

# Modelling, Simulation and Testing of EIRSAT-1 Attitude Determination and Control System

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## 1 Introduction

This document details a study of the modelling and simulation of the Attitude Determination and Control System (ADCS) for the EIRSAT-1 cubesat. This includes the satellite attitude dynamics, orbit propagation after release from the ISS; disturbances torques, position and attitude sensor models, magnetic actuator models and attitude feedback control system. The simulation environment was developed using MATLAB/Simulink.

Simulation results are presented for several different test cases. These test cases include demonstrations of different control modes, different control systems and different actuator and sensor configurations including failure of one or more devices.

## 2 Objectives

- Create a simulation environment for the EIRSAT-1 ADCS subsystem including:
  - Satellite attitude dynamics, modelled as a single rigid body with 6DOF
  - Orbit propagation of the satellite after release from the ISS
  - Disturbance torques including: gravity gradient, solar radiation pressure, aerodynamic, residual magnetic field
  - orbit propagation after release from ISS
- Define the different ADCS operational modes and functional requirements for each mode
- Design feedback controllers for each mode of the ADCS:
  - first assuming ideal 3-axis actuators
  - then with two magnetorquer actuators
- Design algorithm for safely switching between control modes
- Design controllers for failure modes including:
  - failure of one magnetorquer
  - failure of attitude sensors (needs to be more clearly defined)

## 3 ADCS Modes and Functional Requirements

- Detumbling
- Sun/object pointing
- Nadir/zenith pointing

### 3.1 Detumbling

Max rate to detumble? Time taken to detumble? Power?

### 3.2 Sun/distant object pointing

Required pointing accuracy? Settling time from one place to another? Power?

### 3.3 Nadir/zenith pointing

Required pointing accuracy? Time? Power?

### 3.4 Switching between modes

Which switches can happen? When can they happen ? (where in orbit, in eclipse?)  
Time taken?

## 4 Mathematical Modelling

### 4.1 Co-ordinate systems and Reference Frames

The spacecraft moves in an inertial reference frame  $N$  with associated cartesian coordinate system  $OXYZ$  where  $\mathbf{n}_1$ ,  $\mathbf{n}_2$  and  $\mathbf{n}_3$  are unit vectors along the  $X$ ,  $Y$  and  $Z$  axes respectively. This coordinate system is centred at the center of the earth,  $\mathbf{n}_3$  is normal to the equatorial plane and parallel to the earth rotational axis, pointing towards north pole,  $\mathbf{n}_1$  points towards the vernal equinox, and  $\mathbf{n}_2$  completes the right-handed orthonormal frame.

A second reference frame  $A$  is attached to the spacecraft rigid body with coordinate system  $Cxyz$  where  $C$  is the mass centre of the rigid body and  $\mathbf{a}_1$ ,  $\mathbf{a}_2$  and  $\mathbf{a}_3$  are unit vectors along the  $x$ ,  $y$  and  $z$  axes respectively.

A third reference frame, named orbital frame, and here denoted  $O$ , is spanned by the unit vectors  $\mathbf{o}_1$ ,  $\mathbf{o}_2$  and  $\mathbf{o}_3$ , and centred at the satellite's center of mass. The vector  $\mathbf{o}_3$  constantly points to the Earth's center, whereas  $\mathbf{o}_2$  aligns with the orbit angular momentum.  $\mathbf{o}_2$  is the cross product of  $\mathbf{o}_3$  and  $\mathbf{o}_1$ , completing the three axis orthonormal frame.

### 4.2 Satellite Dynamics

EIRSAT-1 is modelled as a single rigid body with 6-DOF (degrees of freedom). The satellite has mass  $m$  and inertia tensor  $I$  in the body fixed coordinate system

### 4.3 Rotational Kinematics

The attitude of the spacecraft is described by quaternions.

#### 4.3.1 Alternative representations of attitude and conversions

The orientation of the spacecraft may also be described using a Direction Cosine Matrix.

The orientation of the spacecraft may also be described using Euler angles. Starting with the body-fixed  $oxyz$  axes aligned with the  $OXYZ$  axes, the body undergoes a sequence of rotations by angles  $\theta_3$ ,  $\theta_2$  and  $\theta_1$  about the body-fixed  $z$  (yaw),  $y$  (pitch) and  $x$  (roll) axes respectively to reach its final orientation.

The DCM (Direction Cosine Matrix) is then described by equation 1.

$$C = \tag{1}$$

### 4.4 Orbit Propagation

### 4.5 Environment

#### Spherical Harmonics

The set of spherical harmonics is an orthonormal set of functions defined in the unit sphere. They are given as the angular part of the solution to the Laplace Equation in spherical coordinates. Analogously to how sines and cosines are used in harmonic analysis to represent periodic functions through Fourier Series, Spherical Harmonics may be used to define functions in the surface of a sphere.

#### Spherical Coordinates

It is important to identify the coordinates to be used, as, for instance, spherical coordinates have often two different interpretations: one, which is mainly used by mathematicians, uses the latitude, being the angle between the position vector and the equatorial plane, as one of the angular coordinates; while the other, broadly employed by physicists, uses the co-latitude, defined as the angle between the position vector and the  $z$  axis, thus being the complementary of the latitude.

Here, the spherical coordinates are taken to be

$r$ , the radial distance from Earth's center

$\phi$ , the East longitude, bounded between  $-\pi$  and  $\pi$

$\theta$ , the co-latitude, defined to be  $\theta = \frac{\pi}{2} - \theta'$ , with  $\theta'$  the latitude. Bounded between 0 and  $\pi$

The magnetic field is given by the negative gradient of the potential  $V$  defined for  $r \geq R_e$ , and given by the spherical harmonic approximation

$$V = r \sum_{n=1}^L \left( \frac{R_e}{r} \right)^{n+1} \sum_{m=0}^n (g_n^m \cos(m\phi) + h_n^m \sin(m\phi)) P_n^m(\cos\theta) \quad (2)$$

That is

$$\boldsymbol{\beta} = -\nabla V \quad (3)$$

Where  $g_n^m$  and  $h_n^m$  are the set of IGRF gaussian coefficients, published and revised every five years by the participating members of the IAGA (International Association of Geomagnetism and Aeronomy). The 12<sup>th</sup> generation is here used, as the last revision by the time of this work. These coefficient also include the secular variation (SV), which bestow the model on the proper time dependence. Keping track of the change in the coefficients in  $nT$  per year. The coefficients for some epoch year are referred to as IGRF. When real data about the geomagnetic field becomes available so that some adjustments can be made, the model becomes definitive and changes its name to DGRF (Definitive geomagnetic reference field).

$P_n^m(\cos\theta)$  are the Schmidt normalized associated Legendre polynomials of degree  $n$  and order  $m$

One can find very effective, recursive ways of computing the Schmidt quasi-normalization factors, the Associated Legendre polynomials and its derivatives, as can be found on [J.Davis - Mathematical Modeling of Earth's Magnetic Field]

The three componentens of the magnetic field in  $r$ ,  $\phi$  and  $\theta$  directions can be found in terms of the partial derivatives of 2

$$\beta_r = -\frac{\partial V}{\partial r} = \sum_{n=1}^L (n+1) \left( \frac{R_e}{r} \right)^{n+2} \sum_{m=0}^n (g_n^m \cos(m\phi) + h_n^m \sin(m\phi)) P_n^m(\cos\theta) \quad (4)$$

$$\beta_\theta = -\frac{1}{r} \frac{\partial V}{\partial \theta} = -\sum_{n=1}^L \left( \frac{R_e}{r} \right)^{n+2} \sum_{m=0}^n (g_n^m \cos(m\phi) + h_n^m \sin(m\phi)) \frac{\partial P_n^m(\cos\theta)}{\partial \theta} \quad (5)$$

$$\beta_\phi = -\frac{1}{r \sin\theta} \frac{\partial V}{\partial \phi} = \frac{1}{\sin\phi} \sum_{n=1}^L \left( \frac{R_e}{r} \right)^{n+2} \sum_{m=0}^n m (g_n^m \sin(m\phi) - h_n^m \cos(m\phi)) P_n^m(\cos\theta) \quad (6)$$

## Legendre Polynomials

The set of regular Legendre Polynomials is given by the solutions  $P_n(\nu)$  to the equation

$$\frac{1}{\sqrt{1-2\nu x+x^2}} = \sum_{n=0}^{\infty} x^n P_n(\nu) \quad (7)$$

Which gives

$$P_n(\nu) = \frac{1}{n!2^n} \frac{d^n}{d\nu^n} (\nu^2 - 1)^n \quad (8)$$

### Associated Legendre Polynomials

The associated Legendre polynomials  $P_n^m(\nu)$  of degree  $n$  and order  $m$  relate to  $P_n(\nu)$  by

$$P_{n,m}(\nu) = (1-\nu^2)^{\frac{1}{2}m} \frac{d^m}{d\nu^m} (P_n(\nu)) \quad (9)$$

These polynomials are not normalized in any way.

### Gaussian normalization and Schmidt quasi-normalization

To compute the geomagnetic field, the Schmidt quasi-normalization is used, and is given by

$$P_n^m(\nu) = \sqrt{\frac{2(n-m)!}{(n+m)!}} P_{n,m}(\nu) \quad (10)$$

The Gaussian normalized Legendre polynomials, whose interest lies in the existence of a recursive formula for their effective computation are given by

$$P^{n,m}(\nu) = \frac{2^n!(n-m)!}{(2n)!} P_{n,m}(\nu) \quad (11)$$

One can relate the Gaussian normalization and the Schmidt quasi-normalization by using

$$P_n^m(\nu) = \Gamma_{n,m} P^{n,m}(\nu) \quad (12)$$

Where

$$\begin{aligned} \Gamma_{n,m} &= \sqrt{\frac{(n-m)!}{(n+m)!} \frac{(2n-1)!!}{(n-m)!}}, \quad m=0 \\ \Gamma_{n,m} &= \sqrt{\frac{2(n-m)!}{(n+m)!} \frac{(2n-1)!!}{(n-m)!}}, \quad m \neq 0 \end{aligned} \quad (13)$$

It is more convenient to modify the coefficients, as

$$\begin{aligned} g_n^m &= \Gamma_{n,m} g_{n,m} \\ h_n^m &= \Gamma_{n,m} h_{n,m} \end{aligned} \quad (14)$$

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1

<sup>1</sup>Note the use of notation here to distinguish between Regular, Associated, and Schmidt quasi-normalized Legendre Polynomials

#### **4.5.1 Magnetic Field**

#### **4.5.2 Gravitational Field**

#### **4.6 Sensor Models**

#### **4.7 Actuator Models**

### **5 Controller Design**

## **6 Matlab-Simulink Model**

Components:

#### **6.1 Dynamics**

##### **6.1.1 Spacecraft Dynamics Block in Simulink**

Inputs

Outputs



### **6.1.2 Orbital Dynamics**

### **6.2 Environment block**

### **6.3 Disturbance block**

### **6.4 Sensor block**

### **6.5 Actuator block**

### **6.6 Controller block**

### **6.7 Reference block**

### **6.8 Mode Select block**

### **6.9 Dynamics block**

### **6.10 Test case**

Model Setup File Simulation Setup File Output Text file

### **6.11 Post processing**

## **7 Results**

## **A Github**

### **A.1 Structure**

### **A.2 Typical Workflow**

Branch for feature Pull Request

## **B Latex**

## **C**