Learning Nominal Automata

Matteo Sammartino

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Learning Nominal Automata (POPL '17)

Joshua Moerman
Radboud University

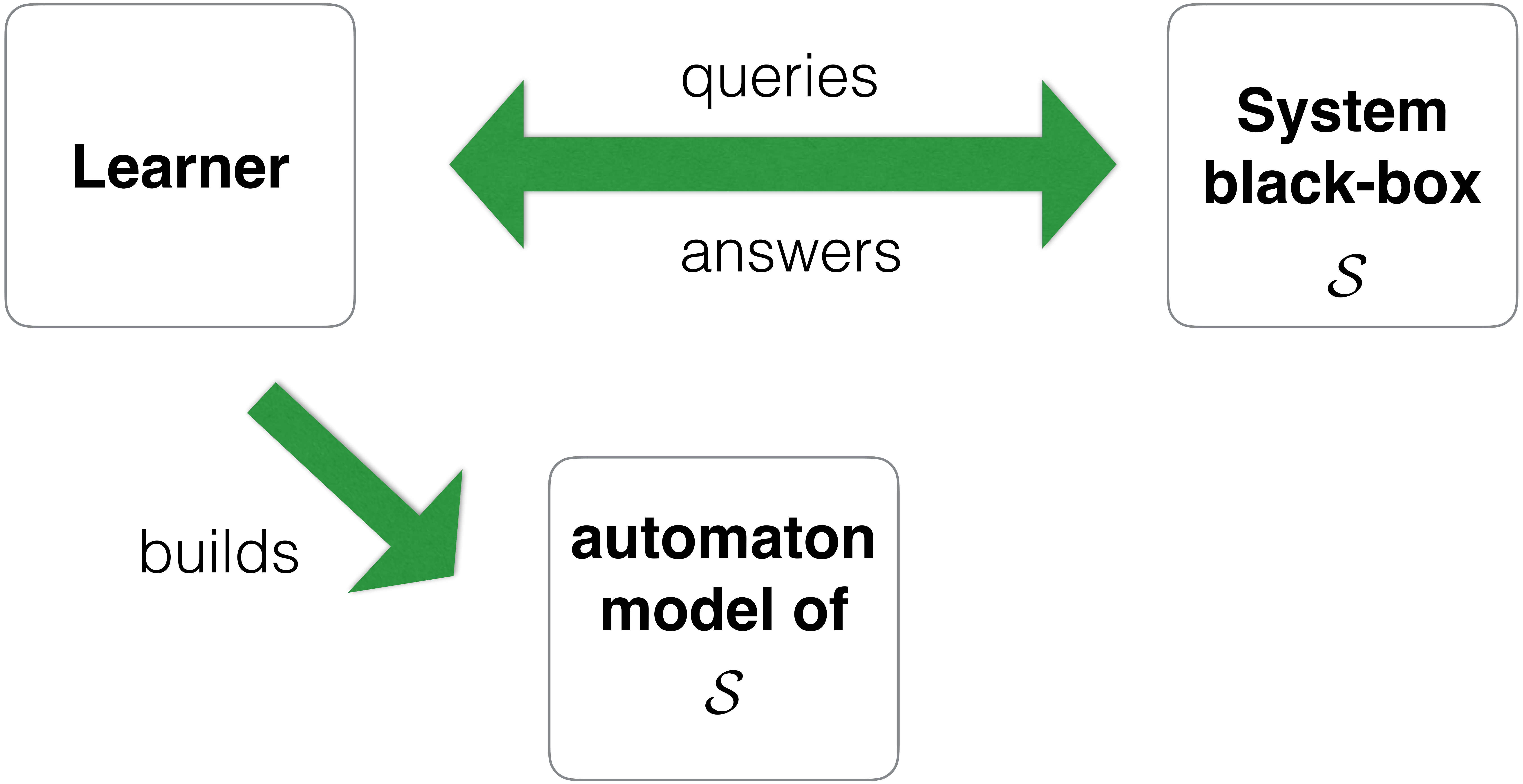
Matteo Sammartino
UCL

Alexandra Silva
UCL

Bartek Klin
Warsaw University

Michal Szynwelski
Warsaw University

Active learning



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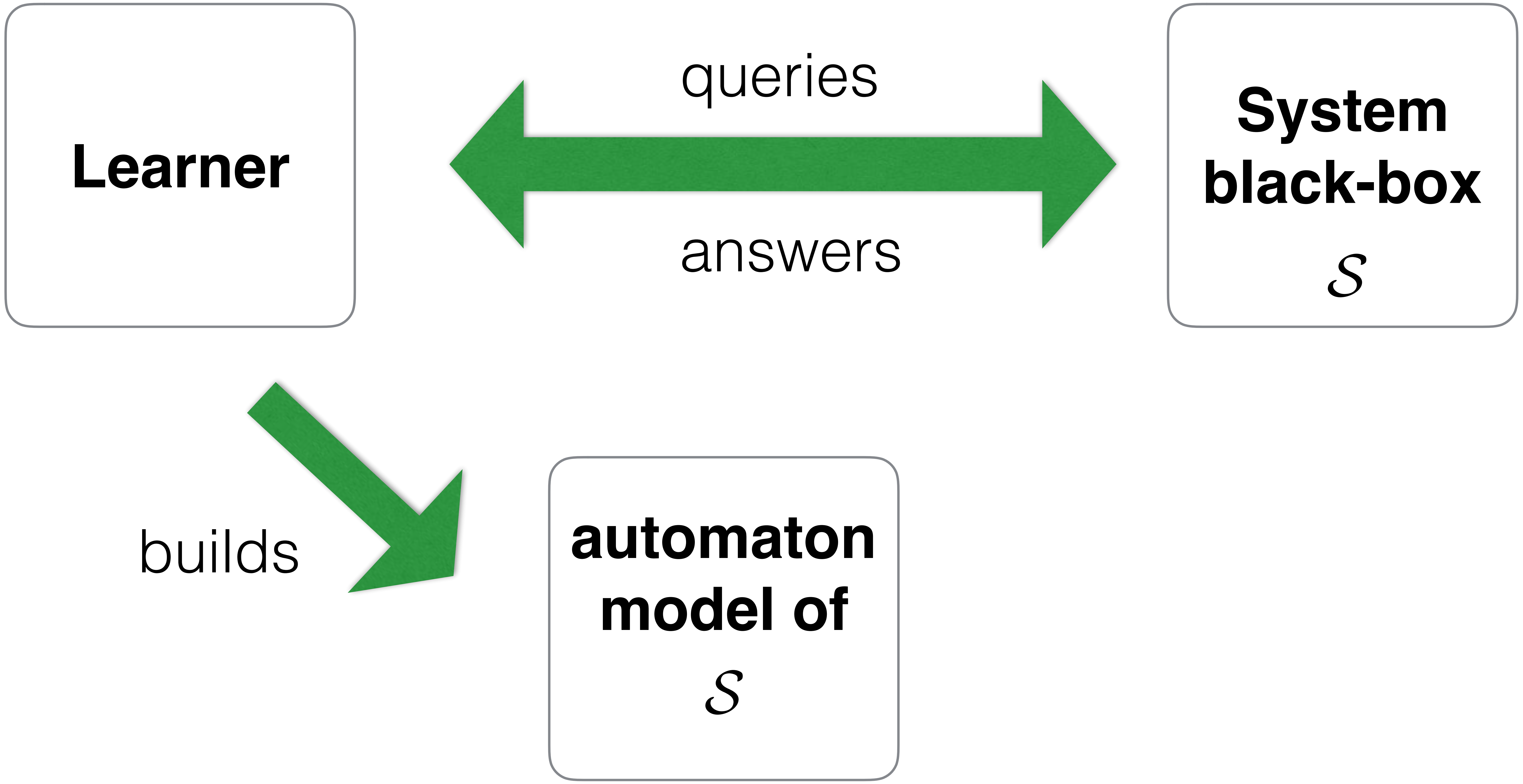

No formal specification available? **Learn it!**

L* algorithm (D.Angluin '87)

Finite alphabet of system's actions A

set of system behaviors is a **regular language** $\mathcal{L} \subseteq A^\star$

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Learner

Teacher

$\mathcal{L}$

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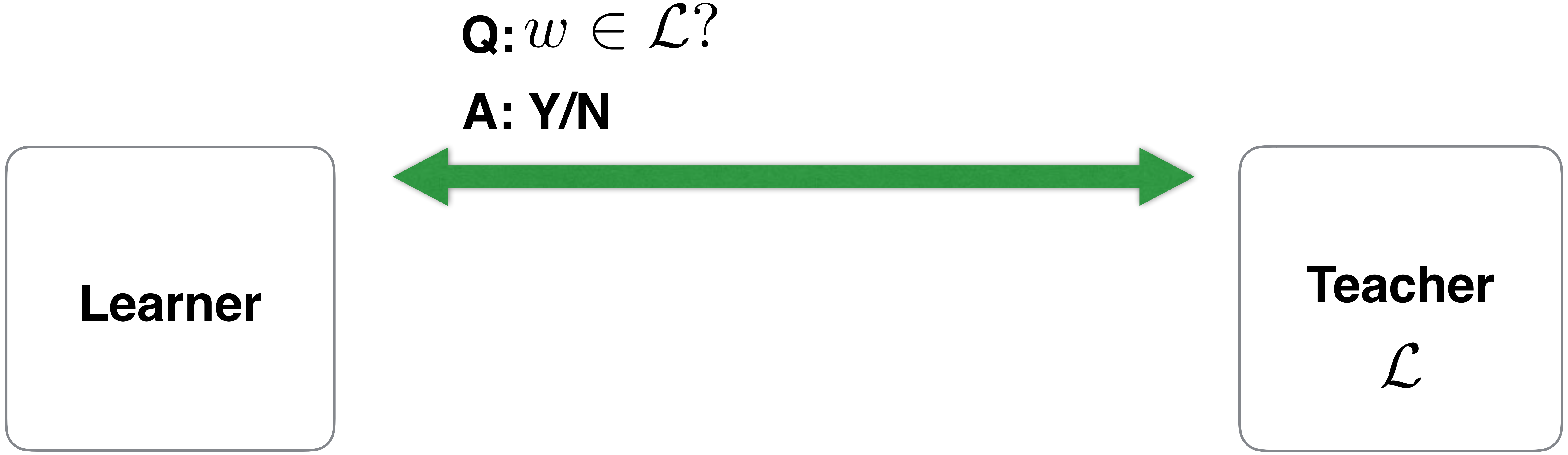

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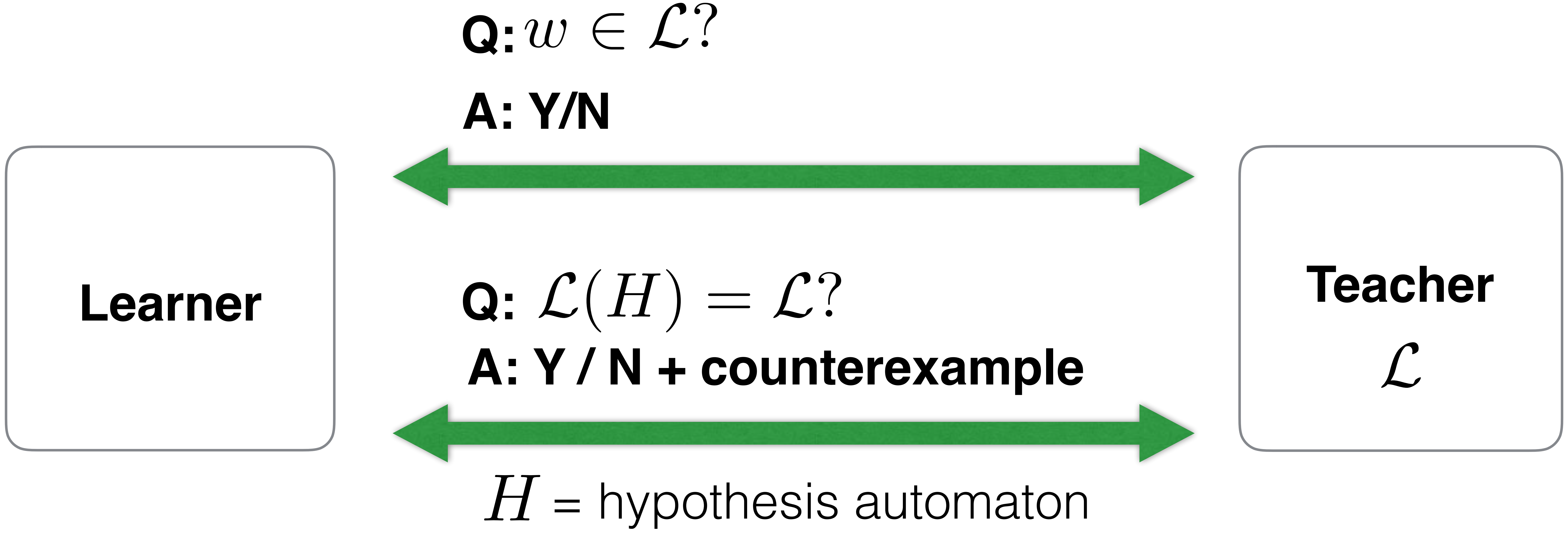

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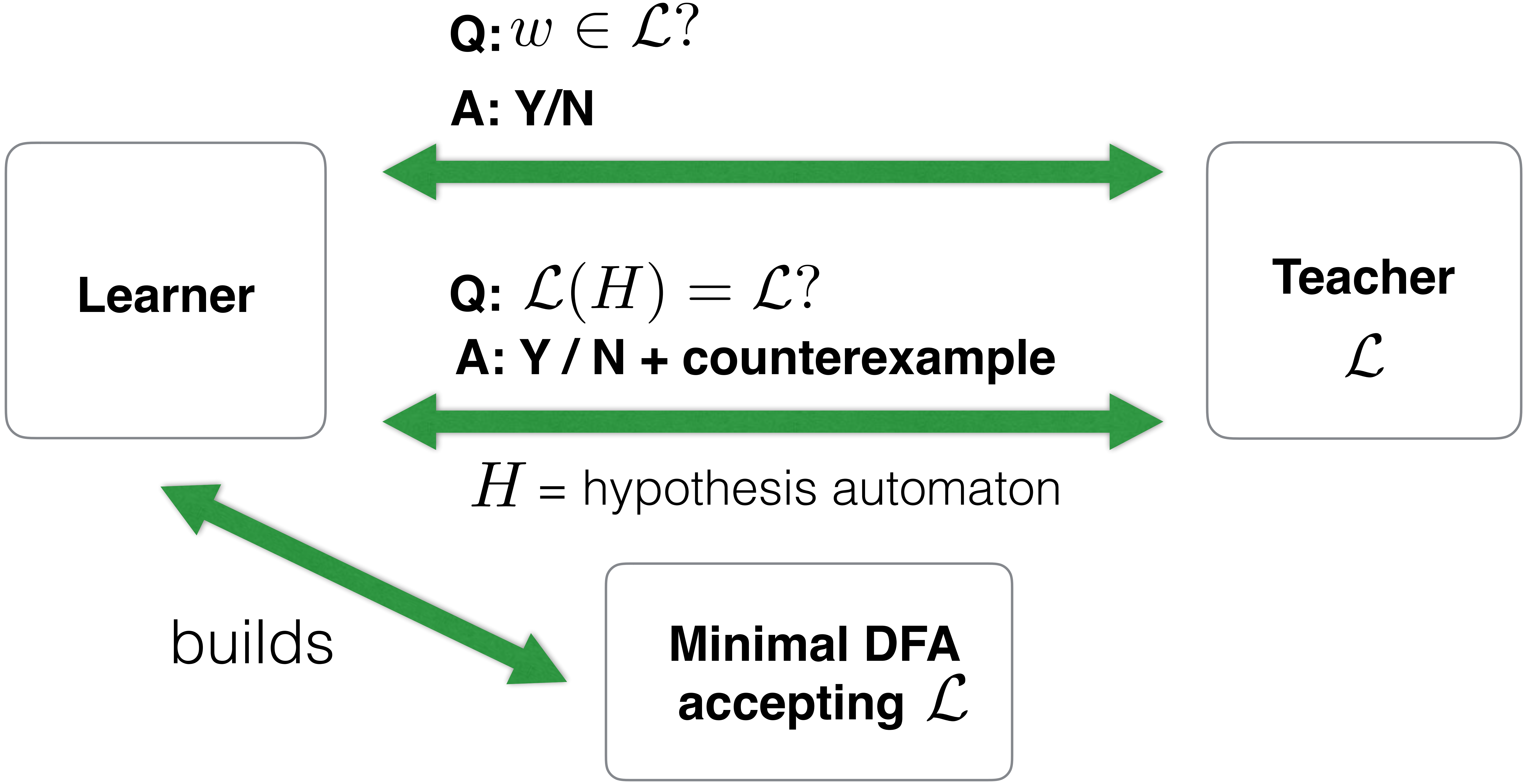

Observation table

E

	ϵ	**a**	**aa**
ϵ	0	0	1
a	0	1	0
b	0	0	0

S: row ϵ; $S{\cdot}A$: rows **a**, **b**

$$row\colon S \cup S{\cdot}A \to 2^E$$

$$row(s)(e) = 1 \iff se \in \mathcal{L}$$

$$S, E \subseteq A^\star \qquad A = \{a, b\}$$

Example

$$\mathcal{L} = \{aa, bb\} \qquad A = \{a, b\}$$

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$S, E \leftarrow \{\epsilon\}$

repeat

  while (S, E) is not closed or not consistent

  if (S, E) is not closed

    find $s_1 \in S$, $a \in A$ such that

      $row(s_1 a) \neq row(s)$, for all $s \in S$

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Hypothesis

states =
$\{row(s) \mid s \in S\}$

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$\boxed{row(\epsilon)}$

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$$\forall t \in S \cdot A \quad \exists s \in S \quad row(t) = row(s)$$

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initial state = $row(\epsilon)$

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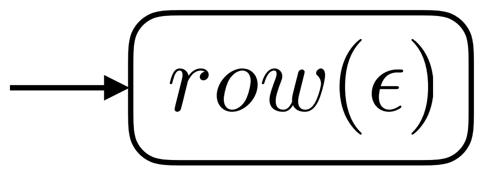

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Hypothesis

$$\rightarrow \boxed{row(\epsilon)}$$

final states =
$\{row(s) \mid row(s)(\epsilon) = 1\}$

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transitions =
$row(s) \xrightarrow{a} row(sa)$

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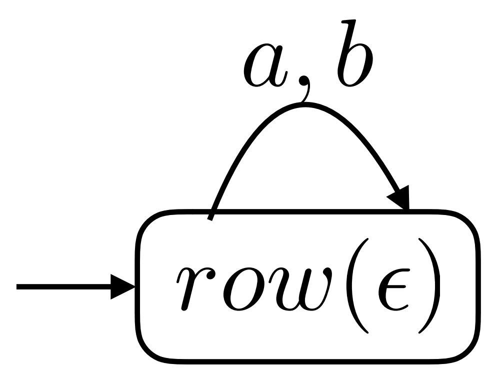
Hypothesis



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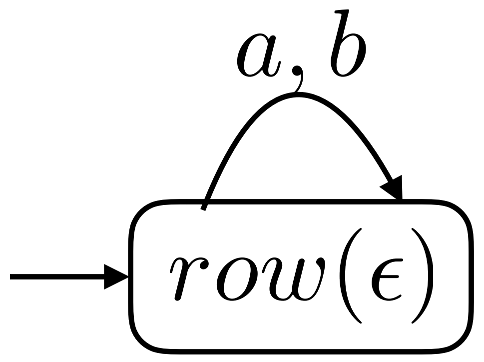
Closed ✓ **transitions are well-defined**

$\forall t \in S \cdot A \quad \exists s \in S \quad row(t) = row(s)$

Consistent ✓

$\forall s_1, s_2 \in S \quad row(s_1) = row(s_2) \implies \forall a \in A \;\; row(s_1 a) = row(s_2 a)$

Hypothesis



transitions =
$row(s) \xrightarrow{a} row(sa)$

$$\mathcal{L} = \{aa, bb\} \qquad A = \{a, b\}$$

$S, E \leftarrow \{\epsilon\}$
repeat
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$row =$

	ϵ
ϵ	0
a	0
b	0

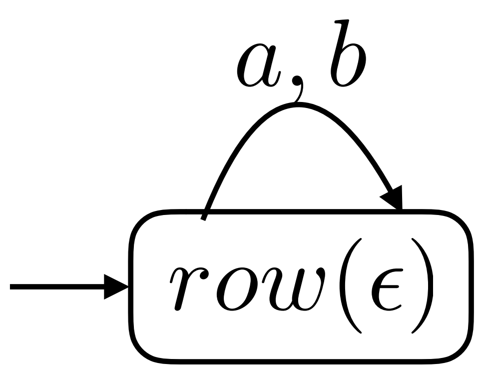
Closed ✓ **transitions are well-defined**

$$\forall t \in S \cdot A \quad \exists s \in S \quad row(t) = row(s)$$

Consistent ✓ **determinism**

$$\forall s_1, s_2 \in S \quad row(s_1) = row(s_2) \implies \forall a \in A \;\; row(s_1 a) = row(s_2 a)$$

Hypothesis



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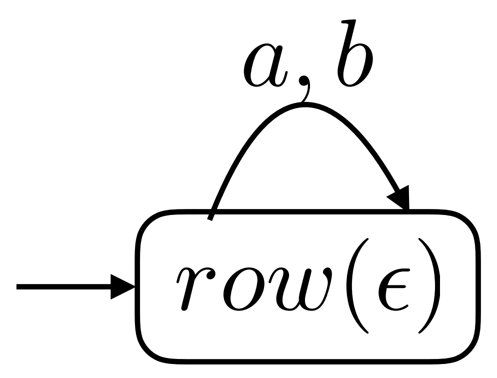

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Counterexample = *aa*

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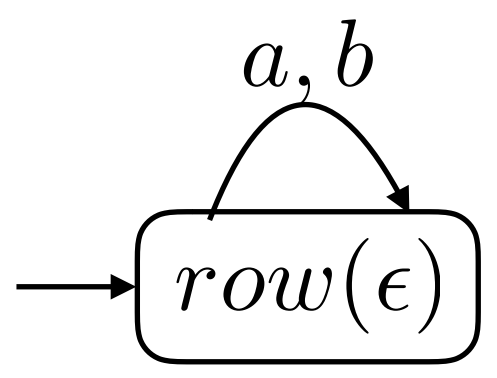

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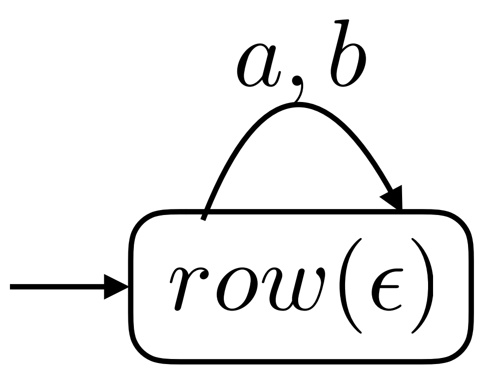

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ϵ	0
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aa	1
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a a

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$row =$

	ϵ	a
ϵ	0	0
a	0	1
aa	1	0
b	0	0
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$row =$

	ϵ	*a*
ϵ	0	0
a	0	1
aa	1	0
b	0	0
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    $S \leftarrow S \cup \texttt{prefixes}(t)$

until the Teacher replies **yes** to H

return H

$row =$

	ϵ	*a*
ϵ	0	0
a	0	1
aa	1	0
b	0	0
ab	0	0
aaa	0	0
aab	0	0

Closed ✓

Consistent ✓

$$\mathcal{L} = \{aa, bb\} \qquad A = \{a, b\}$$

$S, E \leftarrow \{\epsilon\}$

repeat

  while (S, E) is not closed or not consistent

  if (S, E) is not closed

    find $s_1 \in S, a \in A$ such that

      $row(s_1 a) \neq row(s), \text{for all } s \in S$

    $S \leftarrow S \cup \{s_1 a\}$

  if (S, E) is not consistent

    find $s_1, s_2 \in S, a \in A$, and $e \in E$ such that

      $row(s_1) = row(s_2)$

      and $row(s_1 a)(e) \neq row(s_2 a)(e)$

    $E \leftarrow E \cup \{ae\}$

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return H

$row =$

	ϵ	a
ϵ	0	0
a	0	1
aa	1	0
b	0	0
ab	0	0
aaa	0	0
aab	0	0

Closed ✓

Consistent ✓

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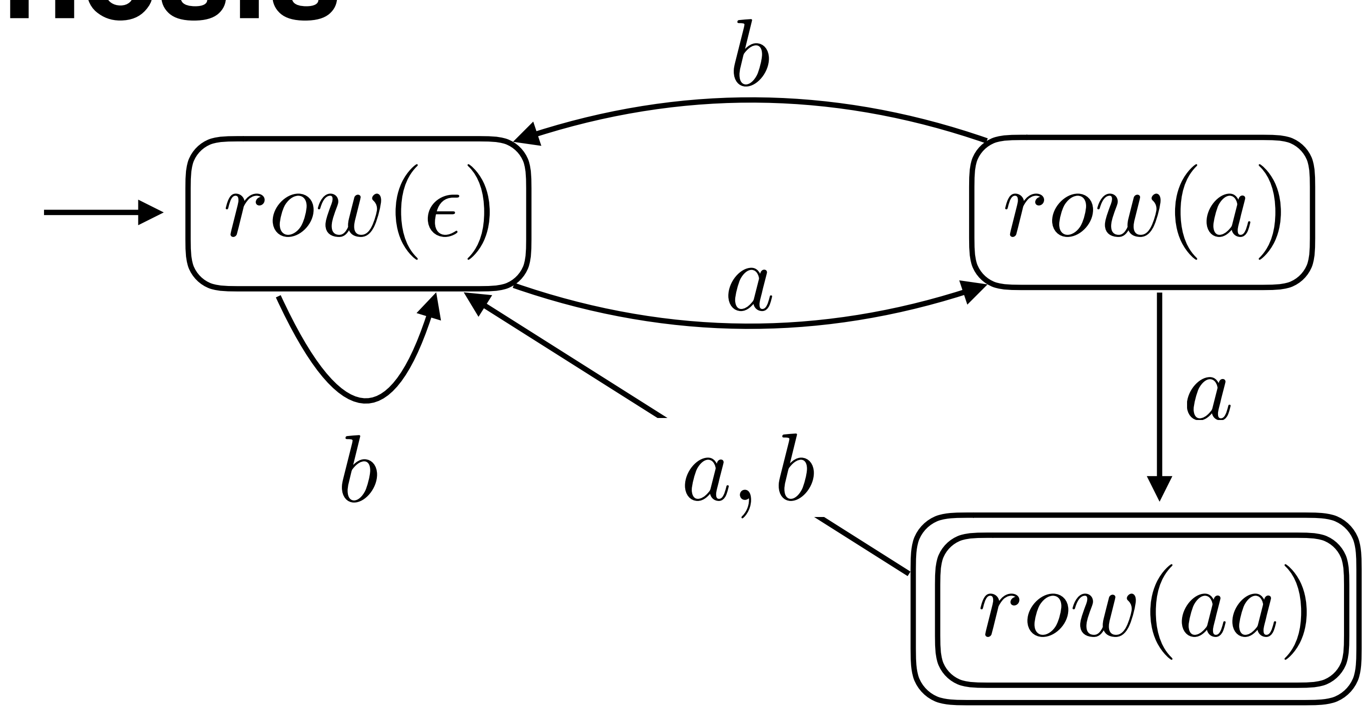
	ϵ	*a*
ϵ	0	0
a	0	1
aa	1	0
b	0	0
ab	0	0
aaa	0	0
aab	0	0

Closed ✓

Consistent ✓

Hypothesis



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$row =$

	ϵ	a
ϵ	0	0
a	0	1
aa	1	0
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aab	0	0

Closed ✓

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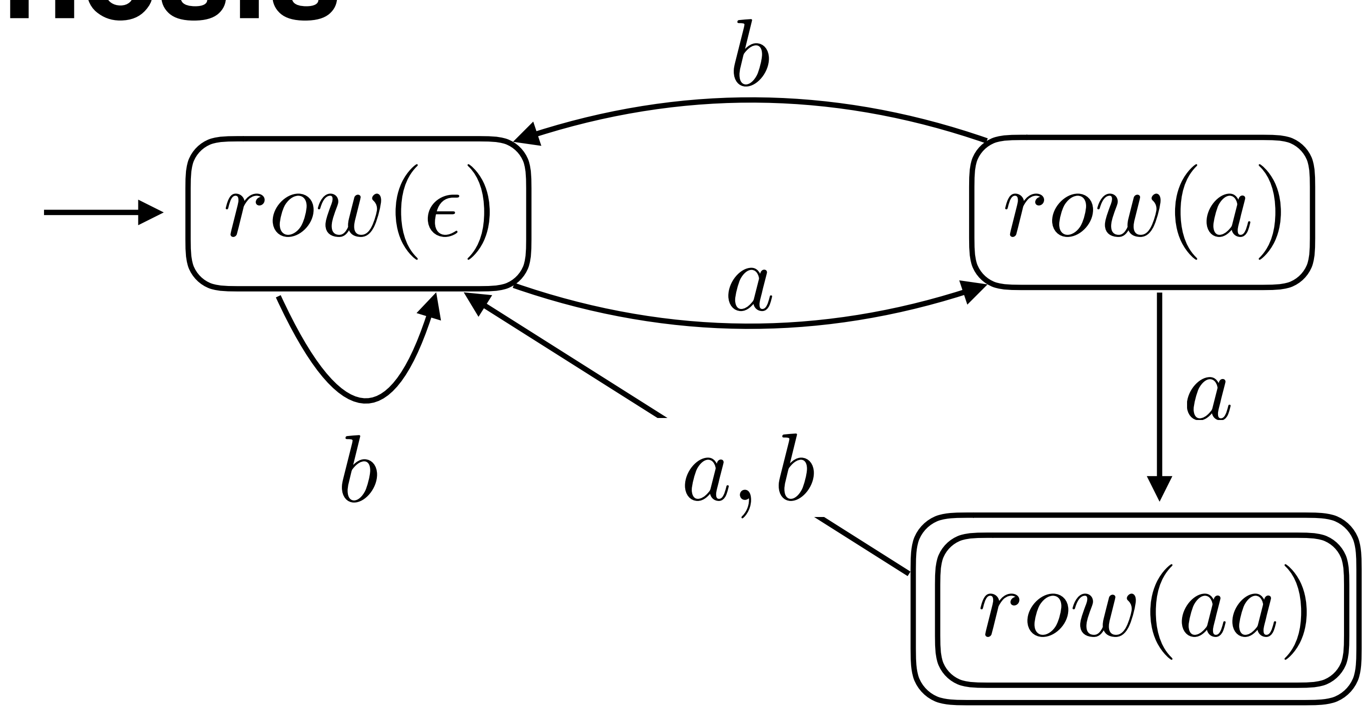
Hypothesis



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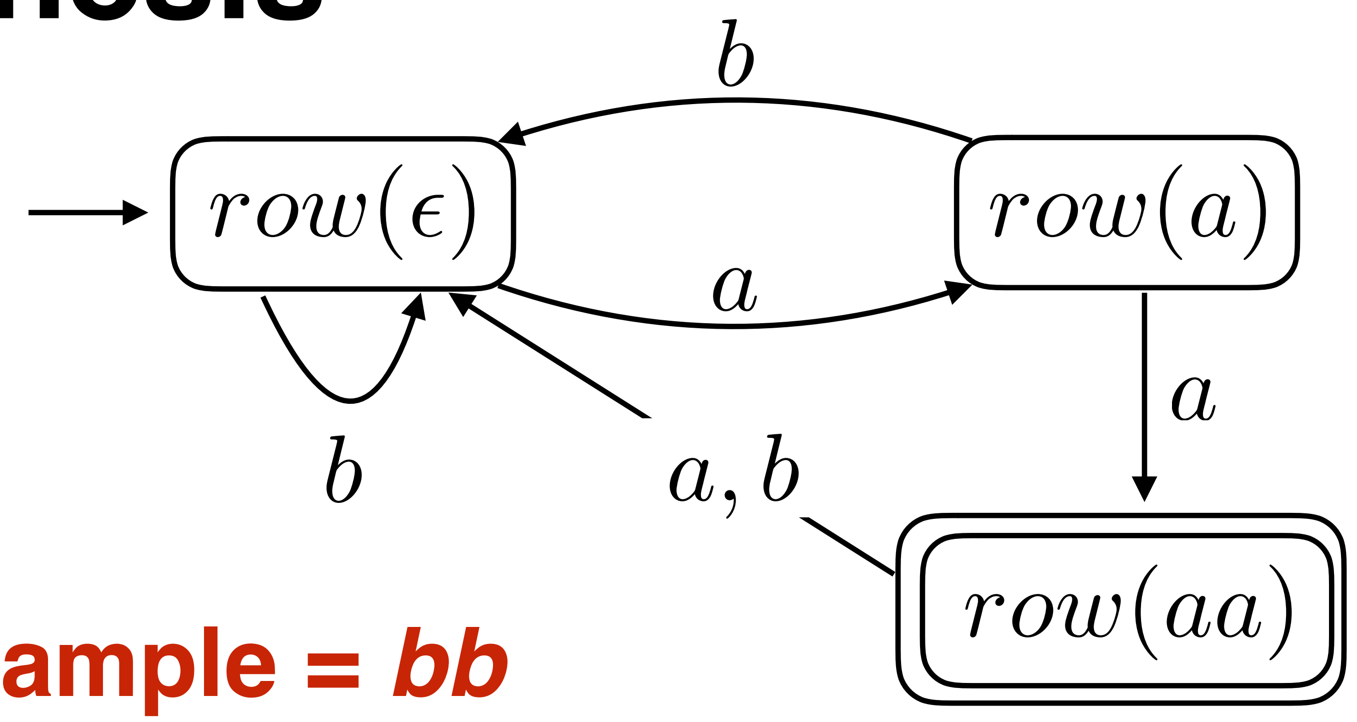
$row =$

	ϵ	*a*
ϵ	0	0
a	0	1
aa	1	0
b	0	0
ab	0	0
aaa	0	0
aab	0	0

Closed ✓

Consistent ✓

Hypothesis



Counterexample = *bb*

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until the Teacher replies **yes** to H
return H

$row =$

	ϵ	a
ϵ	0	0
a	0	1
aa	1	0
b	0	0
bb	1	0
ab	0	0
aaa	0	0
aab	0	0
ba	0	0
bba	0	0
bbb	0	0

$$\mathcal{L} = \{aa, bb\} \qquad A = \{a, b\}$$

$S, E \leftarrow \{\epsilon\}$

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aa	1	0
b	0	0
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$row =$

	ϵ	a
ϵ	0	0
a	0	1
aa	1	0
b	0	0
bb	1	0
ab	0	0
aaa	0	0
aab	0	0
ba	0	0
bba	0	0
bbb	0	0

Closed ✓

Consistent ✗

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$row =$

	ϵ	a
ϵ	0	0
a	0	1
aa	1	0
b	0	0
bb	1	0
ab	0	0
aaa	0	0
aab	0	0
ba	0	0
bba	0	0
bbb	0	0

b

b

Closed ✓

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ϵ	0	0
a	0	1
aa	1	0
b	0	0
bb	1	0
ab	0	0
aaa	0	0
aab	0	0
ba	0	0
bba	0	0
bbb	0	0

b

b

Closed ✓

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	ϵ	**a**
ϵ	0	0
a	0	1
aa	1	0
b	0	0
bb	1	0
ab	0	0
aaa	0	0
aab	0	0
ba	0	0
bba	0	0
bbb	0	0

b b

Closed ✓

Consistent ✗

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$row =$

	ϵ	a	b
ϵ	0	0	0
a	0	1	0
aa	1	0	0
b	0	0	1
bb	1	0	0
ab	0	0	0
aaa	0	0	0
aab	0	0	0
ba	0	0	0
bba	0	0	0
bbb	0	0	0

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$row =$

	ϵ	*a*	*b*
ϵ	0	0	0
a	0	1	0
aa	1	0	0
b	0	0	1
bb	1	0	0
ab	0	0	0
aaa	0	0	0
aab	0	0	0
ba	0	0	0
bba	0	0	0
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ϵ	0	0	0
a	0	1	0
aa	1	0	0
b	0	0	1
bb	1	0	0
ab	0	0	0
aaa	0	0	0
aab	0	0	0
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ϵ	0	0	0
a	0	1	0
aa	1	0	0
b	0	0	1
bb	1	0	0
ab	0	0	0
aaa	0	0	0
aab	0	0	0
ba	0	0	0
bba	0	0	0
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	ϵ	a	b
ϵ	0	0	0
a	0	1	0
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b	0	0	1
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ab	0	0	0
aaa	0	0	0
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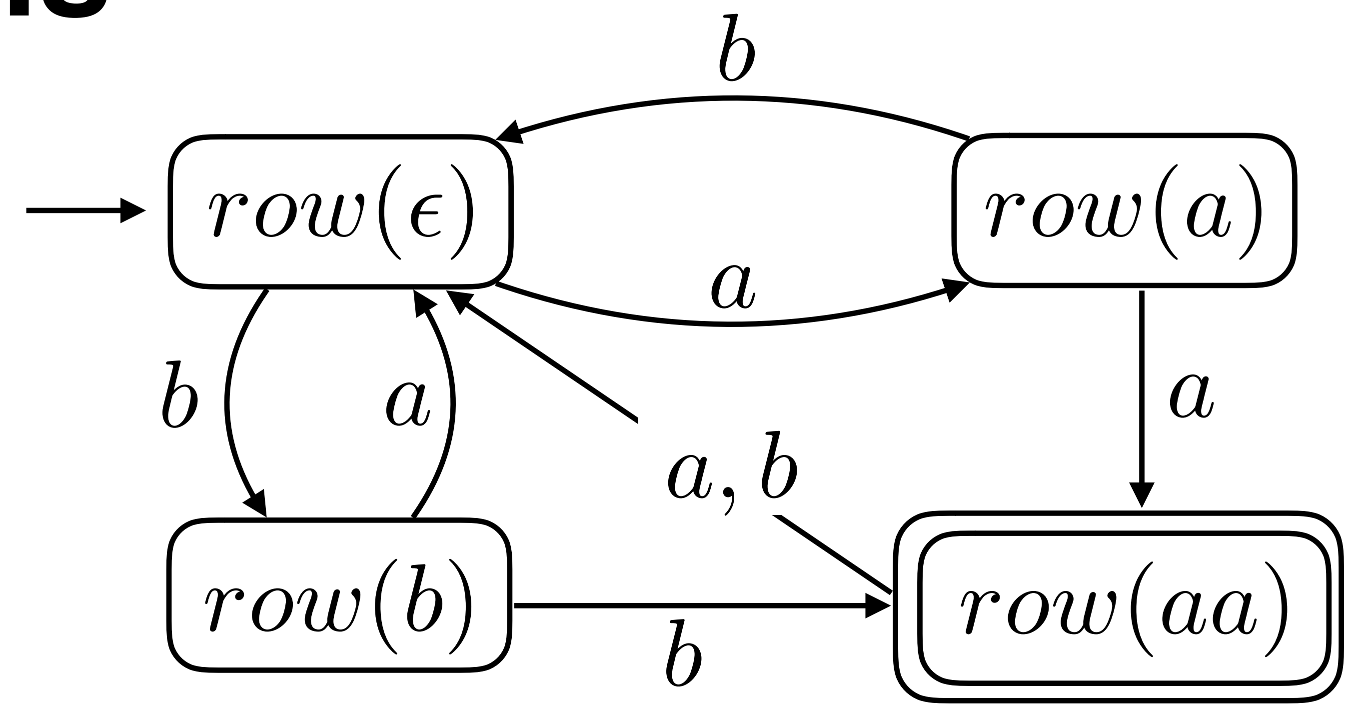
bba	0	0	0
bbb	0	0	0

Closed ✓

Consistent ✓

Hypothesis



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$row =$

	ϵ	a	b
ϵ	0	0	0
a	0	1	0
aa	1	0	0
b	0	0	1
bb	1	0	0
ab	0	0	0
aaa	0	0	0
aab	0	0	0
ba	0	0	0
bba	0	0	0
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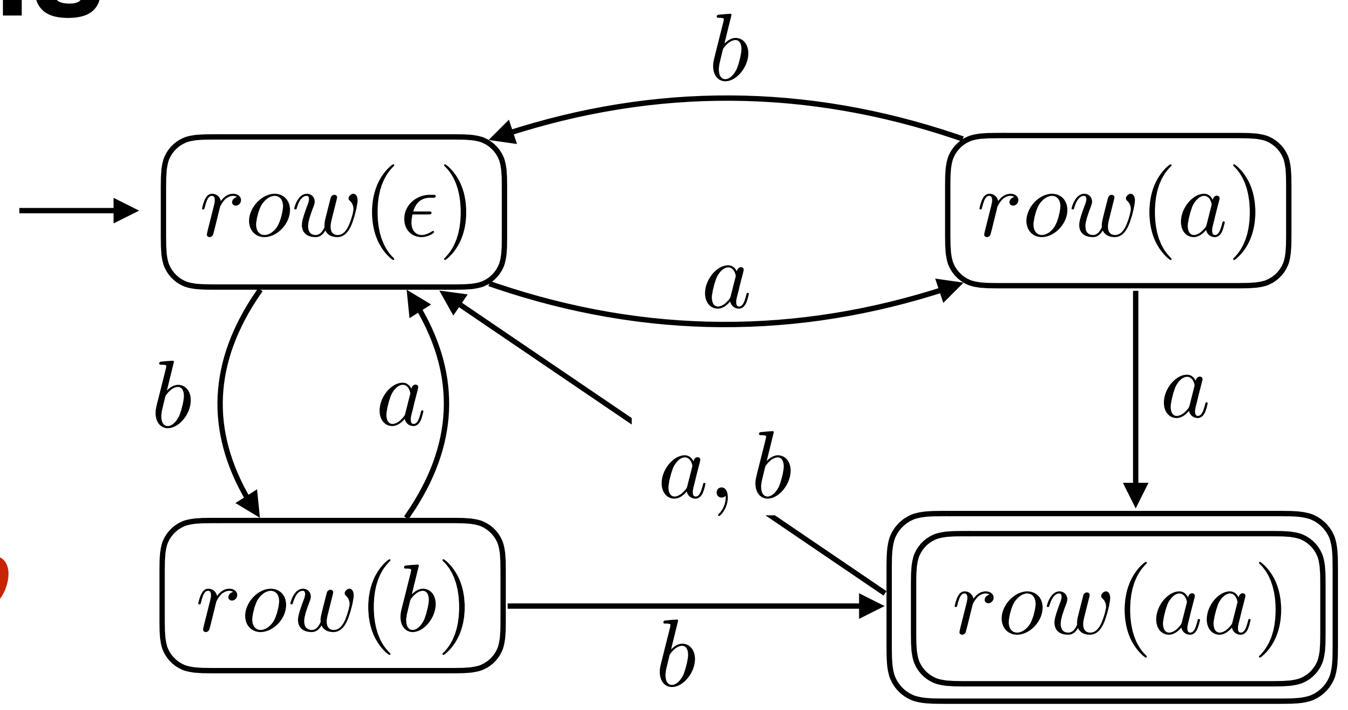
Hypothesis



Counterexample = *babb*

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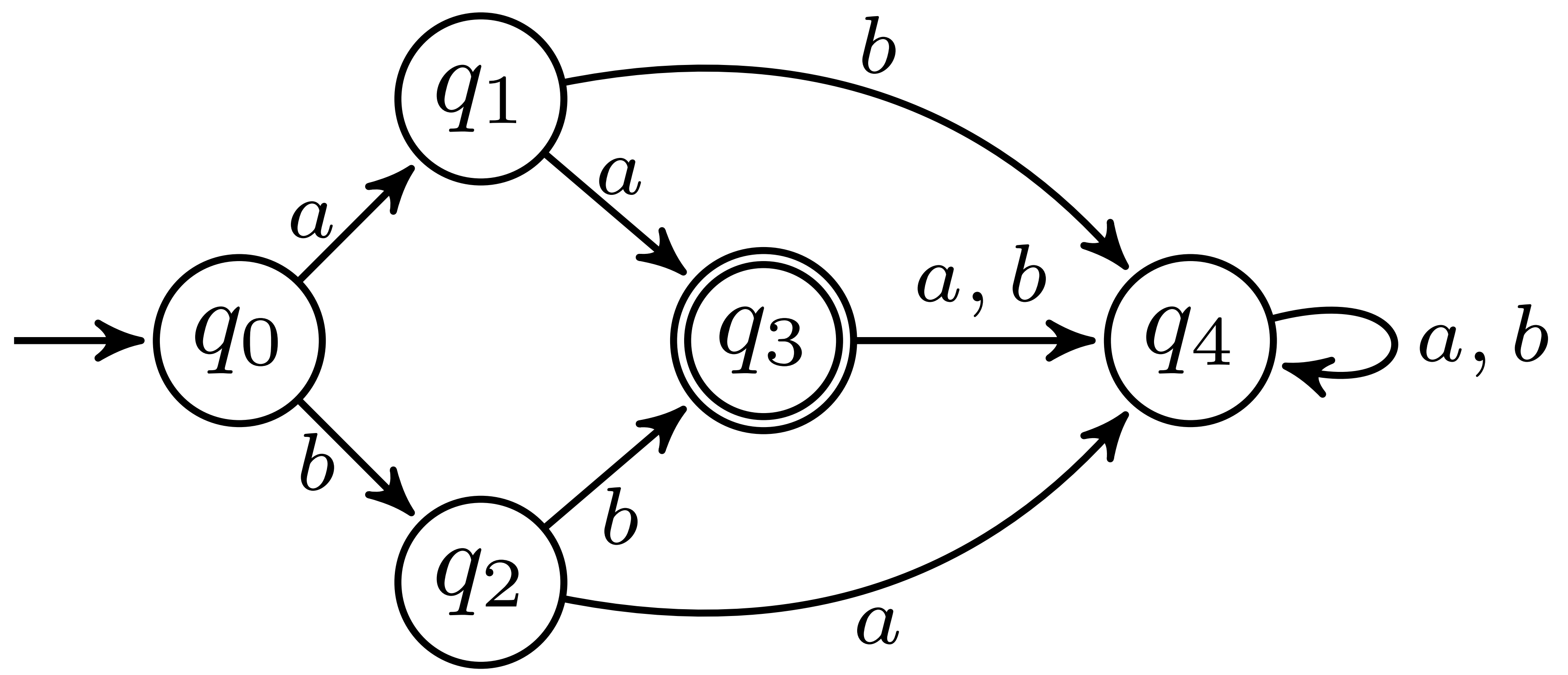


Pros of L* …

simple is beautiful

&

POWERFUL

Applications : Hardware verification, security/network protocols…

Generalizations : Mealy machines, I/O automata, …

… and shortcomings

L* learns **control-flow**

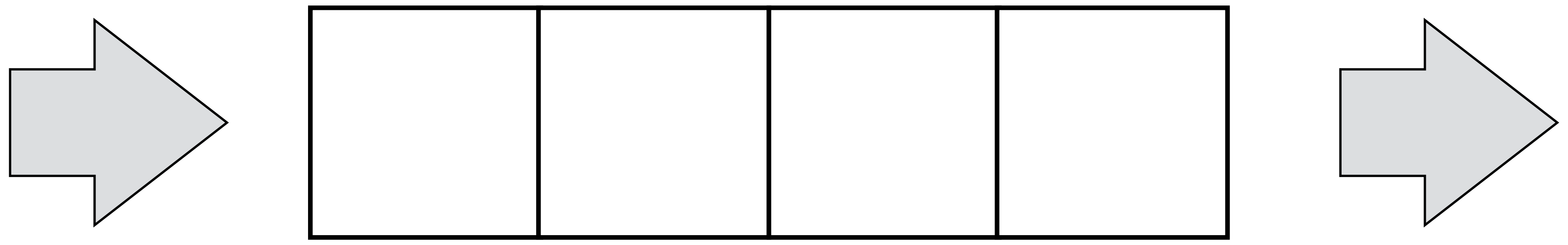
What if program model needs to express **data-flow?**

operations on **data values**

comparisons between data values

FIFO

push(x)



pop(y)

y = front element

Automata over infinite alphabets
(nominal automata)

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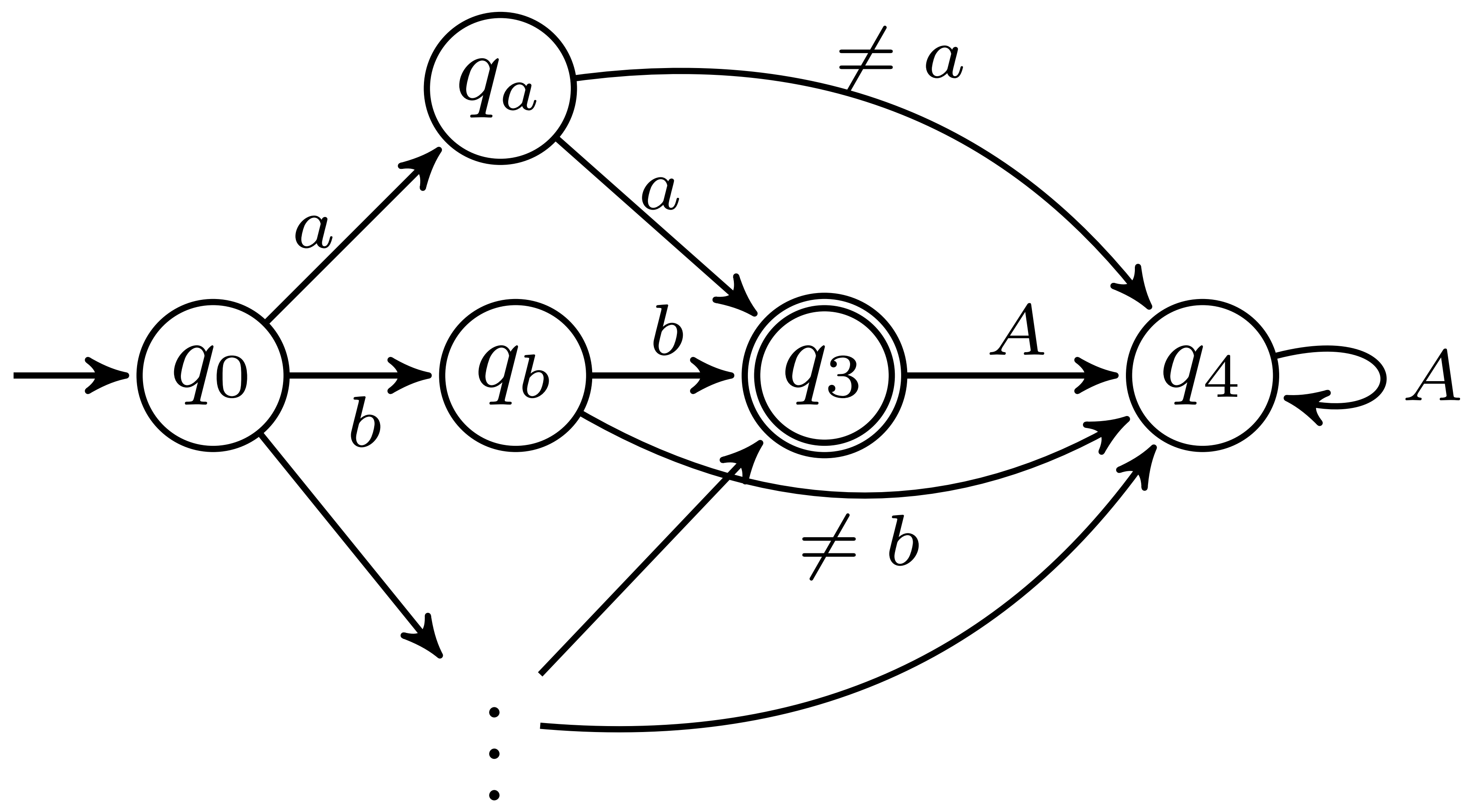
$A = \{a, b, c, d, \dots\}$ **infinite alphabet**

$$\mathcal{L} = \{aa, bb, cc, dd, \dots\}$$

Automata over infinite alphabets (nominal automata)

$A = \{a, b, c, d, \ldots\}$ **infinite alphabet**

$\mathcal{L} = \{aa, bb, cc, dd, \ldots\}$



infinite automaton

Automata over infinite alphabets (nominal automata)

$$A = \{a, b, c, d, \dots\}$$ **infinite alphabet**

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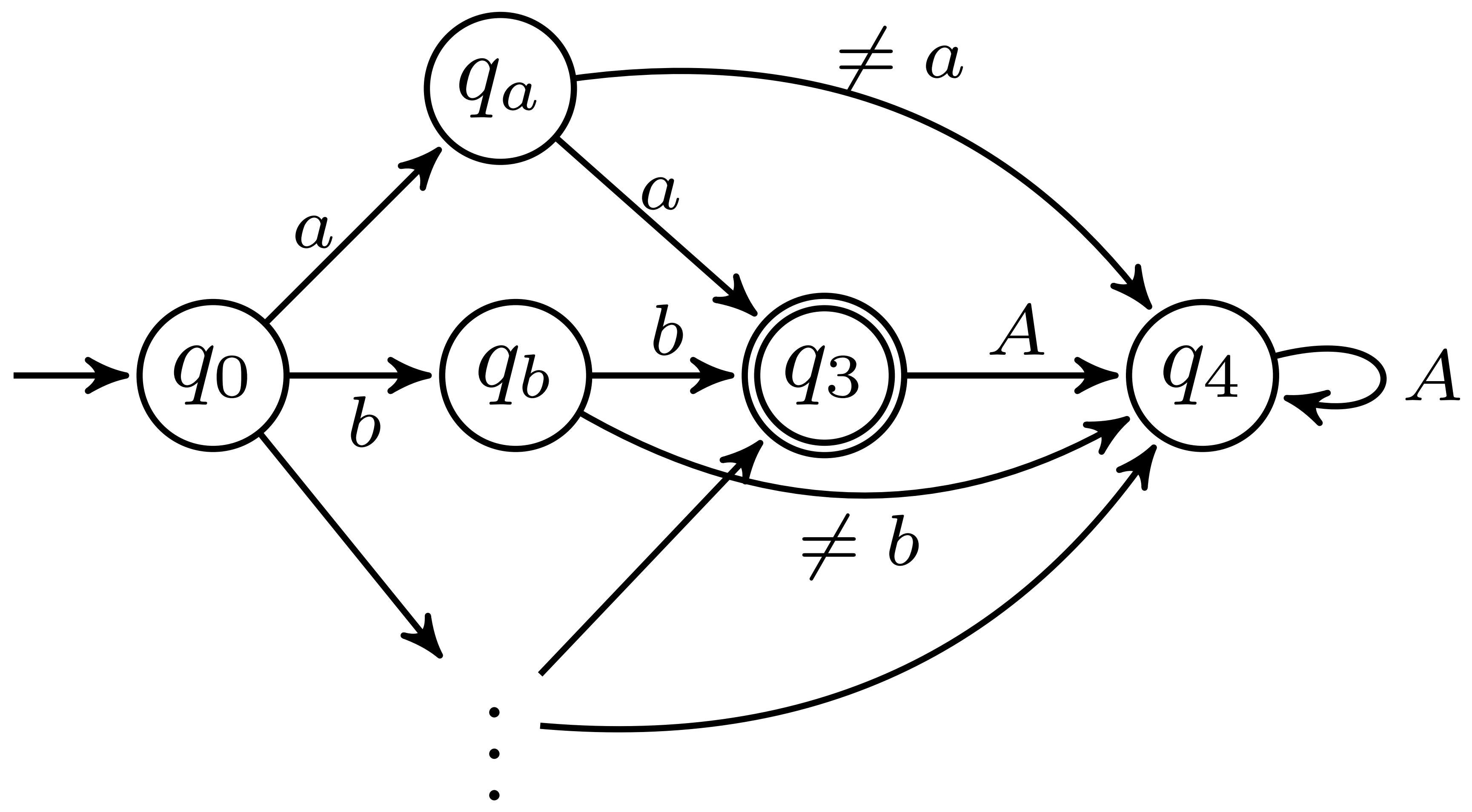
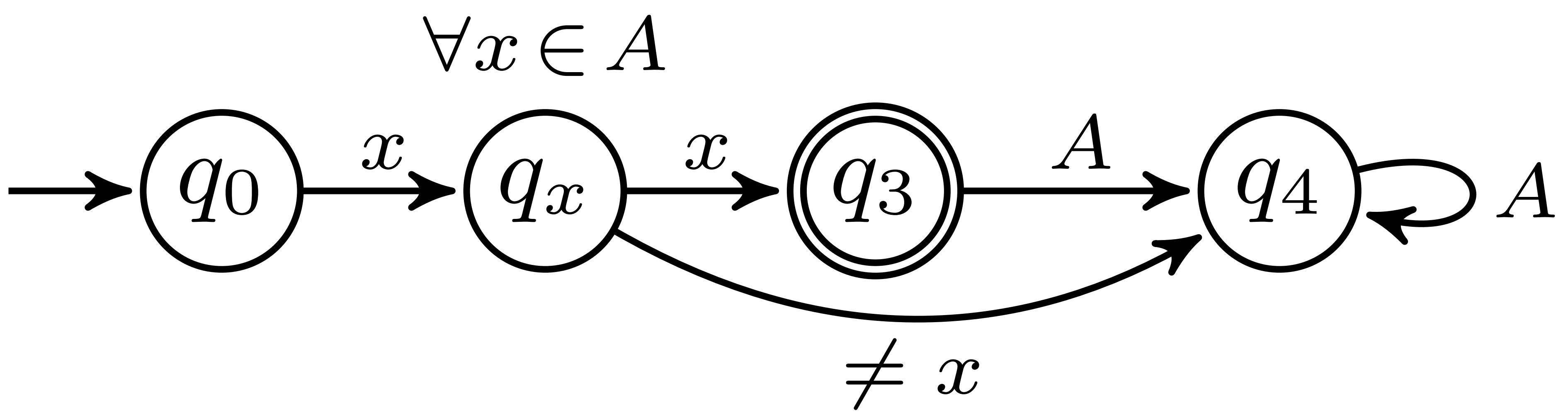


infinite automaton



but with a finite representation

How to learn them?

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Ad-hoc algorithm? **NO!**

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Ad-hoc algorithm? **NO!**

Challenges:

- table needs to be **infinite**
- code operates on **infinite sets**

$$\boxed{\forall t \in S \cdot A \quad \exists s \in S \qquad row(t) = row(s).}$$

$$\boxed{\forall s_1, s_2 \in S \quad row(s_1) = row(s_2) \implies \forall a \in A \;\; row(s_1 a) = row(s_2 a)}$$

How to learn them?

Ad-hoc algorithm? **NO!**

Challenges:

- table needs to be **infinite**
- code operates on **infinite sets**

$$\forall t \in S \cdot A \quad \exists s \in S \qquad row(t) = row(s).$$

Everything is "finitely representable"

$$\forall s_1, s_2 \in S \quad row(s_1) = row(s_2) \implies \forall a \in A \;\; row(s_1 a) = row(s_2 a)$$

Theory of nominal sets

Model of data

$\mathbb{D}$ **Domain of data + relations**

$Sym(\mathbb{D})$ **group of bijective** $\pi : \mathbb{D} \to \mathbb{D}$ **(permutations)**

E.g.

$(A, =)$ **+** **bijective functions**

$(\mathbb{Q}, <)$ **+** **bijective monotonic functions**

Nominal set

$$(X , \cdot)$$

Nominal set

$$(\,X\,,\,\underline{\;\cdot\;}\,)$$

group action of $Sym(\mathbb{D})$ **on** X

$$Sym(\mathbb{D}) \times X \to X$$

$$\pi \cdot x = x'$$

Nominal set

$$(\; X \; , \; \underline{\;\cdot\;} \;)$$

group action of $Sym(\mathbb{D})$ **on** X

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with **finite support** $\mathtt{supp}(x) \subseteq \mathbb{D}$

π acts as the identity on $\mathtt{supp}(x) \implies \pi \cdot x = x$

Nominal set

$$(X , \underline{\ \cdot\ })$$

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$$\pi \cdot x = x'$$

with **finite support** $\mathtt{supp}(x) \subseteq \mathbb{D}$

π acts as the identity on $\mathtt{supp}(x) \implies \pi \cdot x = x$

Data values "stored" in x

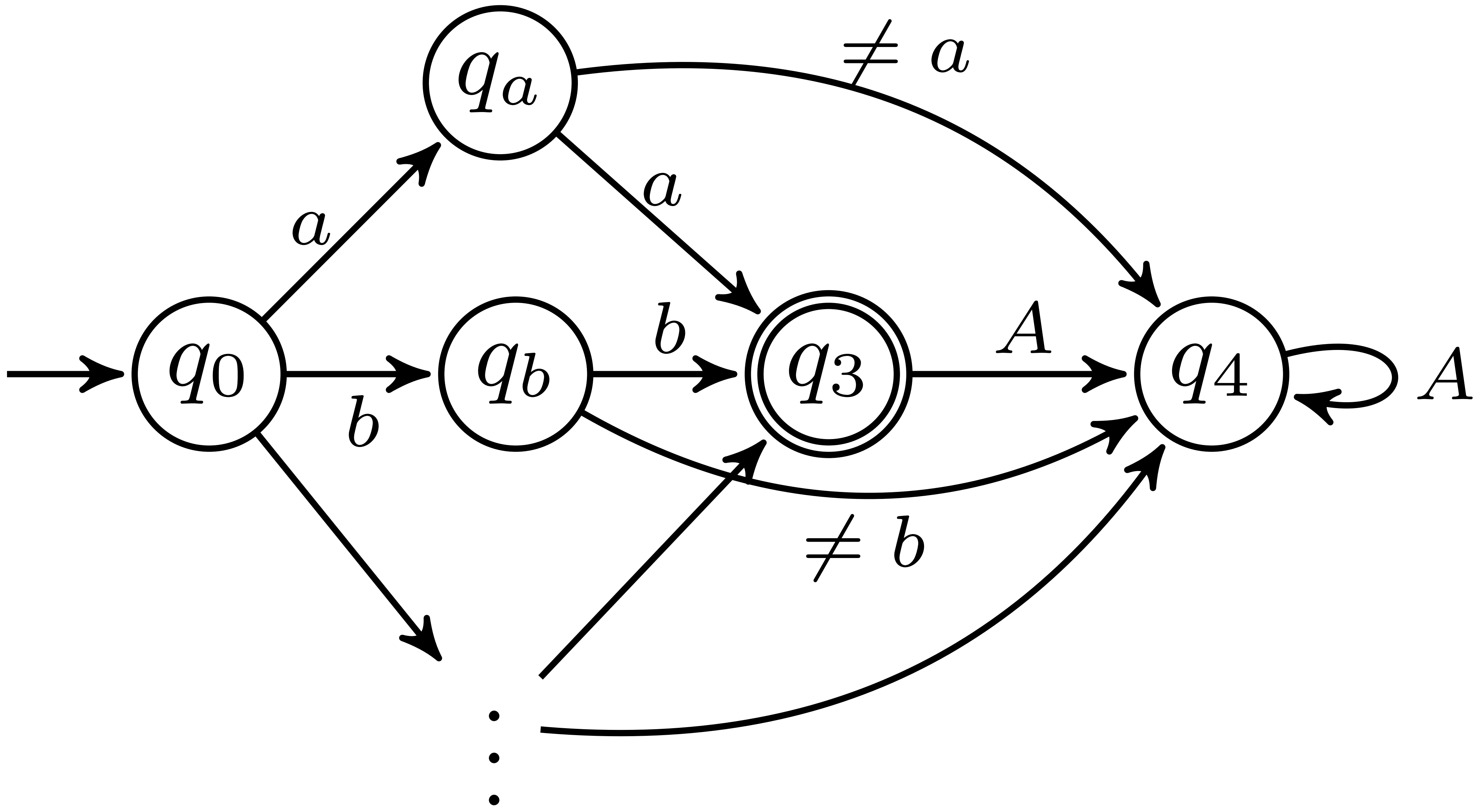
Example: nominal set of strings

$$(A^*, \cdot)$$

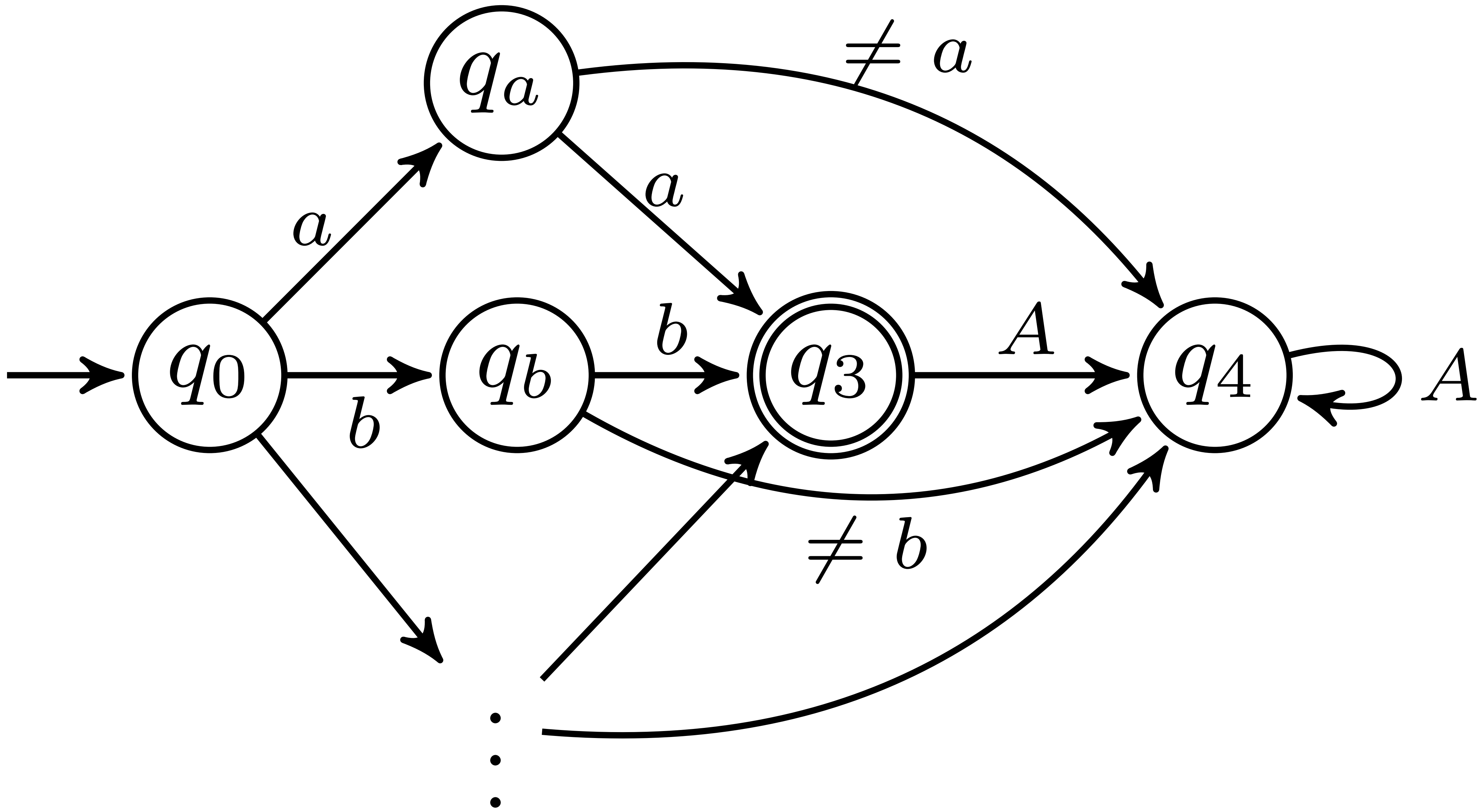
$$\pi \quad \cdot \quad abcd = \pi(a)\pi(b)\pi(c)\pi(d)$$

$$\mathtt{supp}(abcd) = \{a, b, c, d\}$$

Nominal automaton



Nominal automaton

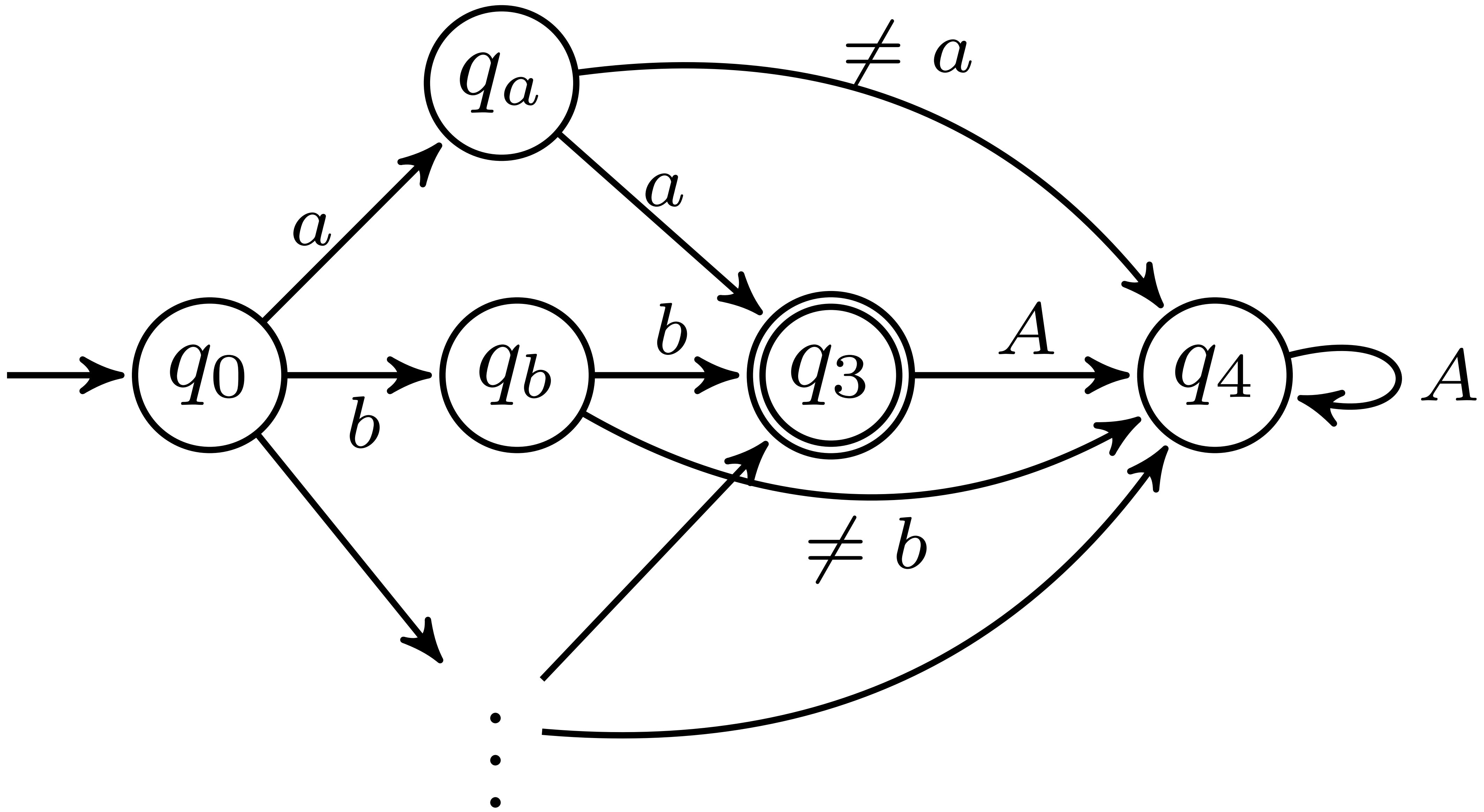


Nominal set of states $(Q, \cdot)$

$$\{q_0\} \cup \{q_a \mid a \in A\} \cup \{q_3\} \cup \{q_4\}$$

$$\pi \cdot q_0 = q_0$$
$$\pi \cdot q_3 = q_3$$
$$\pi \cdot q_4 = q_4$$
$$\pi \cdot q_x = q_{\pi(x)} \quad x \in A$$

Nominal automaton



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$$\mathtt{supp}(q_x) = \{x\} \qquad x \in A$$

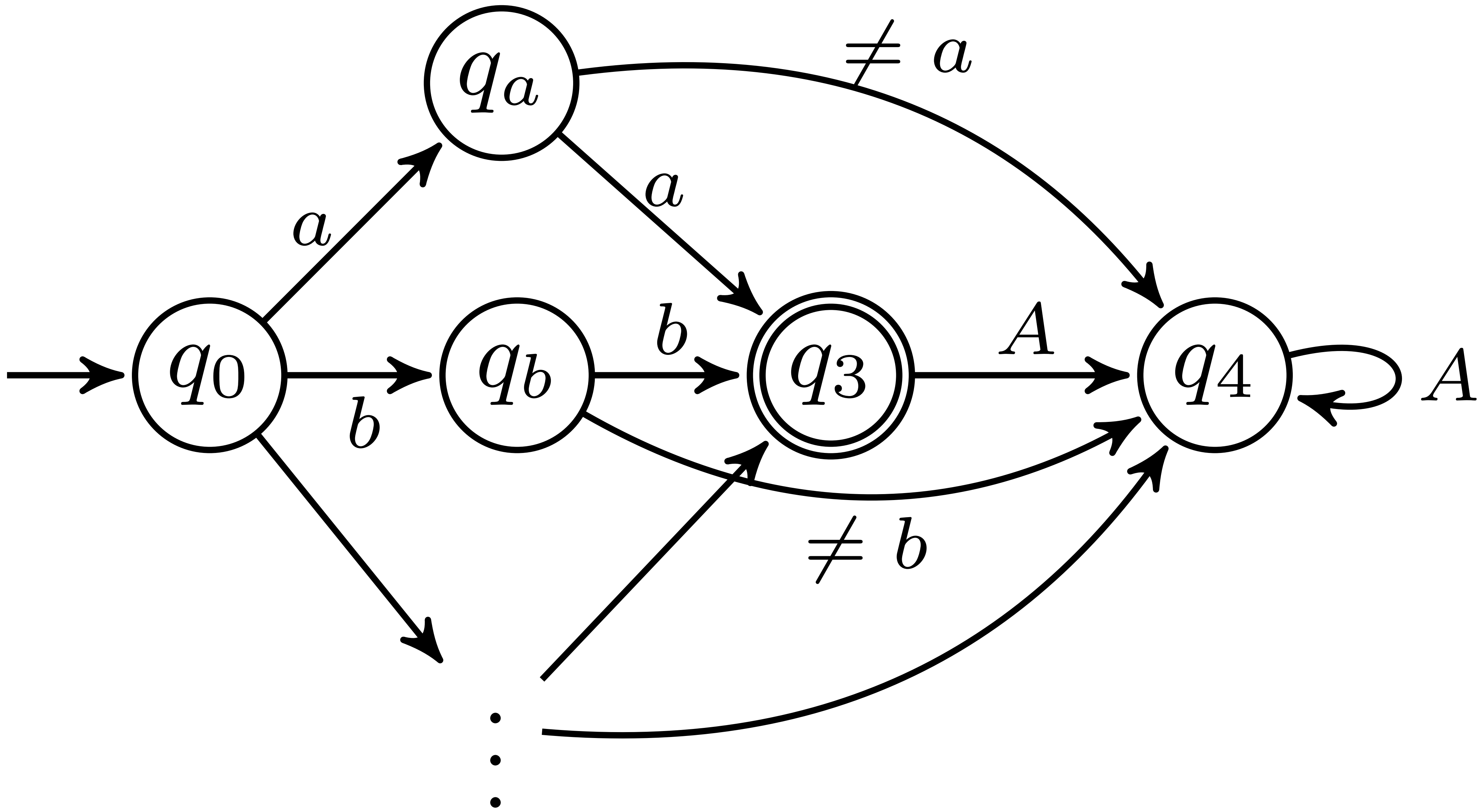
$$\mathtt{supp}(q_0) = \mathtt{supp}(q_3) = \mathtt{supp}(q_4) = \emptyset$$

Equivariant functions

$$f : (\boldsymbol{X}, \cdot) \to (\boldsymbol{Y}, \cdot)$$

$$f(\pi \cdot x) = \pi \cdot f(x)$$

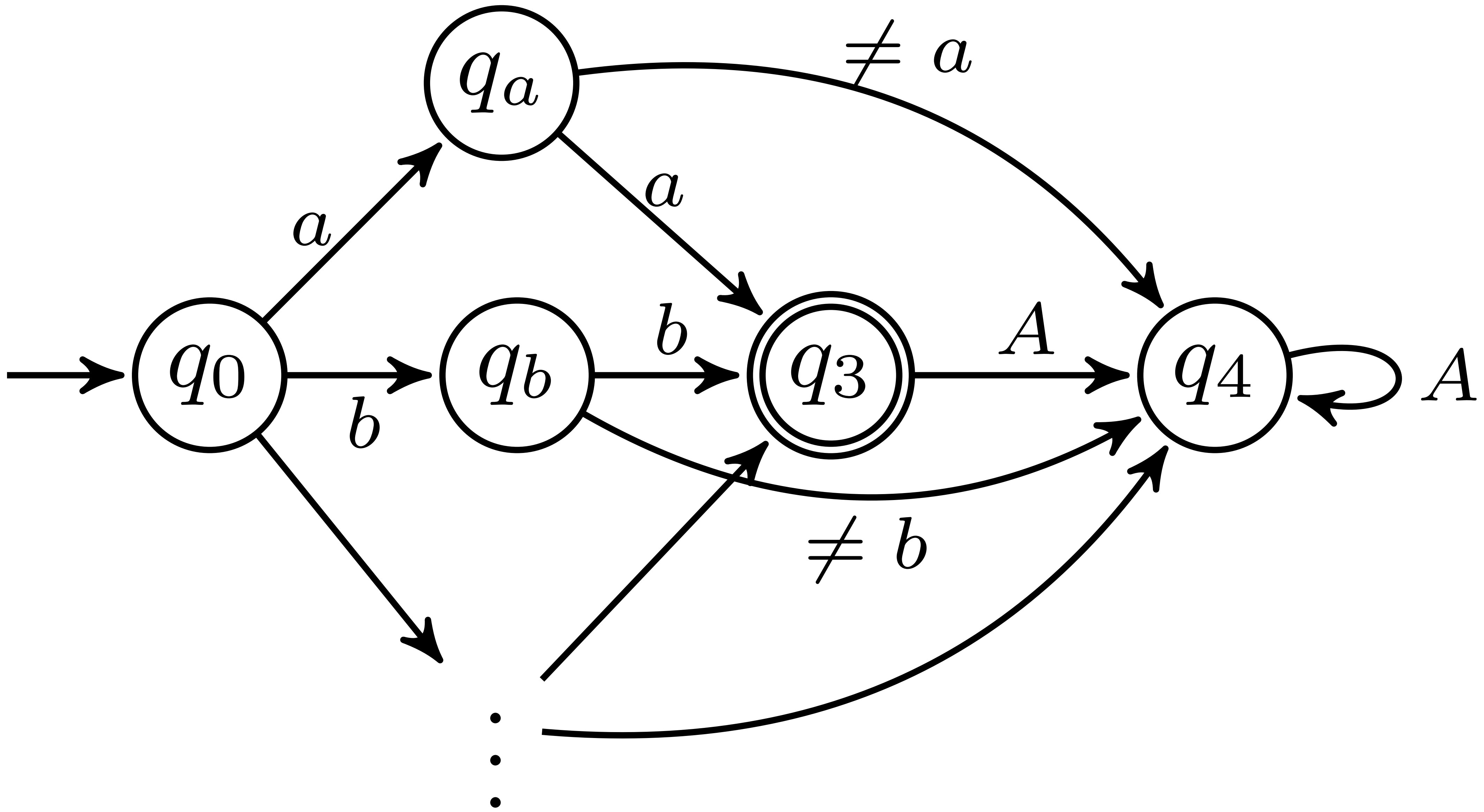
Nominal automaton



Equivariant transition function

$$\delta\colon (Q \times A, \cdot) \to (Q, \cdot)$$

Nominal automaton



Equivariant transition function

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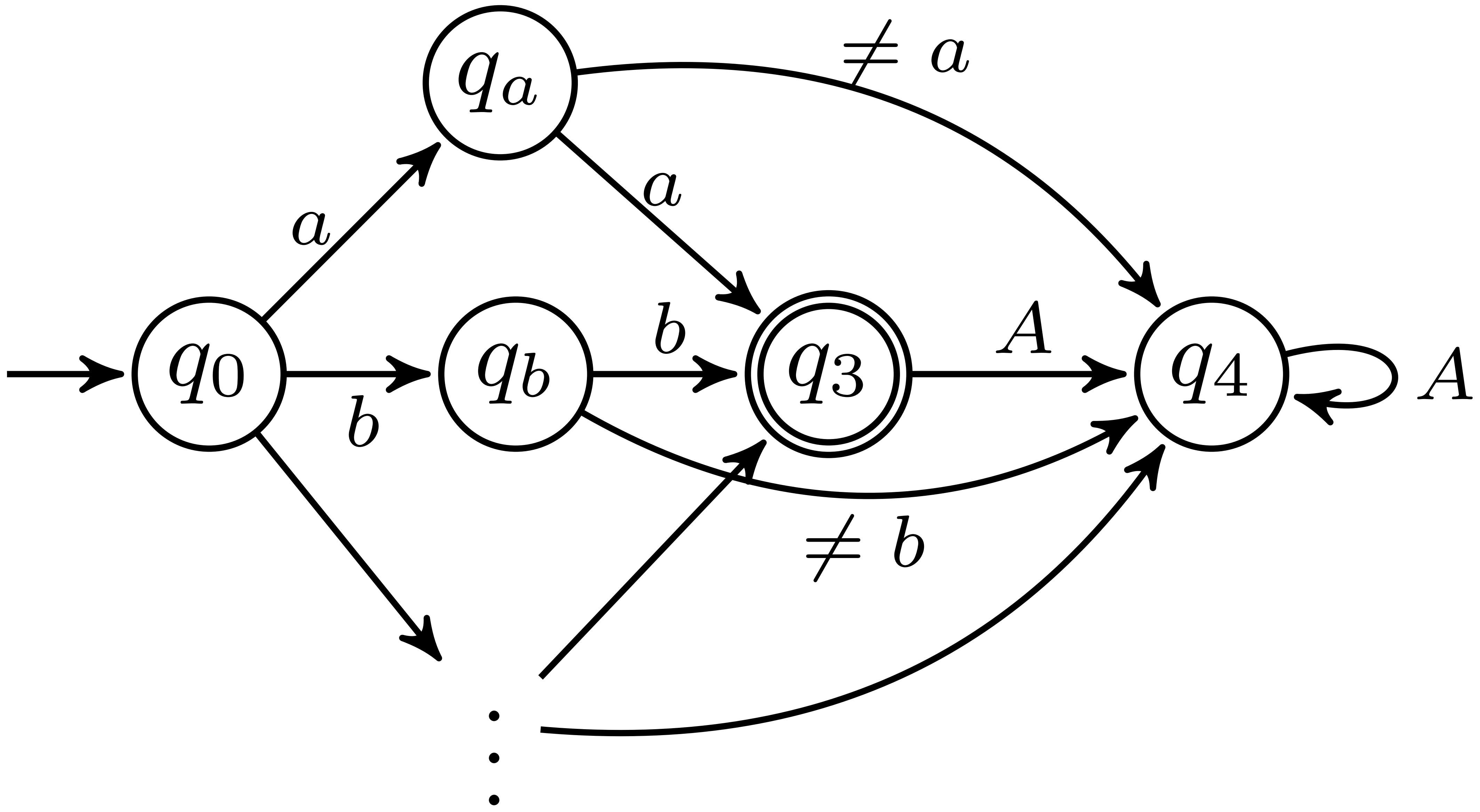
$$q_0 \xrightarrow{a} q_a$$

Nominal automaton



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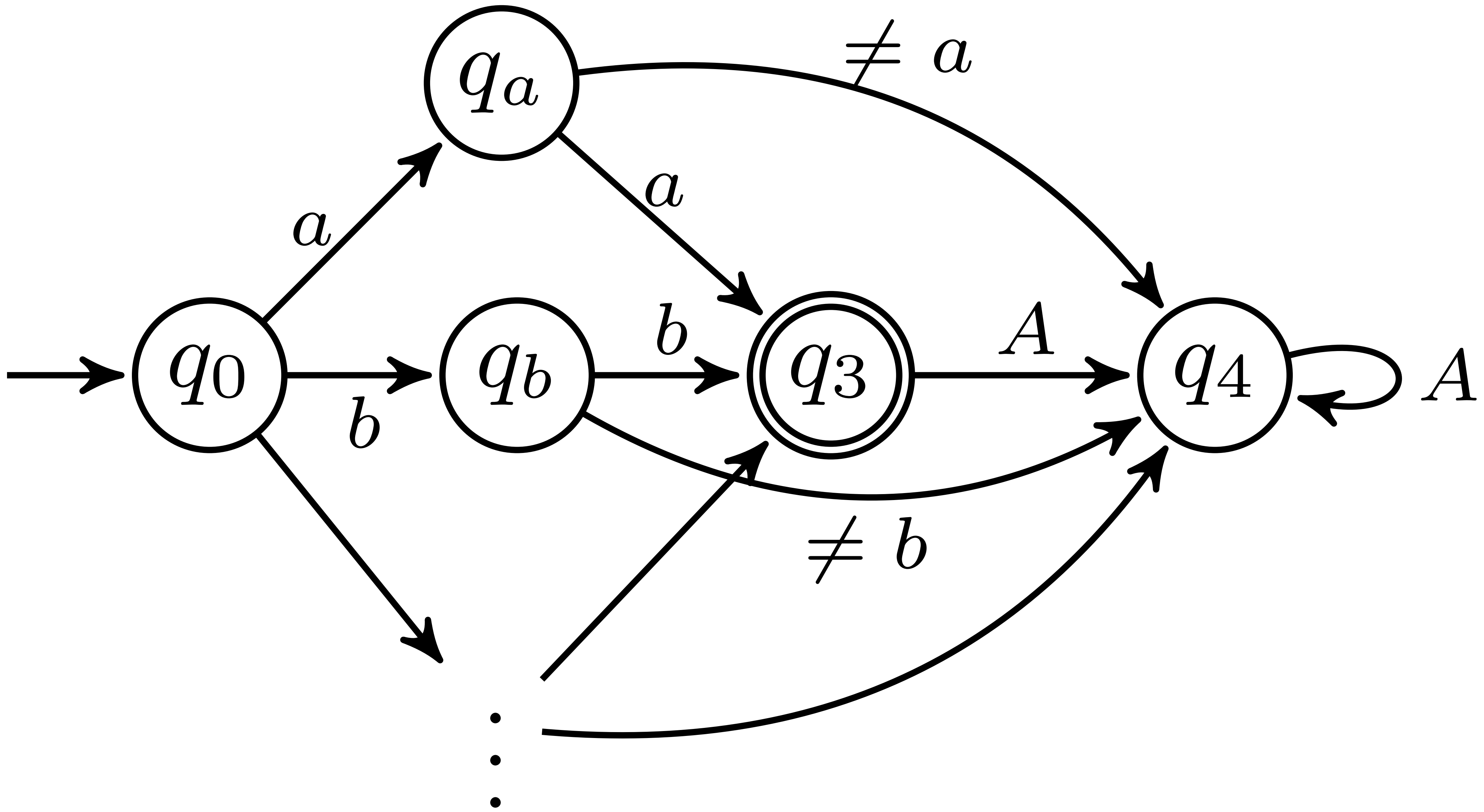
$$(a\ b)\ q_0 \xrightarrow{(a\ b)a} q_a$$

Nominal automaton



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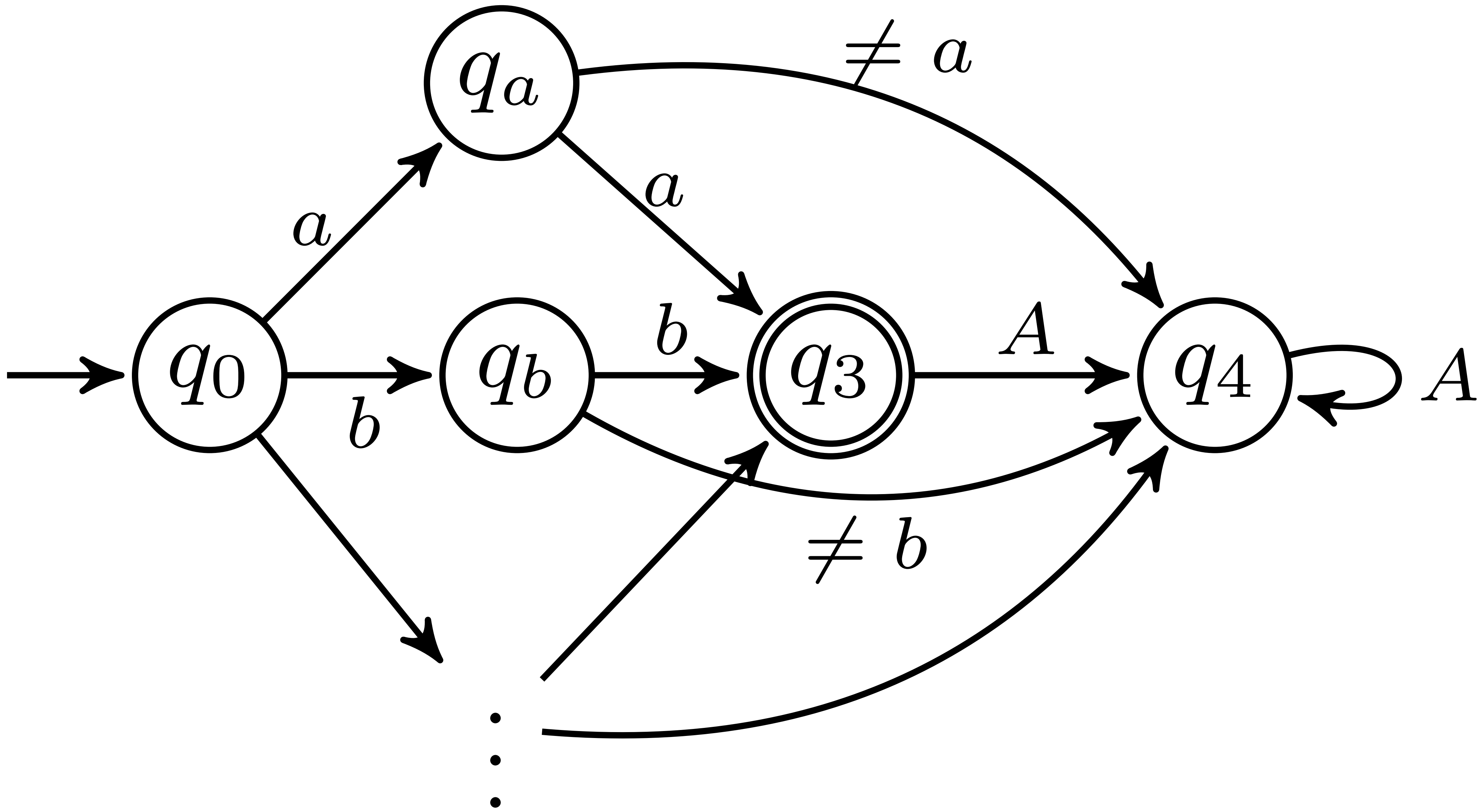

Equivariant transition function

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$$(a\ b)\ q_0 \xrightarrow{(a\ b)a} (a\ b) q_a$$

$$=$$

$$q_0 \xrightarrow{b} q_b$$

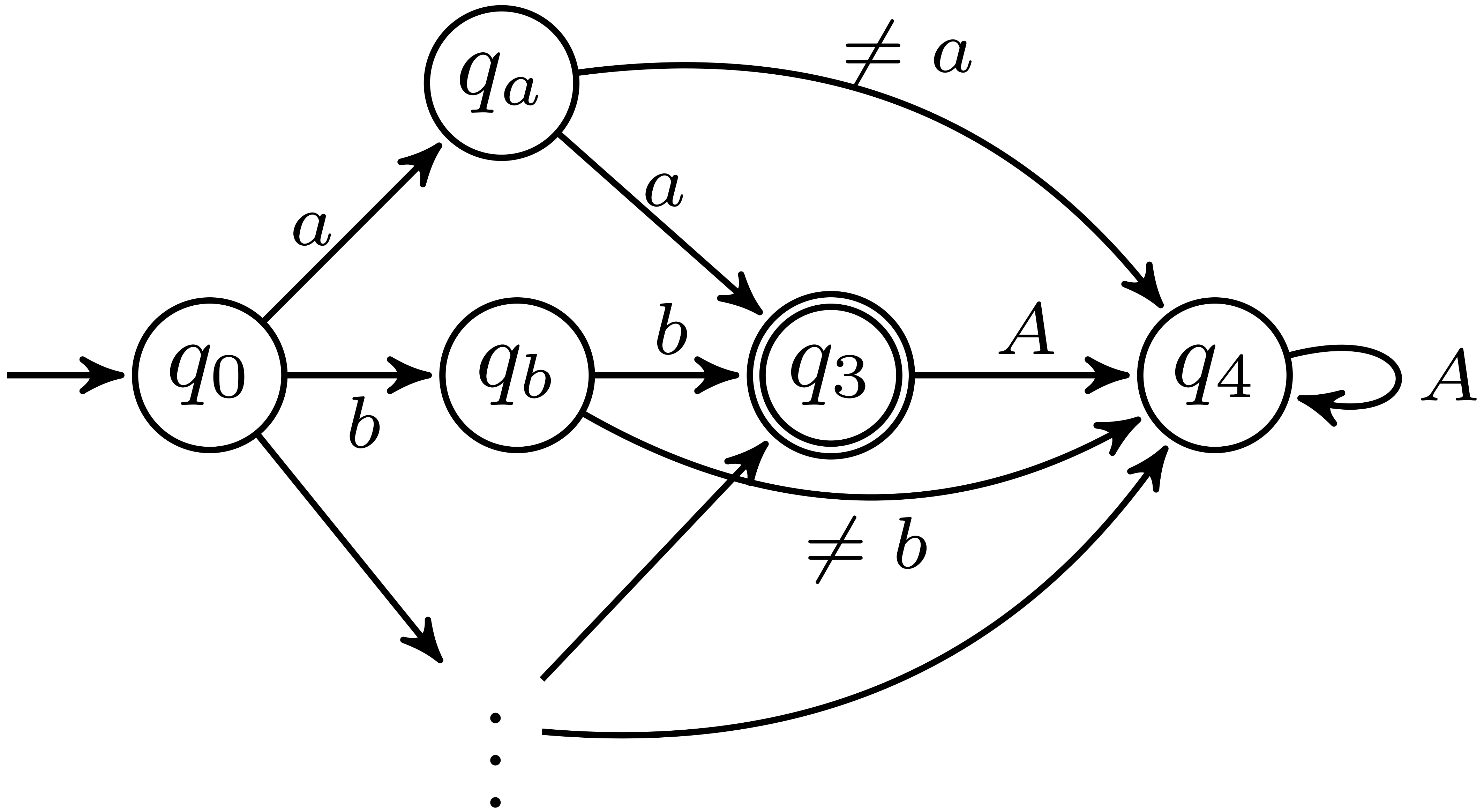
"closed under permutations"

Orbits

$$\mathtt{orb}(x) = \{\pi \cdot x \mid \pi \in Sym(\mathbb{D})\}$$

$(X, \cdot)$ **orbit-finite** whenever $\{\mathtt{orb}(x) \mid x \in X\}$ is finite

Nominal automaton



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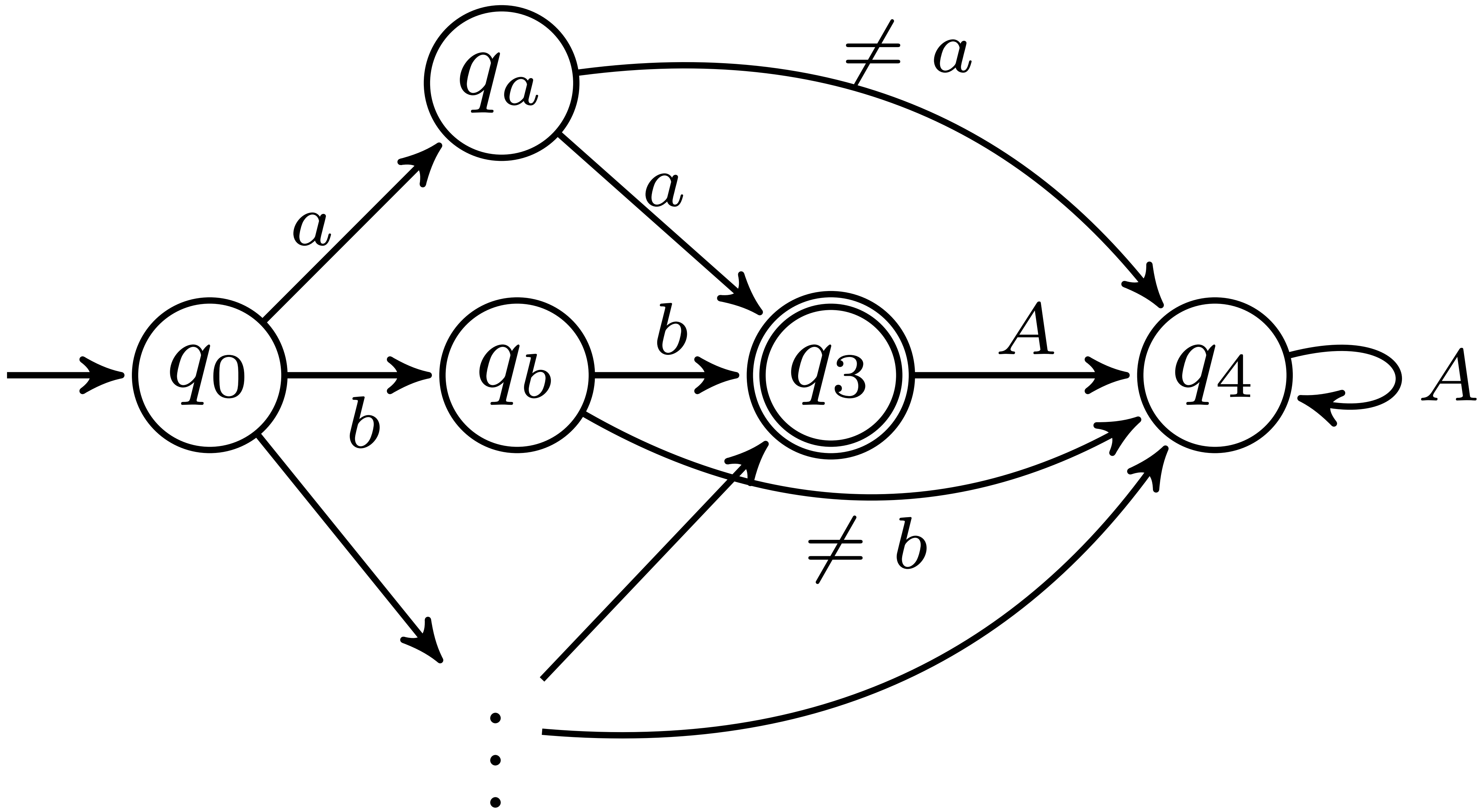
Nominal automaton



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4 orbits

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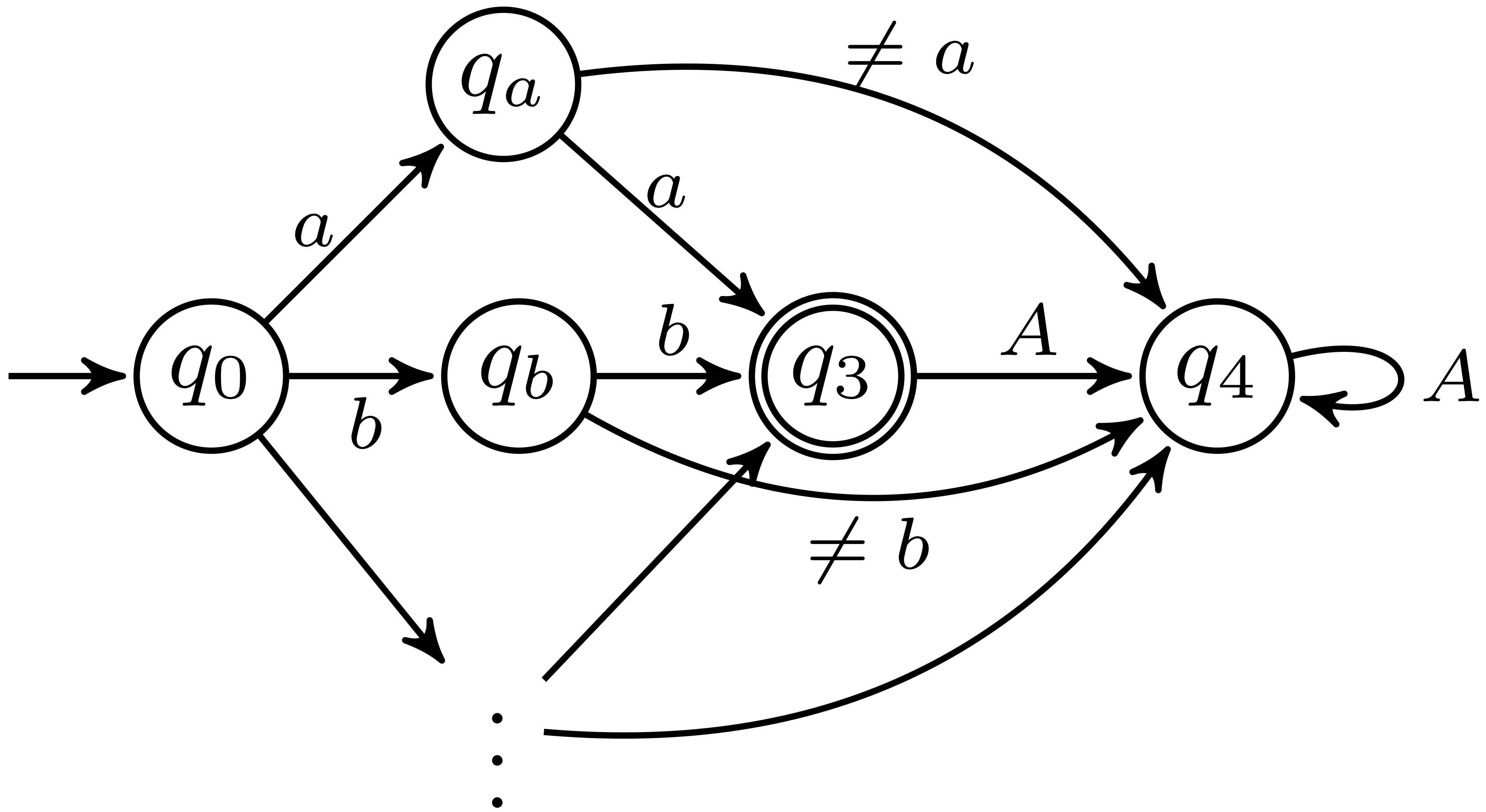

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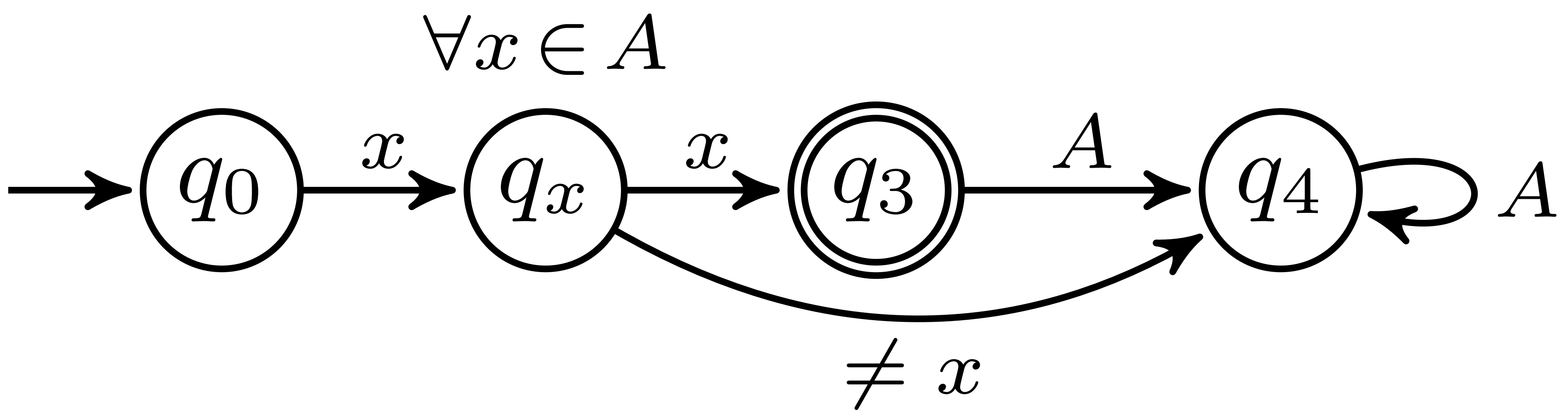
4 orbits

one representative per orbit



Nominal L*

S, E finite sets

$row\colon S \cup S \times A \to 2^{E}$ function

Nominal L*

S, E

orbit-finite **nominal** sets

$row\colon S \cup S \times A \to 2^E$

equivariant function

Example

$$\mathcal{L} = \{aa, bb, cc, dd, \ldots\} \qquad A = \{a, b, c, d, \ldots\}$$

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$S, E \leftarrow \{\epsilon\}$

repeat

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  if (S, E) is not closed

    find $s_1 \in S$, $a \in A$ such that

      $row(s_1 a) \neq row(s)$, for all $s \in S$

    $S \leftarrow S \cup \{s_1 a\}$

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    find $s_1, s_2 \in S$, $a \in A$, and $e \in E$ such that

      $row(s_1) = row(s_2)$

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    $E \leftarrow E \cup \{ae\}$

  Submit hypothesis H to the Teacher

  if the Teacher replies **no**, with a counter-example t

    $S \leftarrow S \cup \texttt{prefixes}(t)$

until the Teacher replies **yes** to H

return H

$\mathcal{L} = \{aa, bb, cc, dd, \dots\}$ $\quad$ $A = \{a, b, c, d, \dots\}$

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$row =$

	ϵ
ϵ	0
a	0
b	0
c	0
d	0
...	...

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...	$\dots$

infinitely-many rows!

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$$row(\pi \cdot s)(\pi \cdot e) = row(s)(e)$$

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$row(\,(a\ b)a\,)(\,(a\ b)\epsilon\,)$

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infinitely-many rows!

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infinitely-many rows!

$$row(\pi \cdot s)(\pi \cdot e) = row(s)(e)$$

finite representation by keeping only **orbit representatives** of rows

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Closed ✓

Consistent ✓

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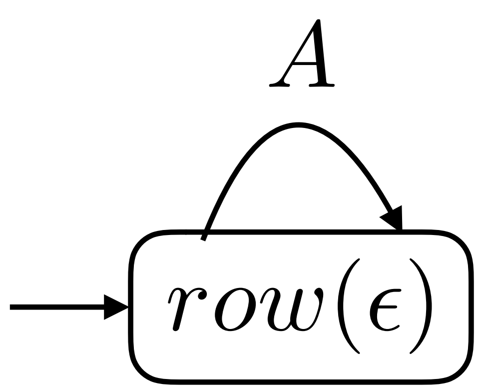
Hypothesis



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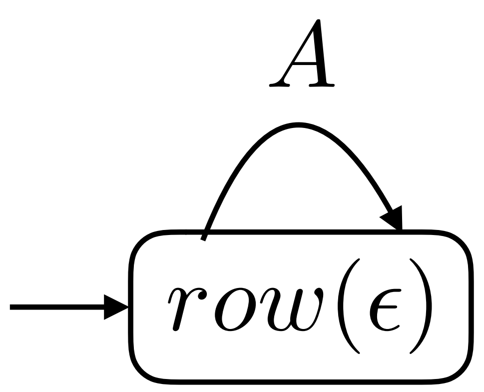
Hypothesis



counterexample = *aa*

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until the Teacher replies **yes** to H
return H

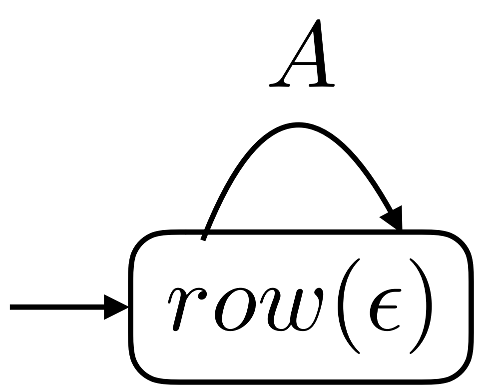
$row =$

	ϵ
ϵ	0
a	0

Closed ✓

Consistent ✓

Hypothesis



counterexample = *aa*

$\mathcal{L} = \{aa, bb, cc, dd, \dots\}$ $\quad A = \{a, b, c, d, \dots\}$

$S, E \leftarrow \{\epsilon\}$
repeat
  while (S, E) is not closed or not consistent
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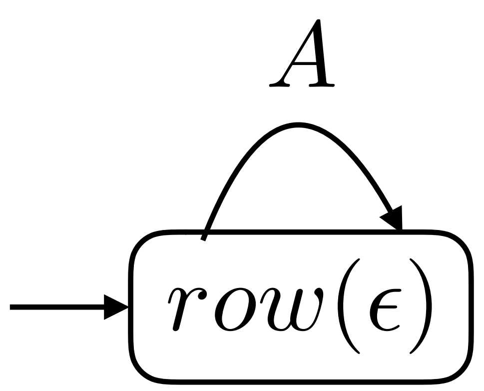

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	ϵ
ϵ	0
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Closed ✓

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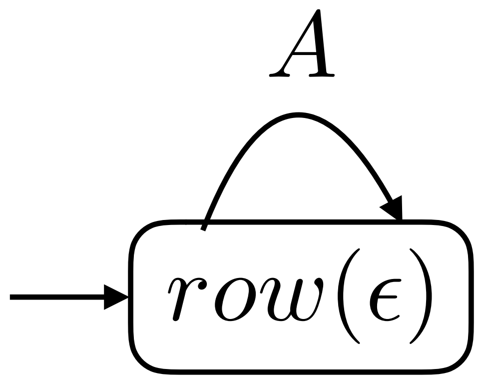
Hypothesis



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ϵ	0
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aaa	0
aab	0

Closed ✓
Consistent ✗

Add all $\pi(a)$ to *E*

$\mathcal{L} = \{aa, bb, cc, dd, \dots\}$ $\qquad$ $A = \{a, b, c, d, \dots\}$

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repeat

  while (S, E) is not closed or not consistent

  if (S, E) is not closed

    find $s_1 \in S, a \in A$ such that

      $row(s_1 a) \neq row(s)$, for all $s \in S$

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$row =$

	ϵ	a	b	c	d	...
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a	0	1	0	0	0	...
aa	1	0	0	0	0	...
ab	0	0	0	0	0	...
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infinitely-many columns!

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$row(s)(c) = row(s)(d) = \cdots$

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$row(s)(c) = row(s)(d) = \cdots$

$c, d \notin \mathtt{supp}(row(s)) \subseteq |s|$

infinitely-many columns!

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infinitely-many columns!

we can just keep one column

$c \notin |s|$

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	ϵ	**a**	**b**	**c**
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Nominal L*

$S, E \leftarrow \{\epsilon\}$
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we only changed 3 lines

Implementation

$$\mathtt{N}\lambda$$

Extension of Haskell with **nominal data-types** and **primitives**

`atoms : Set Atoms` -- the infinite (but orbit-finite) set of all atoms

Internal representation as **FO formulae**, resolved via **Z3**

https://www.mimuw.edu.pl/~szynwelski/nlambda/

Correctness from scratch?

Correctness from scratch?

Not really

Set-based proofs as **guidelines**

L* enjoys a nice **category-theoretic** generalization

Categorical
Automata
Learning
Framework

`calf-project.org`

The general picture

(finite) sets	➡	(orbit-finite) nominal sets
functions	➡	equivariant functions

(change category from **Set** to **Nom**)

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Nominal L*

Works with any (suitable) data domain

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In the paper…

- Nominal L*
- Variations, Nominal NL*
- Implementation
- Experimental results

https://github.com/Jaxan/nominal-lstar

Ongoing and future work

Maverick Chardet
ENS Lyon

- **Library & tool**
- Applications:
    - Specification mining
    - Network verification, with amazon
    - Verification of cryptographic protocols
    - Ransomware detection

Demo