Anisotropic actions for openQCD

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1 Parameters for the anisotropic action

The anisotropy is activated by adding the following section to your program configuration files for the `qcd1, ym1, ms1, ms2` & `ms4` executables.

```
[Anisotropy parameters]
use_tts    0
nu         1.5
xi         4.3
cR         1.5
cT         0.9
us_gauge   0.7
ut_gauge   1.0
us_fermion 1.0
ut_fermion 1.0
```

The block has been made optional to be compatible with openQCD 1.6 configuration files. If no such section is specified, the resulting action will be the isotropic action as specified by the other options. The parameters have the following interpretations:

`use_tts(boolean)`
Determines whether the gauge action should include rectangles that extend twice into the temporal direction. See section 2.1

`nu (double)`
Sets the ν parameter as defined in section 2.2

`xi (double)`
Sets the ξ_0 parameter as defined in section 2.1

`cR (double)`
Sets the anisotropic clover parameter c_R as defined in eq. (12)

`cT (double)`
Sets the anisotropic clover parameter c_T as defined in eq. (12)

`us_gauge (double)` (*optional*, `=1.0`)
Sets the tadpole improvement factor for spatial links in the gauge action (u_s) as defined in eqs. (1,3)

`ut_gauge (double)` (*optional*, `=1.0`)
Sets the tadpole improvement factor for temporal links in the gauge action (u_s) as defined in eqs. (1,3)

`us_fermion (double)` (*optional*, `=1.0`)
Sets the tadpole improvement factor for spatial links in the fermion action ($\tilde{u}_s$) as defined in eq. (12)

`ut_fermion (double)` (*optional*, `=1.0`)
Sets the tadpole improvement factor for temporal links in the fermion action ($\tilde{u}_t$) as defined in eq. (12)

2 The anisotropic action

We use the anisotropic action as defined in [2] with minor alterations to allow for user defined c_0 and c_1 factors. The gauge- and fermion actions are summarised in the next two subsections.

2.1 *Gauge action*

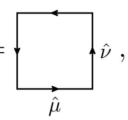
The anisotropic gauge action can take two forms depending on the `use_tts` setting. By setting this flag one determines whether the rectangular plaquette part of the improvement scheme for the gauge action is allowed to extend in all directions, or in the spatial directions only.

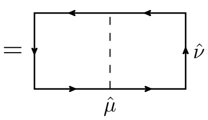
`use_tts = 1`

In this case the gauge action is isotropic besides the different weights applied to the different directions and thus takes the same form in all directions

$$S_G = \frac{\beta}{N_c \gamma_g} \Bigg\{ \sum_{x,s>s'} \left[\frac{c_0}{u_s^4} P_{ss'}(x) + \frac{c_1}{u_s^6} \big(R_{ss'}(x) + R_{s's}(x) \big) \right] + \gamma_g^2 \sum_{x,s} \left[\frac{c_0}{u_s^2 u_t^2} P_{st} + \frac{c_1}{u_s^4 u_t^2} R_{st}(x) - \frac{c_1}{u_s^2 u_t^4} R_{ts}(x) \right] \Bigg\}. \quad (1)$$

In this equation u_s and u_t are the tadpole improvement factors for the spatial- and temporal links in the gauge action respectively. $\gamma_g = 1/\xi_0$ where ξ_0 is the bare gauge anisotropy, and the plaquettes are defined as

$P_{\mu\nu} =$ 

, $R_{\mu\nu} =$ 

. (2)

Specifically this action includes both R_{st} and R_{ts}.

`use_tts = 0`

For this second case we exclude rectangular loops that extend twice into the temporal direction. This is the most common choice for improved anisotropic gauge actions. The action then takes the form

$$S_G = \frac{\beta}{N_c \gamma_g} \Bigg\{ \sum_{x,s>s'} \left[\frac{c_0}{u_s^4} P_{ss'}(x) + \frac{c_1}{u_s^6} \big(R_{ss'}(x) + R_{s's}(x) \big) \right] + \gamma_g^2 \sum_{x,s} \left[\frac{c_0 + 4c_1}{c_0} \frac{c_0}{u_s^2 u_t^2} P_{st} + \frac{c_1}{u_s^4 u_t^2} R_{st}(x) \right] \Bigg\}, \quad (3)$$

which one can see lacks the R_{ts} term. The additional prefactor in front of P_{st} is chosen in such a way that the series expansion in a for the spatial and temporal components match up. Specifically if we write down this series expansion[3]

$$P_{\mu\nu} = 1 - \frac{1}{6} a^4 \,\mathrm{tr}\, \big(gF_{\mu\nu}\big)^2 - \frac{1}{72} a^6 \,\mathrm{tr}\, \Big(gF_{\mu\nu} \big(D_\mu^2 + D_\nu^2\big) gF_{\mu\nu} \Big) + \mathcal{O}\big(a^8\big), \quad (4)$$

$$R_{\mu\nu} = 1 - \frac{4}{6} a^4 \,\mathrm{tr}\, \big(gF_{\mu\nu}\big)^2 - \frac{4}{72} a^6 \,\mathrm{tr}\, \Big(gF_{\mu\nu} \big(4D_\mu^2 + D_\nu^2\big) gF_{\mu\nu} \Big) + \mathcal{O}\big(a^8\big), \quad (5)$$

we see that the spatial contributions are (ignoring tadpole improvement)

$$S_{G,ss'} \sim \frac{1}{6}(c_0 + 8c_1) a^4 \,\mathrm{tr}\, \big(gF_{ss'}\big)^2 + \frac{1}{72}(c_0 + 20c_1) a^6 \,\mathrm{tr}\, \Big(gF_{ss'} \big(D_s^2 + D_{s'}^2\big) gF_{ss'} \Big). \quad (6)$$

The temporal contribution on the other hand is

$$S_{G,st} \sim \frac{1}{6}(c_0 + 4c_1) a^4 \,\mathrm{tr}\, \big(gF_{st}\big)^2 + \frac{1}{72}(c_0 + 16c_1) a^6 \,\mathrm{tr}\, \Big(gF_{ss'} D_s^2 gF_{ss'} \Big) + \frac{1}{72}(c_0 + 4c_1) a^6 \,\mathrm{tr}\, \Big(gF_{ss'} D_t^2 gF_{ss'} \Big). \quad (7)$$

If we consider a gauge action in which we scale the temporal contributions independently

$$S_G \sim c_0 P_{ss'} + c_1 \big(R_{ss'} + R_{s's}\big) + x c_0 P_{st} + y c_1 R_{st}, \tag{8}$$

then we can match the first two terms in the series expansions in the lattice spacing a by setting

$$x = \frac{c_0 + 4c_1}{c_0}, \qquad y = 1. \tag{9}$$

For a Symanzik improved action

$$c_0 = \frac{5}{3}, \qquad c_1 = -\frac{1}{12}, \tag{10}$$

we recover the action in [2]

$$S_G = \frac{\beta}{N_c \gamma_g} \Bigg\{ \sum_{x,s>s'} \left[\frac{5}{3u_s^4} P_{ss'}(x) - \frac{1}{12u_s^6} \big(R_{ss'}(x) + R_{s's}(x)\big) \right] + \gamma_g^2 \sum_{x,s} \left[\frac{4c_0}{3u_s^2 u_t^2} P_{st} - \frac{1}{12 u_s^4 u_t^2} R_{st}(x) \right] \Bigg\}. \tag{11}$$

2.2 *Fermion action*

The fermionic action is as usual defined in terms of the Dirac operator

$$D = \hat{m}_0 + \frac{1}{\tilde{u}_t} D_{W,t} + \frac{1}{\gamma_f \tilde{u}_t} D_{W,s} - \frac{1}{2} \left[\frac{c_T}{\tilde{u}_t^2 \tilde{u}_s^2} \sum_s i\sigma_{ts} \hat{F}_{ts} + \frac{c_R}{\xi_0 \tilde{u}_t \tilde{u}_s^3} \sum_{s<s'} i\sigma_{ss'} \hat{F}_{ss'} \right]. \tag{12}$$

Here $D_{W,t}$ and $D_{W,s}$ are the temporal and spatial hopping terms. $\tilde{u}_s$ and $\tilde{u}_t$ are the fermionic tadpole improvement factors for the spatial- and temporal gauge links respectively. $\gamma_f = \xi_0/\nu$ is the bare fermionic anisotropy; in the input file we specify this through ν, which is the ratio of the gluonic and fermionic anisotropies. Finally c_T and c_R are the temporal and spatial clover coefficients.

The anisotropy also changes the relationship between κ and m_0, which is now [1]

$$\kappa = \frac{1}{2(\hat{m}_0 + 1 + 3\frac{\nu}{\xi_0})} \quad \Leftrightarrow \quad \hat{m}_0 = \frac{1}{2\kappa} - 1 - 3\frac{\nu}{\xi_0}. \tag{13}$$

2.3 *Programming implementation*

For organisational purposes we list all the parts of the code touched by the anisotropy implementation.

module	**file**	**function**	**Comment**
`dirac`	`Dw.c`	`*`	aniso hopping coeffs, tadpole improvement
	`Dw_dble.c`	`*`	—"—
	`Dw_bnd.c`	`*`	—"—
`flags`	`dfl_parms.c`	`set_dfl_gen_parms`	$m_0 \leftrightarrow \kappa$ conversion
	`lat_parms.c`	`set_lat_parms`	—"—
`forces`	`force0.c`	`*`	tadpole factors, aniso gauge coeffs, R_{ts} toggle
	`force3.c`	`sdet`	$m_0 \leftrightarrow \kappa$ conversion
	`force4.c`	—"—	—"—
	`genfrc.c`	`sw_frc`	aniso clover coeffs, tadpole improvement
		`hop_frc`	aniso hopping coeffs, tadpole improvement
`little`	`Aw_ops.c`	`set_Aw`	aniso hopping coeffs, tadpole improvement
`sw_term`	`sw_term.c`	`*`	aniso clover coeffs, tadpole improvement

References

[1] Ping Chen. "Heavy quarks on anisotropic lattices: The Charmonium spectrum". In: *Phys. Rev.* D64 (2001), p. 034509. DOI: 10.1103/PhysRevD.64.034509. arXiv: hep-lat/0006019 [hep-lat].

[2] Robert G. Edwards, Balint Joo, and Huey-Wen Lin. "Tuning for Three-flavors of Anisotropic Clover Fermions with Stout-link Smearing". In: *Phys. Rev.* D78 (2008), p. 054501. DOI: 10.1103/PhysRevD.78.054501. arXiv: 0803.3960 [hep-lat].

[3] G. P. Lepage. "QCD on coarse lattices". In: *Confinement, duality, and nonperturbative aspects of QCD. Proceedings, NATO Advanced Study Institute, Newton Institute Workshop, Cambridge, UK, June 23-July 4, 1997*. 1997, pp. 75–111.