

Core Data Structures in Python

Lists, Implementation, and Hash Tables

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ENGF0034 – Design and Professional Skills

Lecture Overview

Objectives

To understand fundamental Python data structures (Lists, Dictionaries, Sets), explore the Python memory model, delve into the underlying implementations (Dynamic Arrays and Hash Tables), and analyse their performance characteristics (Complexity and Amortization).

- 1 Python Lists: Interface and Usage
- 2 The Python Memory Model and References
- 3 List Implementation: Dynamic Arrays
- 4 Performance Analysis and Amortisation
- 5 Hash Tables: Dictionaries and Sets
- 6 Conclusion

Python Lists: Interface and Usage

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Data Structure of the Week: The List

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- **Ordered:** Elements maintain a defined sequence, accessible via index.
- **Mutable:** The list can be modified in place after creation.
- **Dynamic:** The size can change (grow and shrink).

Creating and Accessing Lists

Syntax and Indexing

Lists are created using square brackets []. Access is via zero-based indexing.

```
1 # Creating a list
2 days = ["mon", "tues", "weds", "thurs", "fri"]
3
4 # Checking the length
5 count = len(days) # 5
6
7 # Accessing elements (Zero-based index)
8 day2 = days[1] # "tues"
9
10 # Negative indexing (from the end)
11 last_day = days[-1] # "fri"
```

Modifying Lists (Mutability)

Lists can be changed in place.

Item Assignment

```
1 days = ["mon", "tues", "weds", "thurs", "fri"]  
2 days[4] = "FRIDAY!!!"  
3 # days is now ["mon", "tues", "weds", "thurs", "FRIDAY!!!"]
```

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```

Adding Elements

- `.append(element)`: Adds to the end.
- `.insert(index, element)`: Inserts at a specific index.

```
1 days.append("sat")
2 # Insert "sun" at the beginning
3 days.insert(0, "sun")
```

Removing Elements

Methods for Removal

- `del list[i]`: Deletes the element at index `i`.
- `.pop()`: Removes and returns the last element.
- `.pop(i)`: Removes and returns the element at index `i`.

```
1 days = ["mon", "tues", "weds", "thurs", "fri"]
2
3 # I don't like mondays
4 del days[0]
5 # days is now ["tues", "weds", "thurs", "fri"]
6
7 # Remove the last element
8 last = days.pop() # last is "fri"
```

Iterating Over Lists I

Iteration using Index (Less Pythonic)

Traditional approach using `range(len(...))`.

```
1 # iteration over a list using an index
2 days = ["mon", "tues", "weds"]
3 for day_num in range(0, len(days)):
4     print(days[day_num])
```

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The preferred method. Directly iterates over the elements.

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1 # native iteration over a list (more pythonic!)
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Iterating Over Lists II

Native Iteration (Pythonic)

The preferred method. Directly iterates over the elements.

```
1 # native iteration over a list (more pythonic!)
2 for day in days:
3     print(day)
```

Using enumerate

If both the index and the item are needed.

```
1 for i, day in enumerate(days):
2     print(f"Index {i}: {day}")
```


The Python Memory Model and References

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Understanding Variables in Python

To understand how lists behave, especially when copied or passed to functions, we must understand the Python memory model.

The "Variables as Boxes" Metaphor (Incorrect for Python)

In languages like C, a variable is often thought of as a memory location that holds a value.

Understanding Variables in Python

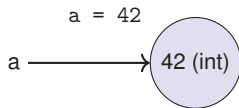
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The "Variables as Labels" Metaphor (Correct)

In Python, variables are **names** (labels) that **refer** to objects in memory. Assignment (=) binds a name to an object.



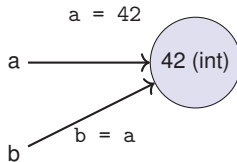
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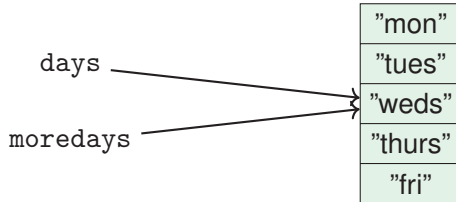
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The Impact of References: Aliasing

When multiple names refer to the same mutable object (like a list), this is called **aliasing**.

```
1 # 1. Create a list object and bind 'days' to it
2 days = ["mon", "tues", "weds", "thurs", "fri"]
3
4 # 2. Bind the name 'moredays' to the SAME object.
5 # This does NOT copy the list!
6 moredays = days
```



The Consequences of Aliasing

Because the object is mutable, changes made through one alias are visible through all other aliases.

```
1 days = ["mon", "tues", "weds", "thurs", "FRIDAY!!!"]
2 moredays = days
3
4 # Modify the list via the 'moredays' alias
5 moredays[4] = "fri"
6
7 # Check the original 'days' variable
8 print(days)
9 # Output: ["mon", "tues", "weds", "thurs", "fri"]
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A Common Source of Bugs

This behaviour is often surprising and is a major source of errors, especially when passing mutable arguments to functions.

How to Copy a List I

If we want an independent copy, we must explicitly create a new list object.

Methods for Shallow Copying

```
1 days = ["mon", "tues", "weds", "thurs", "fri"]
2
3 # 1. Using the .copy() method (Recommended)
4 copy1 = days.copy()
5
6 # 2. Using a full slice (Idiomatic Python)
7 copy2 = days[:]
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Verification

```

1 copy1[0] = "YAWN!"
2 # copy1 is ["YAWN!", "tues", ...]
3 # days is still ["mon", "tues", ...]

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How to Copy a List II

If we want an independent copy, we must explicitly create a new list object.

Verification

```
1 copy1[0] = "YAWN!"
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3 # days is still ["mon", "tues", ...]
```

Shallow vs. Deep Copy

.copy() creates a shallow copy. If the list contains nested mutable objects (e.g., lists within lists), those inner objects are still shared. Use copy.deepcopy() if needed for full independence.

Shallow Copy: The Pitfall

Example with Nested Lists

```
1 list_a = [[1, 2], [3, 4]]
2 # Create a shallow copy
3 list_b = list_a.copy()
4 # Modify an inner list via list_b
5 list_b[0][0] = 99
6 # The change is reflected in list_a!
7 print(list_a) # Output: [[99, 2], [3, 4]]
```

Shallow Copy: The Pitfall

Example with Nested Lists

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1 list_a = [[1, 2], [3, 4]]
2 # Create a shallow copy
3 list_b = list_a.copy()
4 # Modify an inner list via list_b
5 list_b[0][0] = 99
6 # The change is reflected in list_a!
7 print(list_a) # Output: [[99, 2], [3, 4]]
```

Why This Happens

The `.copy()` method created a new outer list for `list_b`, but it filled it with pointers to the **same inner lists** that `list_a` was pointing to.

List Implementation: Dynamic Arrays

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Looking Under the Hood

We know the interface (how to use lists). To understand *why* certain operations are fast and others slow (performance characteristics), we must examine the implementation.

The Underlying Data Structure

Python lists are implemented as **Dynamic Arrays** (similar to C++ `std::vector` or Java `ArrayList`).

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Key Feature: Contiguous Memory

A dynamic array uses a contiguous block of memory to store its data. This is crucial for performance.

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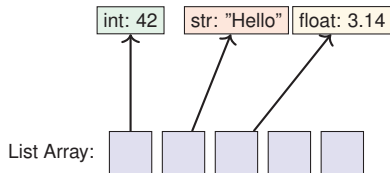
The Heterogeneous Challenge

Python lists can store different types, which have different sizes (e.g., an integer vs. a long string). How can they be stored contiguously?

Implementation: Array of Pointers

The Solution

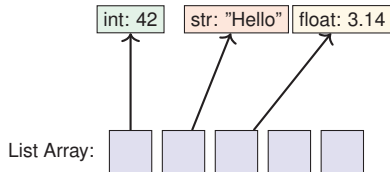
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Implications

All pointers have the same size (e.g., 64 bits), regardless of the object they point to. This maintains contiguity and enables fast indexing.

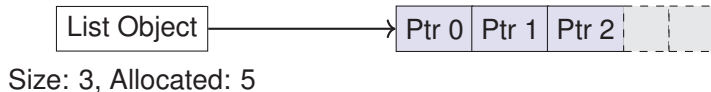
The CPython List Structure

In CPython (the standard implementation), a list object has metadata alongside the pointer array.

Conceptual Structure (Simplified)

A list object contains:

- 1 `ob_size`: The current number of elements (the length, `len(L)`).
- 2 `allocated`: The total capacity of the underlying array.
- 3 `ob_item`: A pointer to the dynamic array of pointers.

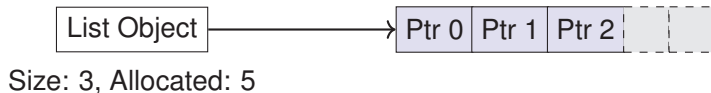


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Key Insight: Over-allocation

Crucially, `allocated` is often greater than `ob_size`. This strategy is called **Over-allocation**.

Performance Analysis and Amortisation

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Analysing Performance (Big O)

We use Big O notation to describe how the time required scales with the length of the list (N).

Constant Time Operations: $\mathcal{O}(1)$

- **Indexing** (`L[i]`): Due to contiguous memory, the address of the *i*-th pointer can be calculated directly: `Base + i * PointerSize`.
- **Length** (`len(L)`): The size is stored in the metadata (`ob_size`) and retrieved instantly.

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Empirical Evidence

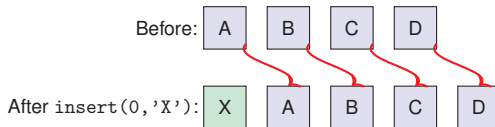
Experiments (e.g., using the logic in `testaccess.py`) confirm that access time is independent of the list size.

The Cost of Insertion and Deletion

The dynamic array implementation involves a trade-off when modifying the structure.

The Problem: Maintaining Contiguity

If we insert or delete an element in the middle, all subsequent elements must be shifted to maintain order and prevent gaps.

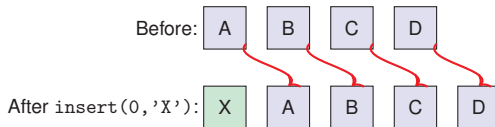


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Computational Cost

Shifting N items requires N memory operations. This is $\mathcal{O}(N)$.

Complexity of Insertion/Deletion I

Worst Case: Beginning of the List

`L.insert(0, x)` or `L.pop(0)`.

- Requires shifting all N elements.
- **Complexity:** $\mathcal{O}(N)$ (**Linear Time**)

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Average Case: Middle of the List

`L.insert(i, x)` or `L.pop(i)`.

- Requires shifting $N - i$ elements (on average $N/2$).
- **Complexity:** $\mathcal{O}(N)$ (**Linear Time**)

Complexity of Insertion/Deletion II

Best Case: End of the List

`L.pop()`.

- No shifting required.
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Warning

Using `pop(0)` repeatedly (e.g., using a list as a queue) leads to $\mathcal{O}(N^2)$ total time. Use `collections.deque` for efficient queues.

The Complexity of Append

What about `L.append(x)`?

The Fast Path (Thanks to Over-allocation)

If `Size < Allocated`, there is free space.

- 1 Place the new pointer in the next slot.
- 2 Increment the Size.

This is clearly $\mathcal{O}(1)$.

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The Slow Path: Resizing

If $\text{Size} == \text{Allocated}$ (The list is full).

- 1 Allocate a new, larger array.
- 2 Copy all N existing pointers to the new array.
- 3 Free the old array.

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- 1 Allocate a new, larger array.
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- 3 Free the old array.

This resize operation takes $\mathcal{O}(N)$ time.

The Dilemma

The worst-case complexity of a single `append()` is $\mathcal{O}(N)$. How can we claim it is efficient?

The Importance of the Growth Strategy

How much extra space should be allocated during a resize?

Naive Strategy: Additive Growth (+C)

Allocate a constant amount of extra space (e.g., +1 slot).

- Problem: Resizing happens too frequently.
- Total time for N appends becomes $\mathcal{O}(N^2)$. (The sum of an arithmetic series).

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Multiply current size by a constant factor (e.g., double the size). This is what Python does.

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CPython's Growth Factor

CPython uses a growth factor of approximately 1.125 (9/8).

Amortised Analysis I

When an operation is usually fast but occasionally slow, we use **amortised analysis** to find the average time taken per operation over a sequence of operations.

Analysis of Geometric Growth

The cost of the expensive $\mathcal{O}(N)$ resize is "amortised" (spread out) over the many cheap $\mathcal{O}(1)$ appends that preceded it.

Consider N appends using the doubling strategy.

- The total cost of copying occurs at sizes $1, g, g^2, \dots, g^k$ where g is the growth factor and $N \approx g^{k+1}$.
- The total number of copies is a geometric series, i.e.

$$C(N) = \sum_{i=0}^k g^i = \frac{g^{k+1} - 1}{g - 1} \approx \frac{N}{g - 1}$$

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- For a doubling strategy ($g = 2$), the total copy cost is $\approx \frac{N}{2-1} = N$.
- For CPython's strategy ($g \approx 1.125$), the cost is $\approx \frac{N}{0.125} = 8N$.
- In both cases, the total copy cost is linear, i.e., $\mathcal{O}(N)$.

Amortised Analysis II

When an operation is usually fast but occasionally slow, we use **amortised analysis** to find the average time taken per operation over a sequence of operations.

Total Cost

Total cost for N operations = Cost of inserts ($\approx N$) + Cost of resizing ($\approx 2N$) = $3N$.

Amortized Complexity

The total time for N appends is $\mathcal{O}(N)$. The average (amortized) cost per append is $\mathcal{O}(N)/N = \mathcal{O}(1)$.

List Performance Summary

Operation	Example	Average Complexity
Indexing / Assignment	<code>L[i]</code>	$\mathcal{O}(1)$
Length	<code>len(L)</code>	$\mathcal{O}(1)$
Append (End)	<code>L.append(x)</code>	$\mathcal{O}(1)$ (Amortised)
Pop (End)	<code>L.pop()</code>	$\mathcal{O}(1)$
Insert (Start/Middle)	<code>L.insert(0, x)</code>	$\mathcal{O}(N)$
Pop (Start/Middle)	<code>L.pop(0)</code>	$\mathcal{O}(N)$
Search (Membership)	<code>x in L</code>	$\mathcal{O}(N)$
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The Limitation

Lists are excellent for ordered sequences, but searching for a value (`x in L`) requires a slow linear scan ($\mathcal{O}(N)$).

Hash Tables: Dictionaries and Sets

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Can we design a data structure that allows lookups in $\mathcal{O}(1)$ time, regardless of the collection size? Yes, using **Hash Tables**.

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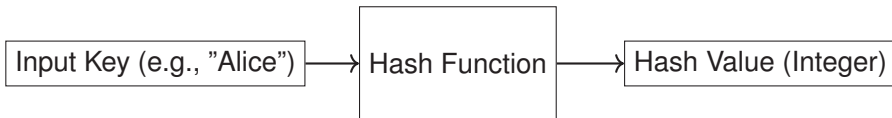
Python Implementations

Hash tables are the foundation for Python's `dict` (Dictionary) and `set` data types.

Hash Functions

Definition

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Properties

- **Deterministic:** Same input always yields the same output.
- **Efficient:** Fast to compute ($\mathcal{O}(1)$).
- **Uniform Distribution:** Spreads inputs evenly to minimise collisions (different inputs mapping to the same hash).

The Hash Table Concept

A hash table uses the hash function to determine where to store data in an underlying array (the "buckets").

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 - Compute the hash and index.
 - Check if K is stored at that location.

Performance

If the hash function is fast and collisions are rare, insertion and lookup are $\mathcal{O}(1)$.

Handling Collisions

The Collision Problem

What happens if two different keys map to the same index? This is a **collision**.

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Collision Resolution Strategies

- **Separate Chaining:** Each bucket stores a linked list of entries.
- **Open Addressing (Probing):** If a bucket is full, the algorithm searches (probes) nearby buckets until an empty one is found. (This is what CPython uses).

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Maintaining Performance: Load Factor

The Load Factor is the ratio of entries to buckets. To keep collisions rare and maintain $\mathcal{O}(1)$ performance, the hash table must be resized (rehashed) when the load factor gets too high (e.g., $> 2/3$). This is similar to dynamic array resizing and also uses amortisation.

Dictionaries (dict) I

Definition

A dictionary is a mutable mapping of unique keys to values. It implements a hash table.

```
1 # Mapping names (keys) to ages (values)
2 ages = {"Alice": 30, "Bob": 25, "Charlie": 35}
3
4 # Accessing a value by key (O(1) average)
5 bobs_age = ages["Bob"] # 25
6
7 # Inserting/Updating (O(1) average, amortized)
8 ages["David"] = 40
9
10 # Membership testing (Checks keys, O(1) average)
11 is_present = "Alice" in ages
```

Dictionaries (dict) II

Definition

A dictionary is a mutable mapping of unique keys to values. It implements a hash table.

Hashability Requirement

Keys must be immutable (hashable) because their hash must remain constant. `int`, `float`, `str`, `tuple` are hashable. `list` and `dict` are mutable and therefore unhashable.

Sets (set) I

Definition

A set is an unordered collection of unique, hashable elements.

```
1 # Creating a set (duplicates removed automatically)
2 colors = {"red", "green", "blue", "red"}
3 # colors is {"red", "green", "blue"}
4
5 # Membership testing (O(1) average)
6 is_present = "red" in colors
7
```

Sets (set) I

Implementation

A set is implemented as a hash table where only the keys are stored (values are ignored). It inherits the performance characteristics of dictionaries.

```
1 # Creating a set (duplicates removed automatically)
2 colors = {"red", "green", "blue", "red"}
3 # colors is {"red", "green", "blue"}
4
5 # Membership testing (O(1) average)
6 is_present = "red" in colors
7
8 # Adding elements (O(1) average, amortized)
9 colors.add("yellow")
```

Sets (set) II

Definition

A set is an unordered collection of unique, hashable elements.

Implementation

A set is implemented as a hash table where only the keys are stored (values are ignored). It inherits the performance characteristics of dictionaries.

Use Cases

Fast deduplication of lists (`list(set(my_list))`) and efficient membership checking.

Conclusion

Conclusion

Summary of Data Structures

Type	Implementation	Ordered?	Mutable?	Key Performance
list	Dynamic Array	Yes	Yes	$\mathcal{O}(1)$ index/append; $\mathcal{O}(N)$ search/insert(0)
dict	Hash Table	Yes (since 3.7)	Yes	$\mathcal{O}(1)$ avg lookup/insert/delete
set	Hash Table	No	Yes	$\mathcal{O}(1)$ avg membership/add/remove
tuple	Fixed Array	Yes	No	$\mathcal{O}(1)$ index; $\mathcal{O}(N)$ search

Key Takeaways

Core Concepts

- **Memory Model:** Python uses references. Understand aliasing and mutability.
- **Implementation Matters:** The underlying data structure determines performance characteristics (Big $\mathcal{O}()$).
- **Dynamic Arrays (Lists):** Optimized for access ($\mathcal{O}(1)$) and operations at the end ($\mathcal{O}(1)$ amortized) via over-allocation. Slow in the middle ($\mathcal{O}(N)$).
- **Amortization:** Geometric growth allows dynamic structures to maintain average $\mathcal{O}(1)$ performance despite occasional $\mathcal{O}(N)$ resizing costs.
- **Hash Tables (Dicts/Sets):** Provide average $\mathcal{O}(1)$ lookups, insertion, and deletion using hashing.

Key Takeaways

The Engineering Trade-off

There is no single best data structure. Choosing the right one depends on the required operations and the expected data patterns.