Suppose we have following examples collected for the car theft problem.

Example No.	Color	Type	Origin	Stolen?
1	Red	Sports	Domestic	Yes
2	Red	Sports	Domestic	No
3	Red	Sports	Domestic	Yes
4	Yellow	Sports	Domestic	No
5	Yellow	Sports	Imported	Yes
6	Yellow	SUV	Imported	No
7	Yellow	SUV	Imported	Yes
8	Yellow	SUV	Domestic	No
9	Red	SUV	Imported	No
10	Red	Sports	Imported	Yes

Question 1. Suppose we want to use the given data to train a Naive Bayes classifier. Fill in the following tables for $P(\text{stolen})$, $P(\text{color}|\text{stolen})$, $P(\text{type}|\text{stolen})$ and $P(\text{origin}|\text{stolen})$.

Stolen=Yes	Stolen=No

Color	Stolen=Yes	Stolen=No
Yellow		
Red		

Type	Stolen=Yes	Stolen=No
Sports		
SUV		

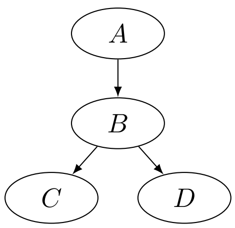
Origin	Stolen=Yes	Stolen=No
Domestic		
Imported		

Question 2. If you have a red domestic SUV, would it be stolen according to your prediction?

In the last lecture, we saw *Naive Bayes*, in which we make the assumption that every attribute is conditionally independent. This method works (perhaps, surprisingly) well, but we can have situations where we want to have a more generic representation of conditional independence.

To do this, we will diagram the dependencies with a *directed graph*[1]. The edges (the lines with an arrow between nodes) represent the direction of influence among variables. The key advantage these *graphical models* give us is the ability to represent conditional independence.

Here is an example of a Bayesian Network:



For this diagram, event A is a cause of event B and event B is a cause of both events C and D.

Question 3. Write the joint probability $P(A, B, C, D)$ for this network as the product of four conditional probabilities.

Question 4. How many independent parameters are needed to fully define this Bayesian network? Why?

Question 5. How many independent parameters would we need to define the joint distribution $P(A, B, C, D)$ if we made no assumptions about independence or conditional independence? Why?

Question 6. Suppose you know that event B has happened (or didn't happen). You then find out that event A happened (or didn't happen). Does this change your belief in whether or not C or D (or both) happened? Why or why not?

[1]A directed graph is similar to a tree; the differences are important in Computer Science in general, but not as much for us today. The particular type of graph we are using is a *Directed Acyclic Graph*.

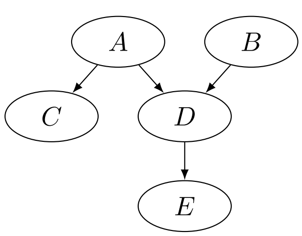
Question 7. Draw a Bayes net over the random variables $\{A, B, C, D\}$ where the following conditional independence assumptions hold. Here, $X \perp Y | Z$ means that X is conditionally independent of Y, given Z.

- $A \perp B | \emptyset$
- $A \not\perp D | B$
- $A \perp D | C$
- $A \not\perp C | \emptyset$
- $B \not\perp C | \emptyset$
- $A \not\perp B | D$
- $B \perp D | A, C$

The most general conditional independence question we might ask is known as **d-separation**, whether two (disjoint) sets of nodes are independent, given a different set of nodes. We can determine if X and Y are independent of Z as follows:

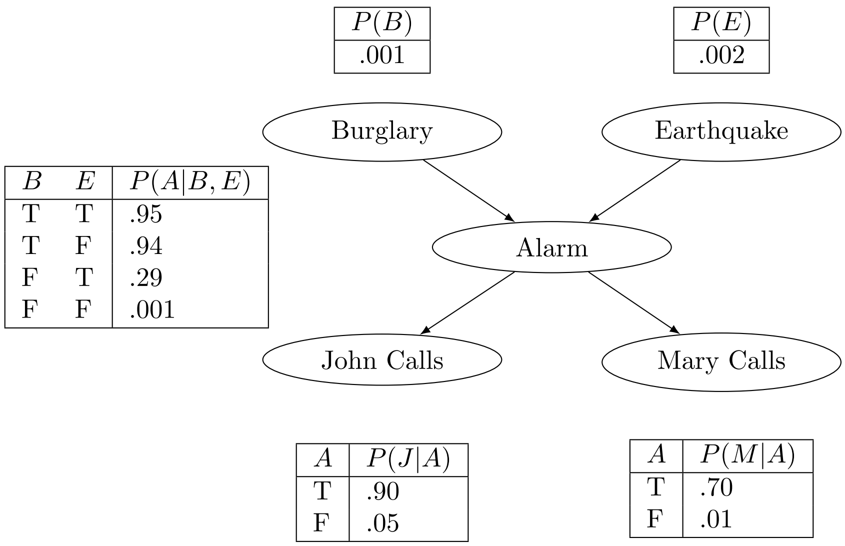
1. Look at the subgraph consisting only of X, Y, Z and their ancestors.
2. (for the sake of this calculation only) add an edge between any two vertices that share a common child.
3. Ignore the direction of all edges.
4. X is conditionally independent of Y given Z if it is impossible to walk in this newly formed graph from a node in X to a node in Y without crossing at least one node from Z.

Question 8. Consider the following graphical model and answer the following questions. Justify each answer.



1. True or False: C and D are conditionally independent given A
2. True or False: C and B are conditionally independent given D
3. True or False: C is conditionally independent of B given A
4. True or False: C is conditionally independent of B given D

Here is an example of a Bayesian network, complete with probabilities. It is the famous alarm example:



$P(B)$
.001

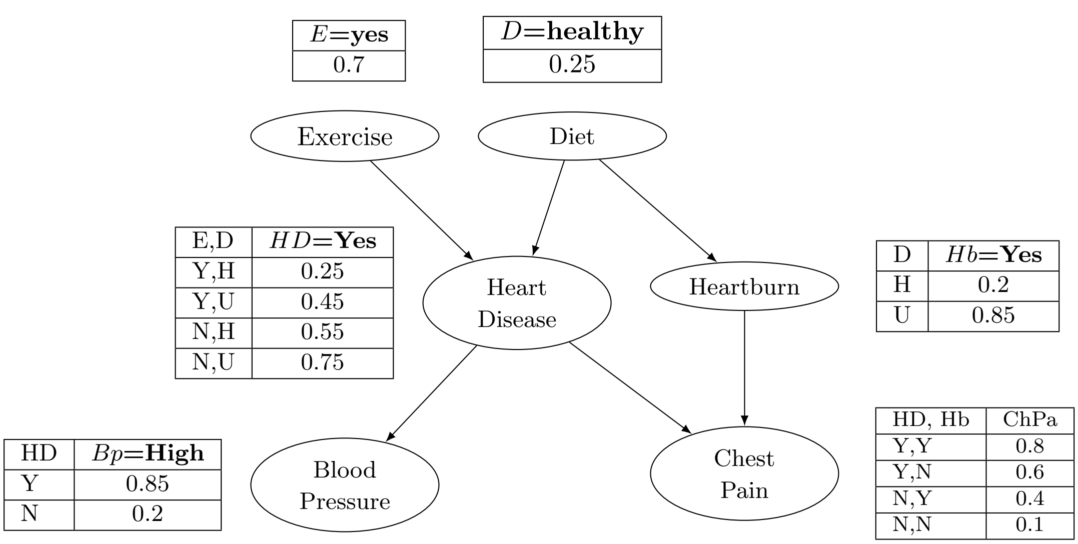
$P(E)$
.002

B	E	$P(A\|B,E)$
T	T	.95
T	F	.94
F	T	.29
F	F	.001

A	$P(J\|A)$
T	.90
F	.05

A	$P(M\|A)$
T	.70
F	.01

Question 9. Give several conditional independences and conditional dependencies in the above graph. Do the same for the heart disease graph below.



E=**yes**
0.7

D=**healthy**
0.25

E,D	HD=**Yes**
Y,H	0.25
Y,U	0.45
N,H	0.55
N,U	0.75

D	Hb=**Yes**
H	0.2
U	0.85

HD	Bp=**High**
Y	0.85
N	0.2

HD, Hb	ChPa
Y,Y	0.8
Y,N	0.6
N,Y	0.4
N,N	0.1