Consider the following problem. We have a friend, Aldo, who has a favorite sport. On any particular day, will Aldo enjoy the sport? Our goal is to be able to determine, from a day's attributes, if Aldo will enjoy the sport on that particular day.

Here are a set of example days, along with whether or not Aldo would enjoy the sport on that particular day. The last column is the *attribute* we would like to be able to predict for arbitrary days.

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Sunny	Warm	Normal	Strong	Warm	Same	Yes
Sunny	Warm	High	Strong	Warm	Same	Yes
Rainy	Cold	High	Strong	Warm	Change	No
Sunny	Warm	High	Strong	Cool	Change	Yes

Question 1. Is this a *classification* or *regression* task?

The representation of a *hypothesis* is a vector of six constraints, specifying the values of the six attributes from each observation. For each attribute, we will have one of the following:

- A question mark to indicate that *any* value is acceptable
- A required value for the attribute
- A $\emptyset$ to indicate that *no value* is acceptable for this attribute.

Question 2. Express the following hypotheses in English:

- $h_3 = \langle ?, \text{Cold}, \text{High}, ?, ?, ? \rangle$
- $h_4 = \langle ?, ?, ?, ?, ?, ? \rangle$
- $h_5 = \langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle$

The formal meaning of the EnjoySport task is as follows.

We are given the following:

- Instances X: possible days, each described by attributes:
  - Sky: possible values Sunny, Cloudy, Rainy
  - AirTemp: possible values Warm and Cold
  - Humidity: Normal, High
  - Wind: Strong, Weak
  - Water: Warm, Cool
  - Forecast: Same, Change
- Hypotheses H: a series of possible hypotheses
- Target concept c
- Training examples D.

We need to determine a hypothesis $h \in H$ such that $\forall x \in X h(x) = c(x)$

Question 3. For the EnjoySport problem, how many possible instances are in X?

Question 4. How many possible distinct hypotheses are in H?

Question 5. Which of the following two hypotheses is more general? Why?

- $h_1 = \langle$Sunny, ?, ?, Strong, ?, ?$\rangle$
- $h_2 = \langle$Sunny, ?, ?, ?, ?, ?$\rangle$

Question 6. Compare the hypotheses from Question 2 for pairwise generality.

Question 7. Compare the following hypotheses for pairwise generality:

- $h_6 = \langle$Sunny, ?, ?, Strong, ?, ?$\rangle$
- $h_7 = \langle$Sunny, ?, ?, ?, ?, ?$\rangle$
- $h_8 = \langle$Sunny, ?, ?, ?, Cool, ? $\rangle$

Question 8. Identify the most general hypothesis that is more specific than any of the ones in the previous question.

Question 9. Suppose we are trying to perform Concept Learning on a dataset of golf clubs with the following attributes:

- *Brand*, which can be Titleist, Ping, or Mizuno (3 choices)
- *Type*, which can be wood, iron, wedge, or putter (4 choices)
- *Shaft material*, which can be graphite or steel (2 choices)

How many syntactically distinct hypotheses are in the hypothesis space? How many semantically distinct?

The find-S Algorithm: finding a maximally specific hypothesis

The idea behind this algorithm is as follows. We start with the most specific possible hypothesis in H. We then generalize this hypothesis whenever we find a positive training example that it fails to correctly classify.

For the EnjoySport task, the most specific possible hypothesis is $h \leftarrow \langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle$
Question 10. Does this correctly classify all of our data?

Let's go through our training data, one row at a time, and adjust the hypothesis incrementally. At each stage, if h does not correctly classify a positive example, we will generalize it.

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Sunny	Warm	Normal	Strong	Warm	Same	Yes
Sunny	Warm	High	Strong	Warm	Same	Yes
Rainy	Cold	High	Strong	Warm	Change	No
Sunny	Warm	High	Strong	Cool	Change	Yes

Question 11. Why do we not change h on a negative example?

Question 12. At the end of the process, did we converge to the correct target concept?

Question 13. Why do we prefer the more specific hypothesis? Should we?

Question 14. What if we have inconsistent training examples? How does find-S behave?

Exercise/Reinforcement

The following example comes from Patrick Henry Winston's *Artificial Intelligence* textbook, third edition and is particularly relevant because we are in the middle of summer. This is a limited version of the problem; we will see a more full example later.

Imagine you are somehow unaware of the factors that leave some people red and in pain after a few hours at the beach, while other people just turn tanned and happy. Being curious, you go to the beach and start jotting down notes. You want to use the observed properties to help you predict whether a new person – one not in the observed set – will turn red.

For purposes of this problem, there are three possible hair colors (blonde, red, and brown), three possible heights (short, average, tall), three possible weights (light, average, heavy), and two possibilities for whether or not someone used lotion (yes or no).

Question 15. How many syntactically distinct hypotheses are in the hypothesis space? How many semantically distinct?

Use the find-S algorithm to find a maximally specific hypothesis for the following training data to determine if someone will suffer sunburn.

Hair	Height	Weight	Lotion	Result
blonde	average	light	no	sunburned
blonde	short	light	yes	none
blonde	short	average	no	sunburned
red	average	heavy	no	sunburned

Version Spaces and the Candidate-Elimination Algorithm

Here we will discuss the CANDIDATE-ELIMINATION algorithm, which will output a description of *the set of all hypotheses consistent with the training examples.*

A hypothesis h is **consistent** with a set of training examples D if and only if $h(x) = c(x)$ for each example in D. If an instance x **satisfies** all the constraints of h, then $h(x) = 1$

Question 16. How are consistency and satisfaction different?

This algorithm searches the **version space**. The **version space**, with respect to the hypothesis space H and training examples D, is the subset of hypotheses from H consistent with training examples in D

Here is an obvious but bad algorithm to solve this problem. List all possible hypotheses. For each possible hypothesis, check all training examples, and if it is inconsistent with any of them, remove that hypothesis from the list.
Question 17. What is good about this algorithm? What is bad about it?

The CANDIDATE-ELIMINATION algorithm stores the most general and the most specific boundary sets. For example, suppose we have the following description of all hypotheses consistent with the training examples:

Most specific: ⟨Sunny, Warm, ?, Strong, ?, ? ⟩

Most general: ⟨Sunny, ?, ?, ?, ?, ? ⟩and ⟨?, Warm, ?, ?, ?, ? ⟩

Question 18. What other hypotheses are implied in the above? Why?

Question 19. We are running the `Candidate-Elimination` algorithm on the dataset of golf clubs from earlier and have S = ⟨Ping, ?, Steel⟩ and G = ⟨Ping, ?, ? ⟩ and ⟨?, ?, Steel ⟩.
Which of the following hypotheses is not under consideration?

(a) ⟨Ping, iron, steel⟩

(b) ⟨Titleist, wedge, graphite⟩

(c) ⟨Ping, iron, graphite⟩

(d) ⟨Titleist, putter, steel⟩

(e) ⟨Ping, wood, steel⟩

Let's, once again, go through our training data, one row at a time, and walk through the algorithm.

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Sunny	Warm	Normal	Strong	Warm	Same	Yes
Sunny	Warm	High	Strong	Warm	Same	Yes
Rainy	Cold	High	Strong	Warm	Change	No
Sunny	Warm	High	Strong	Cool	Change	Yes

Question 20. What is the most general and most specific boundaries we can start with?

Row Specific hypotheses General Hypotheses

Question 21. If our training data is sufficiently large and has no errors, and some correct hypothesis exists, can we "stop short?" That is, is there a point where we know we have our answer?

Question 22. How does this algorithm behave if we have errors in the training data?

Question 23. Suppose you wanted to narrow down the version space from the earlier example and could choose the next row (although not the Yes/No value of EnjoySport). This is similar to setting up an experiment and seeing whether or not our target would say "yes" or "no" to it. What would you choose and why?

Classifying New Instances

The algorithm finished with the following final version space for the training data from earlier:

$S_4 = \langle$Sunny, Warm, ?, Strong, ?, ?$\rangle$

$\langle$Sunny, ?, ?, Strong, ?, ?$\rangle$ $\langle$Sunny, Warm, ?, ?, ?, ?$\rangle$ $\langle$?, Warm, ?, Strong, ?, ? $\rangle$

$G_4 = \langle$Sunny, ?, ?, ?, ?, ? $\rangle$ $\langle$?, Warm, ?, ?, ?, ? $\rangle$

Question 24. For each of the following test instances, how will we predict the yes/no for EnjoySport?

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Sunny	Warm	Normal	Strong	Cool	Change	??
Rainy	Cold	Normal	Light	Warm	Same	??
Sunny	Warm	Normal	Light	Warm	Same	??
Sunny	Cold	Normal	Strong	Warm	Same	??

Biased Hypothesis Spaces

To illustrate this, consider the following training data:

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Sunny	Warm	Normal	Strong	Cool	Change	Yes
Cloudy	Warm	Normal	Strong	Cool	Change	Yes

Question 25. What is the most specific hypothesis consistent with these two?

Question 26. How does that hypothesis handle the following test case?

Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
Rainy	Warm	Normal	Strong	Cool	Change	No

Question 27. When we say this algorithm covers a *biased hypothesis space*, what does that mean?

Exercise/Reinforcement

Use the CANDIDATE-ELIMINATION algorithm to output a description of *all hypotheses consistent with the training examples.* This example is also from Patrick Henry Winston's *Artificial Intelligence* textbook.

Suppose you are a doctor treating a patient who occasionally suffers an allergic reaction. Your intuition tells you that the allergic condition is a direct result of a certain combination of the place where your patient eats, the time of day, the day of the week, and the amount that your patient spends on food. It might be, for example, that eating breakfast on Fridays is the source of your patient's allergy, or perhaps eating something expensive at Sam's.

Restaurant	Meal	Day	Cost	Reaction
Sam's	breakfast	Friday	cheap	yes
Lobdell	lunch	Friday	expensive	no
Sam's	lunch	Saturday	cheap	yes
Sarah's	breakfast	Sunday	cheap	no
Sam's	breakfast	Sunday	expensive	no

Row Specific hypotheses General Hypotheses