Modelling and Simulation of EIRSAT-1 Attitude Determination and Control System

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1 Introduction

This document details a study of the modelling and simulation of the Attitide Determination and Control System (ADCS) for the EIRSAT-1 cubesat. This includes the satellite attitude dynamics, orbit propagation after release from the ISS; disturbances torques, position and attitude sensor models, magnetic actuator models and attitude feedback control system. The simulation environment was developed using MATLAB/Simulink. Simulation results are presented for several different test cases. These test cases include demonstrations of different control modes, different control systems and different actuator and sensor configurations including failure of one or more devices.

2 Objectives

- Create a simulation environment for the EIRSAT-1 ADCS subsystem including:
  - Satellite attitude dynamics, modelled as a single rigid body with 6DOF
  - Orbit propagation of the satellite after release from the ISS
  - Disturbance torques including: gravity gradient, solar radiation pressure, aerodynamic, residual magnetic field

  - orbit propagation after release from ISS

- Define the different ADCS operational modes and functional requirements for each mode

- Design feedback controllers for each mode of the ADCS:

  - first assuming ideal 3-axis actuators
  - then with two magnetorquer actuators

- Design algorithm for safely switching between control modes

- Design controllers for failure modes including:

  - failure of one magnetorquer
  - failure of attitude sensors (needs to be more clearly defined)

3 ADCS Modes and Functional Requirements

- Detumbling

- Sun/object pointing

- Nadir/zenith pointing

3.1 Detumbling

Max rate to detumble? Time taken to detumble? Power?

3.2 Sun/distant object pointing

Required pointing accuracy? Settling time from one place to another? Power?

3.3 Nadir/zenith pointing

Required pointing accuracy? Time? Power?

3.4 Switching between modes

Which switches can happen? When can they happen ? (where in orbit, in eclipse?) Time taken?

4 Mathematical Modelling

4.1 Co-ordinate systems and Reference Frames

The spacecraft moves in an inertial reference frame N with associated cartesian coordinate system $OXYZ$ where $\mathbf{n_1}$, $\mathbf{n_2}$ and $\mathbf{n_3}$ are unit vectors along the X, Y and Z axes respectively. This coordinate system is centred at the center of the earth, $\mathbf{n_3}$ is normal to the equatorial plane and parallel to the earth rotational axis, pointing towards north pole, $\mathbf{n_1}$ points towards the vernal equinox, and $\mathbf{n_2}$ completes the right-handed orthonormal frame.

A second reference frame A is attached to the spacecraft rigid body with coordinate system $Cxyz$ where C is the mass centre of the rigid body and $\mathbf{a_1}$, $\mathbf{a_2}$ and $\mathbf{a_3}$ are unit vectors along the x, y and z axes respectively.

A third reference frame, named orbital frame, and here denoted O, is spanned by the unit vectors $\mathbf{o_1}$, $\mathbf{o_2}$ and $\mathbf{o_3}$, and centred at the satellite's centre of mass. The vector $\mathbf{o_3}$ constantly points to the Earth's center, whereas $\mathbf{o_2}$ aligns with the orbit angular momentum. $\mathbf{o_2}$ is the cross product of $\mathbf{o_3}$ and $\mathbf{o_1}$, completing the three axis orthonormal frame.

4.2 Satellite Dynamics

EIRSAT-1 is modelled as a single rigid body with 6-DOF (degrees of freedom). The satellite has mass m and inertia tensor I in the body fixed coordinate system

4.3 Rotational Kinematics

The attitude of the spacecraft is described by quaternions.

4.3.1 Alternative representations of attitude and conversions

The orientation of the spacecraft may also be described using a Direction Cosine Matrix.

The orientation of the spacecraft may also be described using Euler angles. Starting with the body-fixed $oxyz$ axes aligned with the $OXYZ$ axes, the body undergoes a sequence of rotations by angles θ_3, θ_2 and θ_1 about the body-fixed z (yaw), y (pitch) and x (roll) axes respectively to reach its final orientation.

The DCM (Direction Cosine Matrix) is then described by equation 1.

$$C = \tag{1}$$

4.4 Orbit Propagation

4.5 Environment

4.5.1 Magnetic Field

Spherical Harmonics

The set of shperical harmonics is an orthonormal set of functions defined in the unit sphere. They are given as the angular part of the solution to the Laplace Equation in spherical coordinates. Analogously to how sines and cosines are used in harmonic analysis to represent periodic functions through Fourier Series, Spherical Harmonics may be used to define functions in the surface of a sphere.

Spherical Coordinates

It is important to identify the coordinates to be used, as, for instance, spherical coordinates have often two different interpretations: one, which is mainly used by mathematicians, uses the latitude, being the angle between the position vector and the equatorial plane, as one of the angular coordinates; while the other, broadly employed by physicists, uses the co-latitude, defined as the angle between the position vector and the z axis, thus being the complementary of the latitude.

Here, the spherical coordinates are taken to be

r, the radial distance from Earth's centre

ϕ, the East longitude, bounded between $-\pi$ and π

θ, the co-latitude, defined to be $\theta = \frac{\pi}{2} - \theta'$, with θ' the latitude. $\theta \in [0, \pi]$

The magnetic field is given by the negative gradient of the potential V defined for $r \geq R_e$, and given by the spherical harmonic approximation

$$V = r \sum_{n=1}^{L} \left(\frac{R_e}{r} \right)^{n+1} \sum_{m=0}^{n} \left(g_n^m cos(m\phi) + h_n^m sin(m\phi) \right) P_n^m(cos\theta) \tag{2}$$

That is

$$\boldsymbol{\beta} = -\nabla V \tag{3}$$

Where g_n^m and h_n^m are the set of IGRF gaussian coefficients, published and revised every five years by the participating members of the IAGA (International Association of Geomagnetism and Aeronomy). The 12^{th} generation is here used, as the last revision by the time of this work. These coefficient also include the secular variation (SV), which bestow the model on the proper time dependence. Keping track of the change in the coefficients in nT per year. The coefficients for some epoch year are referred to as IGRF.

When real data about the geomagnetic field becomes available so that some adjustements can be made, the model becomes definitive and changes its name to DGRF (Deffinitive geomagnetic reference field).

$P_n^m(cos\theta)$ are the Schmidt normalized associated Legendre polynomials of degree n and order m

One can find very effective, recursive ways of computing the Schmidt quasi-normalization factors, the Associated Legendre polynomials and its derivatives, as can be found on [J.Davis - Mathematical Modeling of Earth's Magnetic Field]

The three components of the magnetic field in r, ϕ and θ directions can be found in terms of the partial derivatives of 2

$$\begin{aligned}
\boldsymbol{\beta}_r &= -\frac{\partial V}{\partial r} = \sum_{n=1}^{N}(n+1)\left(\frac{R_e}{r}\right)^{n+2}\sum_{m=0}^{n}\left(g_n^m cos(m\phi) + h_n^m sin(m\phi)\right)P_n^m(\theta) \\
\boldsymbol{\beta}_\theta &= -\frac{1}{r}\frac{\partial V}{\partial \theta} = -\sum_{n=1}^{L}\left(\frac{R_e}{r}\right)^{n+2}\sum_{m=0}^{n}\left(g_n^m cos(m\phi) + h_n^m sin(m\phi)\right)\frac{\partial P_n^m(\theta)}{\partial \theta} \\
\boldsymbol{\beta}_\phi &= -\frac{1}{rsin\theta}\frac{\partial V}{\partial \phi} = \frac{1}{sin\phi}\sum_{n=1}^{L}\left(\frac{R_e}{r}\right)^{n+2}\sum_{m=0}^{n} m\left(g_n^m sin(m\phi) - h_n^m cos(m\phi)\right)P_n^m(\theta)
\end{aligned} \tag{4}$$

Legendre Polynomials

The set of regular Legendre Polynomials is given by the solutions $P_n(\nu)$ to the equation

$$\frac{1}{\sqrt{1-2\nu x + x^2}} = \sum_{n=0}^{\infty} x^2 P_n(\nu) \tag{5}$$

Which gives

$$P_n(\nu) = \frac{1}{n!2^n}\frac{d^n}{d\nu^n}(\nu^2-1)^n \tag{6}$$

Associated Legendre Polynomials

The associated Legendre polynomials $P_n^m(\nu)$ of degree n and order m relate to $P_n(\nu)$ by

$$P_{n,m}(\nu) = (1-\nu^2)^{\frac{1}{2m}}\frac{d^m}{d\nu^m}(P_n(\nu)) \tag{7}$$

These polynomials are not normalized in any way. Recall that, in order to use the spherical harmonic approximation, one needs to find the Schmidt quasi-normalization of the corresponding Legendre Polynomials. Such approximation is derived in what follows.

Gaussian normalization and Schmidt quasi-normalization The two most broadly used normalizations are the Gaussian normalization and the Schmidt quasi-normalization (see, for instance [R. A. Langel. Geomagnetism]). To compute the geomagnetic field, the Schmidt quasi-normalization is used, and it relates to the associated Legendre Polynomials by

$$P_n^m(\nu) = \sqrt{\frac{2(n-m)!}{(n+m)!}} P_{n,m}(\nu) \tag{8}$$

However, it is much more efficient to compute the Gaussian normalized Legendre polynomials $P^{n,m}$, whose interest lies in the existence of a recursive formula for their effective computation [James R.Wertz. Spacecraft Attitude Determination and Control]. These are given by

$$P^{n,m}(\nu) = \frac{2^n!(n-m)!}{(2n)!} P_{n,m}(\nu) \tag{9}$$

An can be related to the Schmidt quasi-normalized Legendre Polynomials by using

$$P_n^m(\nu) = \Gamma_{n,m} P^{n,m}(\nu) \tag{10}$$

Where

$$\begin{aligned} \Gamma_{n,m} &= \sqrt{\frac{(n-m)!}{(n+m)!}} \frac{(2n-1)!!}{(n-m)!}, \quad m = 0 \\ \Gamma_{n,m} &= \sqrt{\frac{2(n-m)!}{(n+m)!}} \frac{(2n-1)!!}{(n-m)!}, \quad m \neq 0 \end{aligned} \tag{11}$$

The algorithm will calculate, for a given value of θ, the Gaussian normalized Legendre Polynomials $P^{n,m}(\theta)$ and its derivatives $\frac{d^m}{d\nu^m}(P^{n,m}(\nu))$, using the recursive formulas presented by [James R.Wertz. Spacecraft Attitude Determination and Control], and outlined below.

One step further, it is computationally cheaper to modify the IGRF coefficients, using the relation

$$\begin{aligned} g_{n,m} &= \Gamma_{n,m}\ g_n^m \\ h_{n,m} &= \Gamma_{n,m}\ h_n^m \end{aligned} \tag{12}$$

So that the recursive algorithm for the Schmidt quasi-normalization coefficients ([James R.Wertz. Spacecraft Attitude Determination and Control]) is ran only once for each pair (m, n).

Transformation to inertial reference frame

After the three components of the magnetic field β_r, β_θ and β_ϕ from ()4) in local spherical coordinates are calculated, we aim to express them in terms of an Earth centred inertial reference frame. This transformation is given by 13, and it is illustrated in figure 4.5.1

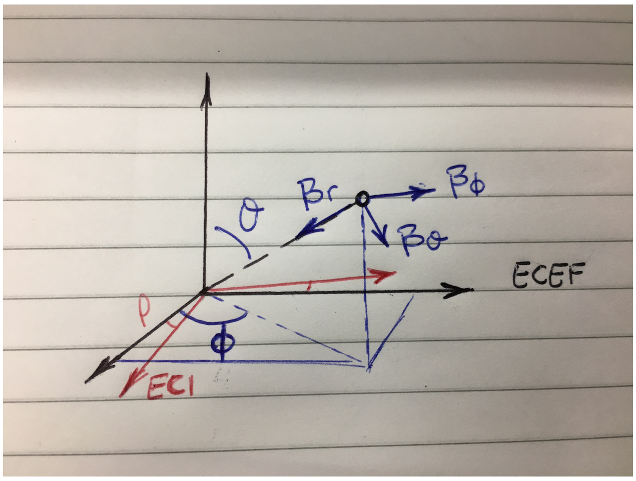

Figure 1: Frame transformation between local spherical and inertial reference frame

$$
\begin{aligned}
\beta_x &= \big(\beta_r cos(\theta') + \beta_\theta sin(\theta')\big)\, cos(l) - \beta_\phi sin(l) \\
\beta_y &= \big(\beta_r cos(\theta') + \beta_\theta sin(\theta')\big)\, sin(l) + \beta_\phi cos(l) \\
\beta_z &= \beta_r sin(\theta') + \beta_\theta cos(l)
\end{aligned} \tag{13}
$$

Recursive formulas for Gaussian normalized Legendre polynomials, its derivatives and Schmidt quasi-normalization factors

Associated Legendre polynomials and derivatives

$$
\begin{aligned}
P^{0,0}(\theta) &= 1 \\
P^{n,m}(\theta) &= sin(\theta) P^{n-1,m-1} \\
P^{n,m}(\theta) &= cos(\theta) P^{n-1,m} - K^{n,m} P^{n-2,m}
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
\frac{\partial P^{0,0}(\theta)}{\partial \theta} &= 0 \\
\frac{\partial P^{n,n}(\theta)}{\partial \theta} &= sin(\theta) \frac{\partial P^{n-1,n-1}(\theta)}{\partial \theta} + cos(\theta) P^{n-1,n-1} \\
\frac{\partial P^{n,m}(\theta)}{\partial \theta} &= cos(\theta) \frac{\partial P^{n-1,m}(\theta)}{\partial \theta} - sin(\theta) P^{n-1,m} - K^{n,m} \frac{\partial P^{n-2,m}(\theta)}{\partial \theta}
\end{aligned} \tag{15}
$$

Schmidt quasi-normalization factors

$$
\begin{aligned}
\Gamma_{0,0} &= 1 \\
\Gamma_{n,0} &= \frac{2n-1}{n} \Gamma_{n-1,0} \\
\Gamma_{n,m} &= \sqrt{\frac{(n-m+1)(\delta_m^1 + 1)}{n+m}} \Gamma_{n,m-1}
\end{aligned} \tag{16}
$$

[1]

4.5.2 Gravitational Field

4.6 Sensor Models

4.7 Actuator Models

5 Controller Design

6 Simulink Model Structure

6.1 Dynamics block

6.1.1 Spacecraft Dynamics

6.1.2 Orbital Dynamics

6.2 Environment block

6.3 Disturbance block

6.4 Sensor block

6.5 Actuator block

6.6 Controller block

6.7 Reference block

6.8 Mode Select block

6.9 Running Simulations

Model Setup File

Simulation Setup File

[1] Note the use of upper and subscript notation here to distinguish between Regular, Associated, and Schmidt quasi-normalized Legendre polynomials

7 Results