ICS Summer Academy Session II
Topic 3: Concept Learning

Michael Shindler

A Task

I want to predict if Aldo will enjoy a sports event.
We have observed:

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Sunny	Warm	Normal	Strong	Warm	Same	Yes
Sunny	Warm	High	Strong	Warm	Same	Yes
Rainy	Cold	High	Strong	Warm	Change	No
Sunny	Warm	High	Strong	Cool	Change	Yes

- Is this a classification or regression task?
- A **hypothesis** looks like:
  - $\langle ?, \text{Cold}, \text{High}, ?, ?, ? \rangle$
  - $\langle ?, ?, ?, ?, ?, ? \rangle$
  - $\langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle$

Notation for Concept Learning

- Items over which concept is defined: *instances*, marked as X
- The concept to be learned, marked as c:
  $c : X \to \{0, 1\}$
- We are presented with training examples $D \subseteq X$
  - Some are positive examples
  - Some are negative examples
- Our goal is to estimate/hypothesize c
- $H =$ set of possible hypotheses
  - Ideally, find $h \in H$ such that $h(x) = c(x)$

Formal Meaning of EnjoySport task

Givens:

- Instances X: possible days, each described by attributes:
  - Sky: possible values Sunny, Cloudy, Rainy
  - AirTemp: possible values Warm and Cold
  - Humidity: Normal, High
  - Wind: Strong, Weak
  - Water: Warm, Cool
  - Forecast: Same, Change
- Hypotheses H: a series of possible hypotheses
- Target concept c
- Training examples D.

Determine a hypothesis $h \in H$ such that $\forall x \in X h(x) = c(x)$

- How many possible instances are in X?

$3 \cdot 2^5$

- How many possible distinct hypotheses are in H?

$5 \cdot 4^5$ by syntax / Semantics: $1 + 4 \cdot 3^5$

General to specific ordering of hypotheses

- We don't want to enumerate every hypothesis
- Concept learning hypotheses have a general-to-specific ordering.
  Example: consider these two:
  - $h_1 = \langle$Sunny, ?, ?, Strong, ?, ?$\rangle$
  - $h_2 = \langle$Sunny, ?, ?, ?, ?, ?$\rangle$

Which is more general?

$h_x = \langle$Sunny, ?, ?, ?, cool, ?$\rangle$

Generality examples

- $h_3 = \langle ?, \text{Cold}, \text{High}, ?, ?, ? \rangle$
- $h_4 = \langle ?, ?, ?, ?, ?, ? \rangle$
- $h_5 = \langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle$

More generality examples

Compare the following hypotheses for pairwise generality:

- $h_6 = \langle$Sunny, ?, ?, Strong, ?, ?$\rangle$
- $h_7 = \langle$Sunny, ?, ?, ?, ?, ?$\rangle$
- $h_8 = \langle$Sunny, ?, ?, ?, Cool, ? $\rangle$

Identify the most general hypothesis that is more specific than these.

Reinforcement: Number of Hypotheses

Golf Club Dataset:

- *Brand*, which can be Titleist, Ping, or Mizuno (3 choices)
- *Type*, which can be wood, iron, wedge, or putter (4 choices)
- *Shaft material*, which can be graphite or steel (2 choices)

- How many syntactically distinct hypotheses?

$5 \cdot 6 \cdot 4$

- How many semantically distinct?

$1 + 4 \cdot 5 \cdot 3$

Finding a maximally specific hypothesis

"find-S"

There is a very simple algorithm.
Start with the most specific hypothesis in H:
$h \leftarrow \langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle$
Does this fit all of our data?

Finding a maximally specific hypothesis

There is a very simple algorithm.
Start with the most specific hypothesis in H:
$h \leftarrow \langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle$
Let's look at example 1:

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Sunny	Warm	Normal	Strong	Warm	Same	Yes

Example continued

Now $h \leftarrow \langle$Sunny, Warm, Normal, Strong, Warm, Same $\rangle$
And we look at instance two:

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Sunny	Warm	High	Strong	Warm	Same	Yes

Does h correctly classify this?

Example continued

Now $h \leftarrow \langle$Sunny, Warm, ?, Strong, Warm, Same $\rangle$
And instance three:

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Rainy	Cold	High	Strong	Warm	Change	No

Should we change h? Why or why not?

Example continued

Now $h \leftarrow \langle$Sunny, Warm, ?, Strong, Warm, Same $\rangle$

And instance four:

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Sunny	Warm	High	Strong	Cool	Change	Yes

Should we change h? Why or why not?

Some remaining questions

- Have we converged to the correct target concept?

- Why do we prefer the more specific hypothesis?

Some remaining questions

- Are the training examples consistent?

- What if there are several maximally specific consistent hypotheses?

Exercise/Reinforcement

Attributes are as follows:

- Hair Color, which could be blonde, brown, or red (3 choices)
- Height, which could be short, average, or tall (3 choices)
- Weight, which could be light, average, or heavy (3 choices)
- Lotion?, did this person apply sunscreen, yes or no (2 choices)

- How many syntactically distinct hypotheses?

$5^3 \cdot 4$

- How many semantically distinct?

$1 + 4^3 \cdot 3$

Exercise/Reinforcement

$h \leftarrow \langle \emptyset, \emptyset, \emptyset, \emptyset \rangle$

Hair	Height	Weight	Lotion	Result
blonde	average	light	no	sunburned

Exercise/Reinforcement

$h \leftarrow \langle$blonde, average, light, no $\rangle$

Hair	Height	Weight	Lotion	Result
blonde	short	light	yes	none

Exercise/Reinforcement

$h \leftarrow \langle$blonde, average, light, no $\rangle$

Hair	Height	Weight	Lotion	Result
blonde	short	average	no	sunburned

Exercise/Reinforcement

$h \leftarrow \langle$blonde, ?, ?, no $\rangle$

Hair	Height	Weight	Lotion	Result
red	average	heavy	no	sunburned

Defining consistency

A hypothesis h is **consistent** with a set of training examples D if and only if $h(x) = c(x)$ for each example in D.

Recall the definition of **satisfies**:

If an instance x satisfies all the constraints of h, then $h(x) = 1$

How are consistency and satisfaction different?

Version Space

If we're searching, where are we searching?

We are searching the **version space**;

The **version space**, with respect to the hypothesis space H and training examples D, is the subset of hypotheses from H consistent with training examples in D

An obvious, but bad, algorithm

- List all possible hypotheses
- For each possible hypothesis, check all training examples
- Whenever you find one that is inconsistent, remove it.

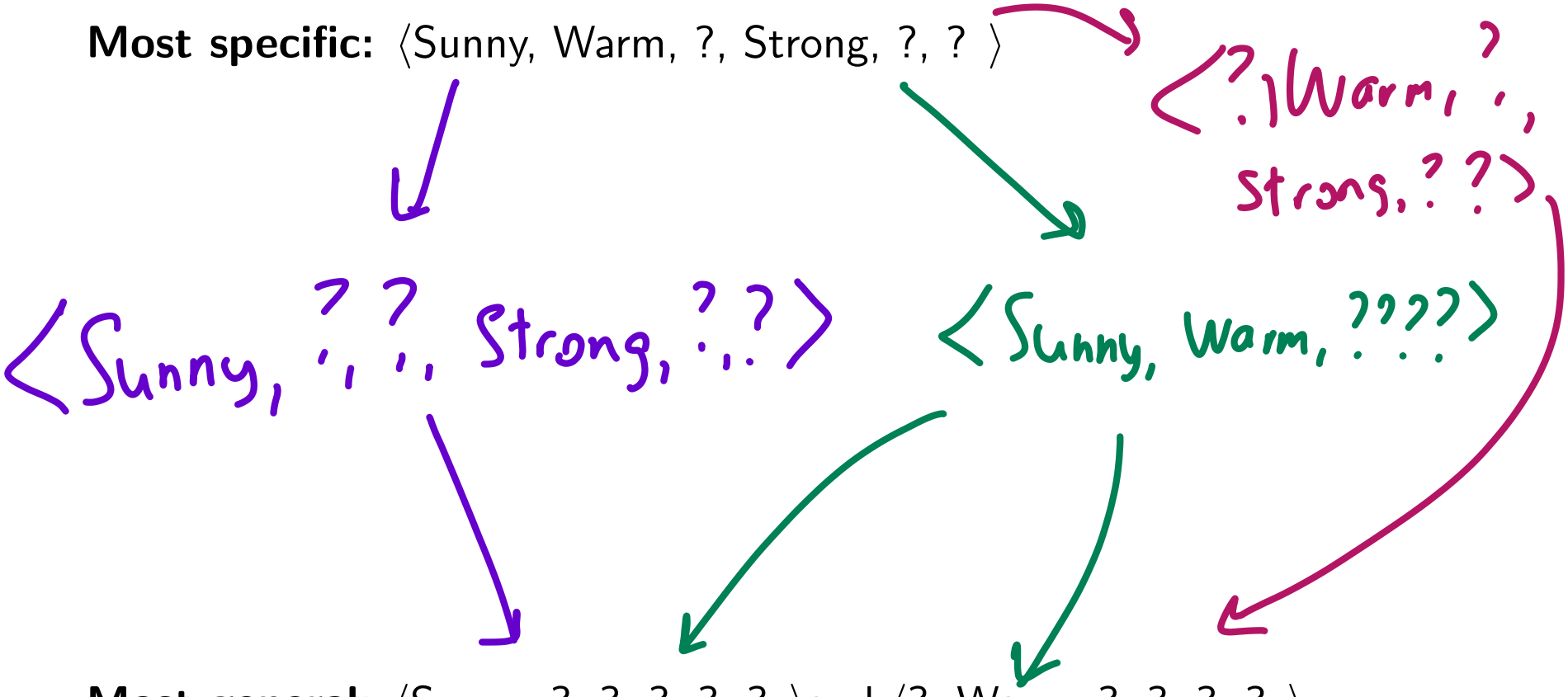
What is good about this algorithm?

What is bad about this algorithm?

More compact representation

Idea: store the most general and most specific boundary sets.
Example:

Most specific: ⟨Sunny, Warm, ?, Strong, ?, ? ⟩



Most general: ⟨Sunny, ?, ?, ?, ?, ? ⟩and ⟨?, Warm, ?, ?, ?, ? ⟩

Hypotheses under consideration

S = ⟨Ping, ?, Steel⟩ and G = ⟨Ping, ?, ? ⟩ and ⟨?, ?, Steel ⟩.
Which of the following hypotheses is not under consideration?

(a) ⟨Ping, iron, steel⟩

(b) ⟨Titleist, wedge, graphite⟩

(c) ⟨Ping, iron, graphite⟩

(d) ⟨Titleist, putter, steel⟩

(e) ⟨Ping, wood, steel⟩

Candidate-Elimination Algorithm

- Most general and most specific boundaries we can start with?

Candidate-Elimination Algorithm

- Most general and most specific boundaries we can start with?

- Starting with those, let's examine how the algorithm behaves with the training data from earlier:

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Sunny	Warm	Normal	Strong	Warm	Same	Yes

$S_0 = \langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle$

$G_0 = \langle ?, ?, ?, ?, ?, ? \rangle$

Candidate-Elimination Algorithm

- Starting with those, let's examine how the algorithm behaves with the training data from earlier:

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Sunny	Warm	High	Strong	Warm	Same	Yes

$S_1 = \langle$Sunny, Warm, Normal, Strong, Warm, Same $\rangle$ $G_1 = \langle$?, ?, ?, ?, ?, ? $\rangle$

Candidate-Elimination Algorithm

- Starting with those, let's examine how the algorithm behaves with the training data from earlier:

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Rainy	Cold	High	Strong	Warm	Change	No

$S_2 = \langle$Sunny, Warm, ?, Strong, Warm, Same $\rangle$ $\quad$ $G_2 = \langle$?, ?, ?, ?, ?, ? $\rangle$

Candidate-Elimination Algorithm

- Starting with those, let's examine how the algorithm behaves with the training data from earlier:

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Sunny	Warm	High	Strong	Cool	Change	Yes

$S_3 = \langle$Sunny, Warm, ?, Strong, Warm, Same $\rangle$

$G_3 = \langle$Sunny, ?, ?, ?, ?, ? $\rangle$ $\langle$?, Warm, ?, ?, ?, ? $\rangle$ $\langle$?, ?, ?, ?, ?, Same $\rangle$

Correctness of Candidate-Elimination

- Case 1: no errors in training, some correct hypothesis exists.
  - As new training examples observed?
  - When has it correctly converged?
- Case 2: Errors in training data

Ordering of training examples

New model of input: *query* answered by *oracle*

Earlier, our algorithm finished with:
$S_4 = \langle$Sunny, Warm, ?, Strong, ?, ?$\rangle$

$\langle$Sunny, ?, ?, Strong, ?, ?$\rangle\langle$Sunny, Warm, ?, ?, ?, ?$\rangle\langle$?, Warm, ?, Strong, ?, ? $\rangle$

$G_4 = \langle$Sunny, ?, ?, ?, ?, ? $\rangle$ $\langle$?, Warm, ?, ?, ?, ? $\rangle$

What should we request for the next query and why?

Classifying New Instances

Our algorithm finished the training data with:
$S_4 = \langle$Sunny, Warm, ?, Strong, ?, ?$\rangle$

$\langle$Sunny, ?, ?, Strong, ?, ?$\rangle\langle$Sunny, Warm, ?, ?, ?, ?$\rangle\langle$?, Warm, ?, Strong, ?, ? $\rangle$

$G_4 = \langle$Sunny, ?, ?, ?, ?, ? $\rangle$ $\langle$?, Warm, ?, ?, ?, ? $\rangle$

How will we classify the following?

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Sunny	Warm	Normal	Strong	Cool	Change	??

Classifying New Instances

Our algorithm finished the training data with:
$S_4 = \langle$Sunny, Warm, ?, Strong, ?, ?$\rangle$

$\langle$Sunny, ?, ?, Strong, ?, ?$\rangle\langle$Sunny, Warm, ?, ?, ?, ?$\rangle\langle$?, Warm, ?, Strong, ?, ? $\rangle$

$G_4 = \langle$Sunny, ?, ?, ?, ?, ? $\rangle$ $\langle$?, Warm, ?, ?, ?, ? $\rangle$

How will we classify the following?

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Rainy	Cold	Normal	Light	Warm	Same	??

Classifying New Instances

Our algorithm finished the training data with:
$S_4 = \langle$Sunny, Warm, ?, Strong, ?, ?$\rangle$

$\langle$Sunny, ?, ?, Strong, ?, ?$\rangle\langle$Sunny, Warm, ?, ?, ?, ?$\rangle\langle$?, Warm, ?, Strong, ?, ? $\rangle$

$G_4 = \langle$Sunny, ?, ?, ?, ?, ? $\rangle$ $\langle$?, Warm, ?, ?, ?, ? $\rangle$

How will we classify the following?

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Sunny	Warm	Normal	Light	Warm	Same	??

Classifying New Instances

Our algorithm finished the training data with:
$S_4 = \langle$Sunny, Warm, ?, Strong, ?, ?$\rangle$

$\langle$Sunny, ?, ?, Strong, ?, ?$\rangle\langle$Sunny, Warm, ?, ?, ?, ?$\rangle\langle$?, Warm, ?, Strong, ?, ? $\rangle$

$G_4 = \langle$Sunny, ?, ?, ?, ?, ? $\rangle$ $\langle$?, Warm, ?, ?, ?, ? $\rangle$

How will we classify the following?

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Sunny	Cold	Normal	Strong	Warm	Same	??

A biased hypothesis space

We have only considered conjunctive hypotheses so far;

What is the most specific hypothesis consistent with these two?

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Sunny	Warm	Normal	Strong	Cool	Change	Yes
Cloudy	Warm	Normal	Strong	Cool	Change	Yes

A biased hypothesis space

We have only considered conjunctive hypotheses so far;

What is the most specific hypothesis consistent with these two?

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Sunny	Warm	Normal	Strong	Cool	Change	Yes
Cloudy	Warm	Normal	Strong	Cool	Change	Yes

$S_2 = \langle$?, Warm, Normal, Strong, Cool, Change$\rangle$

Now classify:

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Rainy	Warm	Normal	Strong	Cool	Change	No

Exercise / Reinforcement

Restaurant	Meal	Day	Cost	Reaction
Sam's	breakfast	Friday	cheap	yes

$S_0 = \langle \emptyset, \emptyset, \emptyset, \emptyset \rangle$

$G_0 = \langle ?, ?, ?, ? \rangle$

Exercise / Reinforcement

Restaurant	Meal	Day	Cost	Reaction
Lobdell	lunch	Friday	expensive	no

$S_1 = \langle$Sam's, breakfast, Friday, cheap$\rangle$

$G_1 = \langle$?, ?, ?, ? $\rangle$

Exercise / Reinforcement

Restaurant	Meal	Day	Cost	Reaction
Sam's	lunch	Saturday	cheap	yes

$S_2 = \langle$Sam's, breakfast, Friday, cheap$\rangle$

$G_2 = \langle$Sam's, ?, ?, ? $\rangle$ $\langle$?, breakfast, ?, ? $\rangle$ $\langle$?, ?, ?, cheap $\rangle$

Exercise / Reinforcement

Restaurant	Meal	Day	Cost	Reaction
Sarah's	breakfast	Sunday	cheap	no

$S_3 = \langle$Sam's, ?, ?, cheap$\rangle$

$G_3 = \langle$Sam's, ?, ?, ? $\rangle$ $\langle$?, ?, ?, cheap $\rangle$

Exercise / Reinforcement

Restaurant	Meal	Day	Cost	Reaction
Sam's	breakfast	Sunday	expensive	no

$S_4 = \langle$Sam's, ?, ?, cheap$\rangle$

$G_4 = \langle$Sam's, ?, ?, ~~?~~ cheap$\rangle$ ← (after this row, we need this change)