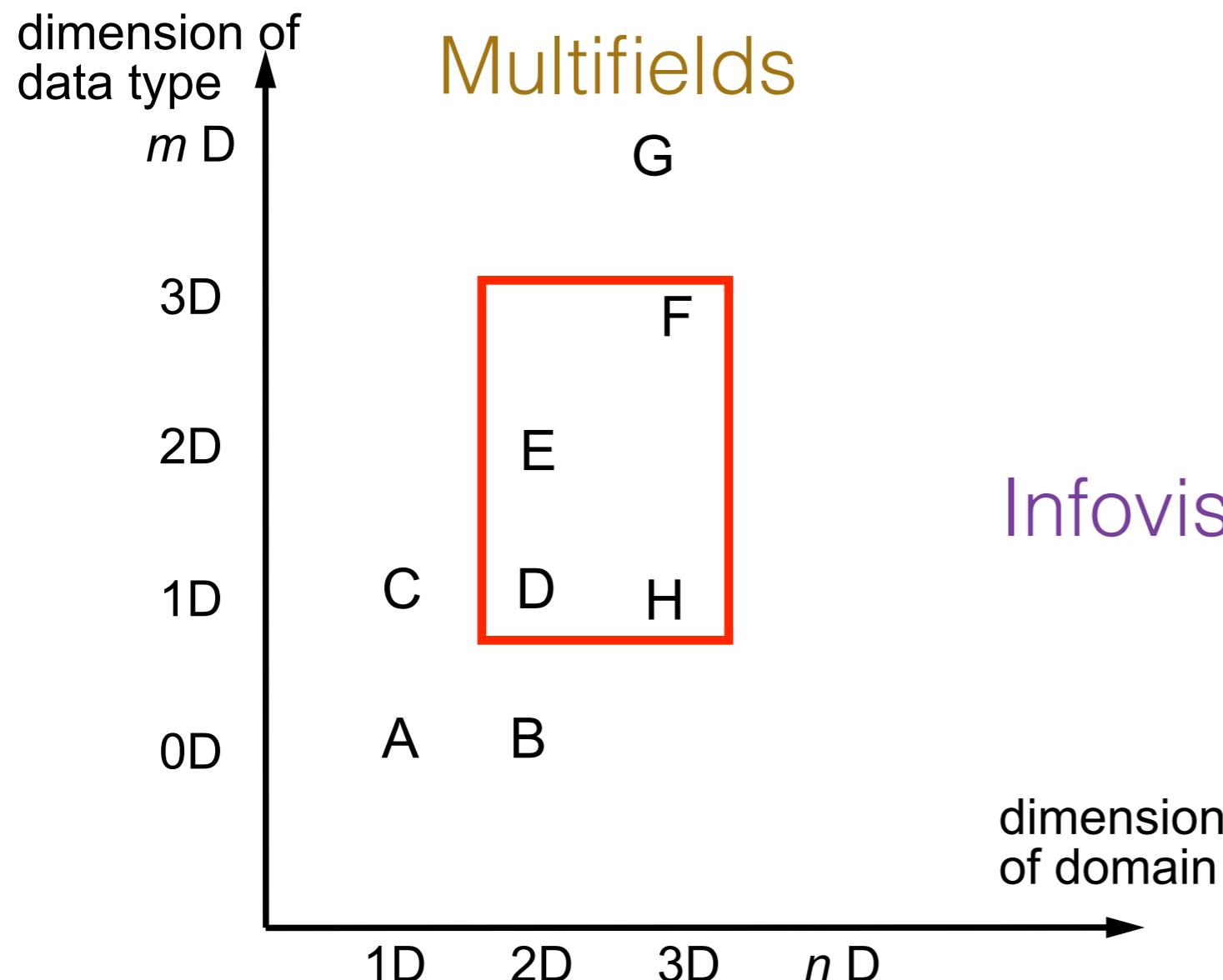


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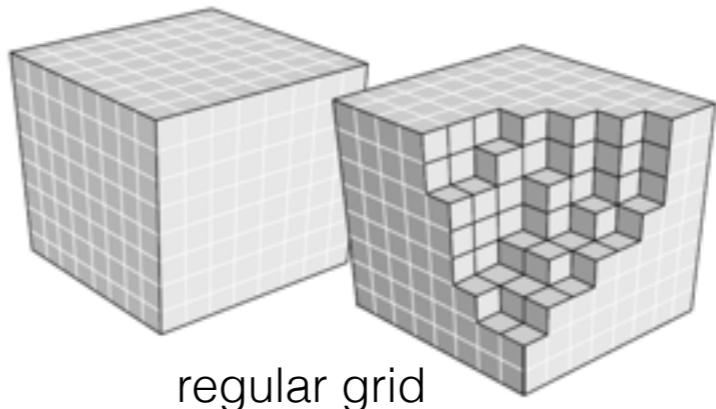


Examples:

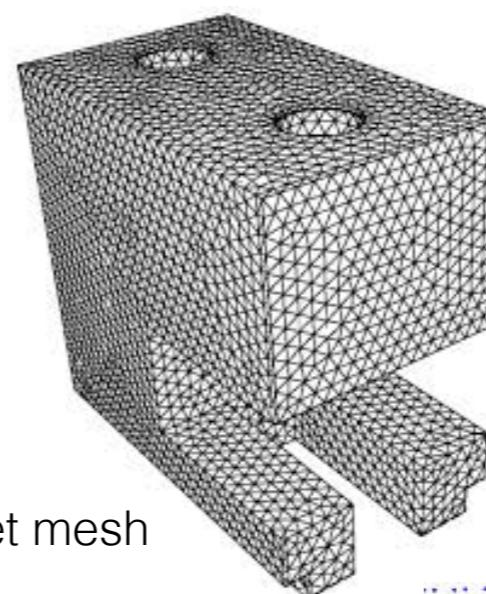
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# Grids

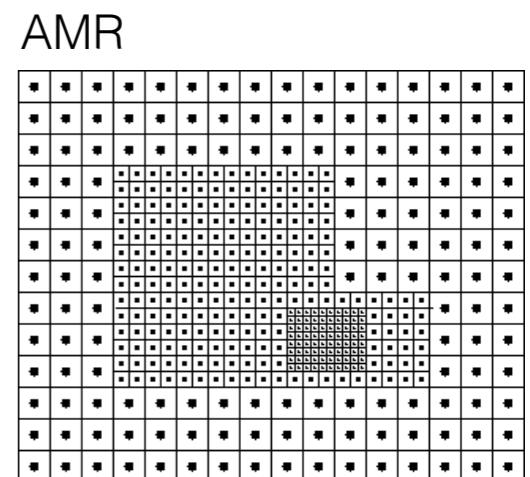
- Continuous fields are an illusion
- All data are discrete
- Meshes are chosen based on what is computationally efficient for the
- Visualization software must implement data models to handle a wide range of field and non-field data



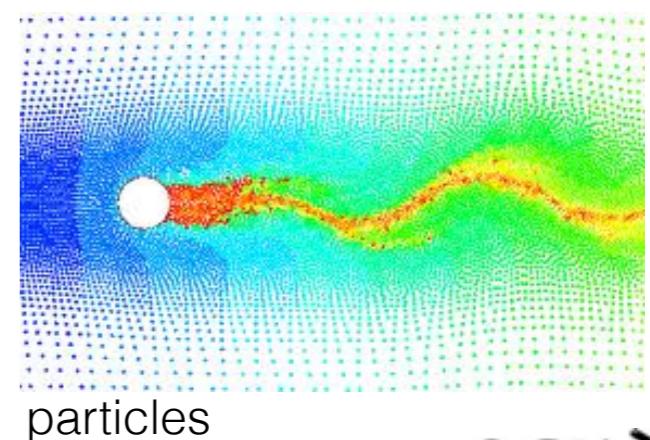
regular grid



tet mesh



AMR



particles

# Structured vs Unstructured

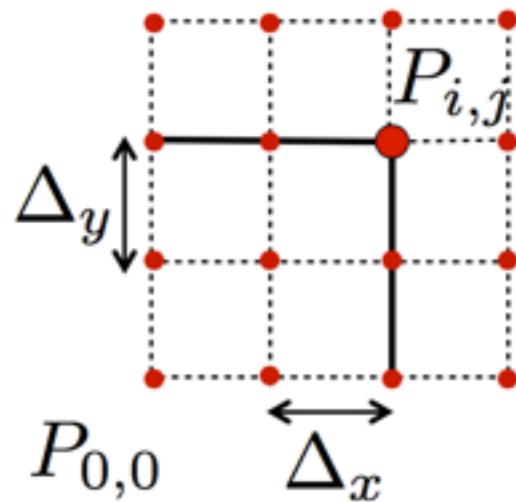
- In general, from the relational database world:
  - **Structured data** are data that are indexed, and can be accessed via a hash, array, or other query.
    - I.e., search time  $O(1)$  or  $O(\log N)$ .
  - **Unstructured data** are not indexed — you have to brute-force search to find them.
    - I.e. search time  $O(N)$
- In information/data visualization:
  - **structured** means data you've already indexed, organized (for example, in D3).
  - **unstructured** is everything else (i.e. text, imagery, video, foo) you have to search through.
- In scientific visualization, this can get a bit confusing...
- First we need to differentiate between geometry and topology.

# Geometry vs Topology

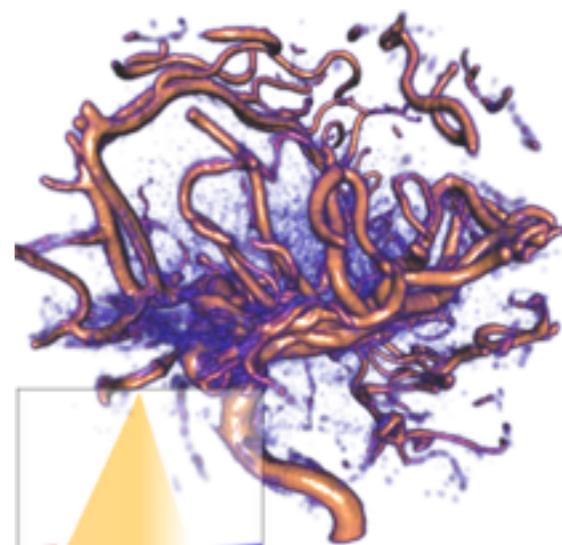
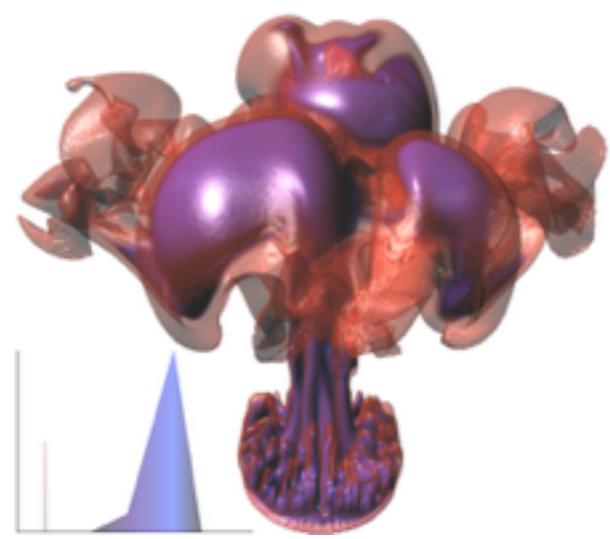
- Geometry
  - Position of vertices in Euclidean space
  - Can be uniform, structured or unstructured.
- Topology
  - Defines the “cells”, or connectivity of the vertices.
  - Can also be structured or unstructured.

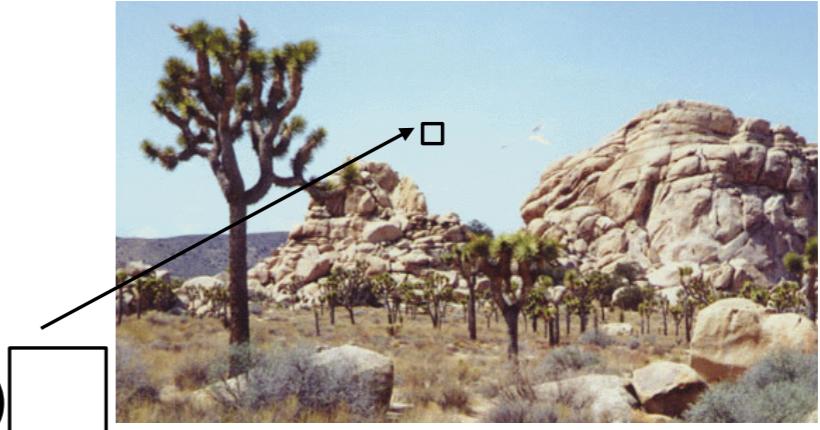
# Uniform grid geometry

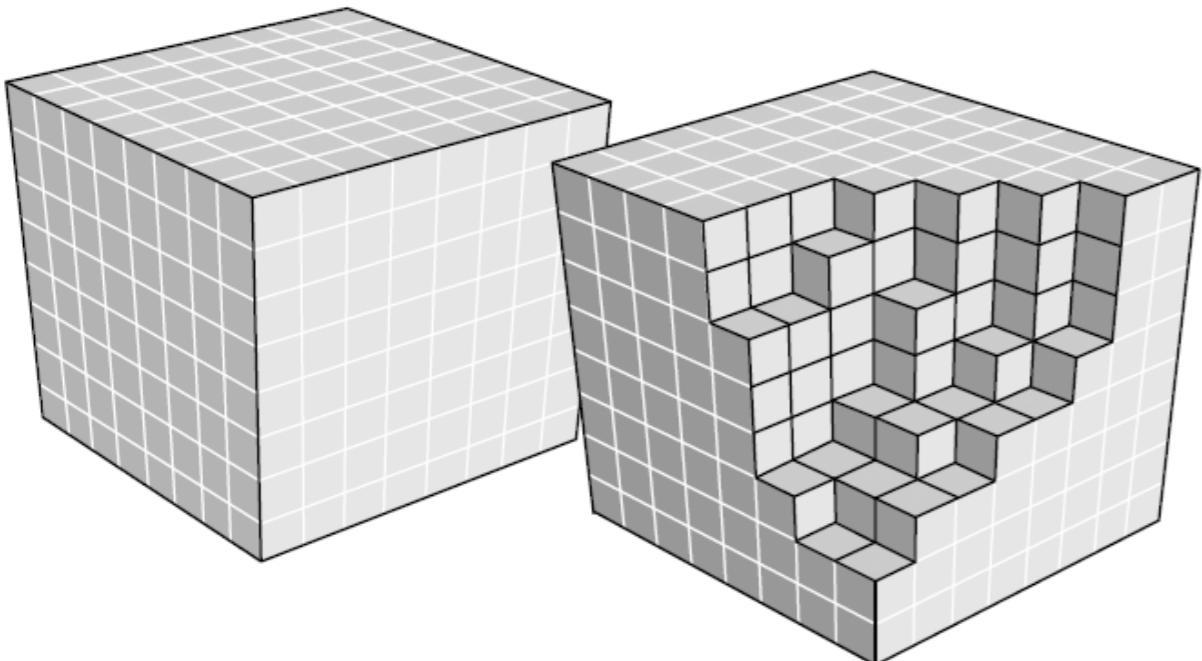
- Uniform spacing along the axes, also known as “raster data”.
- Most volume data look like this; structured data usually means this.
- You still need metadata to know the size of the axes!



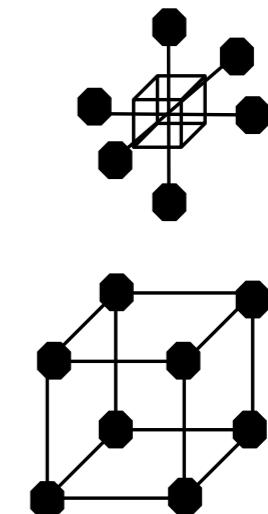
$$P_{i,j,k} = P_{0,0} + i\Delta_x \vec{e}_x + j\Delta_y \vec{e}_y$$



- Representation of scalar 3D data set  $\Omega \in R^3 \rightarrow R$
- Analogy: pixel (picture element) 
- Voxel (volume element), with two interpretations:
  - Values between grid points are resampled by interpolation



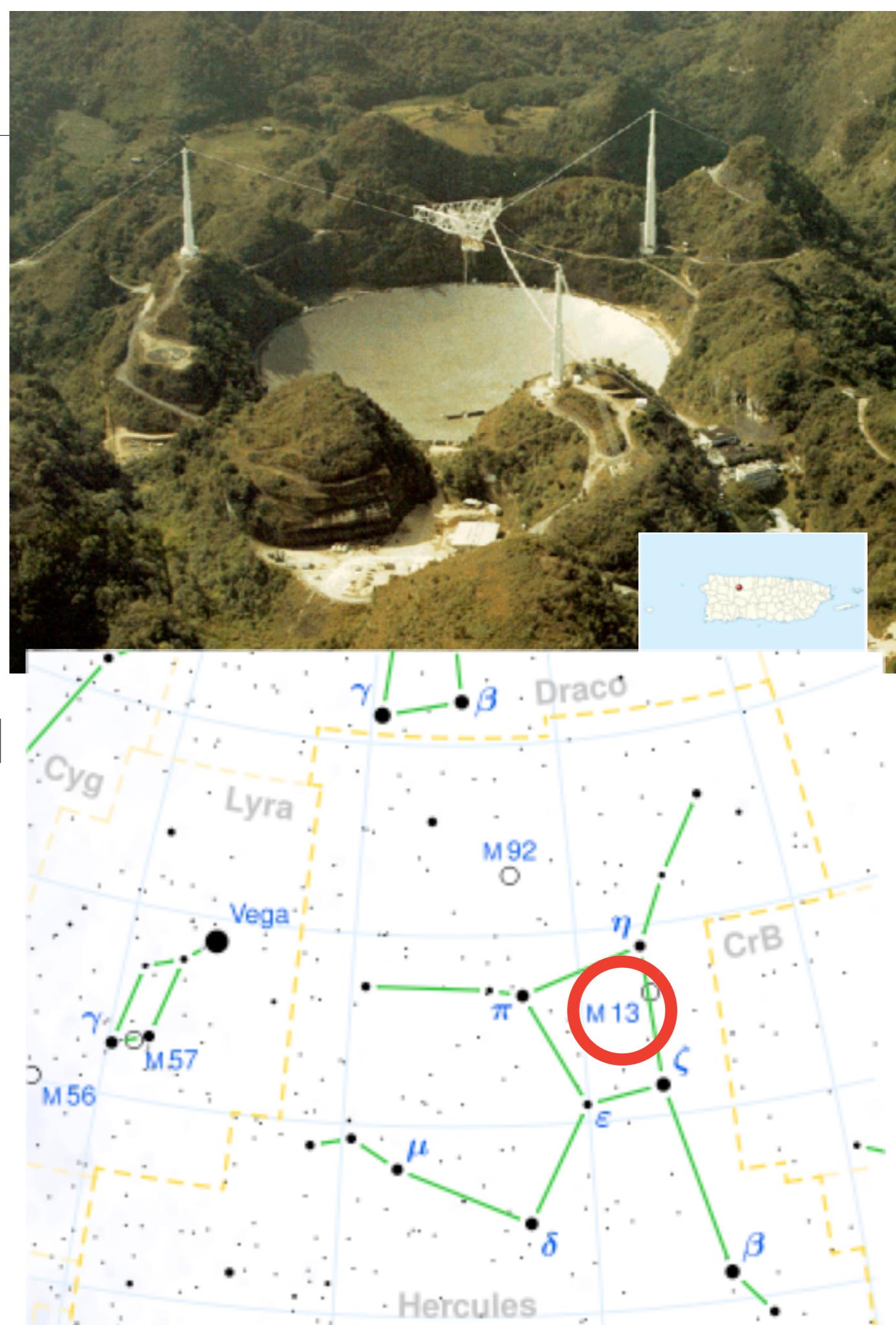
- Collection of voxels
- Uniform grid



# Arecibo Message

[http://en.wikipedia.org/wiki/Arecibo\\_message](http://en.wikipedia.org/wiki/Arecibo_message)

- Way of understanding mechanics of raster image representation
- Radio telescope in Puerto Rico
- built in 1964, renovated in 1974
- To celebrate: Frank Drake and Carl Sagan (Cornell University) sent message to M13 in Hercules (25,000 light years away)
- 1679 bits, frequency modulate 2380 MHz



# The Message

<http://www.physics.utah.edu/~cassiday/p1080/lec06.html>

1679 bits were encoded as 2380MHz plus and minus some frequency

```
000000101010100000000000101000010100000010010001000100010010010110010101  
0101010101010010010000000000000000000000000000000000000000000000000000000000  
000011010000000000000000000000000000000000000000000000000000000000000000000000  
111110000000000000000000000000000000000000000000000000000000000000000000000000  
001100100001101000110001100001101011110111101111000000000000000000000000000000  
00000000000100000000000000000000000000000000000000000000000000000000000000000000  
00001111100000000000000000000000000000000000000000000000000000000000000000000000  
00100000001000000000000000000000000000000000000000000000000000000000000000000000  
00000000000000000000000000000000000000000000000000000000000000000000000000000000  
10000011000000000000000000000000000000000000000000000000000000000000000000000000  
00001000000001000000000000000000000000000000000000000000000000000000000000000000  
01000011000000000000000000000000000000000000000000000000000000000000000000000000  
00000100000000100000000000000000000000000000000000000000000000000000000000000000  
00001000100000000000000000000000000000000000000000000000000000000000000000000000  
00000000110000000000000000000000000000000000000000000000000000000000000000000000  
00000000110000000000000000000000000000000000000000000000000000000000000000000000  
00010000011111000001100000000000000000000000000000000000000000000000000000000000  
11100001110000011011000000000000000000000000000000000000000000000000000000000000  
00001010000011000000000000000000000000000000000000000000000000000000000000000000  
00100000000000000000000000000000000000000000000000000000000000000000000000000000  
00010100000000000000000000000000000000000000000000000000000000000000000000000000  
11100000000000000000000000000000000000000000000000000000000000000000000000000000  
01100001000101000000000000000000000000000000000000000000000000000000000000000000  
00000000100001000000000000000000000000000000000000000000000000000000000000000000  
0111100111101001111000
```

This is a **1-D** sequence of bits in time  
How will an alien understand this list of bits?  
(will have different symbols than “0” “1”)  
No meta-information!

# Understanding the message

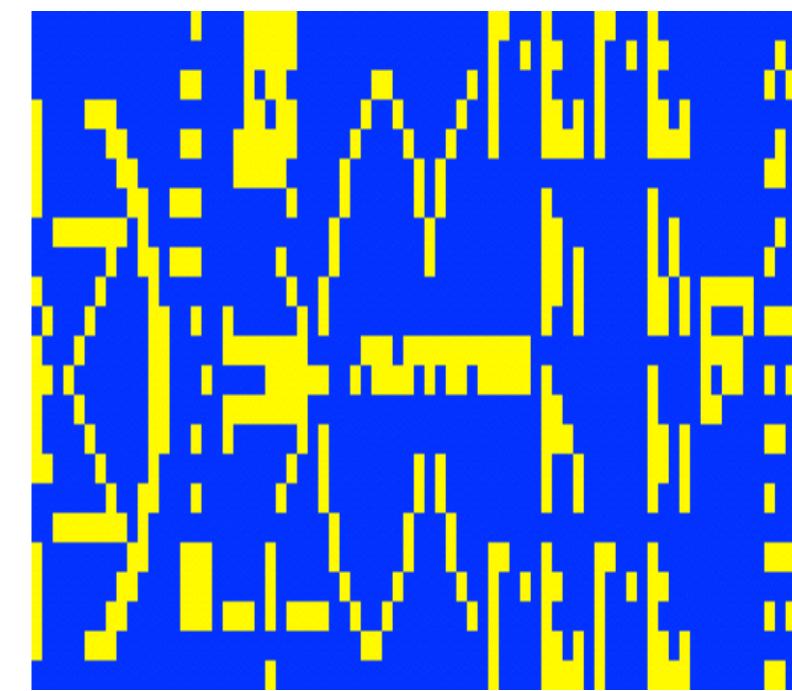
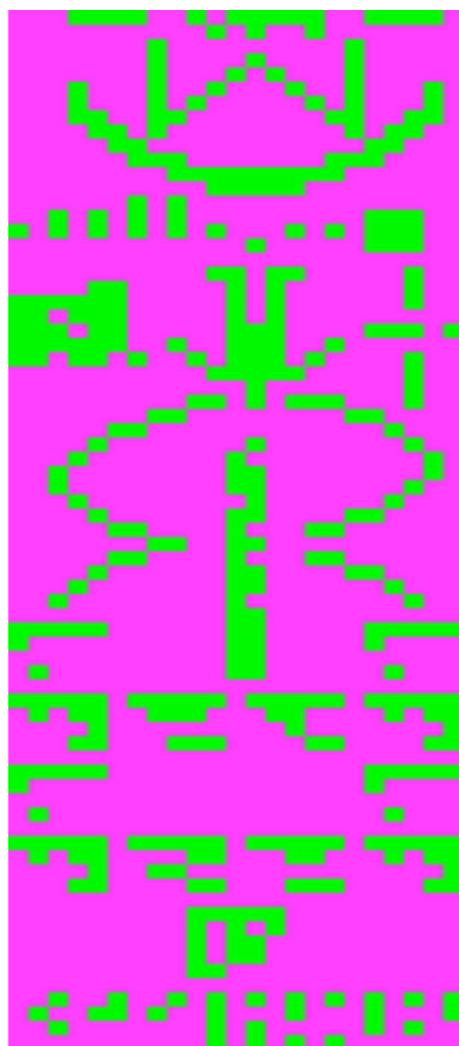
- Perhaps some “visual” representation of bits



- (what is black vs white?)
- Aliens notice  $1679 = 23 \times 73$  (product of two primes)
- Perhaps its not a linear sequence: 2-D array
- Two ways of sequencing values in 2D array
- Various ways of laying them out in 2D space
- Then: have to decipher it!



73 x 23



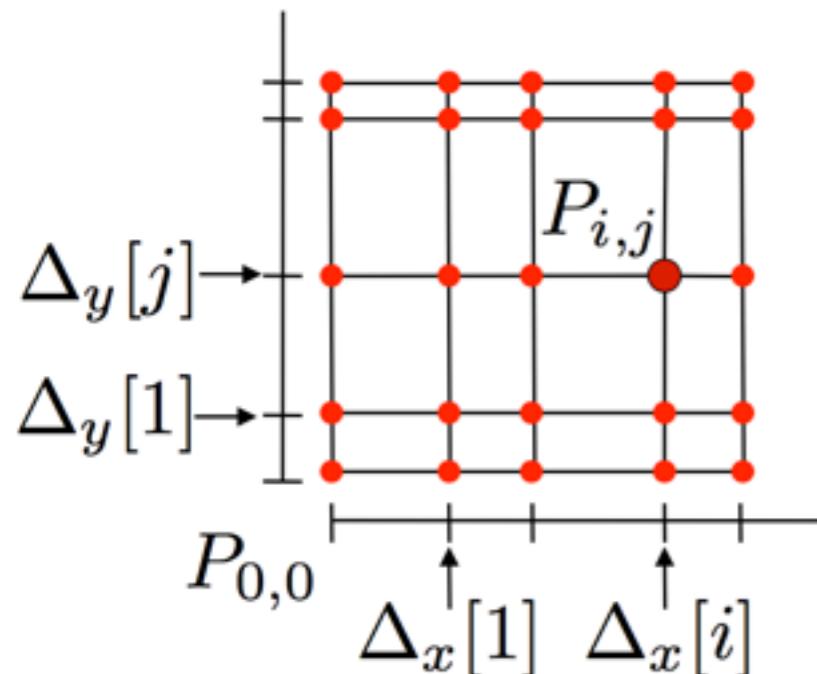
compare to:  
[http://en.wikipedia.org/wiki/Arecibo\\_message](http://en.wikipedia.org/wiki/Arecibo_message)

23 x 73: what was different?

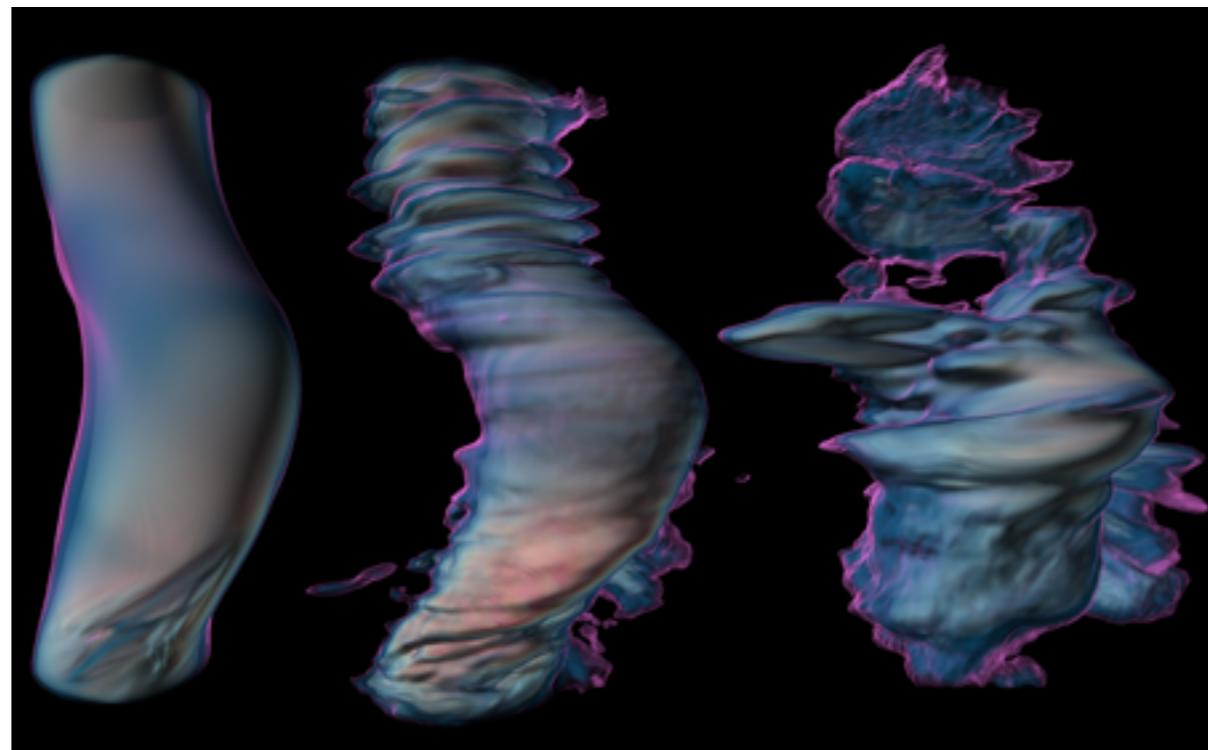


# Structured (rectilinear) grid geometry

- Still structured, but with non-uniform spacing along the axes.
- Positions can still be computed procedurally
- Some meteorology, climate CFD data like this.



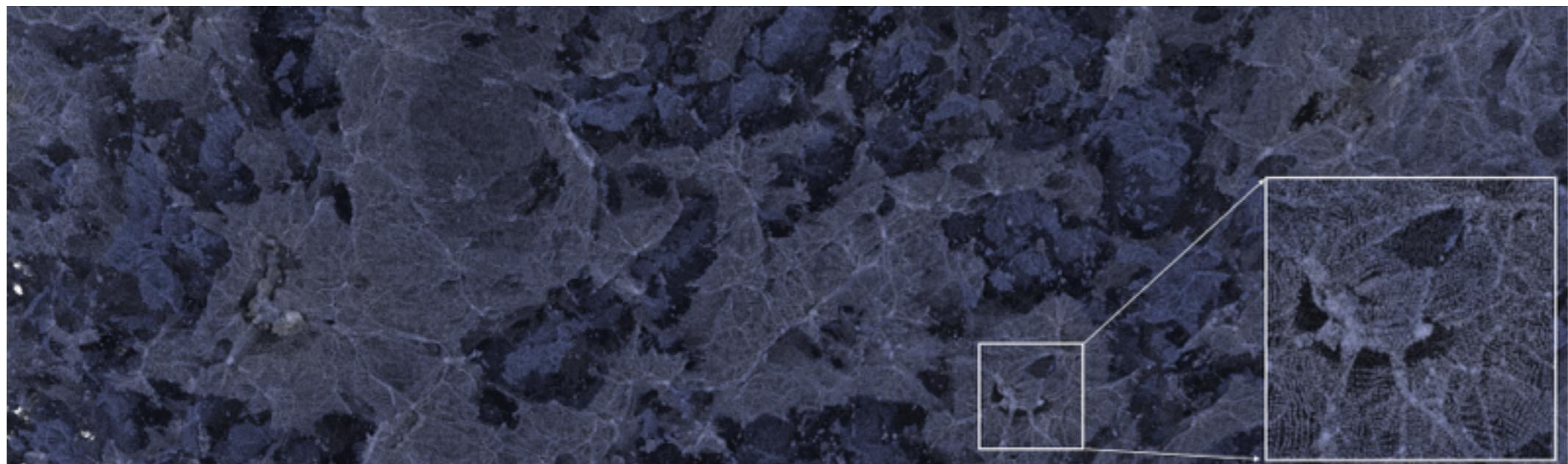
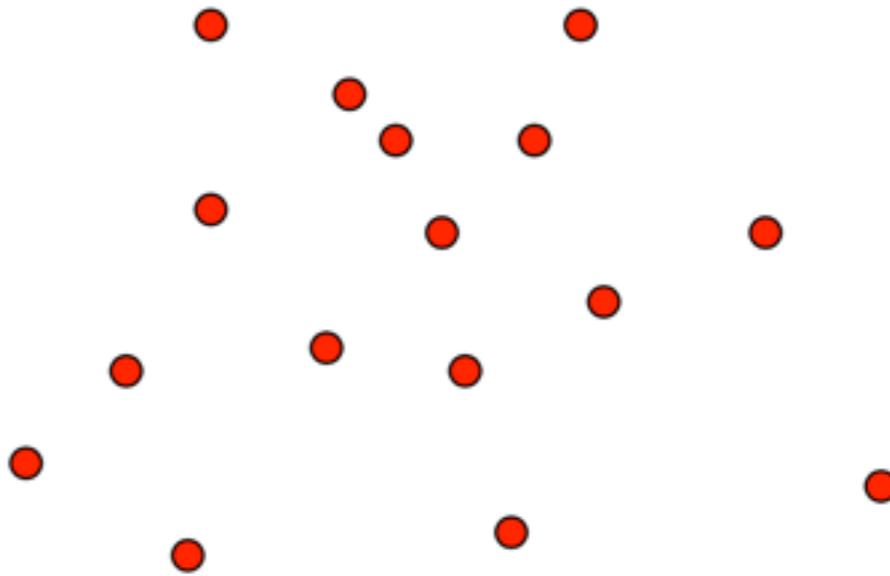
$$P_{i,j,k} = P_{0,0} + \Delta_x[i]\vec{e}_x + \Delta_y[j]\vec{e}_y$$



Turbulence in the Ionosphere - Greg Foss, TACC

# Unstructured geometry

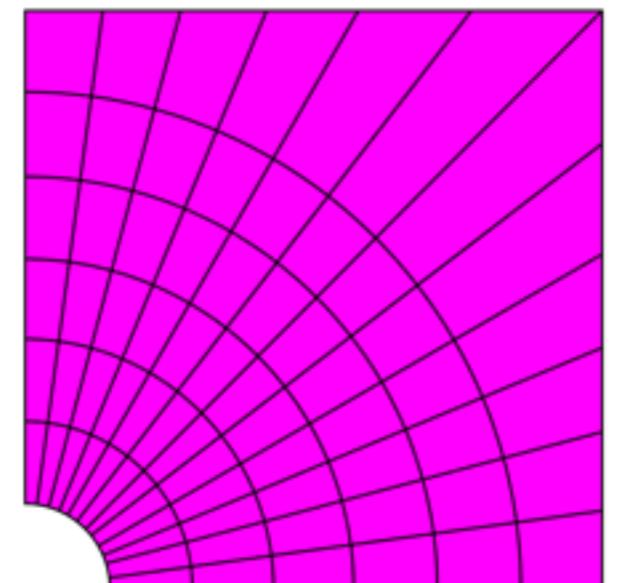
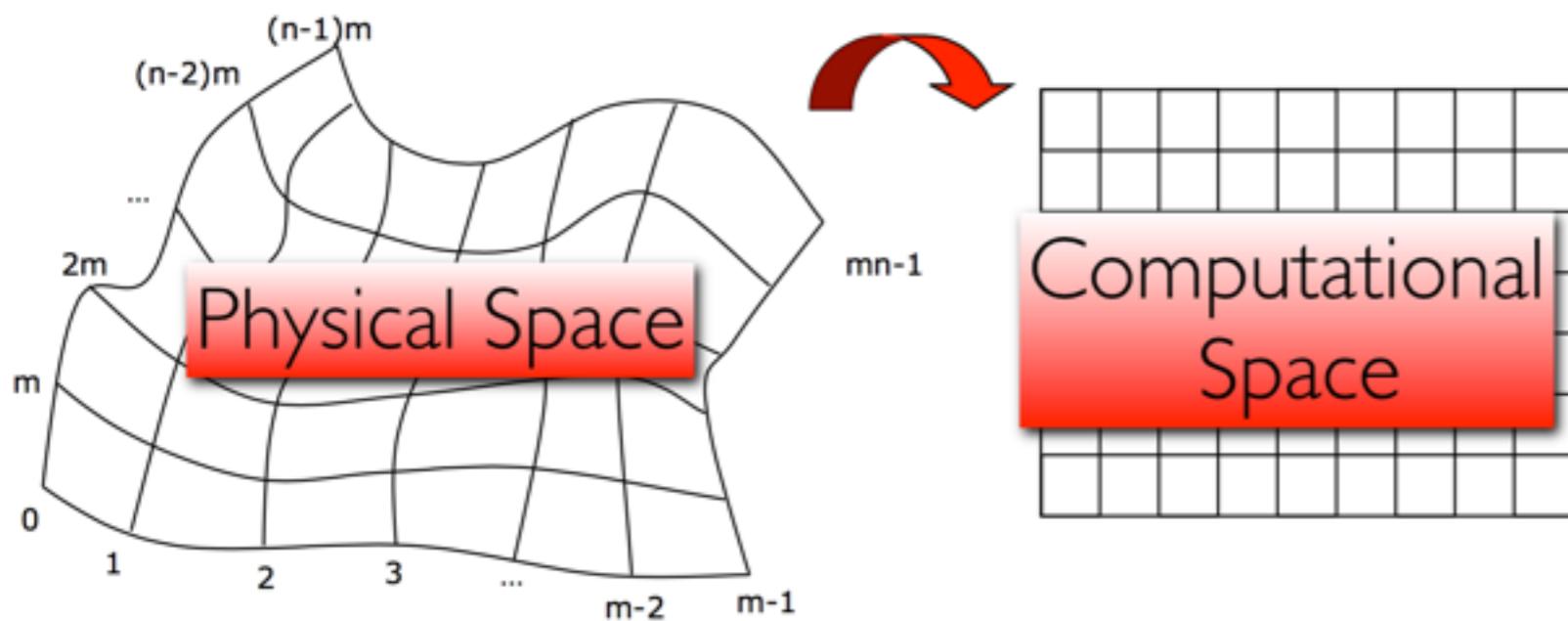
- Raw, unstructured point data.
- You actually need to store the x,y,z positions of vertices.
- Some of the largest computational and scanned data
  - LiDAR, RGB-D point clouds, range scans
  - n-body codes — molecular dynamics, cosmology
- Note: this is unstructured topology, too!



30-billion particle "Cosmic Web" data — Paul Shapiro, University of Texas at Austin  
I Wald, A Knoll, G.P. Johnson, W. Usher, V. Pasccci & M.E. Papka: "CPU Ray Tracing Large Particle Data with P-k-d Trees" IEEE Visualization 2015

# Structured grid topology

- You can have unstructured geometry but structured topology
  - Implicit definition of cells
  - Implicit connectivity between vertices
- More exotic options with structured grid topology:
  - Finite elements, finite differences on curvilinear grids
  - spectral F/E, some spline-based finite elements simulations
  - Good for precision-critical flow computations (blood flow, CFD)



# Curvilinear grids

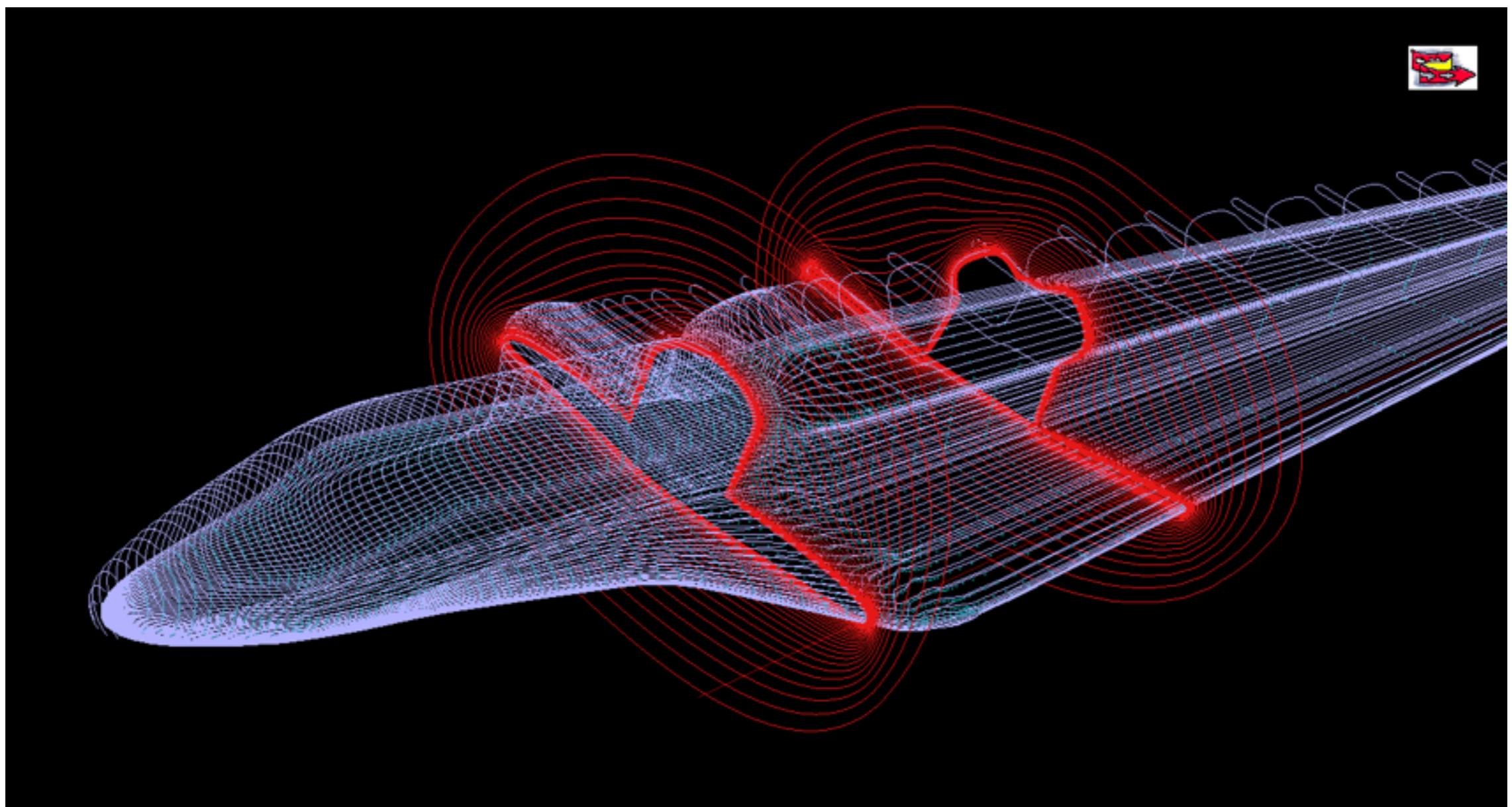
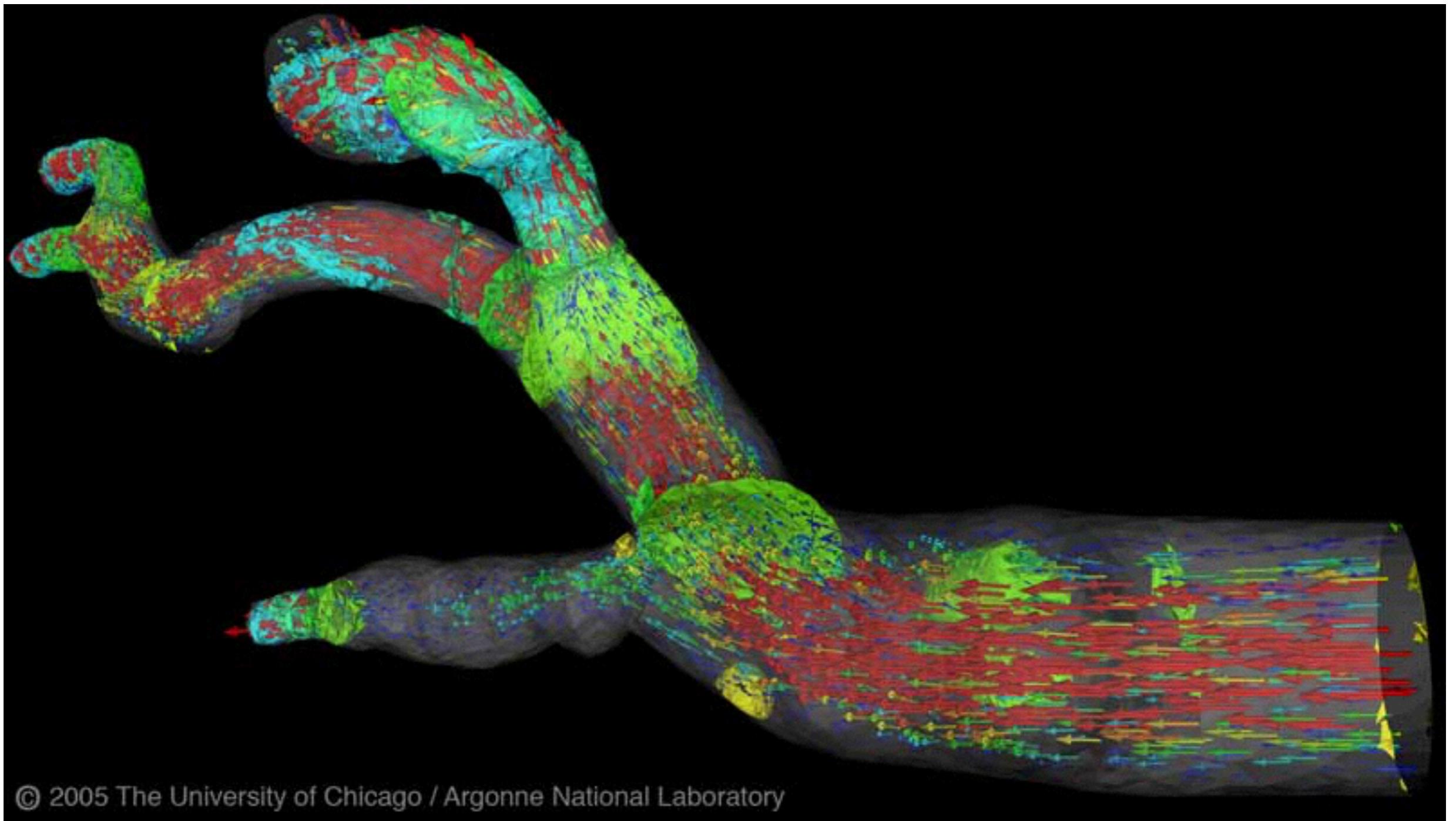


Image:T.U. Graz

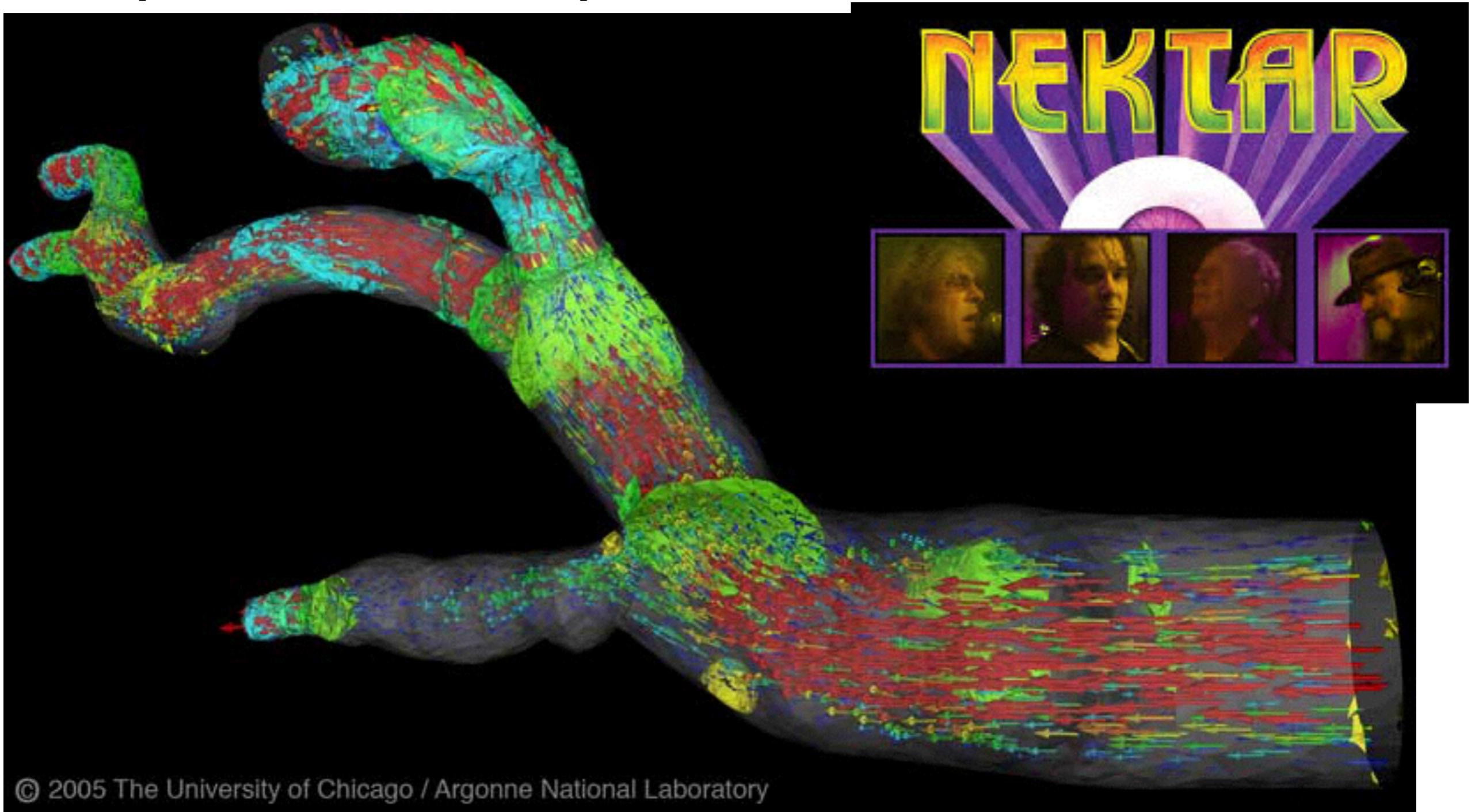
# Spectral/hp finite elements



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Data: George Karniadakis, Brown University. Visualization: Joe Insley, ANL

# Spectral/hp finite elements

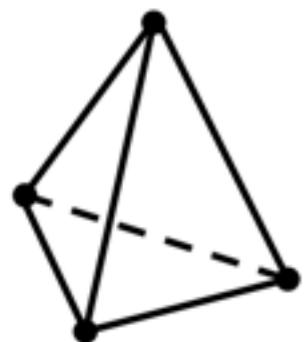


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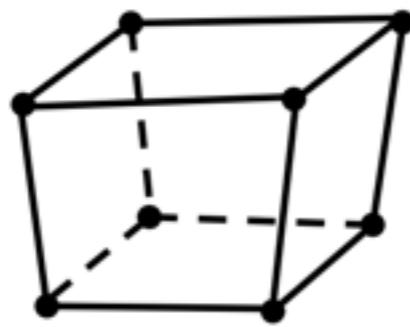
Data: George Karniadakis, Brown University. Visualization: Joe Insley, ANL

# Unstructured grid topology

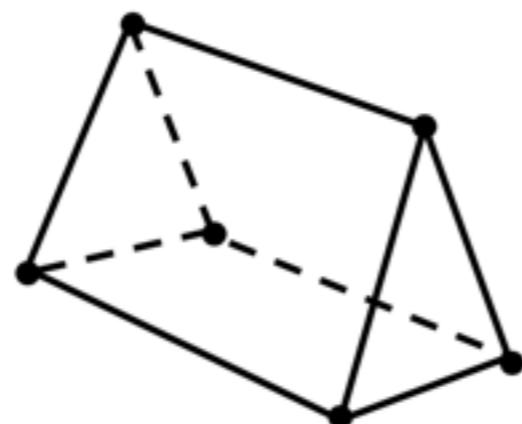
- Both uniform elements and “mixed elements” (allowing any cell type)
- Need to store vertices and indices separately
  - mixed elements: vertices, indices and count
- Many, many finite elements codes.
  - solid mechanics, CAD
  - bioelectric modeling



Tetrahedron



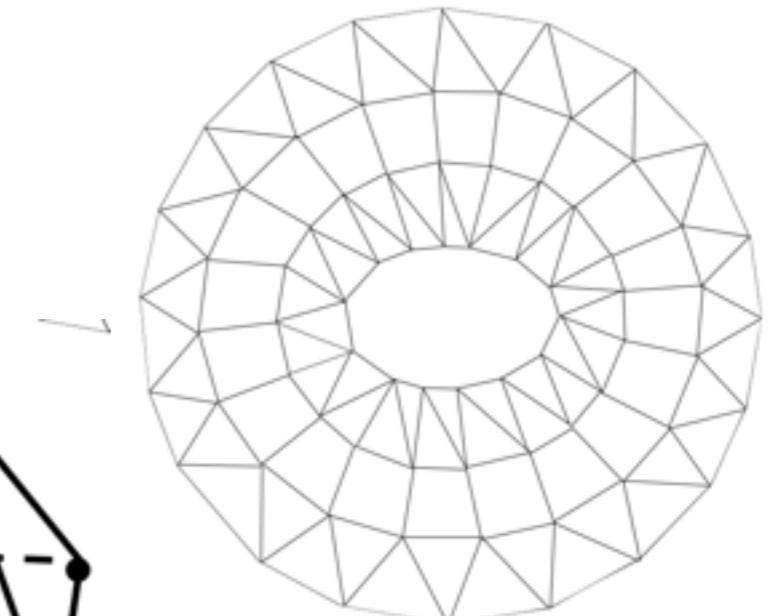
Hexahedron



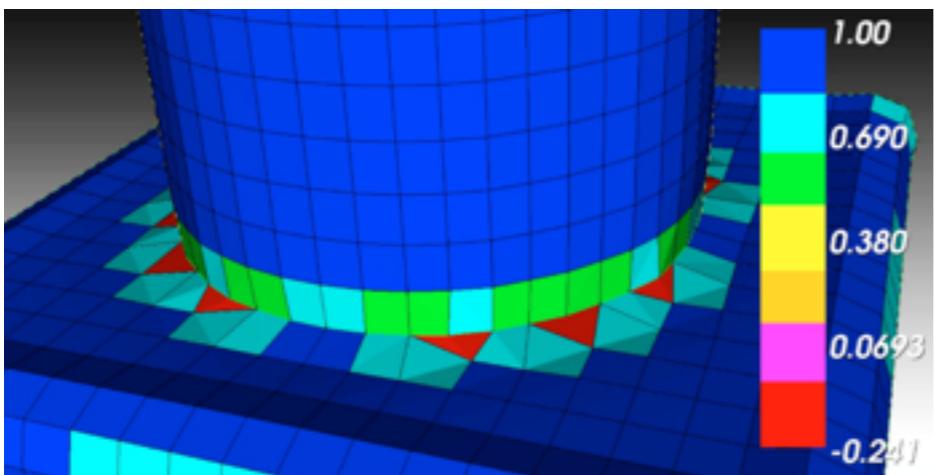
Wedge



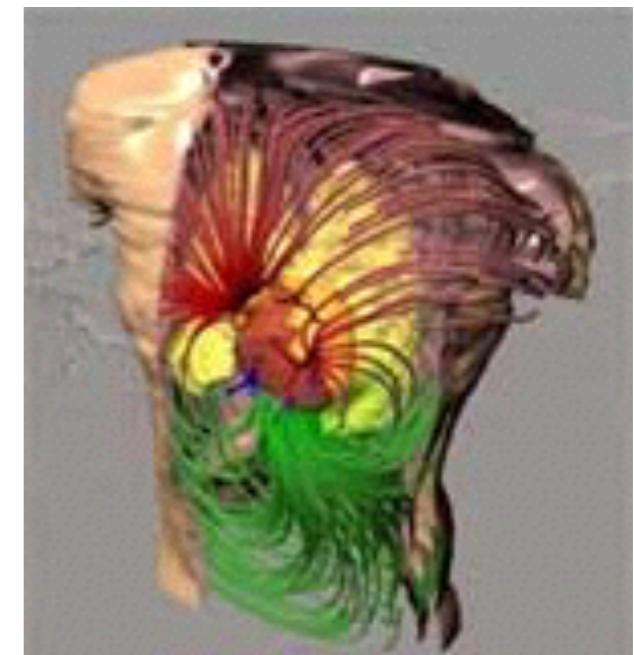
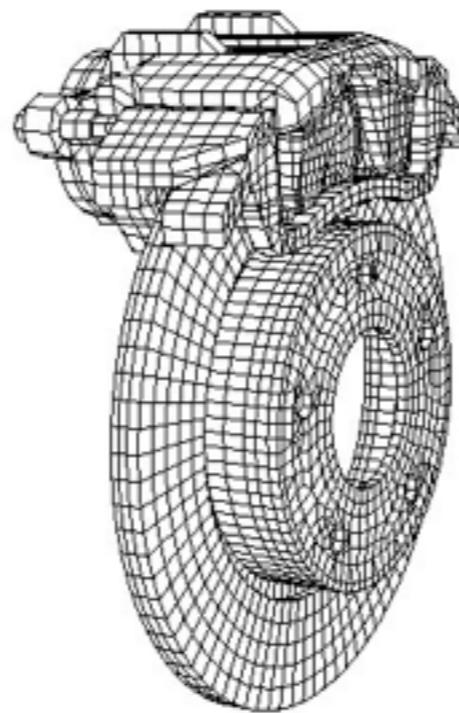
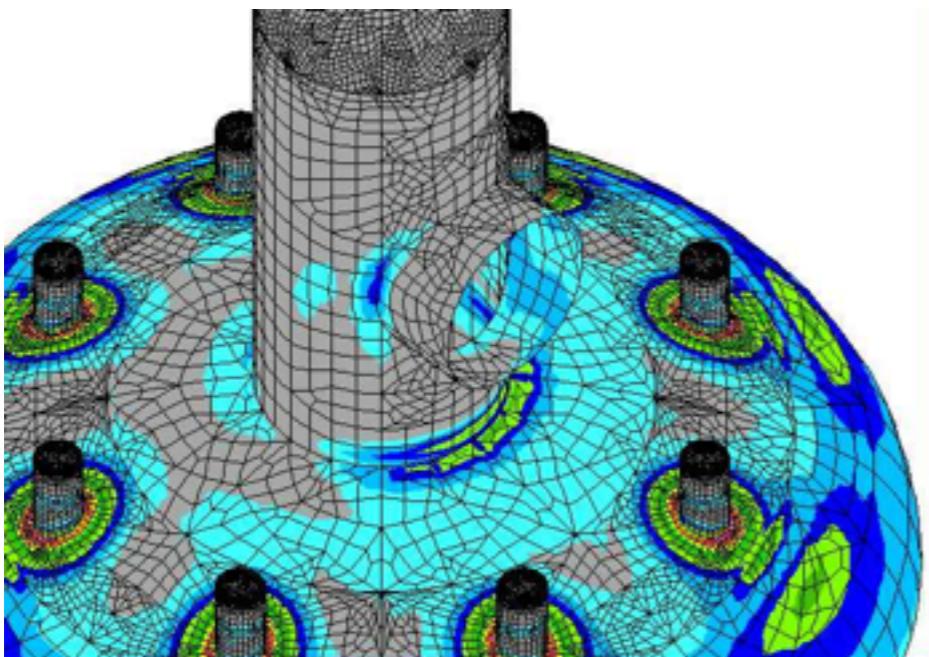
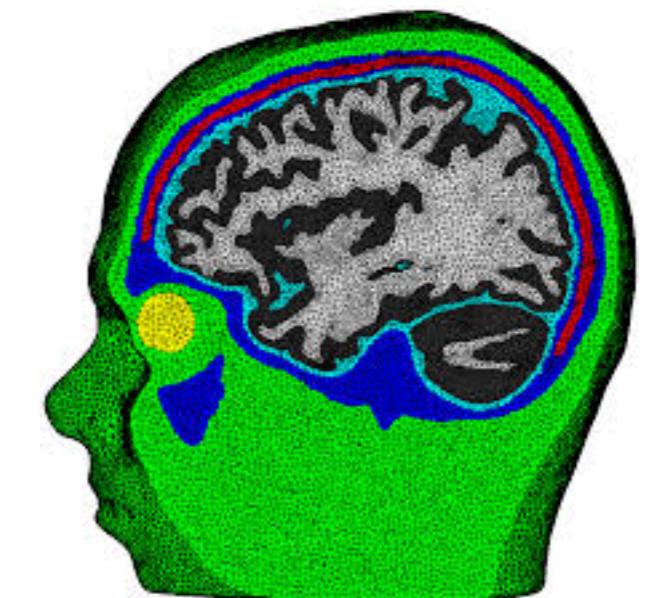
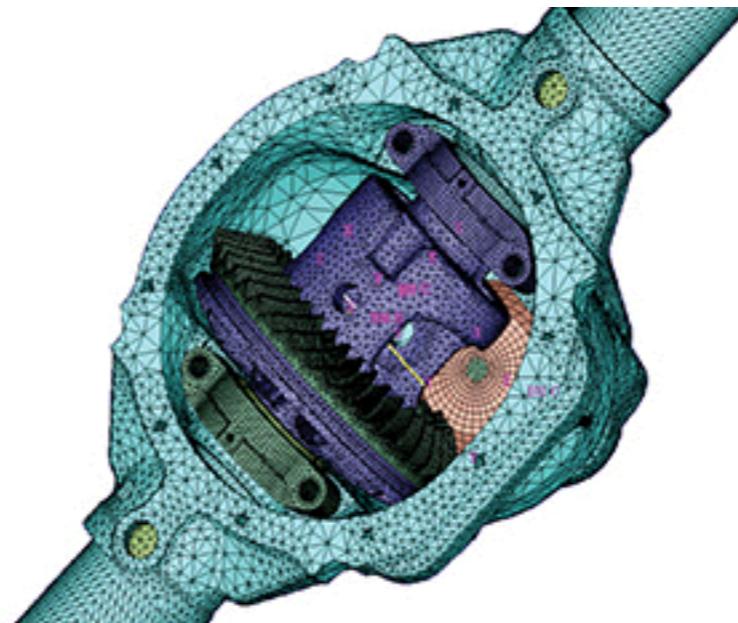
Pyramid



# Finite elements



Visualization: Steve Owen, using Cubit



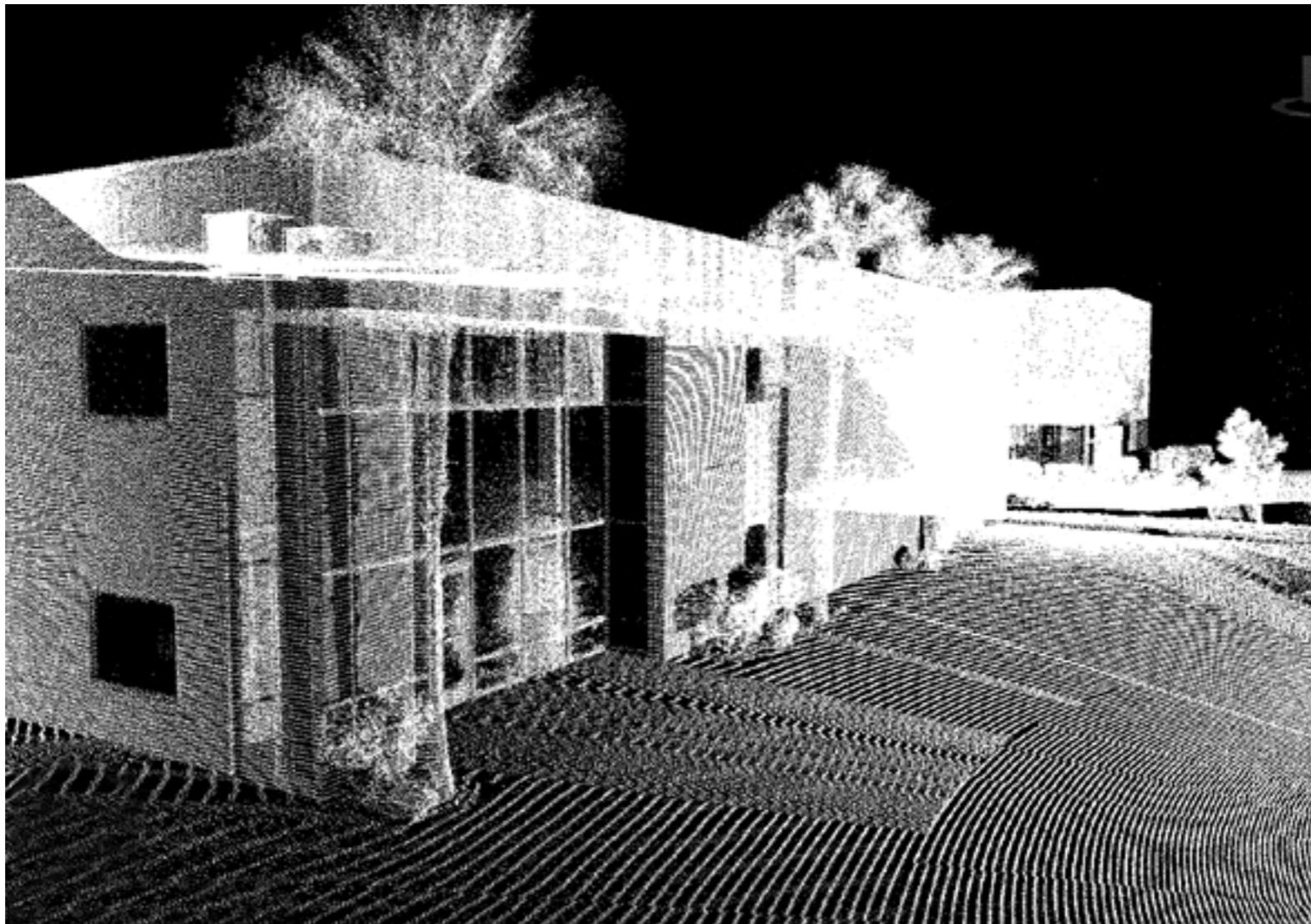
# Geometry vs topology

			Topology
		Structured	Unstructured
Geometry	Uniform	Image	Unstructured
	Structured	Rectilinear	Unstructured
	Unstructured	Curvilinear	Unstructured

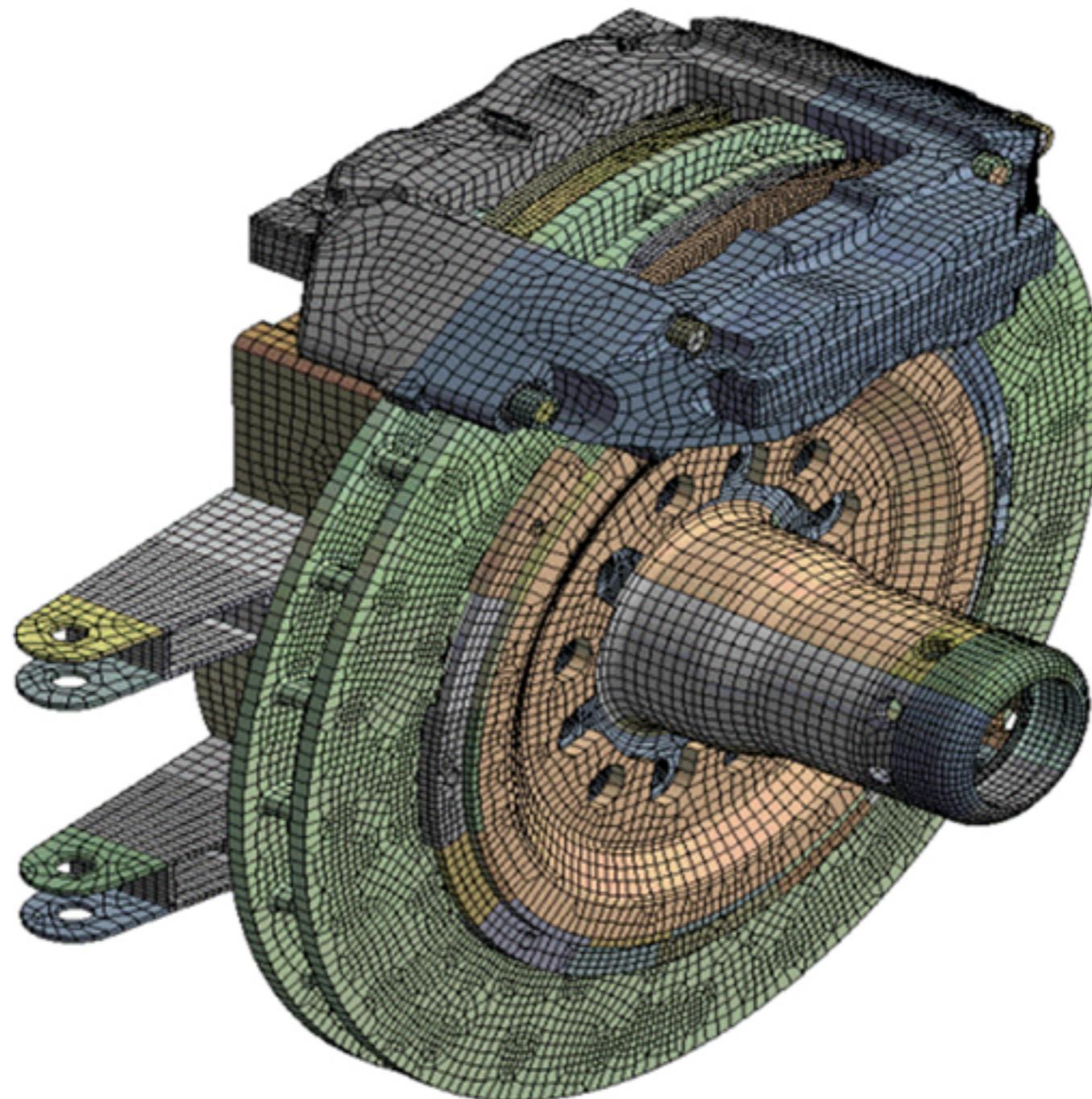
# Colloquially

- In spatial (scientific) vis we usually talk about **geometry**, and:
  - **structured** means rectilinear grid (usually, but not always uniform).
  - **unstructured** means everything else (curvilinear grids, tetrahedra, hexahedra, points, etc.)

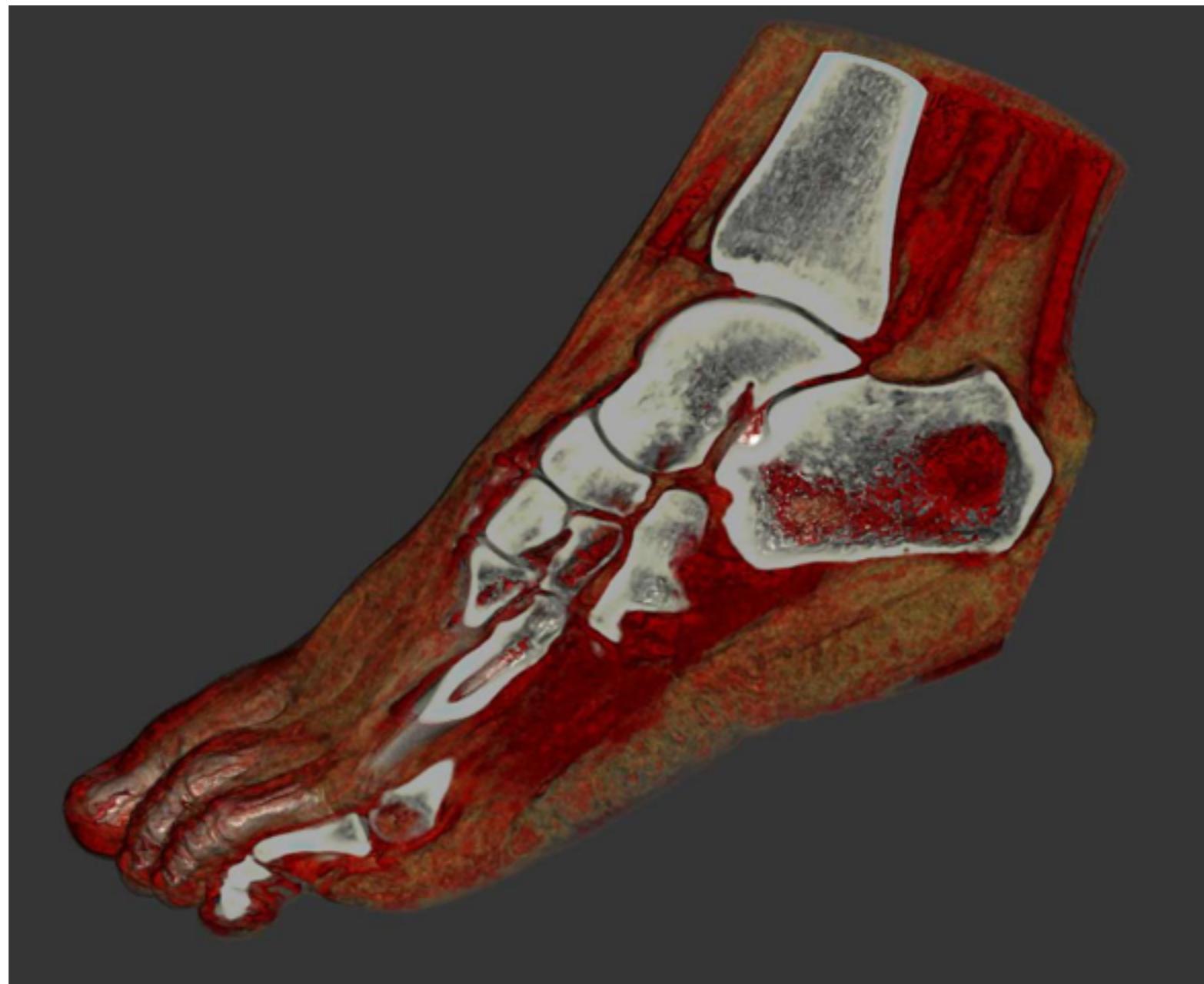
# Is it (geometrically) structured or unstructured?



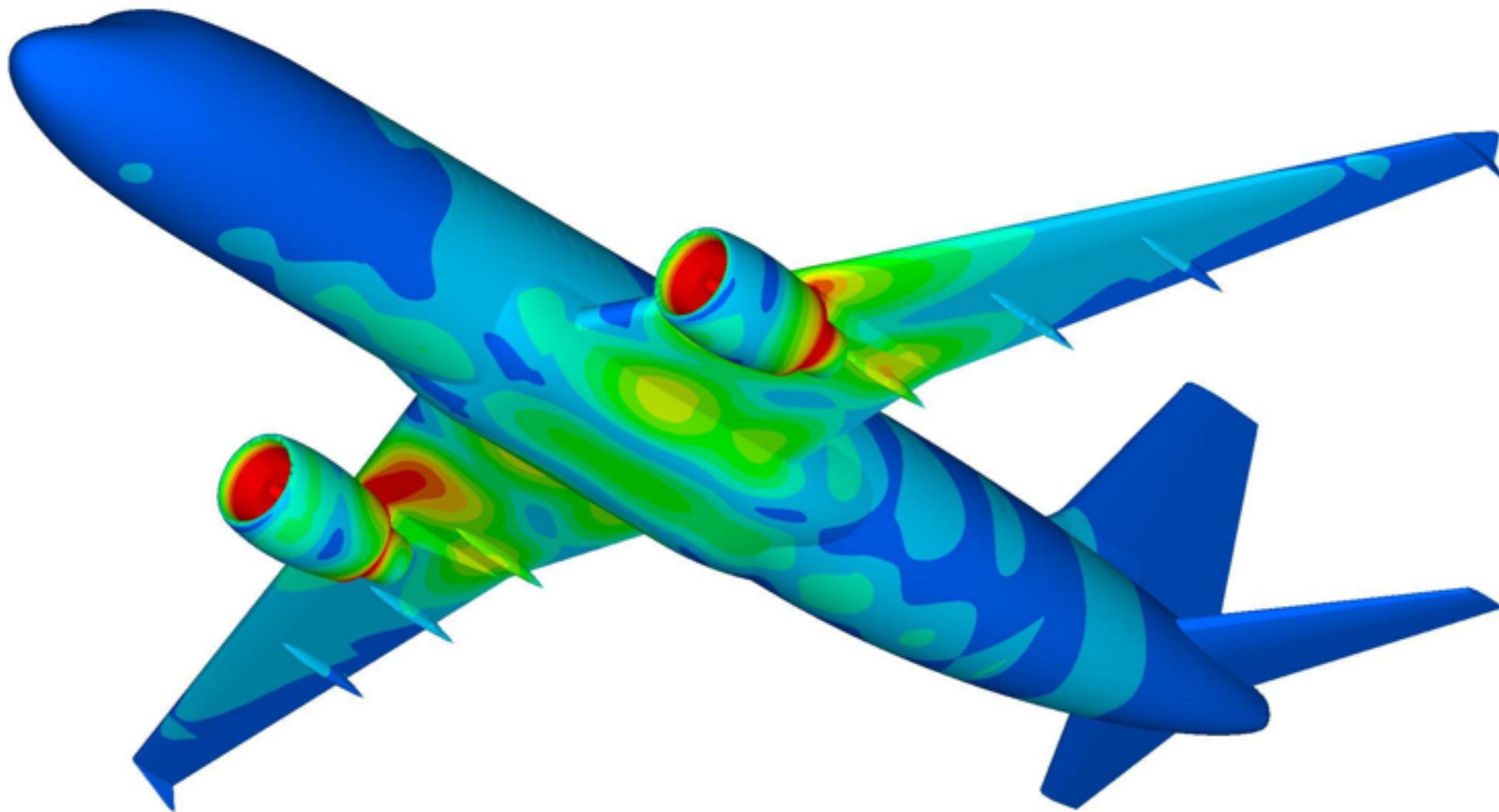
Is it (geometrically)  
structured or unstructured?



Is it (geometrically)  
structured or unstructured?

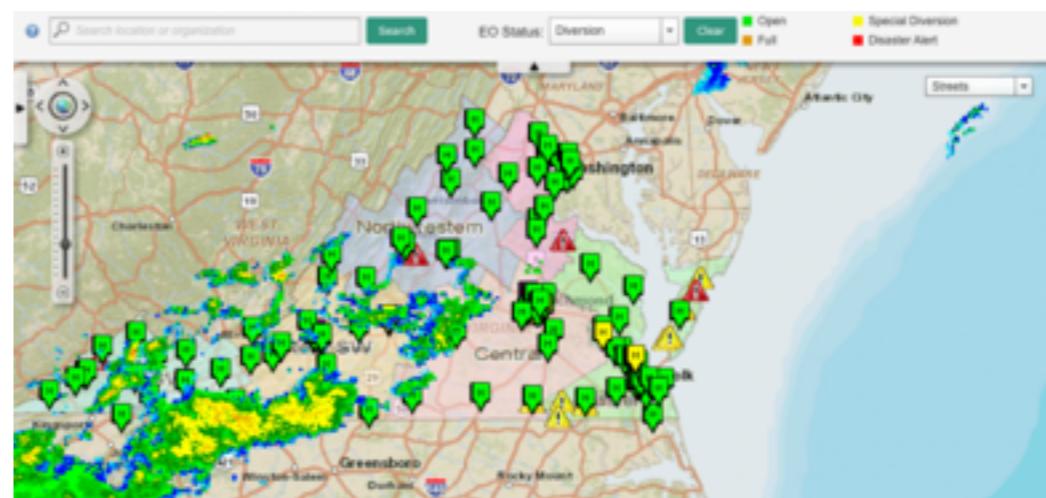
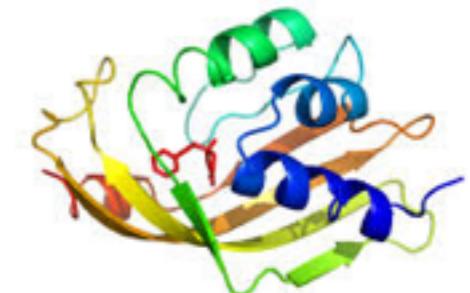
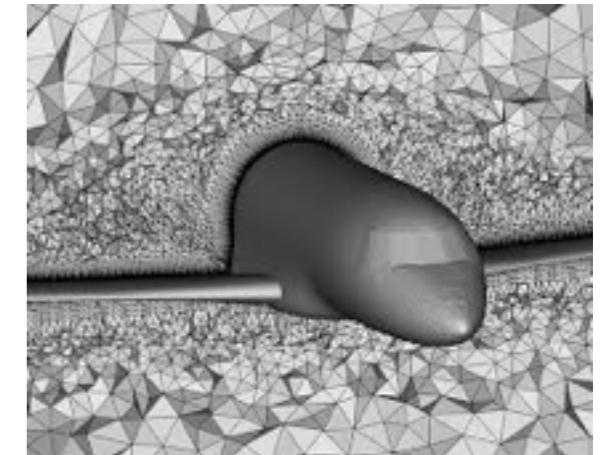


Is it (geometrically)  
structured or unstructured?



# Non-field and other data

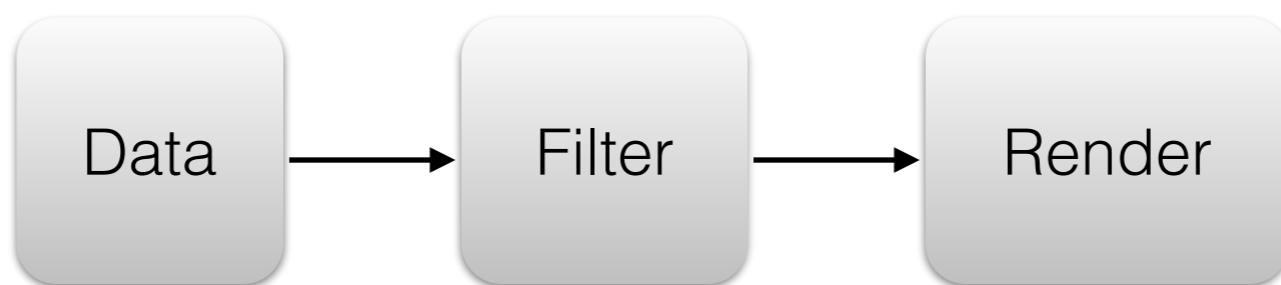
- In addition to structured/unstructured field data, you can have non-field geometry.
  - Boundary surface meshes
  - Atom positions, bonds, ribbons
- Non-geometric annotations
  - Especially in GIS.
  - Visualization data models are complex!



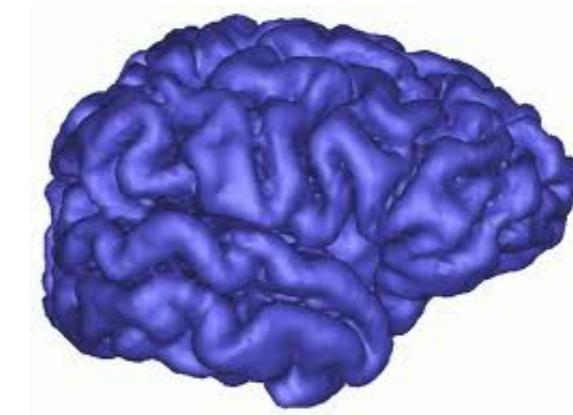
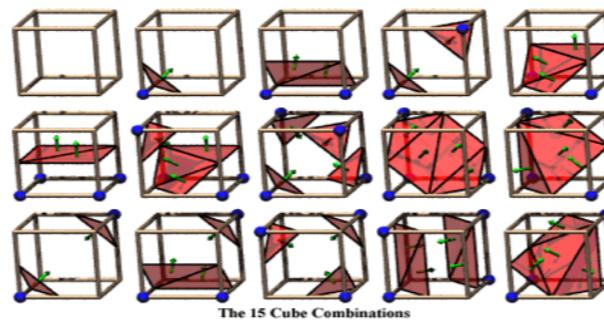
# What do we do with these data?

- In computer graphics, life is “easy”
  - Have a triangle mesh, render it!
- Visualization is more than just rendering.
- Two approaches:
  - direct visualization:  
i.e. render from a (usually 3D) field directly
  - indirect visualization:  
i.e. convert the field to triangles and render those (usually with GPU rasterization)

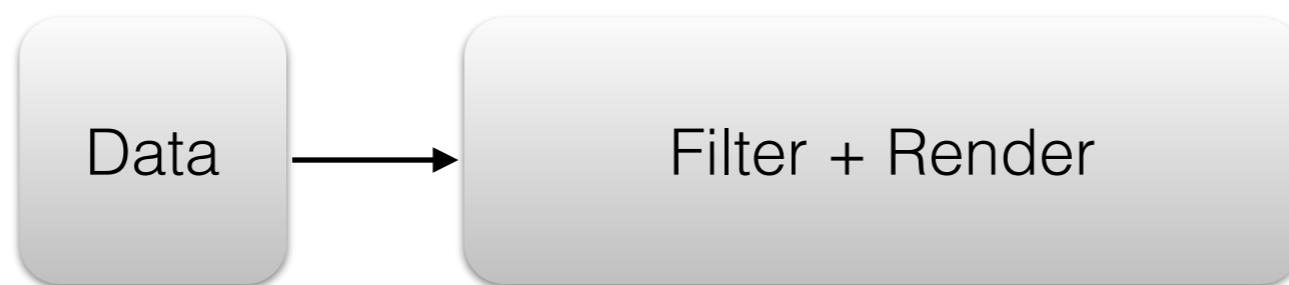
# Indirect visualization



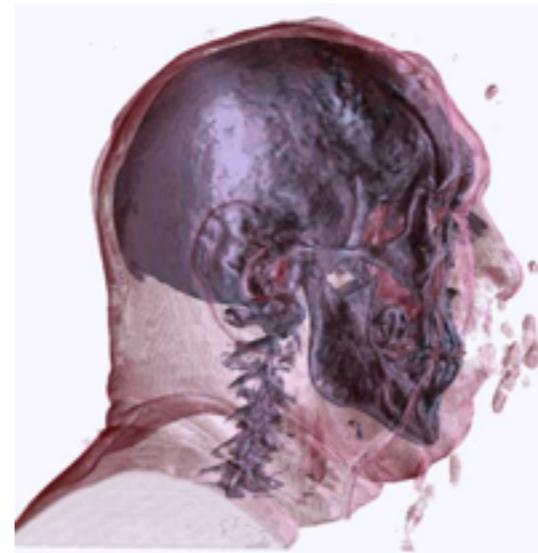
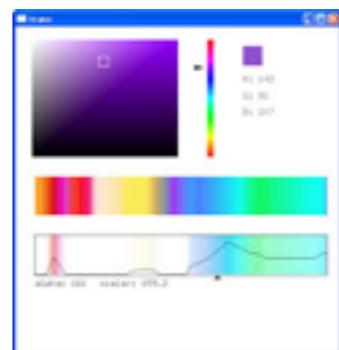
0	4	8	0
4	14	9	0
6	11	1	0
2	1	0	0



# Direct visualization



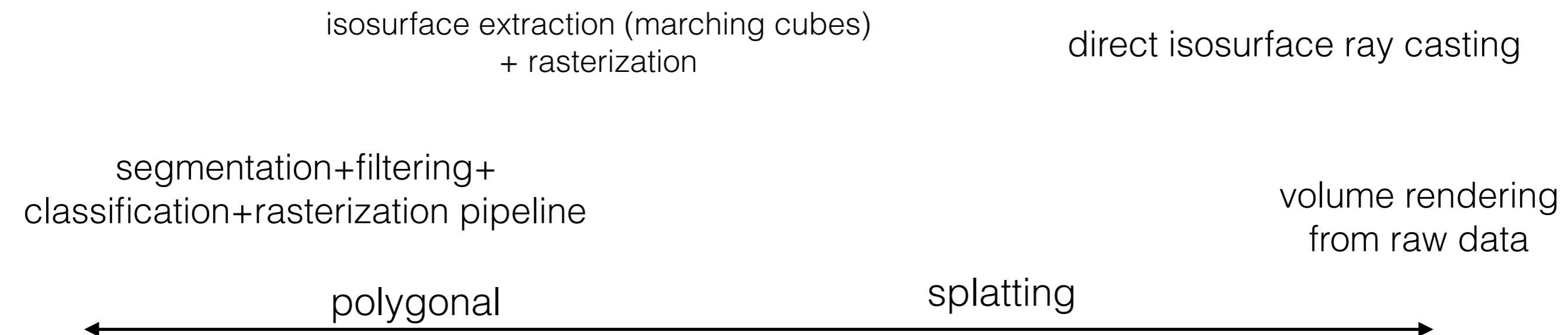
0	4	8	0
4	14	9	0
6	11	1	0
2	1	0	0



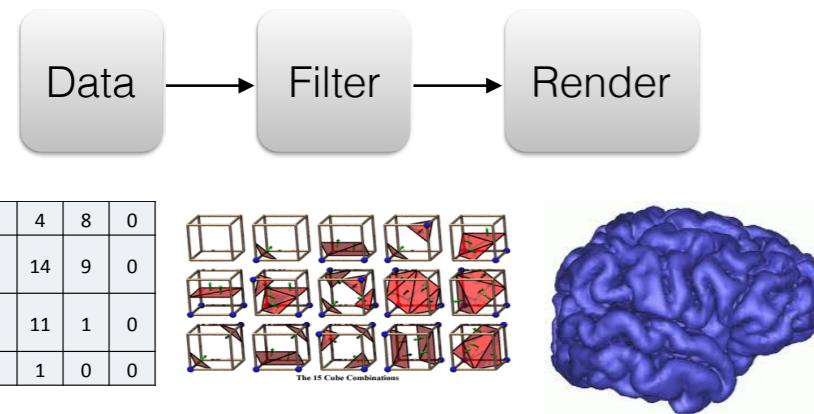
# The indirect-direct spectrum

- Most data must be processed somewhere after acquisition
- Direct/Indirect are not “absolute”, but relative.
  - Direct = less processing prior to rendering
  - Indirect = more processing prior to rendering
- Multiple questions:
  - how do the data need to be transformed for the desired analysis?
  - what does the target rendering engine / API require?
- There is no such thing as pure “direct” or “indirect” visualization; only “more direct” or “more indirect”.

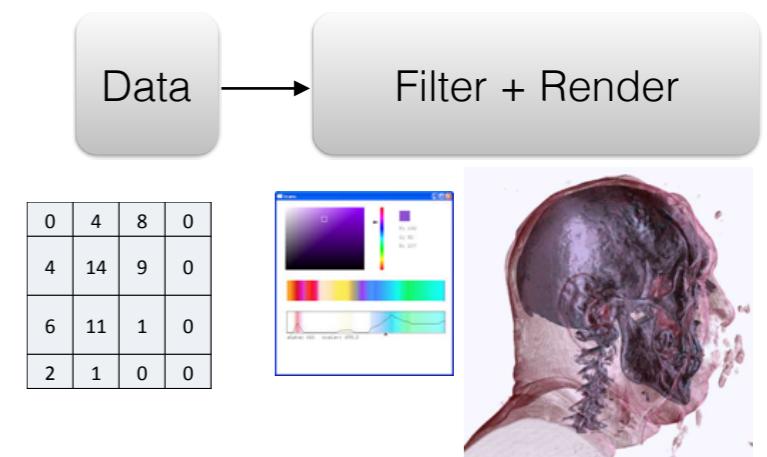
# The indirect-direct spectrum



Indirect

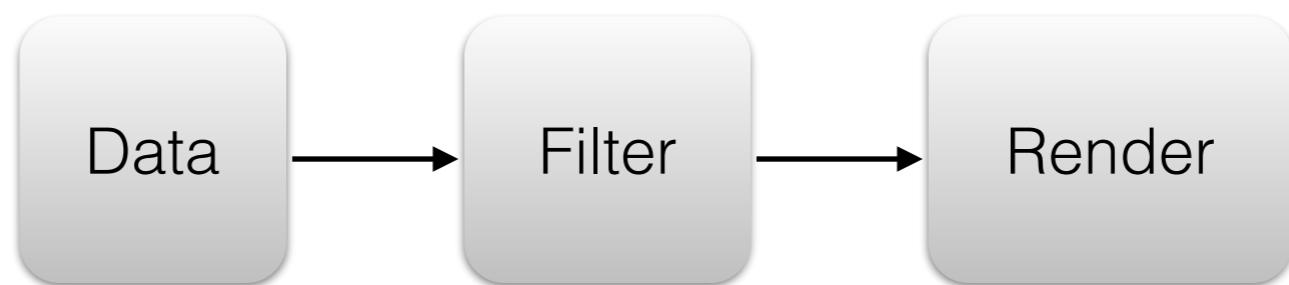


Direct



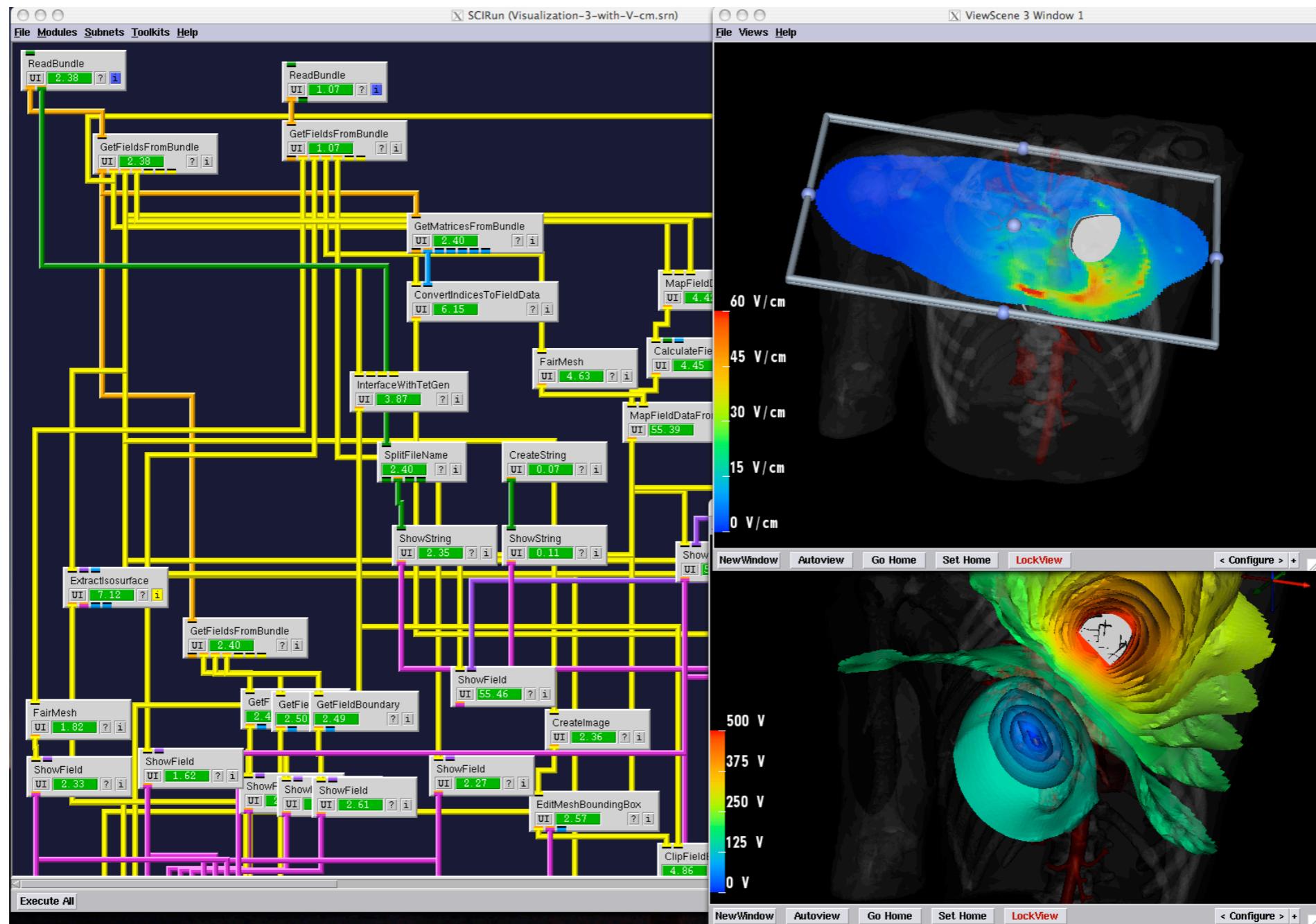
# The visualization pipeline

- Even if we merge filtering and rendering, it is still helpful to think of them as a chain of operations.

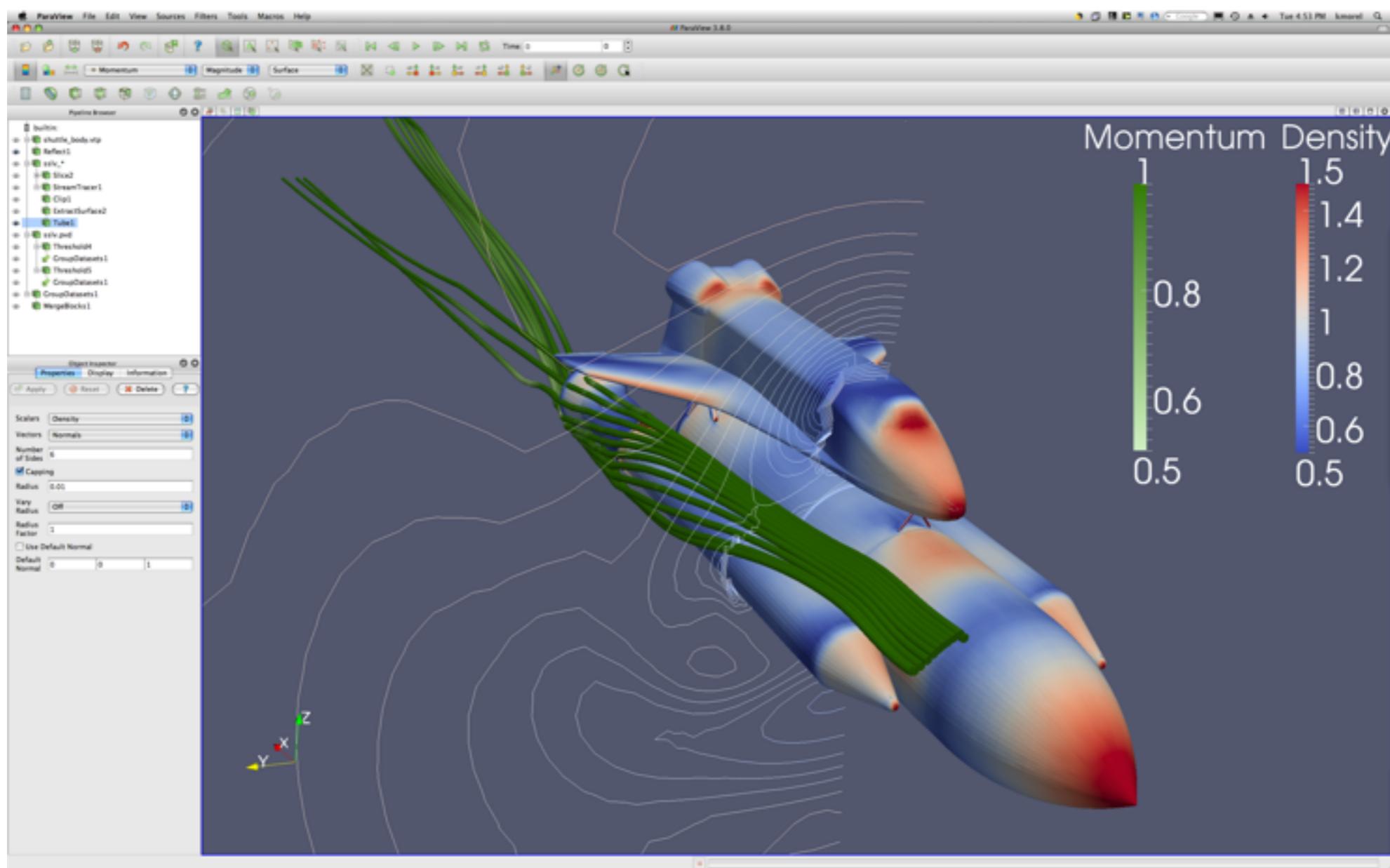


- Visualization workflows take the form of a flow chart, tree or network...

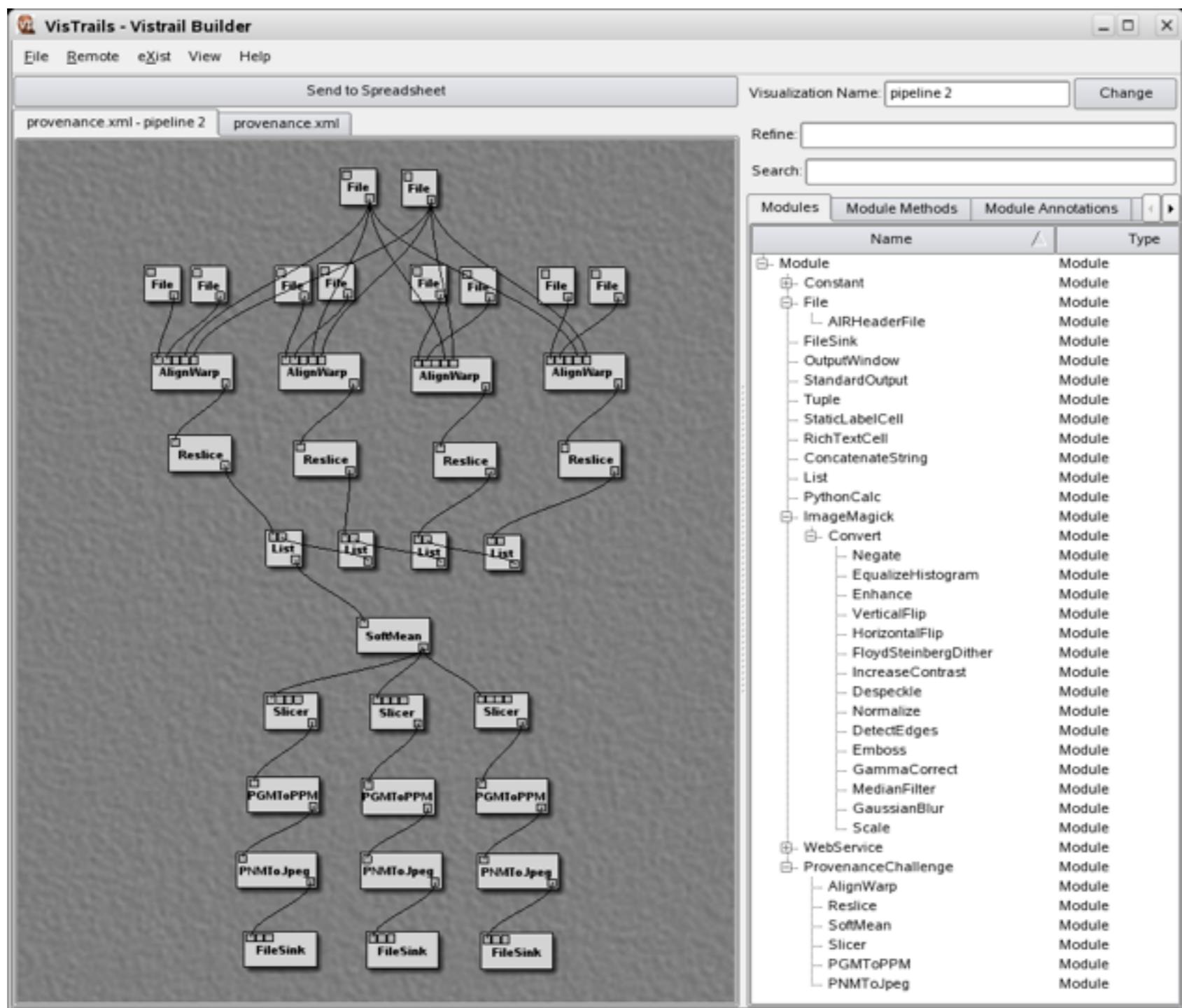
# SCI Run



# ParaView



# VisTrails



# VTK

- <http://www.vtk.org>, open-source, developed and maintained by Kitware.
- The standard-bearer API for general-purpose scientific visualization
  - Full-fledged data model for structured, unstructured, particle data
  - Marching cubes, cut/clip planes, streamlines, etc.
  - Hundreds of other analysis filters
  - Numerous readers for common scientific formats
- Call as a library from C++, Java, Python, Tcl/Tk
- Limitations:
  - no UI — you need to code (or at least, script) your workflows.
  - data model can be “heavy”, memory-inefficient — but it nearly always works!

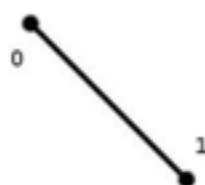
# Cell types in VTK



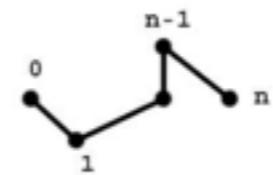
(1) Vertex



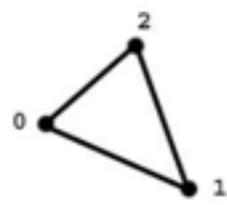
(2) Poly-vertex



(3) Line



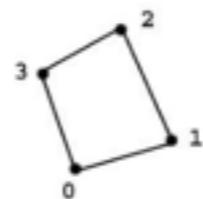
(4) Poly-line



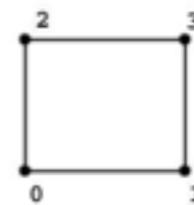
(5) Triangle



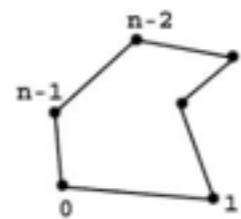
(6) Triangle strip



(7) Quadrilateral



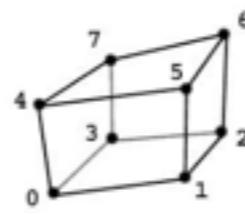
(8) Pixel



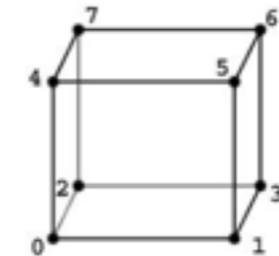
(9) Polygon



(10) Tetrahedron



(11) Hexahedron

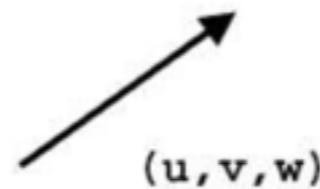


(12) Voxel

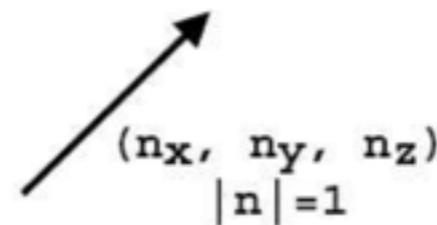
# Attribute types in VTK



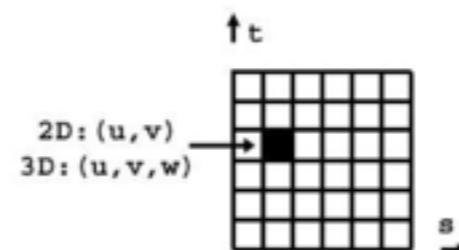
(1) Scalar



(2) Vector



(3) Normal



(4) Texture coordinate

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

(5) Tensor

S. Bruckner, "Data Structures in the Visualization Toolkit."

# Simple data flow in VTK

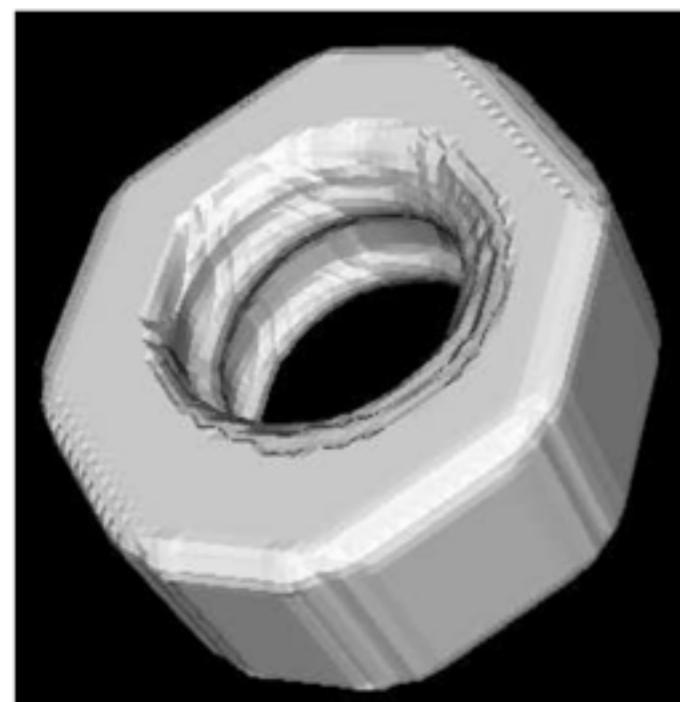


Figure 7: Resulting image for program 3.2

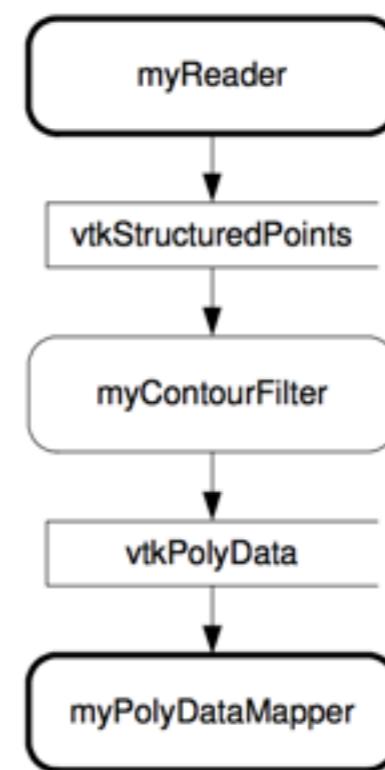


Figure 8: Data-flow chart for program 3.2

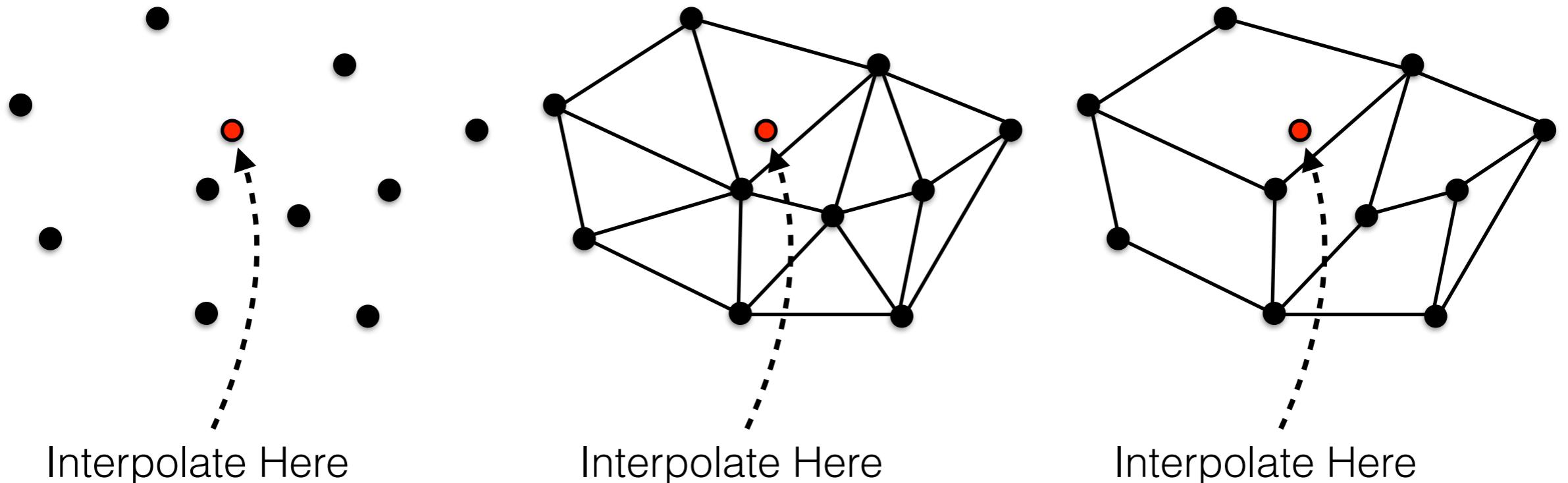
# Interpolation

# Interpolation

- Converting from discrete to continuous — i.e. **grid** to **field**.
- How do we find the values of points “inside” a grid/mesh?
  - Indirect visualization: how to find vertices of triangles
  - Direct visualization: how to find the value of samples in space
- i.e., from an explicit grid, we want to evaluate a field **f** at some point **x**
  - The way we do this interpolation is called the **filter kernel** of the field f.

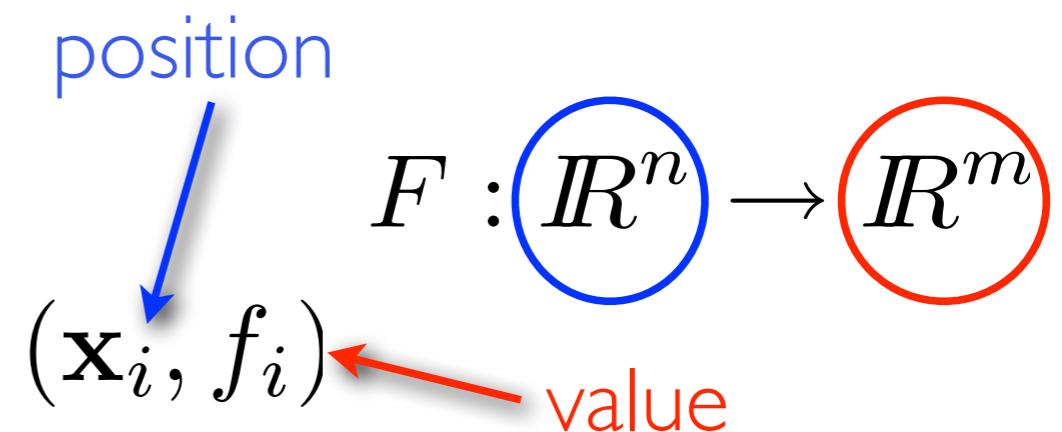
# Mesh Choice Impacts How the Continuous Data is Interpreted

- Two key questions:
  - Sampling, or the choice of where attributes are measured
  - Interpolation, or how to model the attributes in the rest of space



# Interpolation

- **Continuous** reconstruction of **discrete** input data

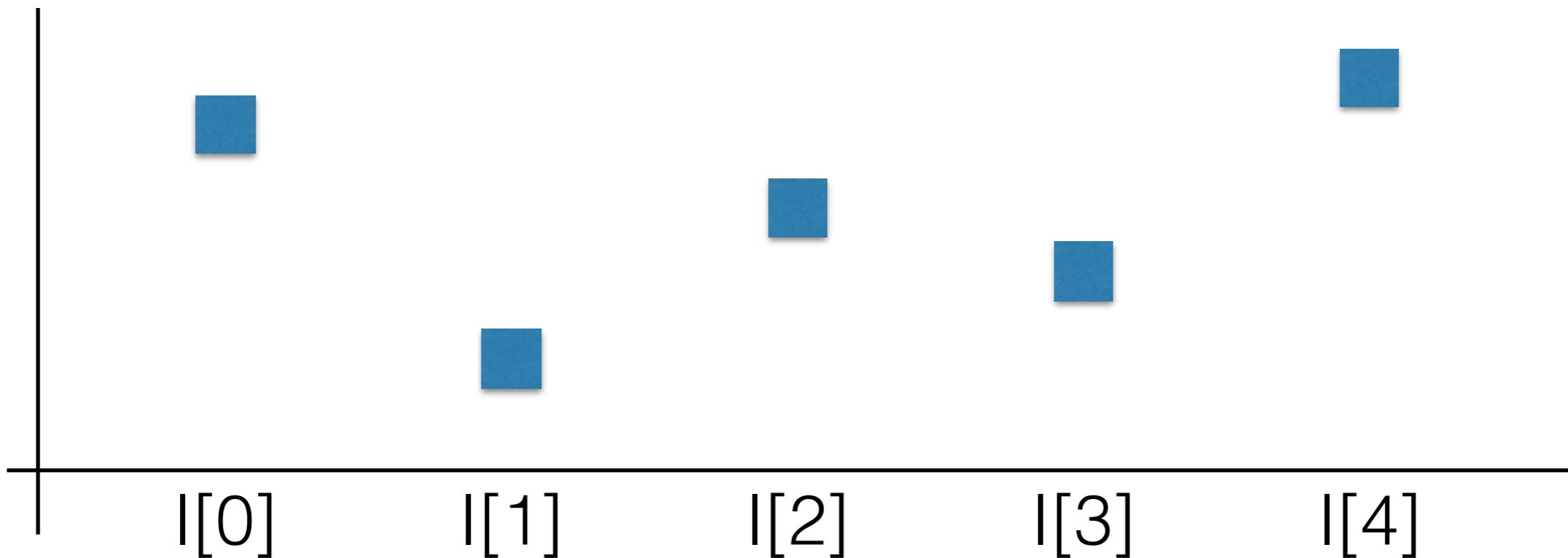


$$\forall i \in \{1, \dots, n\}, F(\mathbf{x}_i) = f_i$$

- Depends on grid structure (when available)
- Interpolation vs. approximation

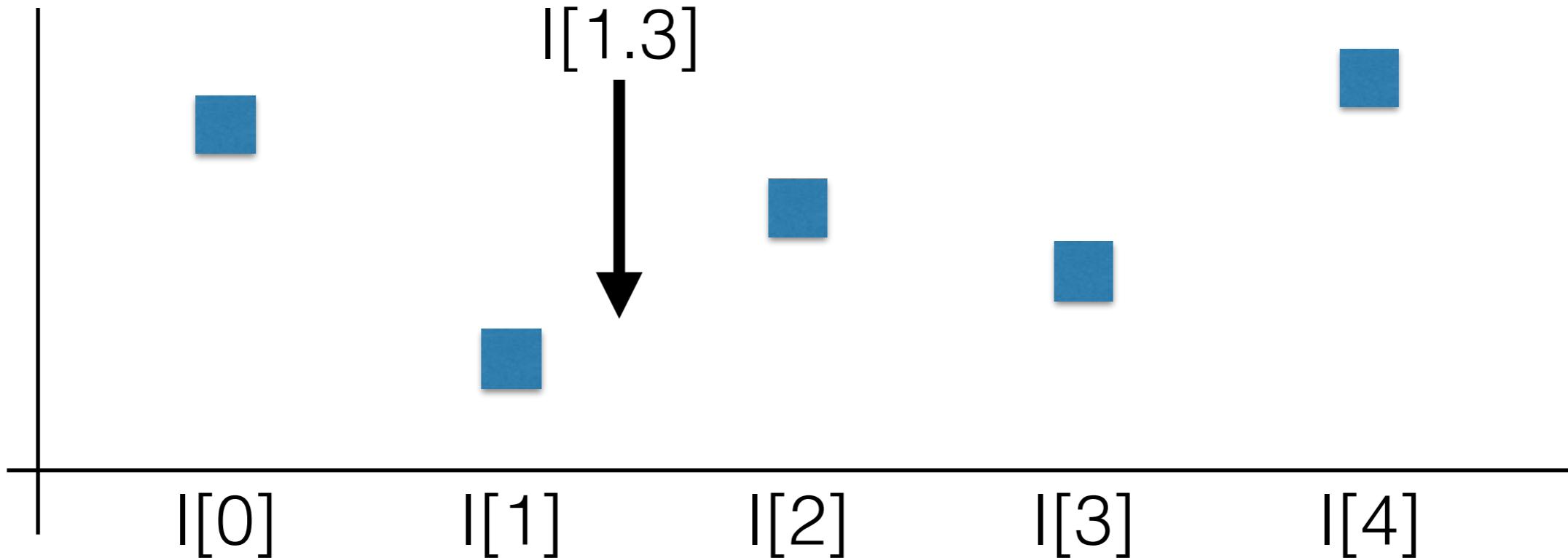
# Nearest Neighbor Interpolation

- Consider a 1-dimensional, grayscale image  $I$  spread horizontally
- What value is  $I[1.3]$  ?



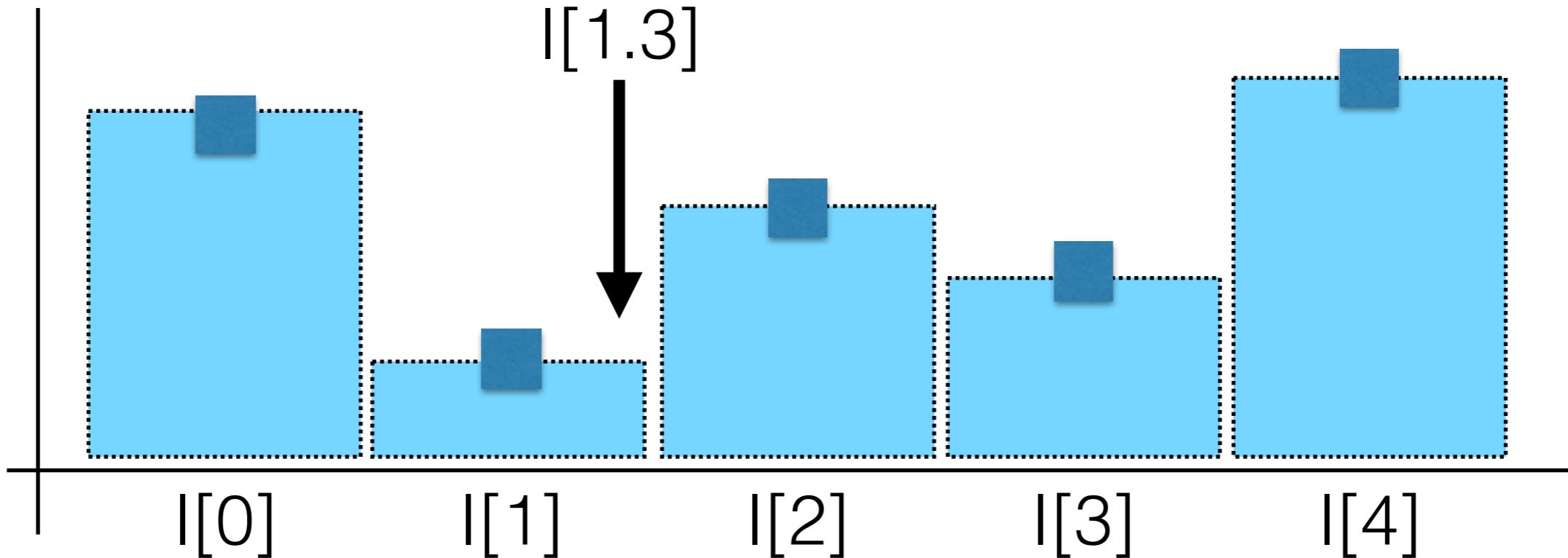
# Nearest Neighbor Interpolation

- Consider a 1-dimensional, grayscale image  $I$  spread horizontally
- What value is  $I[1.3]$  ?



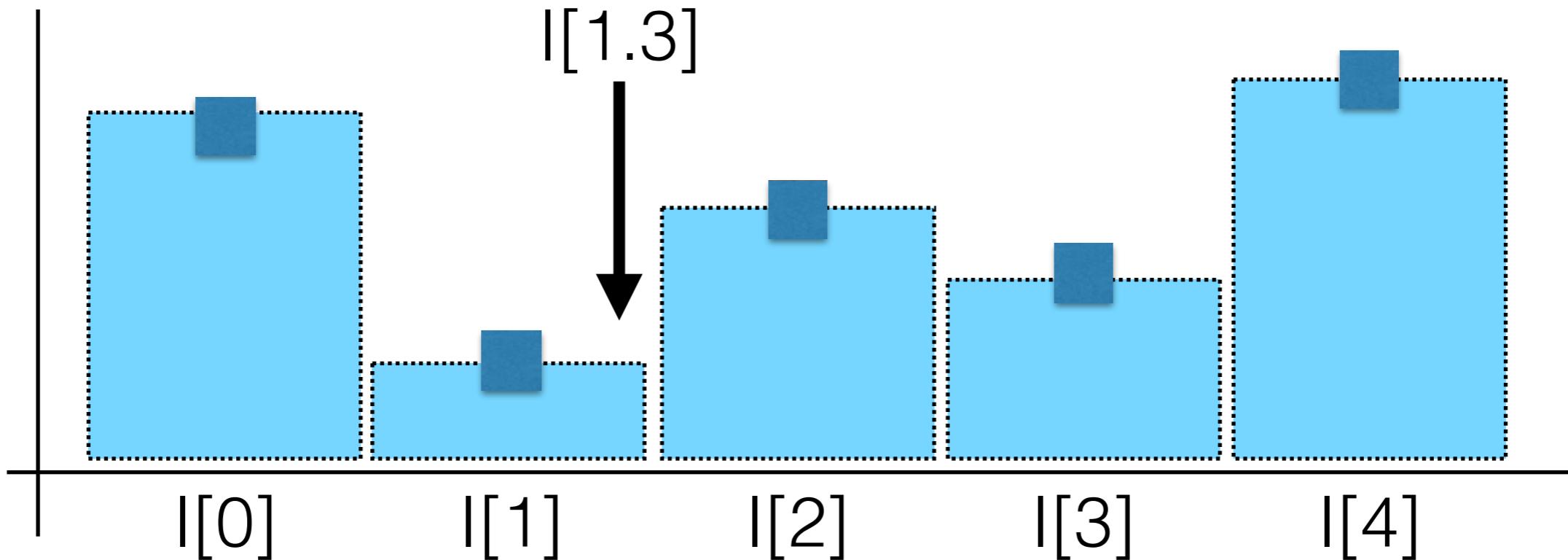
# Nearest Neighbor Interpolation

- Consider a 1-dimensional, grayscale image  $I$  spread horizontally
- What value is  $I[1.3]$  ?



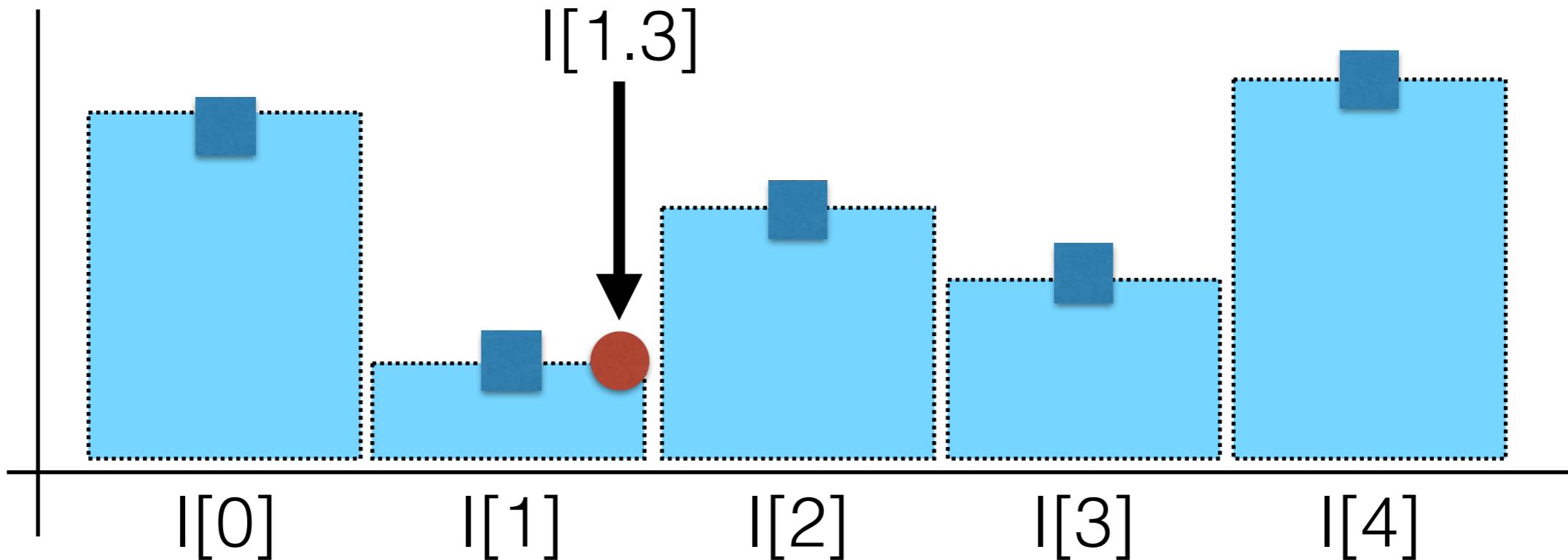
# Nearest Neighbor Interpolation

- Consider a 1-dimensional, grayscale image  $I$  spread horizontally
- What value is  $I[1.3]$  ?
  - $I[1.3] = I[\text{round}(1.3)] = I[1]$



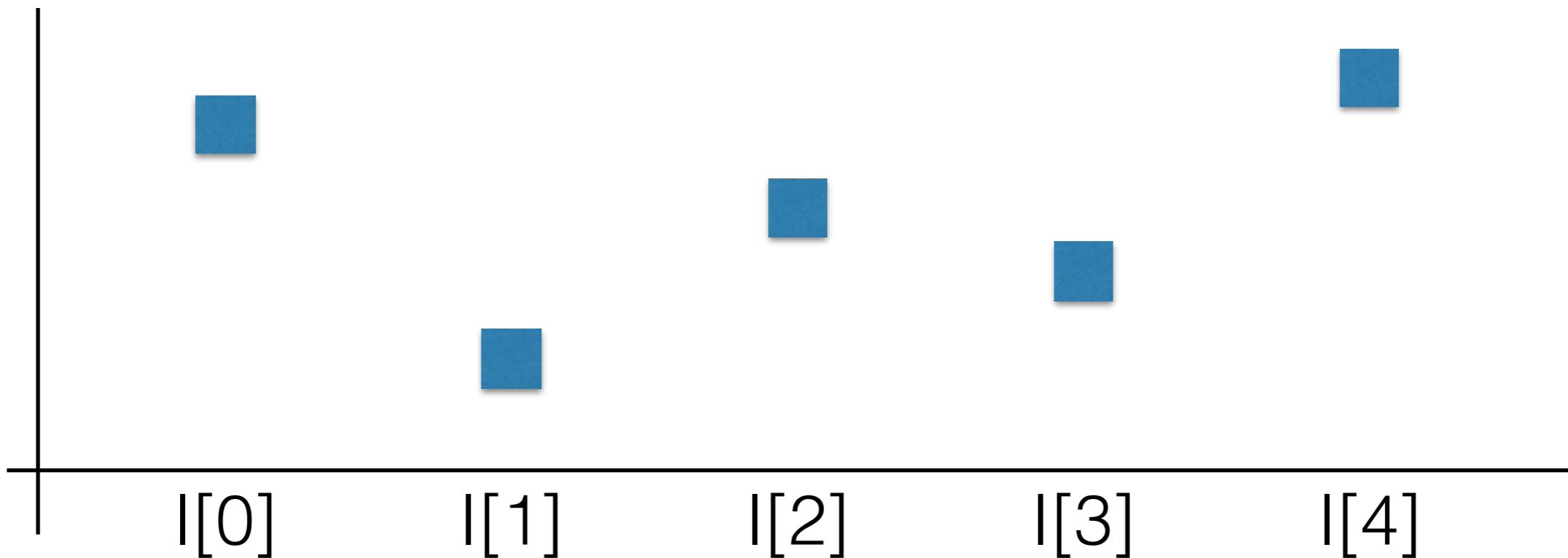
# Nearest Neighbor Interpolation

- Consider a 1-dimensional, grayscale image  $I$  spread horizontally
- What value is  $I[1.3]$  ?
  - $I[1.3] = I[\text{round}(1.3)] = I[1]$



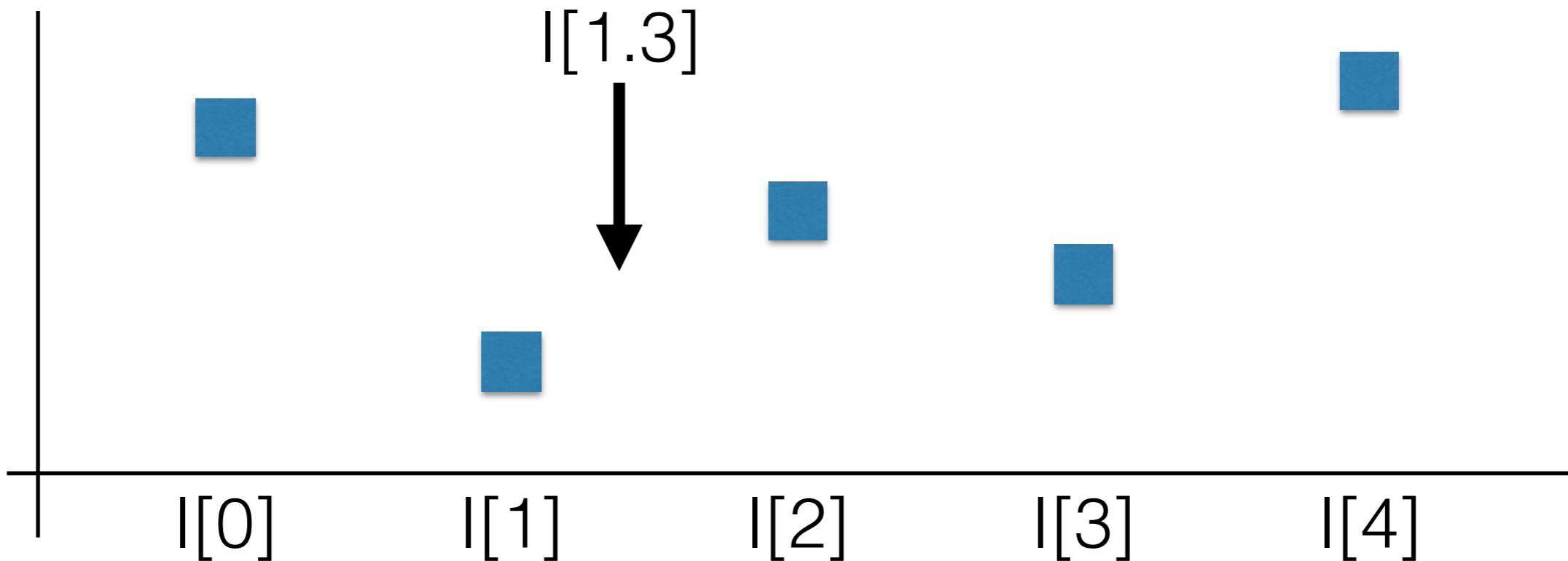
# Linear Interpolation

- Consider a 1-dimensional, grayscale image  $I$  spread horizontally
- What value is  $I[1.3]$  ?



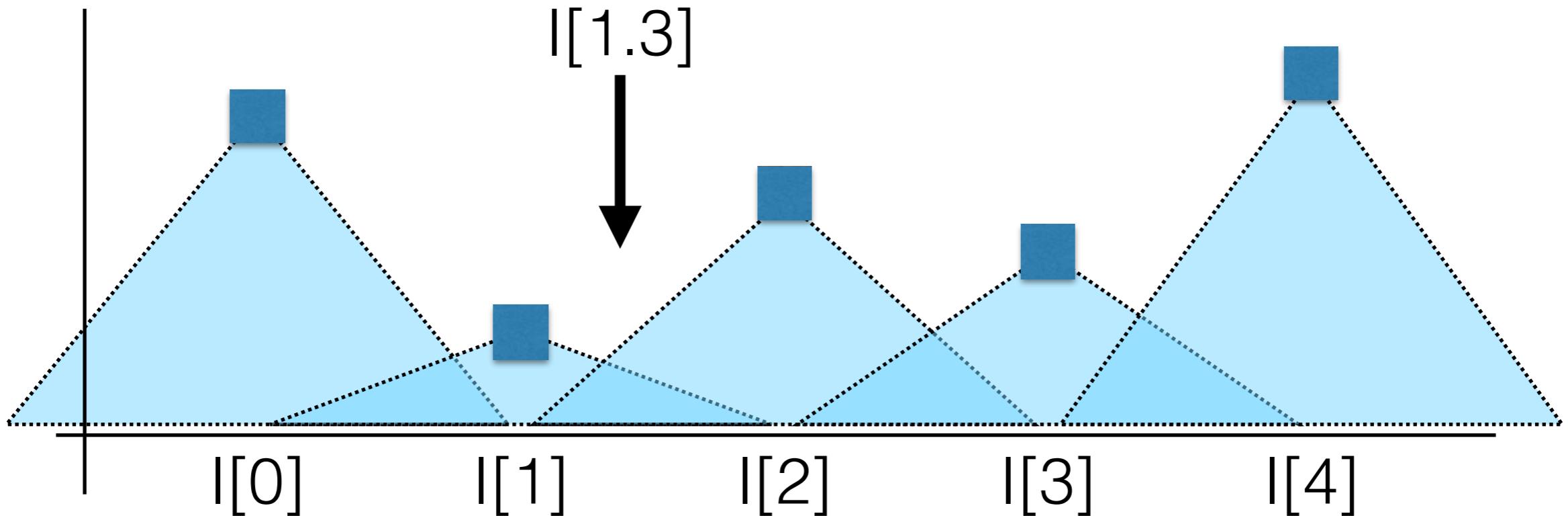
# Linear Interpolation

- Consider a 1-dimensional, grayscale image  $I$  spread horizontally
- What value is  $I[1.3]$  ?



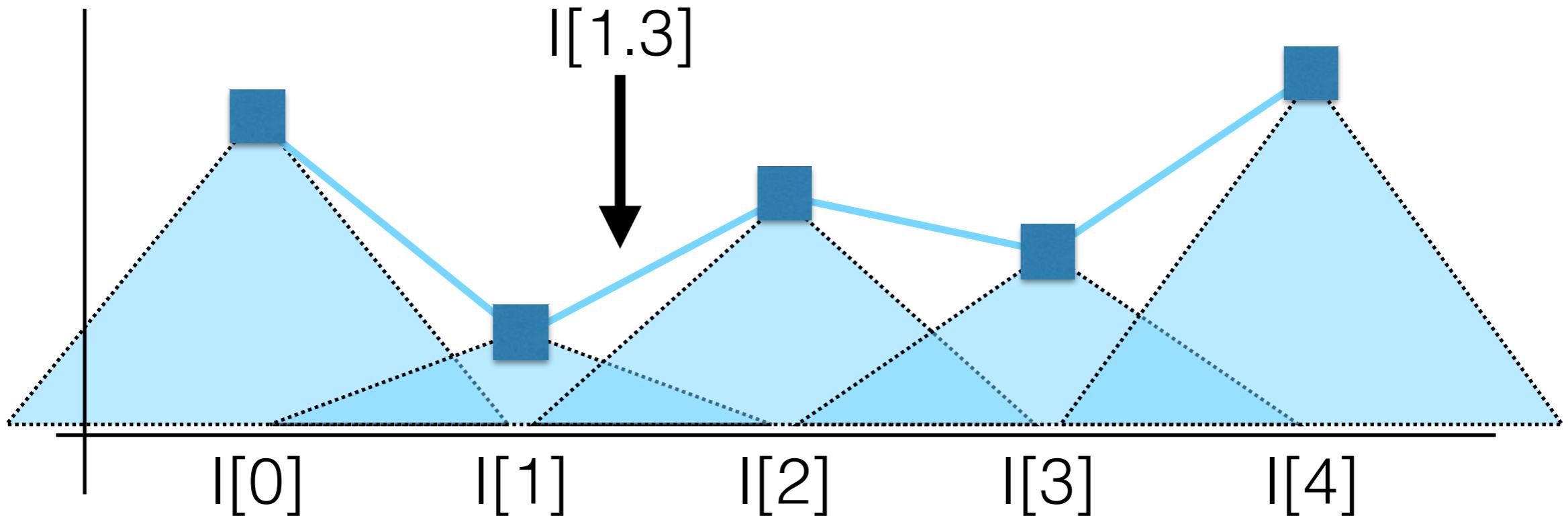
# Linear Interpolation

- Consider a 1-dimensional, grayscale image  $I$  spread horizontally
- What value is  $I[1.3]$  ?



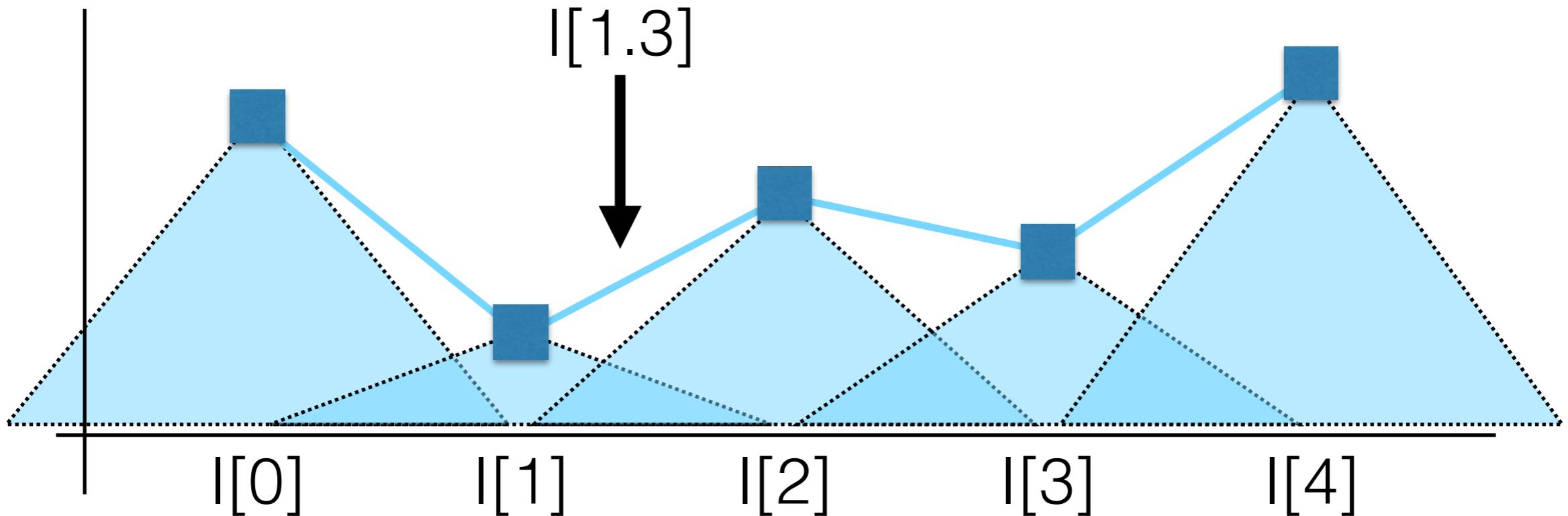
# Linear Interpolation

- Consider a 1-dimensional, grayscale image  $I$  spread horizontally
- What value is  $I[1.3]$  ?



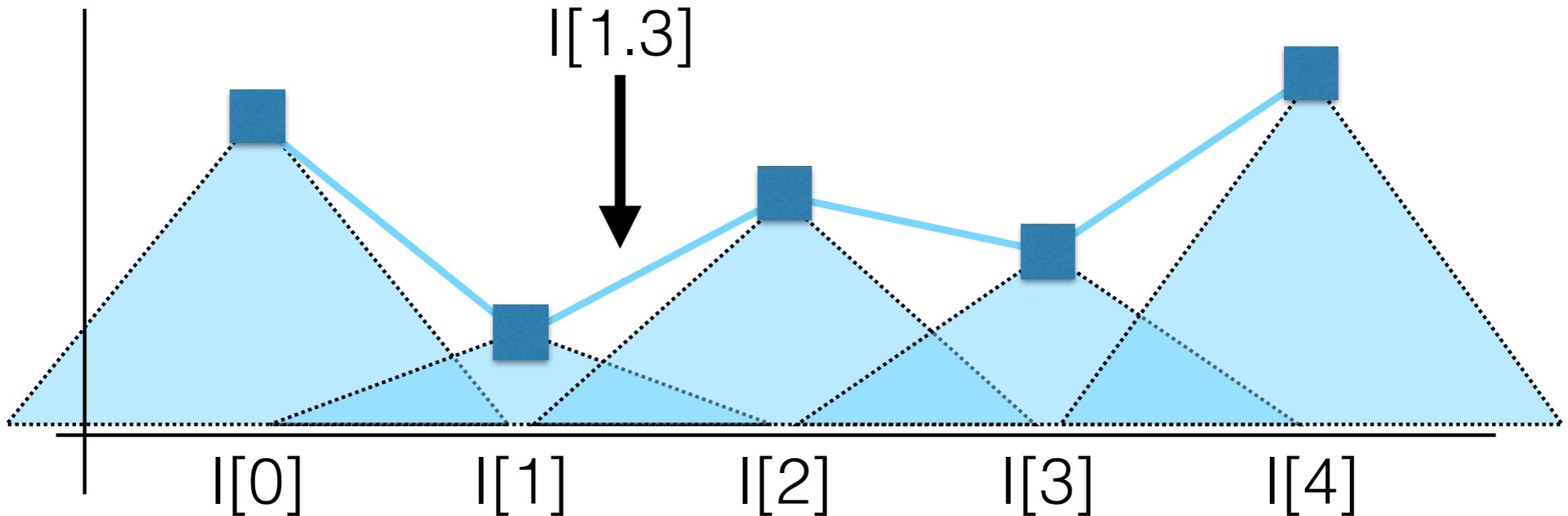
# Linear Interpolation

- Consider a 1-dimensional, grayscale image  $I$  spread horizontally
- What value is  $I[1.3]$ ?
  - Let  $s = 1.3 - \text{round}(1.3)$



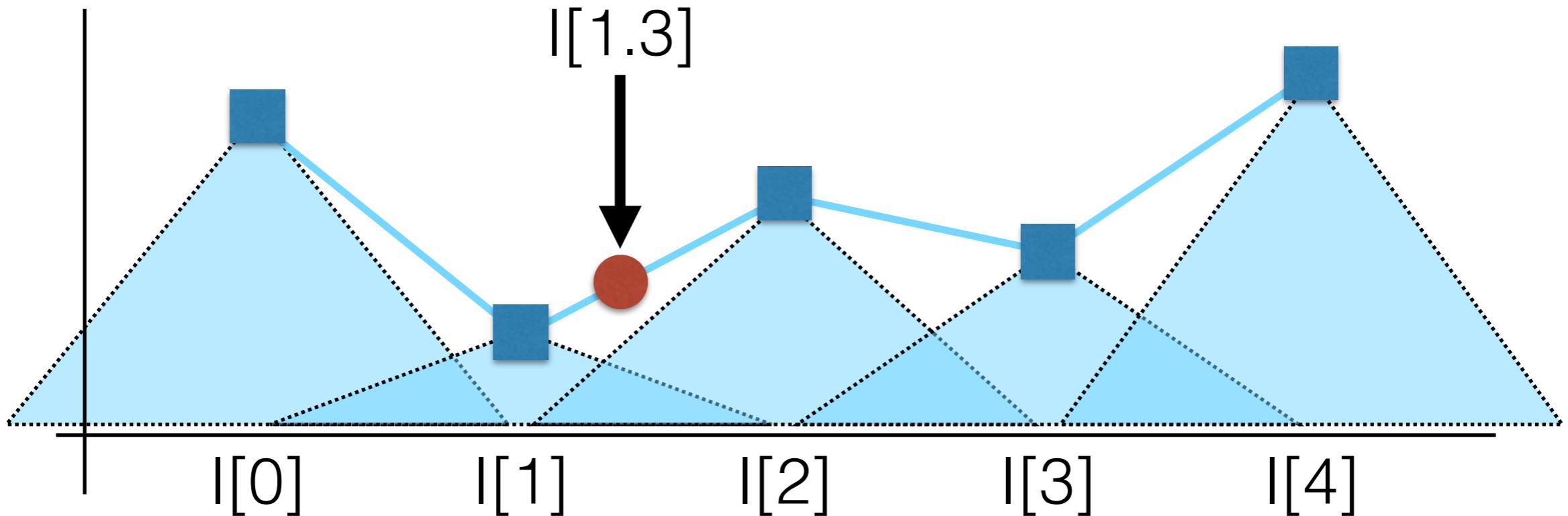
# Linear Interpolation

- Consider a 1-dimensional, grayscale image  $I$  spread horizontally
- What value is  $I[1.3]$ ?
  - Let  $s = 1.3 - \text{round}(1.3)$
  - $I[1.3] = 0.7*I[1] + 0.3*I[2] = (1-s)*I[1] + s*I[2]$



# Linear Interpolation

- Consider a 1-dimensional, grayscale image  $I$  spread horizontally
- What value is  $I[1.3]$ ?
  - Let  $s = 1.3 - \text{round}(1.3)$
  - $I[1.3] = 0.7*I[1] + 0.3*I[2] = (1-s)*I[1] + s*I[2]$



# Linear interpolation

```
#define lerp(a,b,t) (1-t) * a + t*b
```

# Bilinear interpolation

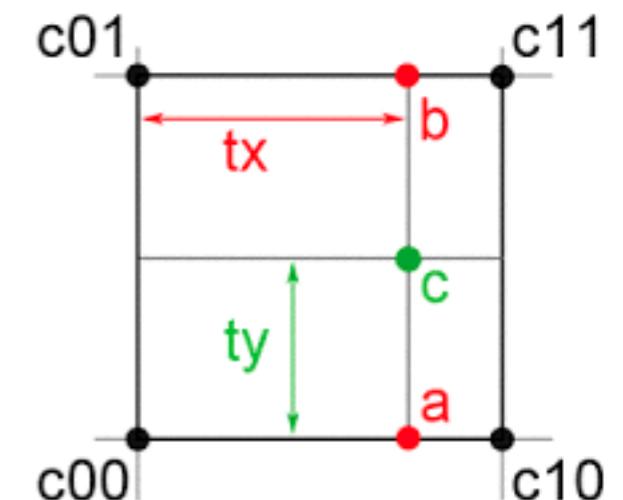
Just 3 linear interpolations

```
#define lerp(a,b,t) (1-t) * a + t*b
```

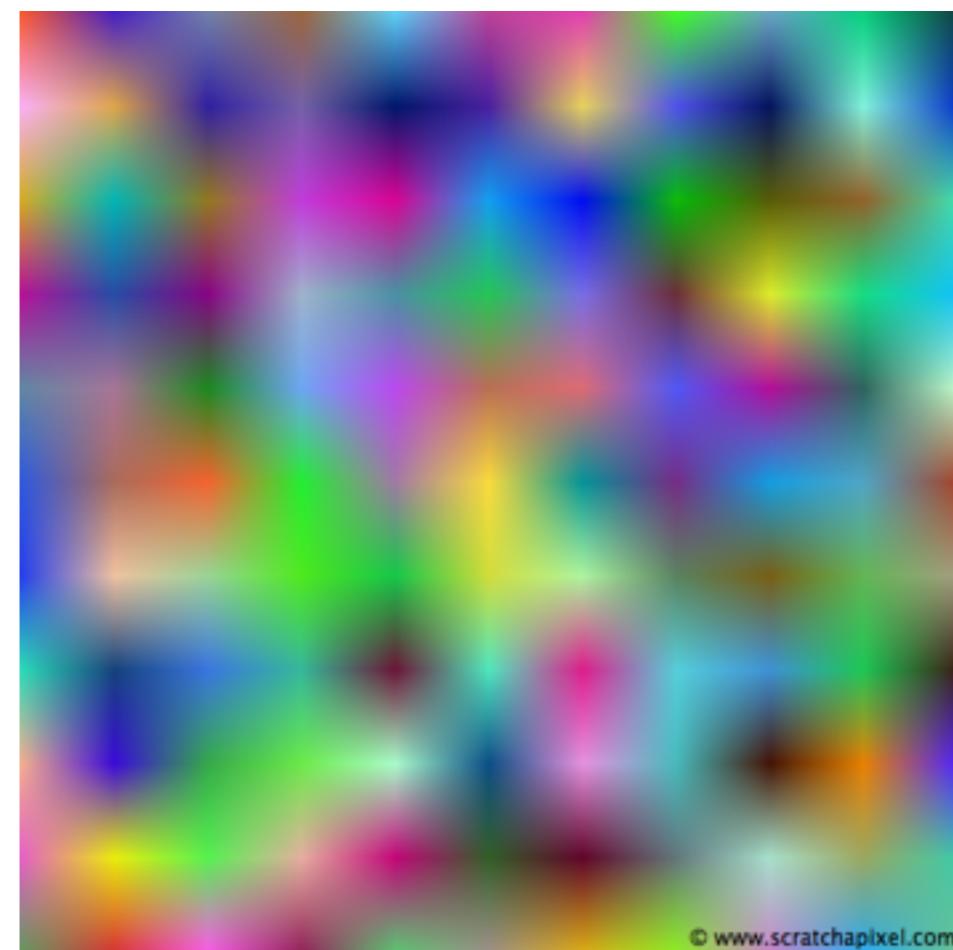
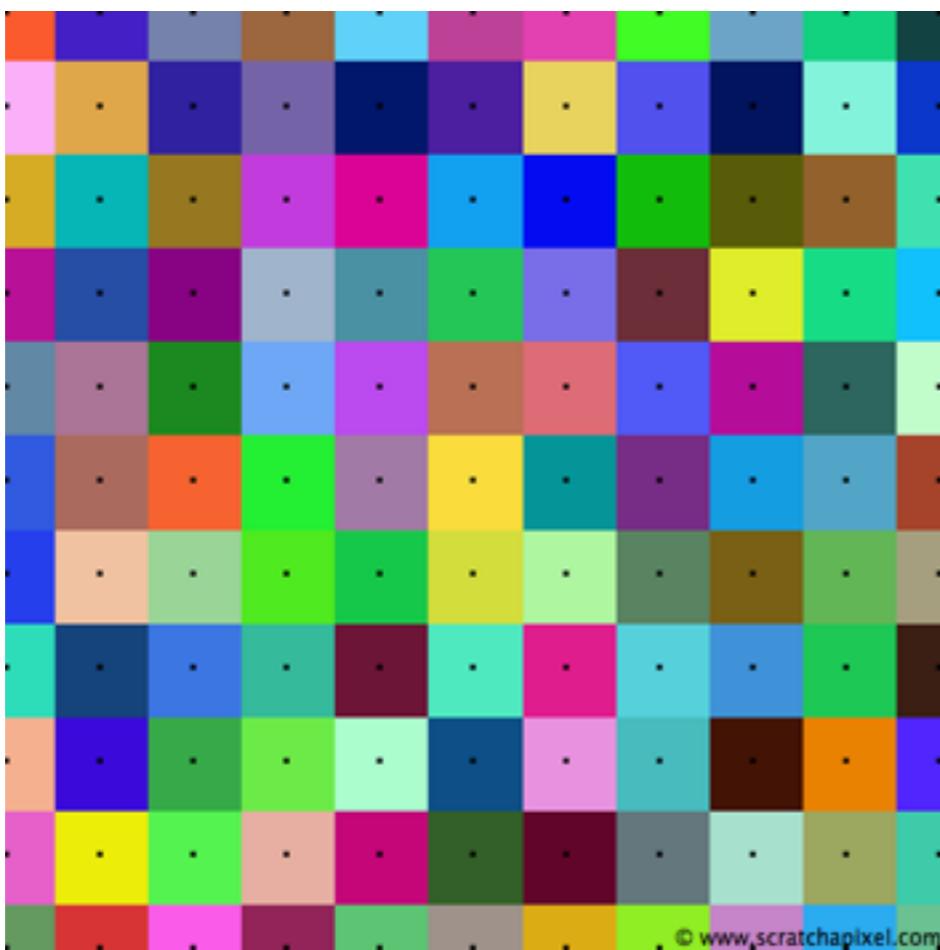
```
//Given voxel vertices cXX and the (tx,ty) position within the voxel [0,1]^2
```

```
//lerp along y direction  
float c00_y = lerp(c00, c01, ty);  
float c10_y = lerp(c10, c11, ty);
```

```
//lerp along x direction  
return lerp(c00_y, c10_y, tx);
```



# Bilinear interpolation



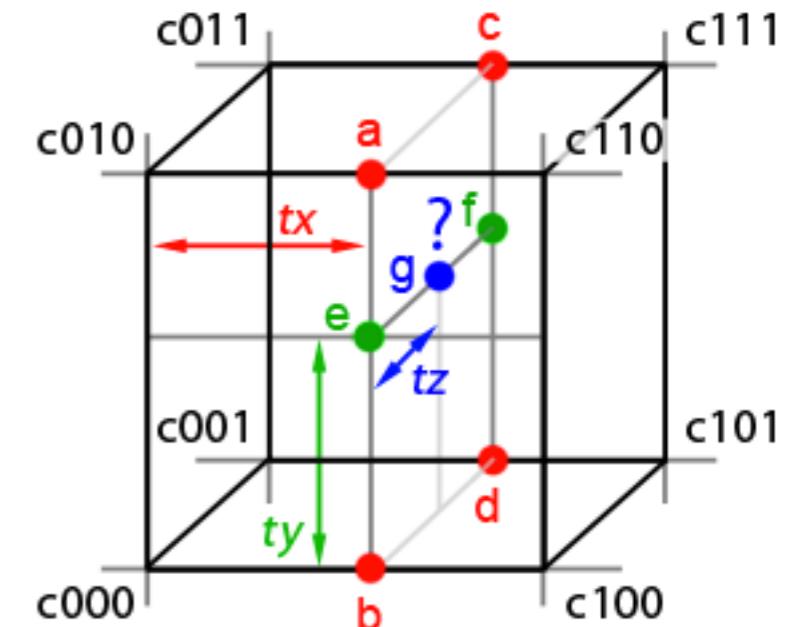
# Trilinear interpolation

Just 7 linear interpolations!

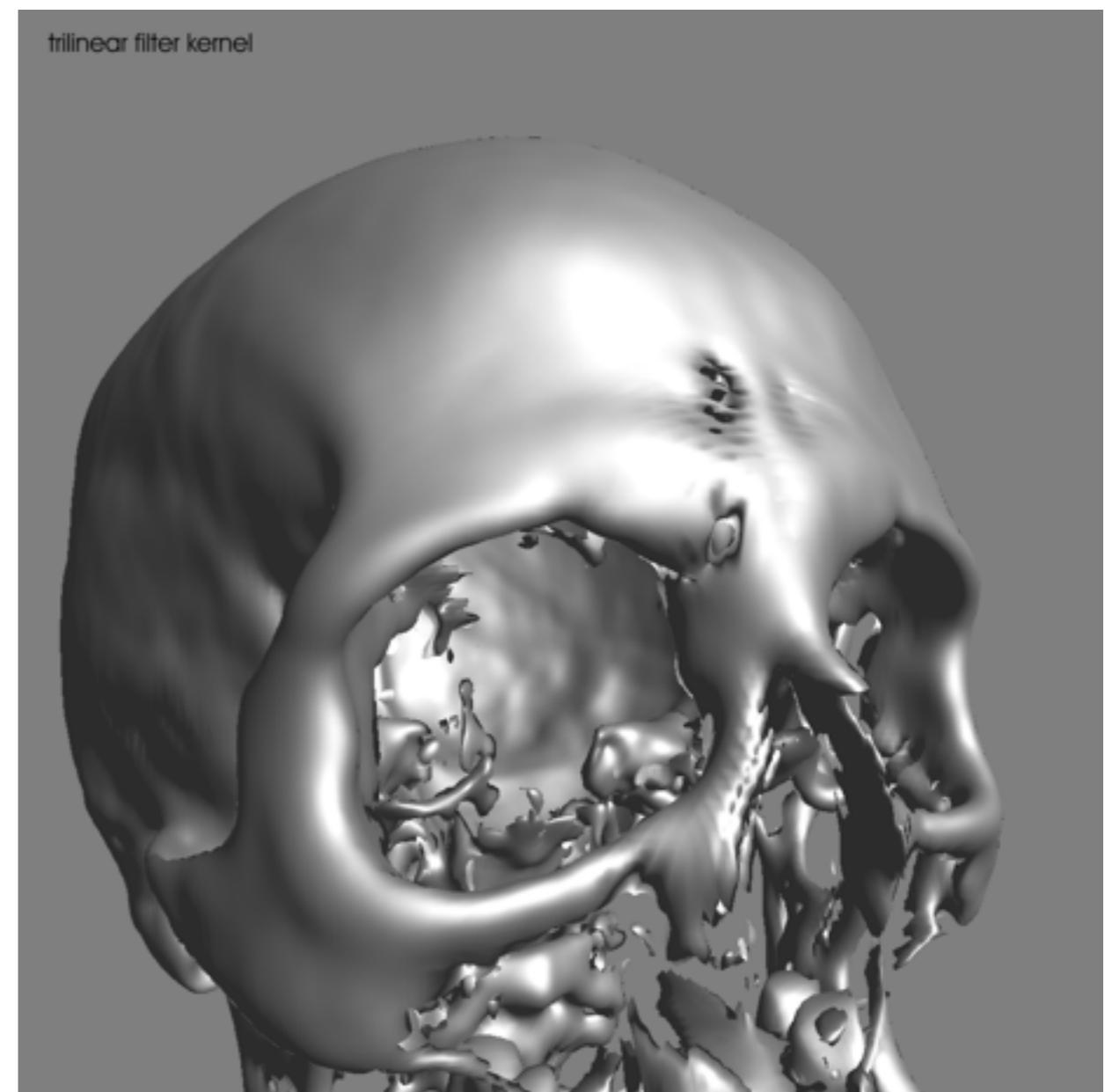
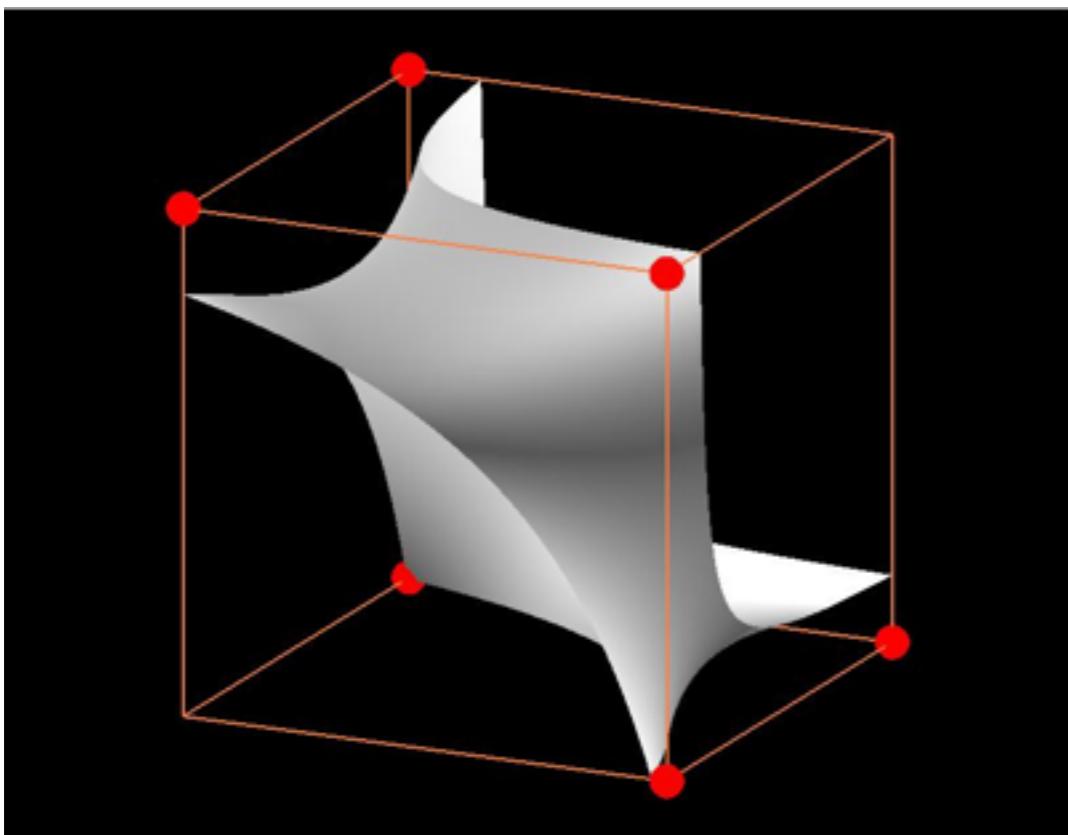
```
#define lerp(a,b,t) (1-t) * a + t*b
```

//Given voxel vertices cXXX and the (tx,ty,tz) position within the voxel [0,1]^3

```
//lerp along z direction.  
float c000_z = lerp(c000, c001, tz);  
float c010_z = lerp(c010, c011, tz);  
float c100_z = lerp(c100, c101, tz);  
float c110_z = lerp(c110, c111, tz);  
  
//lerp along y direction  
float c000_yz = lerp(c000_z, c010_z, ty);  
float c100_yz = lerp(c100_z, c110_z, ty);  
  
//lerp along x direction  
return lerp(c000_yz, c100_yz, tx);
```



# Isosurfaces from trilinear filter kernels



# Another formula for trilinear interpolation (3D)

$$f(x, y, z) = \sum_{i,j,k=\{0,1\}} x_i y_j z_k v_{ijk}$$

Where  $x_0 = i + 1 - x$ ,  $x_1 = x - i$ , ditto for y and z

And  $v_{ijk}$  is the value of the voxel at that vertex.

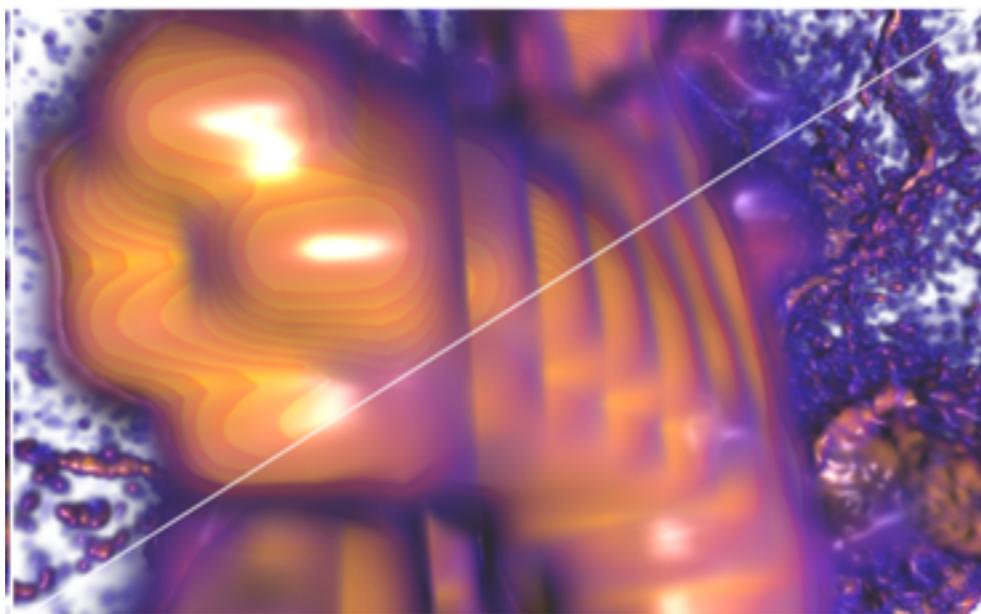
# General interpolation (3D)

$$f(x, y, z) = \sum_{i,j,k} B_i(x)B_j(y)B_k(z)v_{ijk}$$

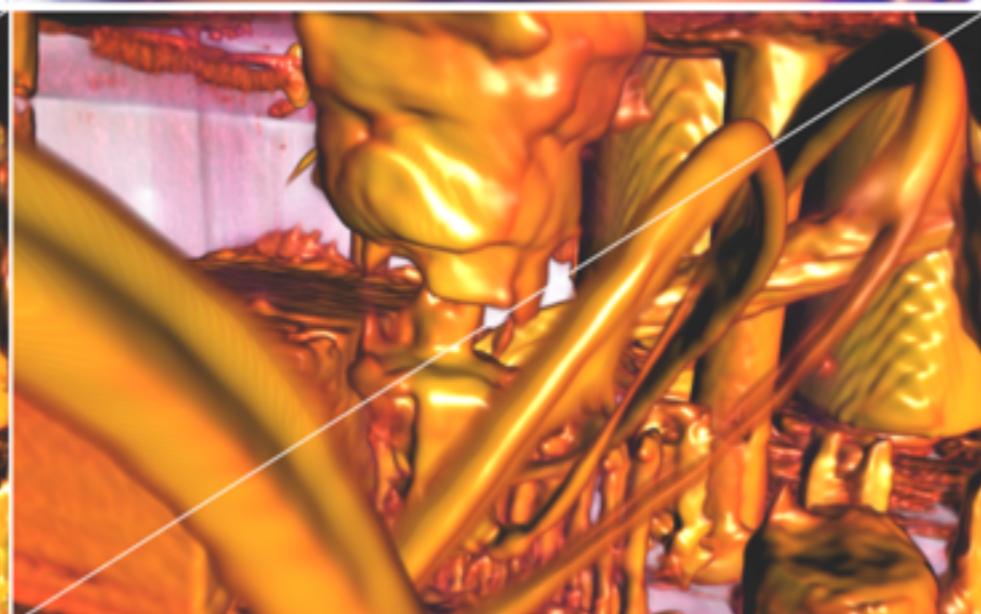
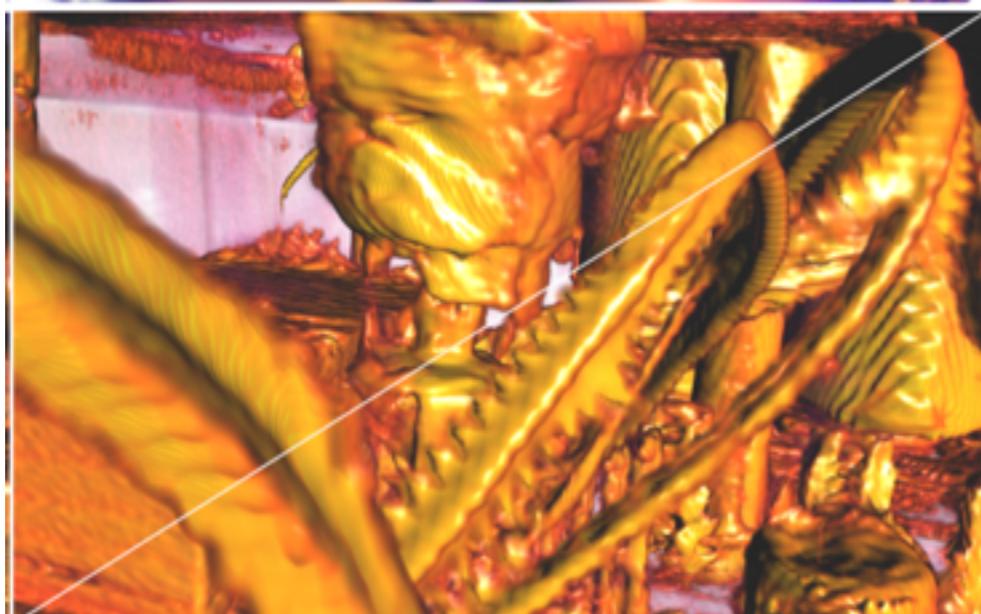
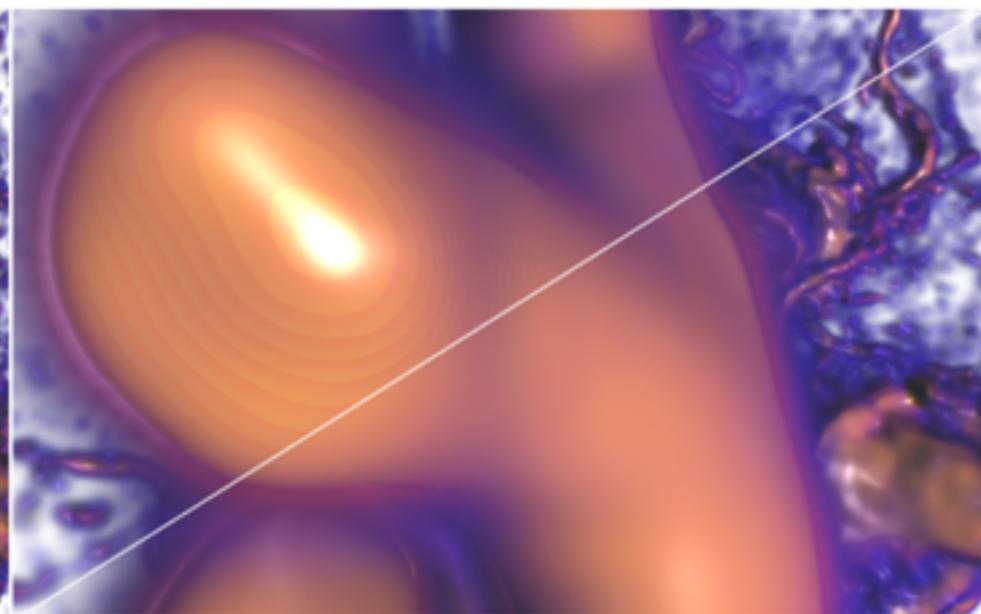
Where the  $B(x)$  is a general basis function,  $v$  is the voxel.

# Trilinear vs B-spline filtering for volume rendering

trilinear



tri - cubic-B-spline



# Next Tuesday lecture

- Volume rendering.