ICS Summer Academy Session II
Topic 6: Bayesian Networks

Michael Shindler

$$P(E \mid F) = \frac{P(F \mid E) \cdot P(E)}{(\text{values})}$$

$$P(\text{yes} \mid \text{suv}, \text{red}) = \frac{P(\text{suv}, \text{red} \mid \text{yes}) \cdot P(\text{yes})}{1, + \; P(\text{suv}, \text{red} \mid \text{no}) \cdot P(\text{no})}$$

Review of Naive Bayes

Example No.	Color	Type	Origin	Stolen?
1	Red	Sports	Domestic	Yes
2	Red	Sports	Domestic	No
3	Red	Sports	Domestic	Yes
4	Yellow	Sports	Domestic	No
5	Yellow	Sports	Imported	Yes
6	Yellow	SUV	Imported	No
7	Yellow	SUV	Imported	Yes
8	Yellow	SUV	Domestic	No
9	Red	SUV	Imported	No
10	Red	Sports	Imported	Yes

Stolen yes: 5/10

Stolen no : 5/10

Training Data Tables

$P(\text{type} = \text{sports} \mid \text{stolen} = \text{yes})$
$= 4/5$

Stolen=Yes	Stolen=No
1/2	1/2

Type	Stolen=Yes	Stolen=No
Sports	4/5	2/5
SUV	1/5	3/5

Color	Stolen=Yes	Stolen=No
Yellow	2/5	3/5
Red	3/5	2/5

Origin	Stolen=Yes	Stolen=No
Domestic	2/5	3/5
Imported	3/5	2/5

Want: $P(\text{yes} \mid \ldots)$

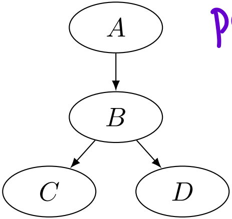
Would a red domestic SUV be stolen according to your prediction?

$P(\text{red}, \text{domestic}, \text{SUV} : \text{yes}) =$

$P(\text{red} \mid \text{yes}) \cdot P(\text{dom} \mid \text{yes}) \cdot P(\text{SUV} \mid \text{yes}) \cdot P(\text{yes})$

$P(\text{red}, \text{domestic}, \text{SUV} : \text{no}) =$

Example Bayesian Network

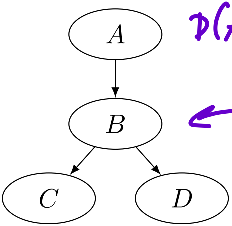


parent
↓
child

Joint probability $P(A, B, C, D)$ as product of conditional probabilities?

$$P(A, B, C, D) = P(A) \cdot P(B|A) \cdot P(C|B) \cdot P(D|B)$$

Example Bayesian Network



p(A)

2

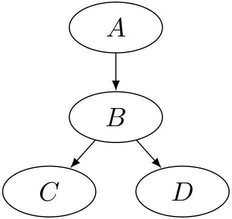
2

A	P(B|A)
T	
F	

How many independent parameters needed to fully define?

7

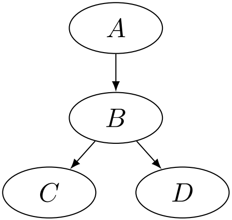
Example Bayesian Network



Define the joint distribution $P(A, B, C, D)$ if no assumptions about independence or conditional independence?

2^4 (or $2^4 - 1$ really)

Example Bayesian Network



- You now know about B happened or not
- Then you find out about A
- Belief changed for C or D?

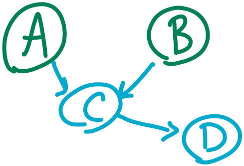
Draw a Bayesian Network

$X \perp Y | Z$ means that X is conditionally independent of Y, given Z.

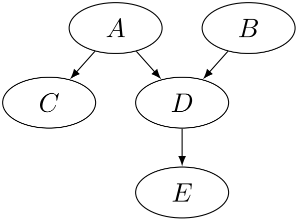
- $A \perp B | \emptyset$
- $A \not\perp D | B$
- $A \perp D | C$
- $A \not\perp C | \emptyset$
- $B \not\perp C | \emptyset$
- $A \not\perp B | D$
- $B \perp D | A, C$

"John Calls" $\perp$ "Earthquake" | "Alarm"

Think of $\perp$ as "does not communicate"



Conditional Independence from Model

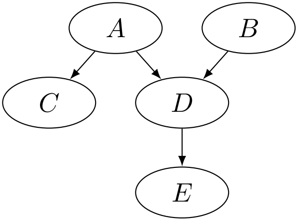


True or False: C and D are conditionally independent given A

Did not discuss this or the other three parts.

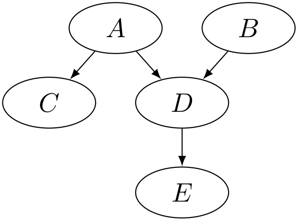
— Material was covered as part of conversations.

Conditional Independence from Model



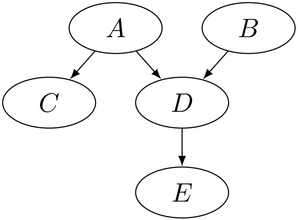
True or False: C and B are conditionally independent given D

Conditional Independence from Model



True or False: C is conditionally independent of B given A

Conditional Independence from Model

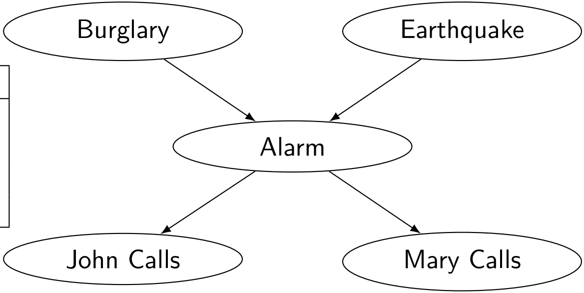


True or False: C is conditionally independent of B given D

Famous Alarm Example

$P(B)$
.001

$P(E)$
.002



B	E	$P(A\|B,E)$
T	T	.95
T	F	.94
F	T	.29
F	F	.001

A	$P(J\|A)$
T	.90
F	.05

A	$P(M\|A)$
T	.70
F	.01

More about Hidden Variables