Question 1. *My company is going to introduce a new product. Every time we release a new product, we do market research to predict if it will be a success or failure. 60% of the products we release have been successes. Of the products that were successful, 70% were predicted to be a success. Of the products that were failures, 40% were predicted to be a success. Our new product has been predicted to be a success.*

Suppose we want to use Bayes' Rule to answer the question: What is the probability our new product is successful?

What are the values we plug into the formula for the following?

We want event F to be the product is successful, and E is that it has been predicted to be a success.

- $p(E|F) = 0.7$, the probability predicted to be a success, given a successful product.
- $p(F) = 0.6$ the probability the product is successful
- $p(E|\overline{F}) = 0.4$ - This is the probability that the product was *predicted to be successful* (event E), given that we know the product was not a success; one way to think of this is "out of all the products that turned out to be a non-success, how many were predicted to be successful?" Except, of course, we're looking at probability, not count, but the idea is similar. This is arguably the most difficult one to get right.
- $p(\overline{F})$ This is, perhaps confusingly, also 0.4. 40% of products released were not successful. Note that while this has the same numeric answer as the previous one, it is for a different reason.

Question 2. There are two boxes. The first contains two gold marbles and seven blue marbles. The second contains four gold marbles and three blue marbles. You choose a box at random, then you choose a marbles at random. You draw out a blue marble. What is the probability that the marble came from the first box?

A different, but related question is: suppose I flip a coin (50/50 chance of each) to decide which box to pull from, and then I take a marble uniformly at random from that box. What is the probability I draw a blue marble? For that, I want $P(\text{blue}) = P(\text{blue} \mid \text{box } 1) * P(\text{box } 1) + P(\text{blue} \mid \text{box } 2) * P(\text{box } 2)$. This is $(7/9) * (1/2) + (3/7) * (1/2)$.

But that isn't what we were asked here; we don't observe the coin toss or the box selection that goes with it; we just see the result is a blue marble. Before we saw the blue marble, we probably thought it was 50/50 that the first box is chosen. Once we observe the blue marble, though, we believe it is more likely that the first box was chosen.

So, again, we want probability of first box (that's event F), given that the marble was blue (that's event E). With Bayes' Rule problems, it is very important to correctly identify what events E and F are.

The following are the probabilities: $P(E|F) = 7/9, P(F) = 1/2, P(E|\overline{F}) = 3/7, P(\overline{F}) = 1/2$. Plugging these values into Bayes' Rule gets us what we want.

Question 3. There is a rare disease which infects only 1 out of $100,000$ people. You can detect it with a very accurate diagnostic test. If someone has the disease, it correctly identifies it 99% of the time. If someone does not have the disease, it correctly states this 99.5% of the time.

- Suppose the test comes out negative. What is the probability the person does not have the disease?

  Event E is a negative test result, while event F is no disease. This gives us values of $P(E|F) = .995, P(F) = .99999, P(E|\overline{F}) = .01, P(\overline{F}) = .00001$.

- Suppose the test comes out positive. What is the probability the person does have the disease?

  Event E is the test result is positive; event F is the probability of having the disease. The probabilities are $P(E|F) = .99, P(F) = .00001, P(E|\overline{F}) = .005, P(\overline{F}) = .99999$

This is an artificial question intended to help you review Naive Bayes in anticipation of more advanced Bayesian techniques. You should feel free to use a computer or calculator (or phone, etc) for this problem.

Consider a binary classification problem with variable $X_1 \in \{0, 1\}$ and label $Y \in \{0, 1\}$. The true generative distribution $P(X_1, Y) = P(Y)P(X_1|Y)$ is shown below:

$Y = 0$	$Y = 1$
0.8	0.2

	$X_1 = 0$	$X_1 = 1$
$Y = 0$	0.7	0.3
$Y = 1$	0.3	0.7

1. Now suppose we have trained a Naive Bayes classifier, using infinite training data generated according to those tables. Now fill in Table 3. In particular, fill in the probabilities in the first two columns, and fill in the prediction of Y in the last column of the table. The process is sufficient (i.e., 0.8×0.7 is fine, as would be 0.56)

	$\hat{P}(X_1, Y = 0)$	$\hat{P}(X_1, Y = 1)$	$\hat{Y}(X_1)$
$X_1 = 0$	$0.8 \times 0.7 = 0.56$	$0.2 \times 0.3 = 0.06$	0
$X_1 = 1$	$0.8 \times 0.3 = 0.24$	$0.2 \times 0.7 = 0.14$	0

2. What is the expected error rate of this classifier on training examples generated according to the first two tables? In other words, what is $P(Y \neq \hat{Y}(X_1))$?

(Hint: $P(Y \neq \hat{Y}(X_1)) = P(Y \neq \hat{Y}(X_1), X_1 = 1) + P(Y \neq \hat{Y}(X_1), X_1 = 0)$)

$P(Y \neq \hat{Y}(X_1)) = P(Y = 1, X_1 = 0) + P(Y = 1, X_1 = 1) = 0.06 + 0.14 = 0.2$

3. Now we add a feature to this data X_2 such that X_2 is an exact duplicate of X_1. Suppose we have trained Naive Bayes classifier using infinite training data that are generated by following the first two tables, and then add the additional duplicate feature X_2. Please fill in the following tables.

	$X_2 = 0$	$X_2 = 1$
$Y = 0$	0.7	0.3
$Y = 1$	0.3	0.7

Fill in the probabilities for the following table and write down the predictions of Y for different X_1 and X_2 value combinations.

	$\hat{P}(X_1, X_2, Y = 0)$	$\hat{P}(X_1, X_2, Y = 1)$	$\hat{Y}(X_1, X_2)$
$X_1 = 0, X_2 = 0$	$0.8 \times 0.7 \times 0.7 = 0.392$	$0.2 \times 0.3 \times 0.3 = 0.018$	0
$X_1 = 1, X_2 = 0$	$0.8 \times 0.7 \times 0.3 = 0.168$	$0.2 \times 0.7 \times 0.3 = 0.042$	0
$X_1 = 0, X_2 = 1$	$0.8 \times 0.7 \times 0.3 = 0.168$	$0.2 \times 0.7 \times 0.3 = 0.042$	0
$X_1 = 1, X_2 = 1$	$0.8 \times 0.3 \times 0.3 = 0.072$	$0.2 \times 0.7 \times 0.7 = 0.098$	1

4. What is the expected error rate of this Naive Bayes classifier on this data?

Note that the testing examples are generated according to the true distribution (i.e., where X_2 is a duplication). We have:

$$P(Y \neq \hat{Y}(X_1, X_2)) = P(Y \neq \hat{Y}(X_1, X_2), X_1 = X_2 = 0) + P(Y \neq \hat{Y}(X_1, X_2), X_1 = X_2 = 1)$$
$$= P(Y = 1, X_1 = 0) + P(Y = 0, X_1 = 1) = 0.06 + 0.24 = 0.3$$

5. Compare the error rate in 4 to the error rate in 2. What is the reason for the difference?

The error rate is increased. This is because X_1 and X_2 are not conditional independent given Y, violating the the Naive Bayes assumption.