Graphs and Social Networks

Roshni Iyer
CS Department
UCLA
roshnigiyer@cs.ucla.edu

Debarnab Mita
ECE Department
UCLA
debarnabucla@ucla.edu

Swapnil Sayan Saha
ECE Department
UCLA
swapnilsayan@g.ucla.edu

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Variability Expedition

Instructors

Roshni Iyer

- Vice President, CS-GSA
- CS PhD student @ UCLA
  & CS Major from UC Berkeley
- Graduate Researcher, ScAi Lab @ UCLA
- Research Interests:
  Graph neural networks, knowledge graphs
  for various applications

Instructors

Swapnil Sayan Saha

- ECE PhD @ UCLA,
  - ECE MS from UCLA
  - EEE Major from UofDhaka
- GSR @ NESL, UCLA
- Entering Event's Director, EGSA
- Research Interests:
  Applied AI, Embedded Systems

Instructors

Debarnab Mitra

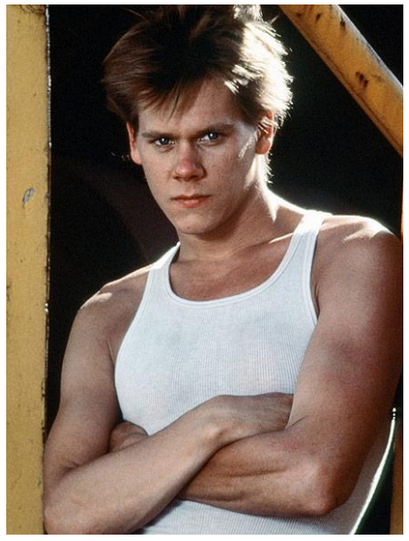
- ECE PhD @ UCLA,
  - ECE MS from UCLA
  - EE Major from IIT Bombay

- GSR @ LORIS, UCLA

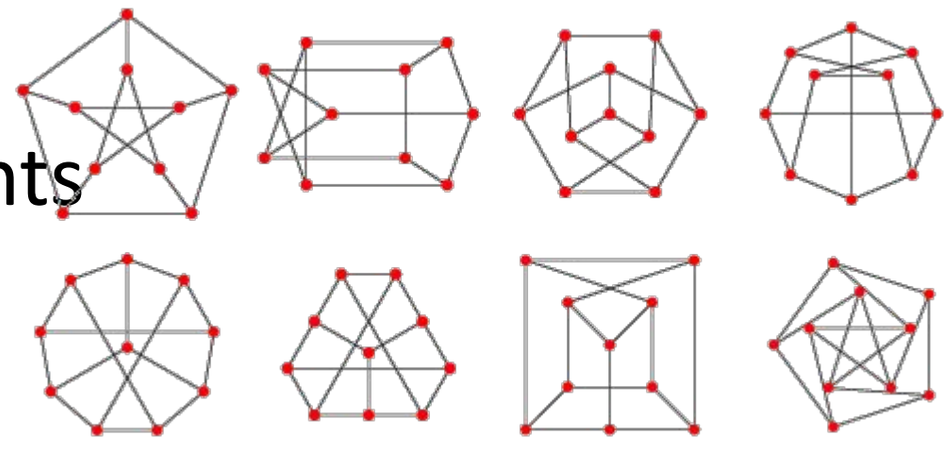
- Research Interests:
  Coding Theory, Blockchains

In This Lecture…

- We are going to:
  - Introduce the concepts of a social network and graph theory
  - Play ‘Six Degrees of Kevin Bacon’
  - Learn some basic graph theory

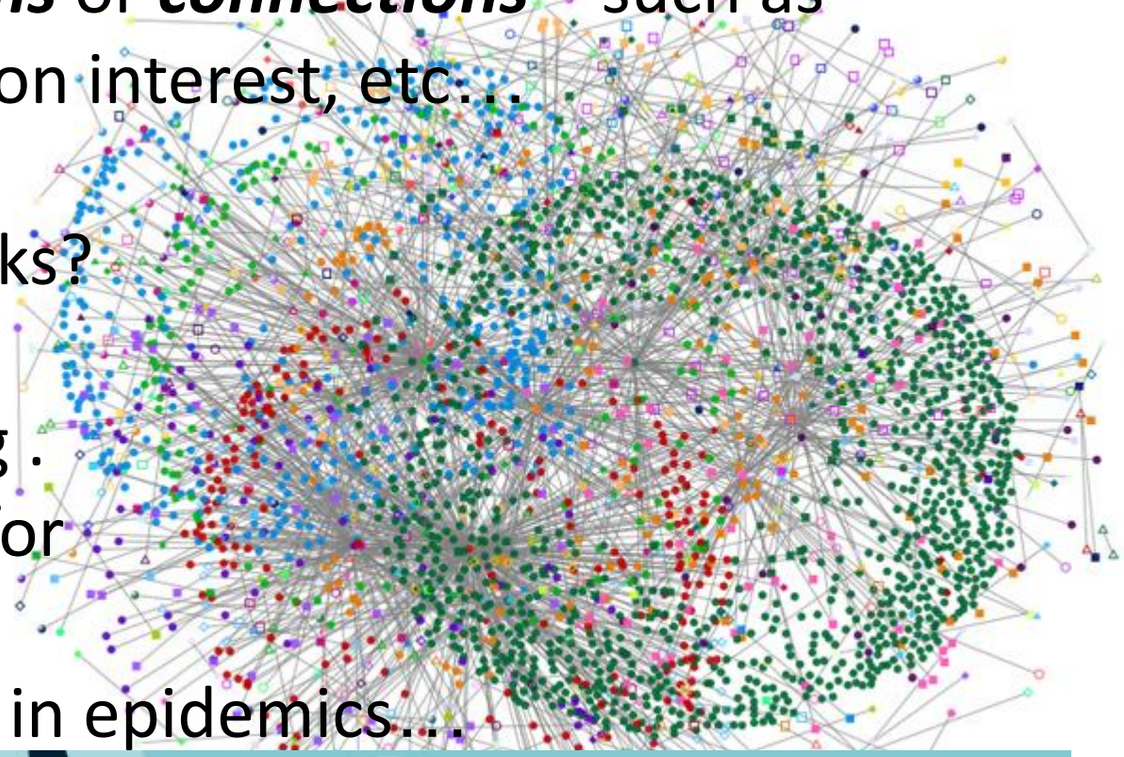


- Then we will do programming exercises:
  - **Easier**: some simple graph theory
  - **Harder**: path and cycle detection
    find connected components



Social Networks

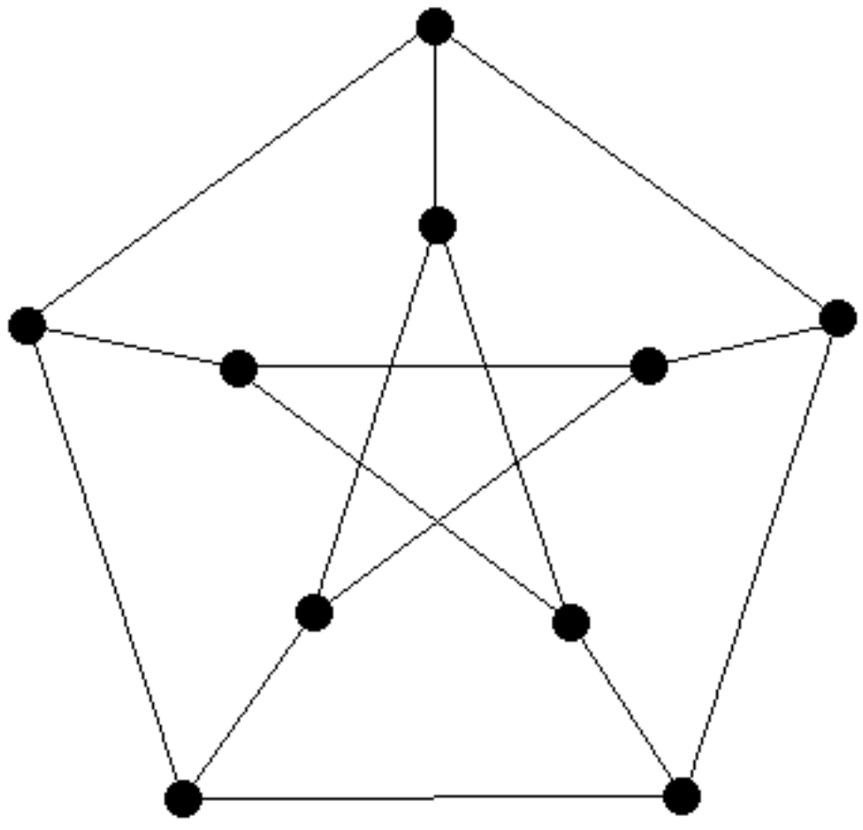
- Q: What's a social network?
- A: It is a structure made up of ***individuals*** (or organizations, or agents, or anything else) tied together by ***relations*** or ***connections*** – such as friendship, alliance, financial or common interest, etc…



- Q: Why should we study social networks?
  - It's useful! Tons of applications:
    - Finding friends and matchmaking .
    - Locating appropriate customers for companies (marketing).
    - Preventing spreading of diseases in epidemics…

Graph Theory

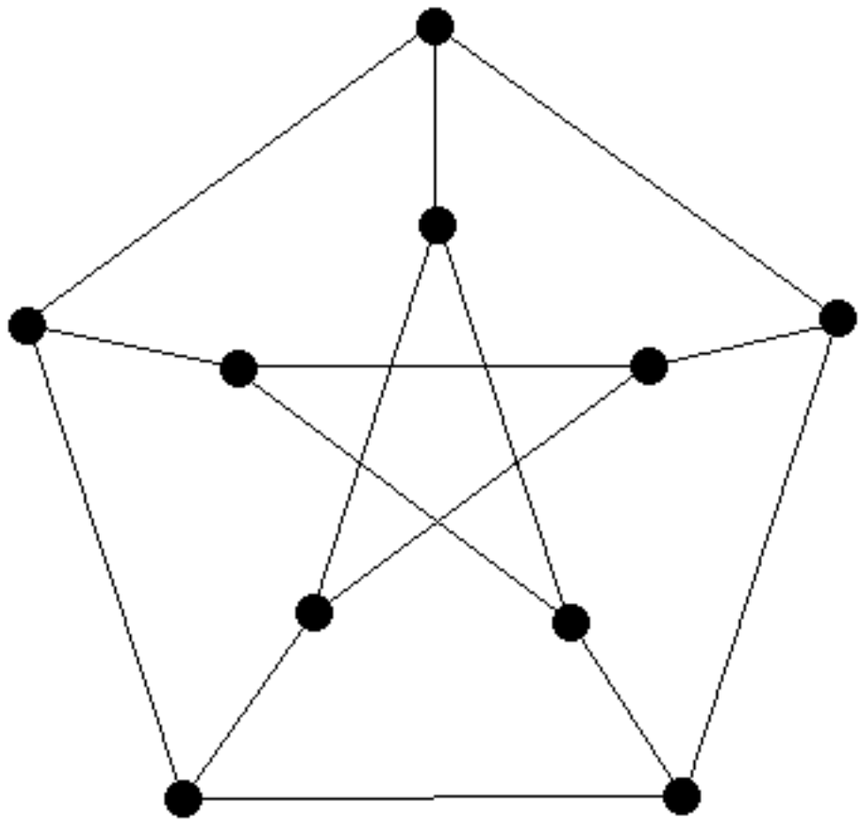
- The main tool used to understand social networks is graph theory.
  - An example of a graph:



Petersen graph

Graph Theory

- A graph is made up of nodes (also called vertices) and edges:
- Which are the nodes and which are the edges in the graph below?

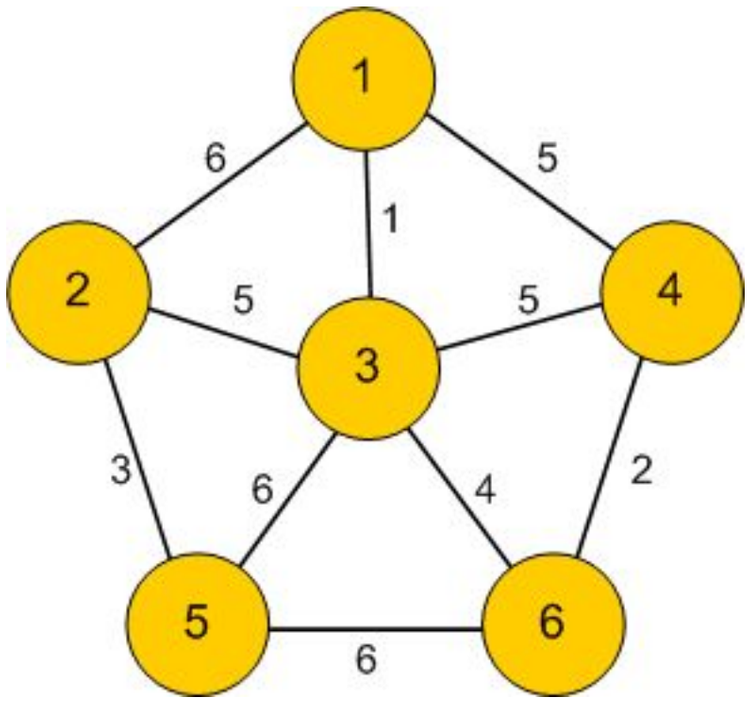


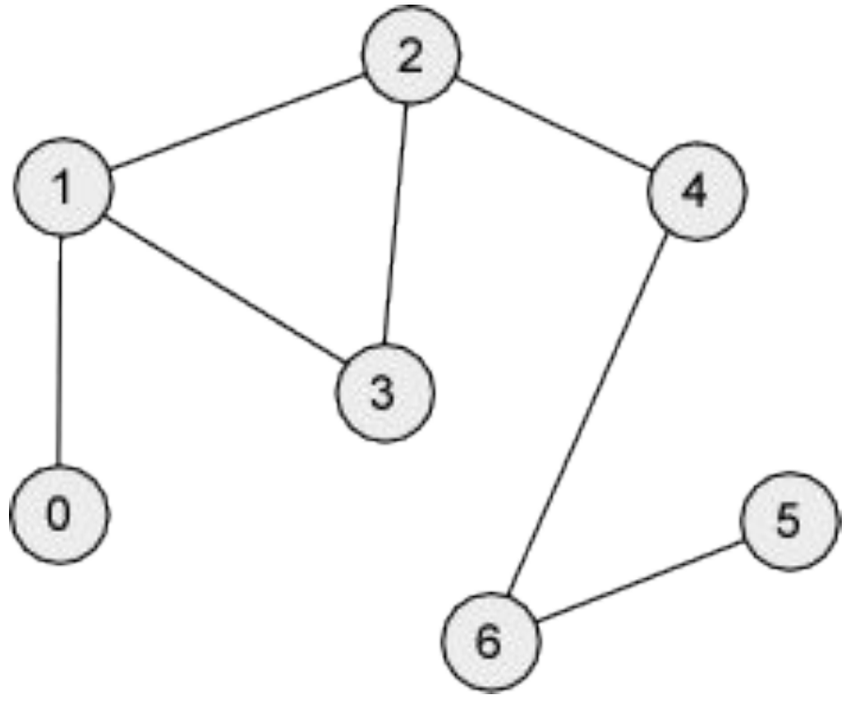
Petersen graph

Graph Theory

- Edges can be **weighted** or **unweighted (= uniformly weighted graph).**
- Weights: allow us to express the importance of a connection.

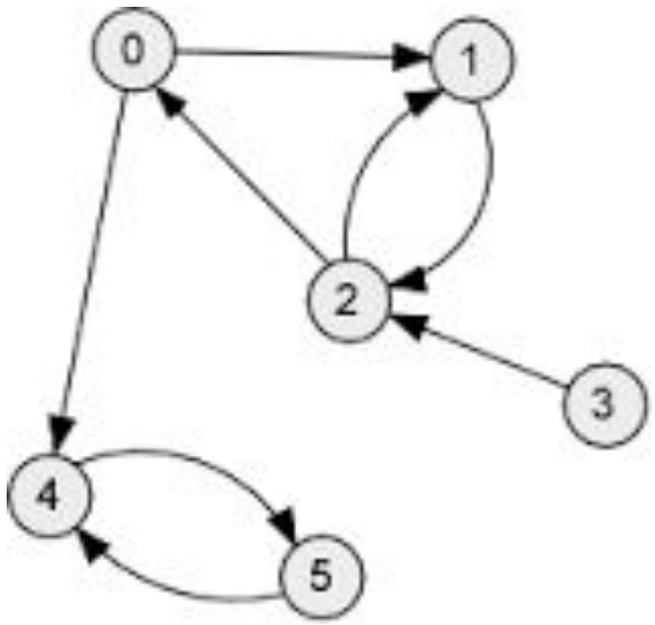
Q: Can you think of some applications where we need edge weights?

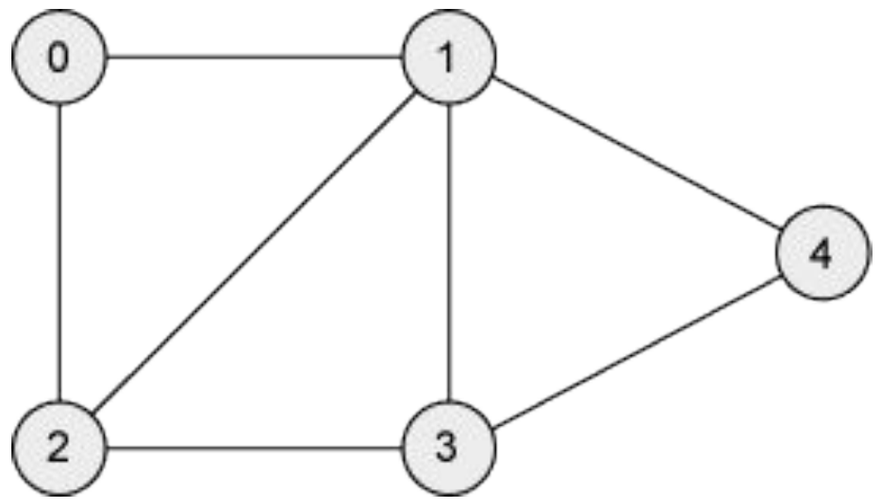




Graph Theory

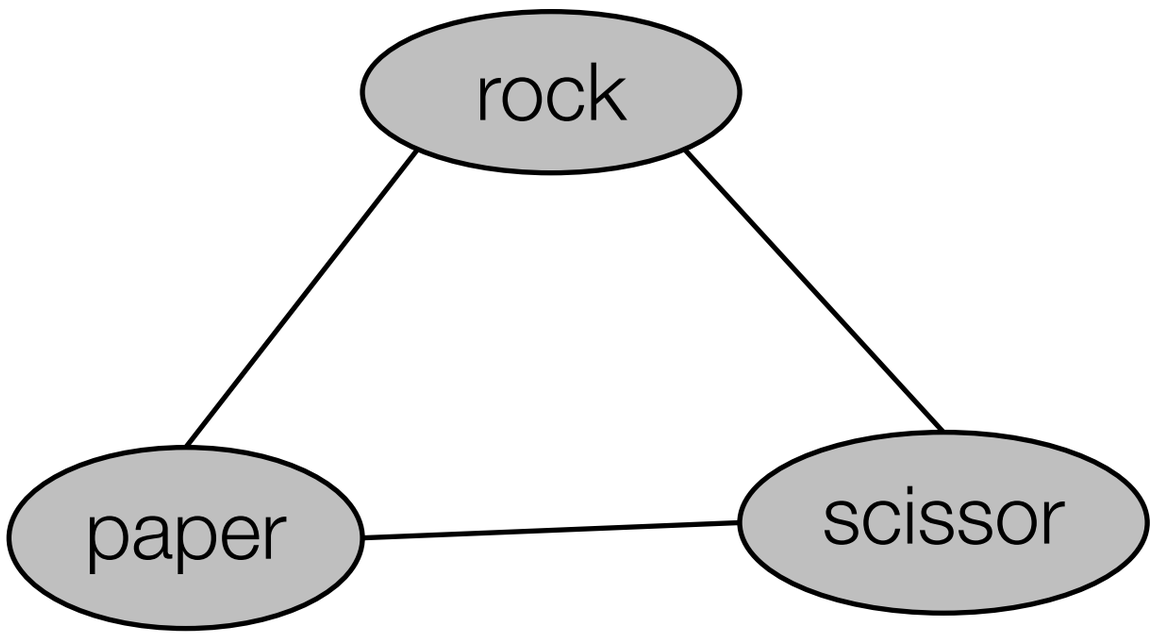
- Edges can be **directed** or **undirected**.
- Directions allow us to have influence go one way (rather than mutual).
  - The type of graph we use is dependent on

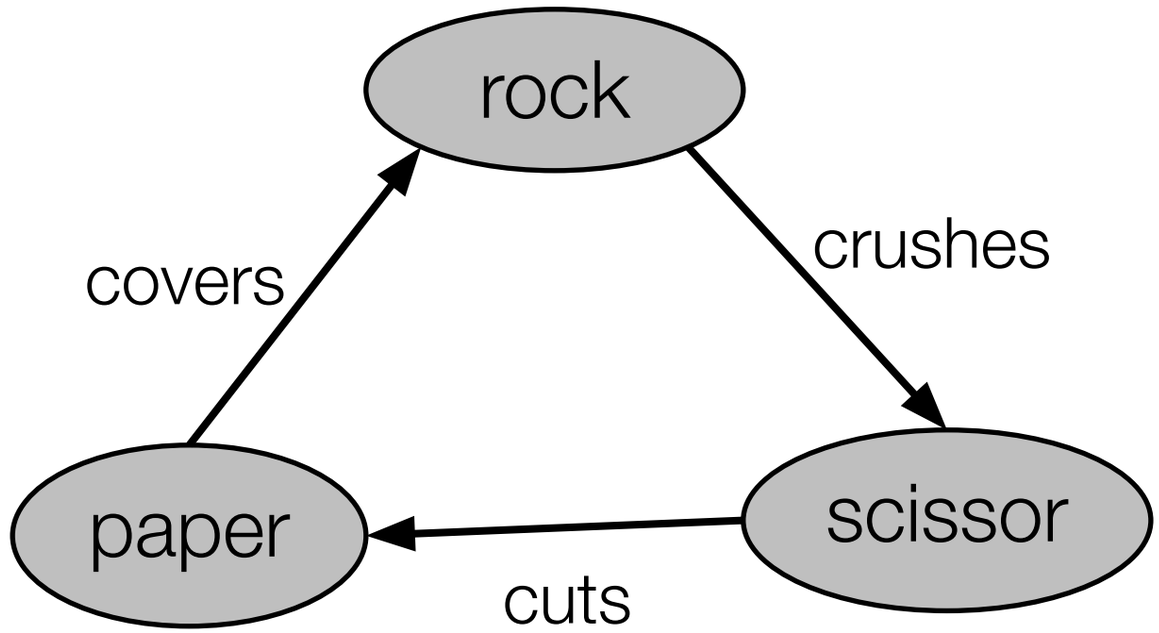




Example of directed edges in a graph

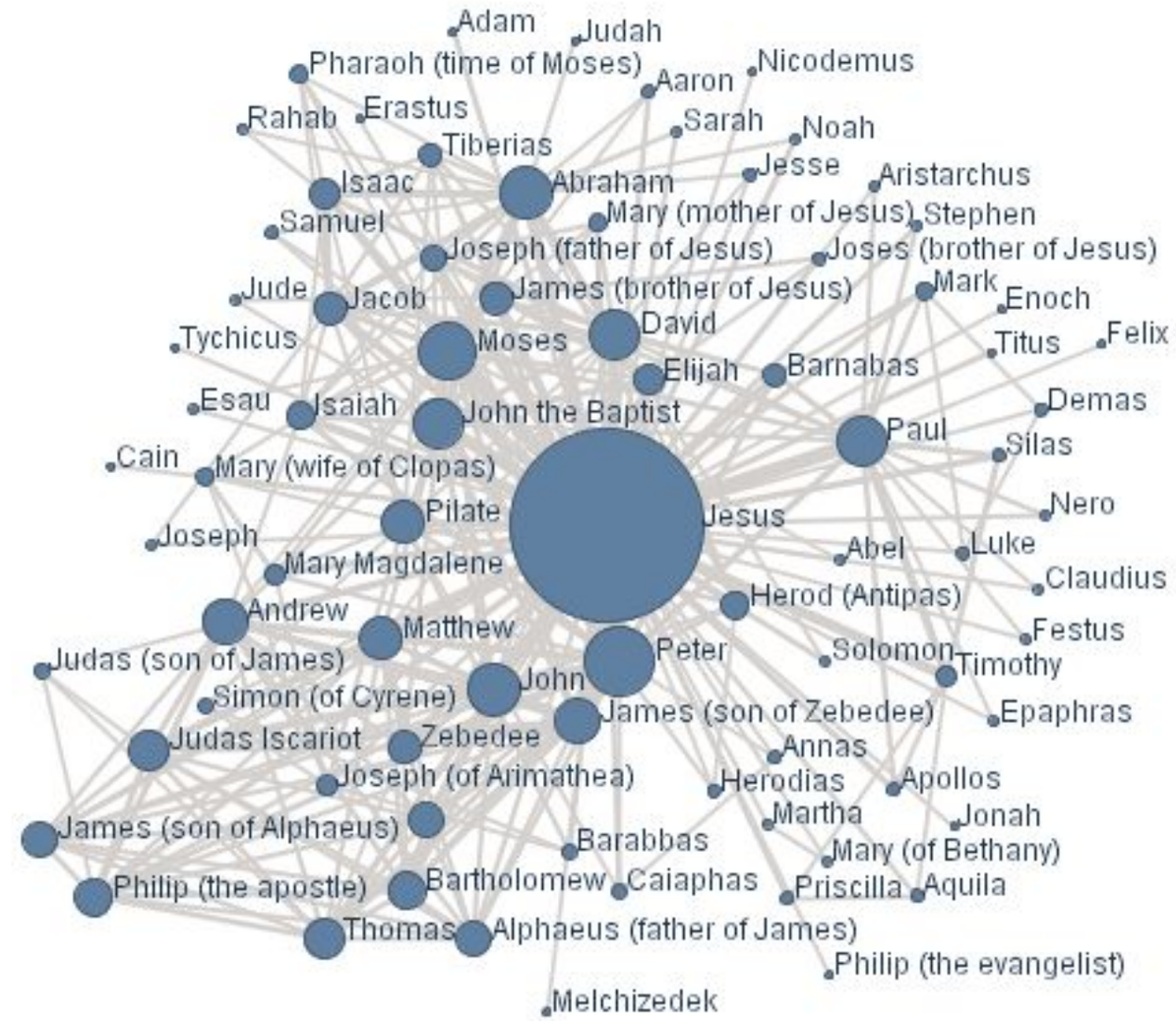
- Rock – Paper – Scissor Game





Graph Theory

- Q: Why do we care about graphs?
- A: We can use them to represent a social network:



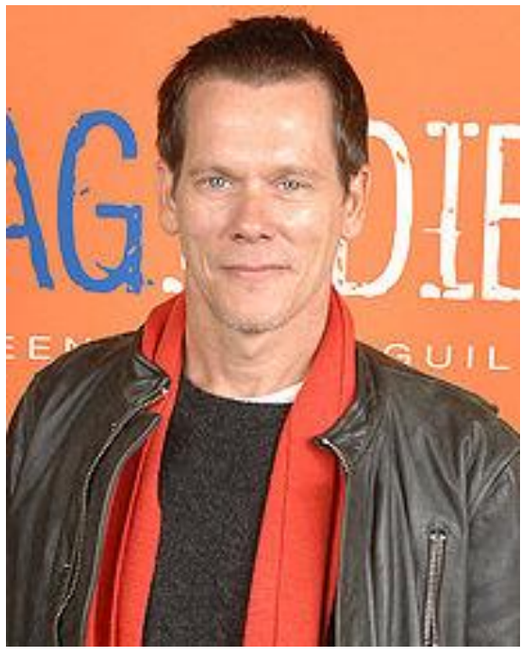
Acting Networks

- A good example of a network is formed by our coworkers, their coworkers, etc…
- We'll look at Hollywood actors.
- We'll say that two actors are connected if they played in the same film.

Kevin Bacon

- Q: Who is the “center” of the movie universe?

- A: Kevin Bacon. Have you heard of him? If not – it’s because he’s known for playing a supporting role in a huge number of films:



Kevin Bacon

Kevin Bacon

- We can play a game called ‘Six Degrees of Kevin Bacon.’

- The rules:
  - Pick an actor or an actress.
  - If they have played in a film with Kevin Bacon, their Bacon number is 1.
  - If they have played in a film with someone with a Bacon number of 1, they have a Bacon number of 2.
  - Etc…

- Example: Bradley Cooper:
  - Bradley Cooper was in “Silver Linings Playbook” (2012) with Jennifer Lawrence.
  - Jennifer Lawrence was in “X-Men: First Class” (2011) with Kevin Bacon.

  - So Bradley Cooper has a Bacon number of 2.

“Six Degrees” Game

- **Exercise:**
- Using IMDB, find the Bacon number (and path) for:
  - Julia Roberts
  - Selena Gomez
  - Christian Bale

“Six Degrees” Game

- It’s a lot easier with an automated tool that searches IMDB:

http://oracleofbacon.org/cgi-bin/movielinks

- For example, it produces this output for Christian Bale:

“Six Degrees” Game

- **Exercise:** Try out the Oracle tool with your favorite actors.
- What’s the largest Bacon number you can get?

“Six Degrees” Game

- We saw a lot of actors with Bacon numbers 1 and 2.
- What about the extremes?
  - What’s the smallest possible Bacon number? Who has it?
  - What’s the largest possible Bacon number?
    - What if someone is not connected to Kevin Bacon at all?
    - Among those who are, what’s the largest Bacon number?

“Six Degrees” Theorem

- The “Six Degrees” idea states that any two people selected at random are connected by at most 6 degrees.
- Of course, there are some counterexamples, but they’re rare.
- Same applies to Hollywood – which is why the game is called “Six Degrees of Kevin Bacon.”

Back to Kevin Bacon

- Here's a statistic about Kevin Bacon numbers for other actors:

Bacon Number	# of People
0	1
1	3434
2	398768
3	1478688
4	382332
5	33473
6	4101
7	679
8	144
9	16
10	7

Back to Kevin Bacon

- What can we see from the table?
- Most people have a Bacon number of 3.
- Slightly less have a Bacon number of 2 and 4.
- Bacon numbers higher than 4 are rare.
- The maximum is 10.

Back to Kevin Bacon

- You can replace Bacon with other actors to see who makes a good “center” for connections.

- Try it out:

http://oracleofbacon.org/cgi-bin/center-cgi

Back to Kevin Bacon

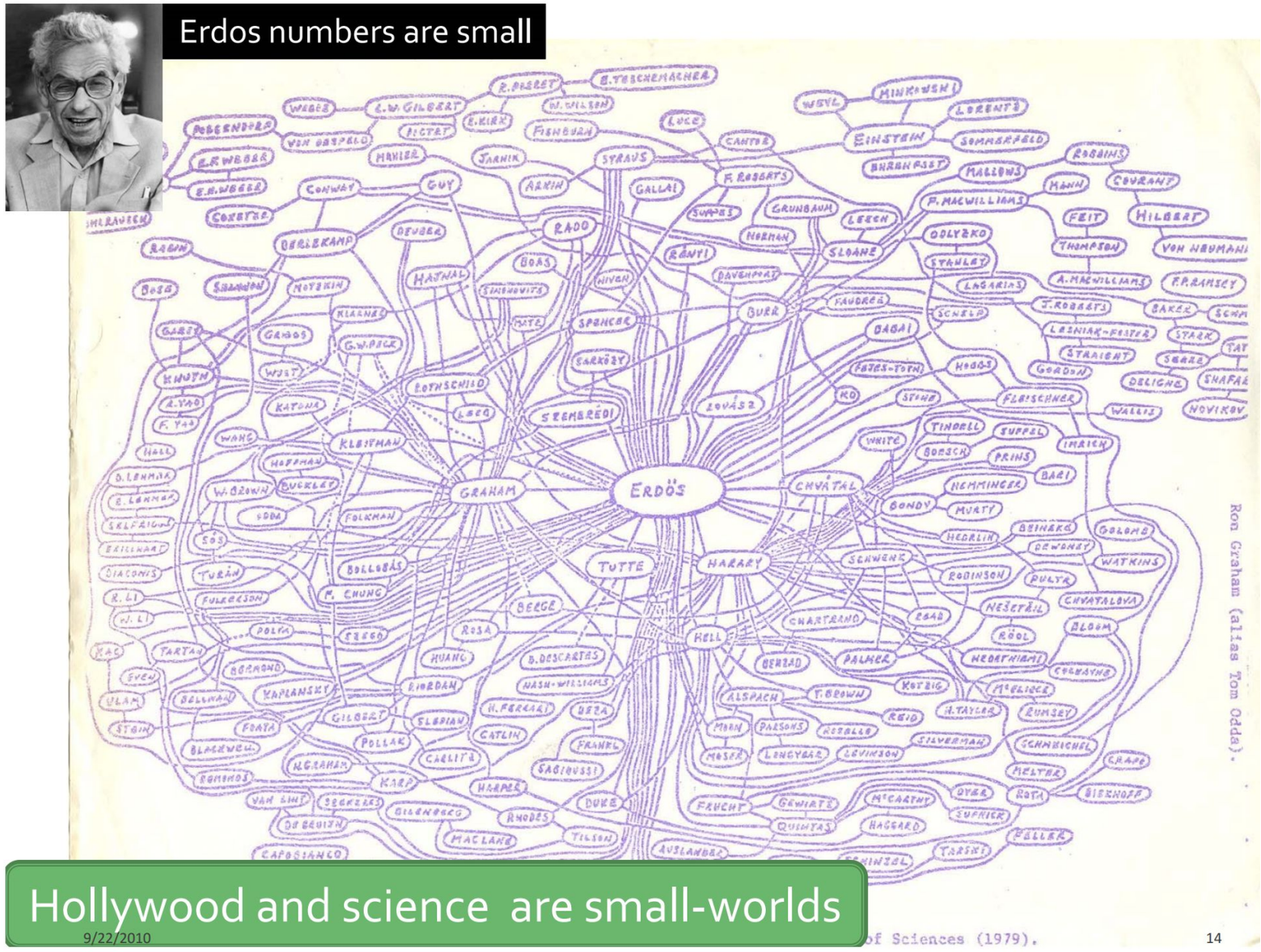
- What does it mean to be a good “center?”
- For example, we see that Tom Cruise has an average number of 2.976 while Bacon has an average of 3.026. Who’s better?
- **Exercise:** Try out your favorite actor. Who picked the best center?

Rachel McAdams Number	# of People
0	1
1	1897
2	300058
3	1505279
4	451047
5	38285
6	4249
7	641
8	163
9	16
10	7

Total number of linkable actors: 2301643
Weighted total of linkable actors: 7144962
Average Rachel McAdams number: 3.104

What about the world of science?

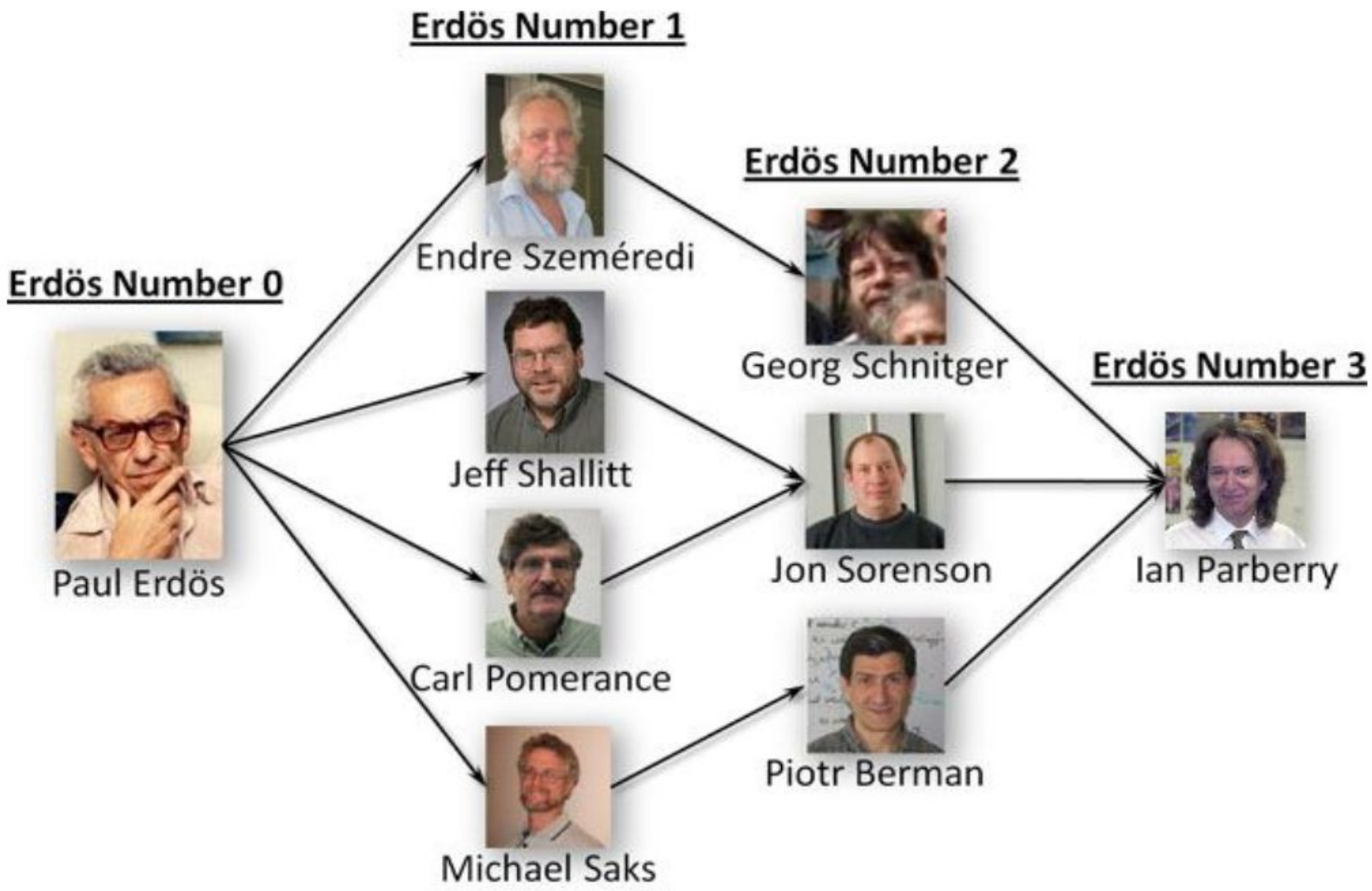
- Erdős numbers:



Hollywood and science are small-worlds

Erdős numbers

- Describes the "collaborative distance" between mathematician Paul **Erdős** and another person, as measured by authorship of mathematical papers.



What’s your Erdos number?

- Roshni Iyer (instructor of this module): 2

Paul Erdős

Satish Rao

Roshni Iyer

Compute:
https://oakland.edu/enp/compute/

Larger networks

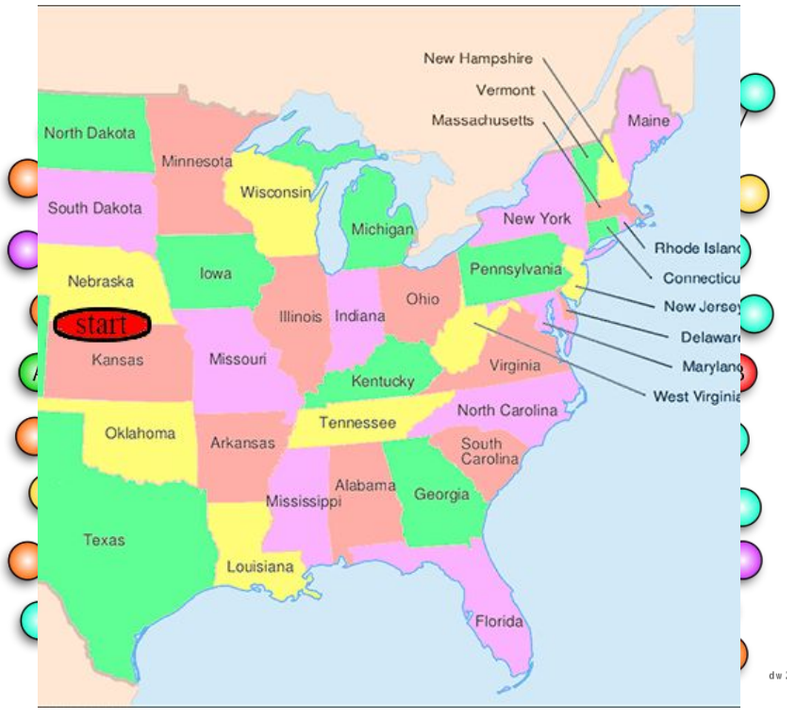
- We saw, from the Kevin Bacon game, that
  - 1. Hollywood has about 1 million actors.
  - 2. Most of them can be related to each other within 4 connections.

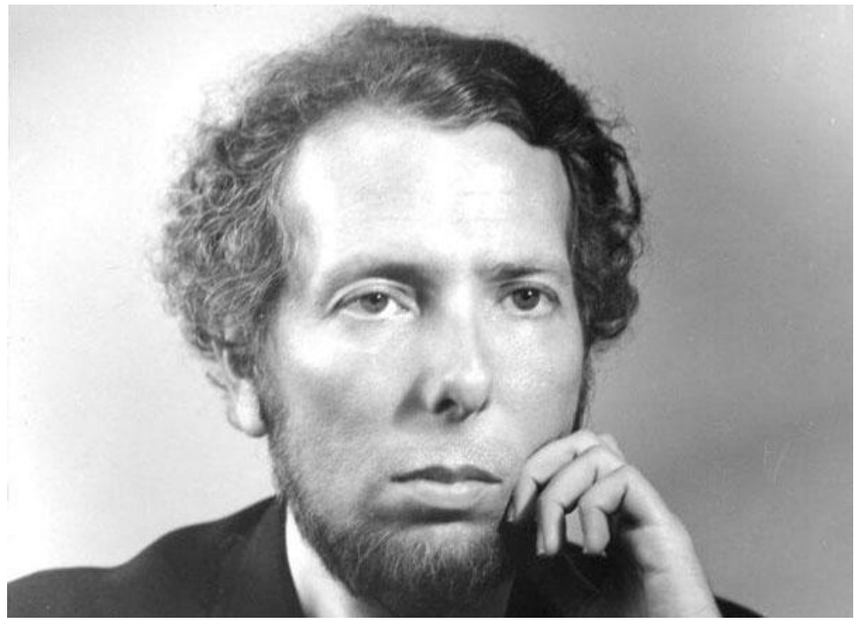
- What about all the people in the US?
  - Population is about 300 million.

- Q: About how many connections on average between two random people?
  - (Note: define connection here as “being acquainted with.”)

Larger Networks

- "Six Degrees" Theorem still applies!
- So, if we pick two random people in the US, using a chain of "friends of friends", we should be able to connect them within 6 connections.
- Studied by the famous psychologist Stanley Milgram in his "Small world experiment" in the 1960's.
- Easily recreated today using Facebook!





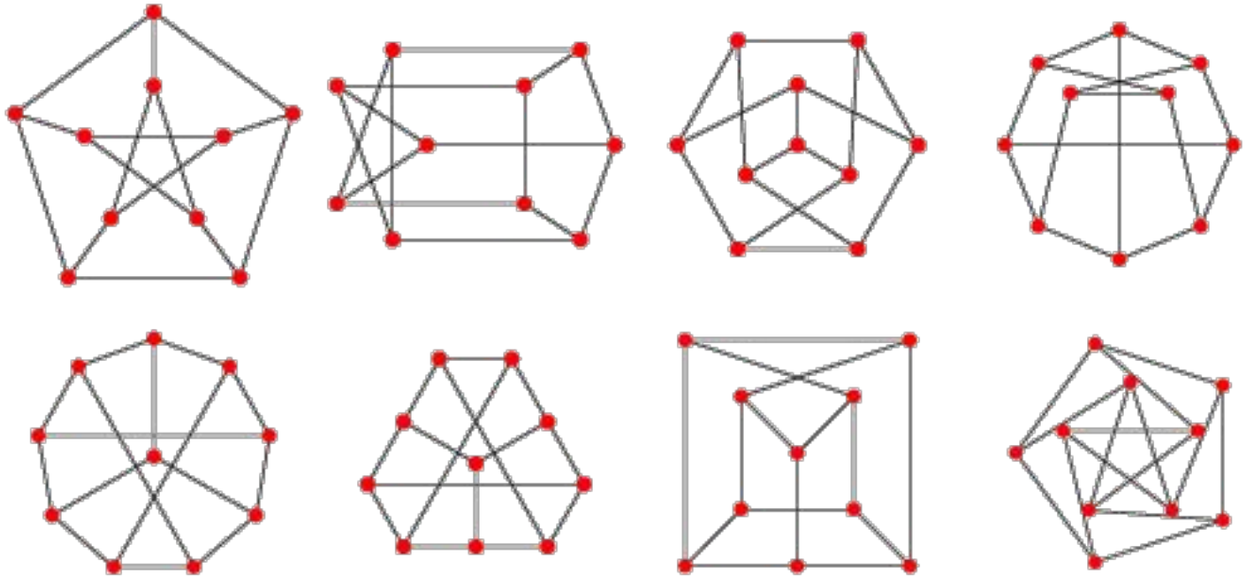
Stanley Milgram

“Six degrees” Theorem

- So what’s useful about it? Applications?
- One example: Insurance companies request customers to provide 5 referrals in order to expand their business.
- **Exercise:** Come up with another example.

Graph Theory

- We've talked about networks, connections, "distances", etc…
- Now it's time to describe the math behind this.
- This domain is called "graph theory."

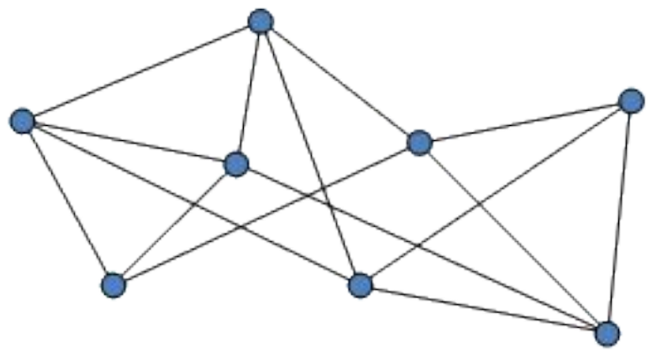


Graphs

- A graph is made up of “nodes” and “edges.”

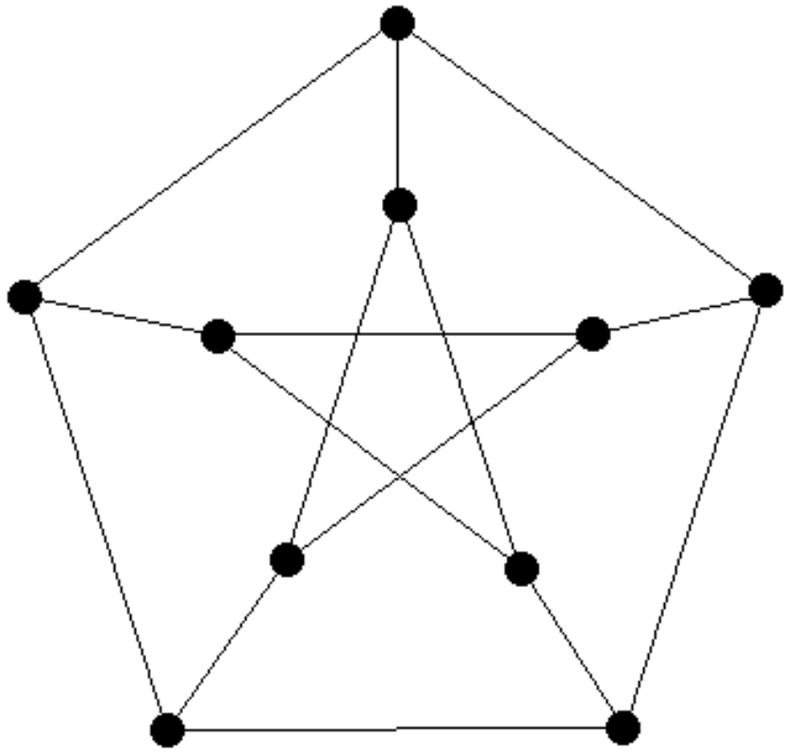
- Nodes are denoted by a dot/vertex.
- Edges connect two nodes.

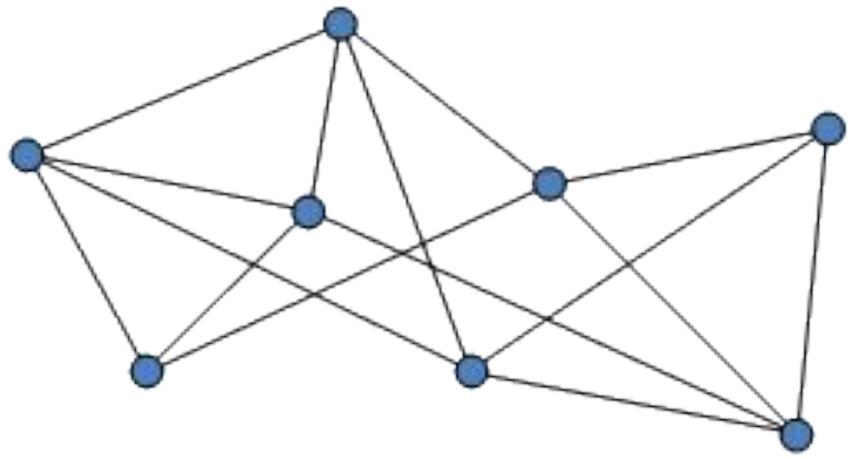
- If two nodes have an edge between them, they are “adjacent” or “neighbors.”



Degrees

- The number of edges coming out of a node is called its “degree.”
  - Sometimes degree is specified to be ***out-degree*** (outgoing edges) or ***in-degree*** (incoming edges).
- What are the in-degrees & out-degrees of the nodes in the graph below?
- All nodes have same degree: **regular** graph. If not, **irregular**.
- Q: Are complete graphs (maximally connected graphs) regular or irregular?

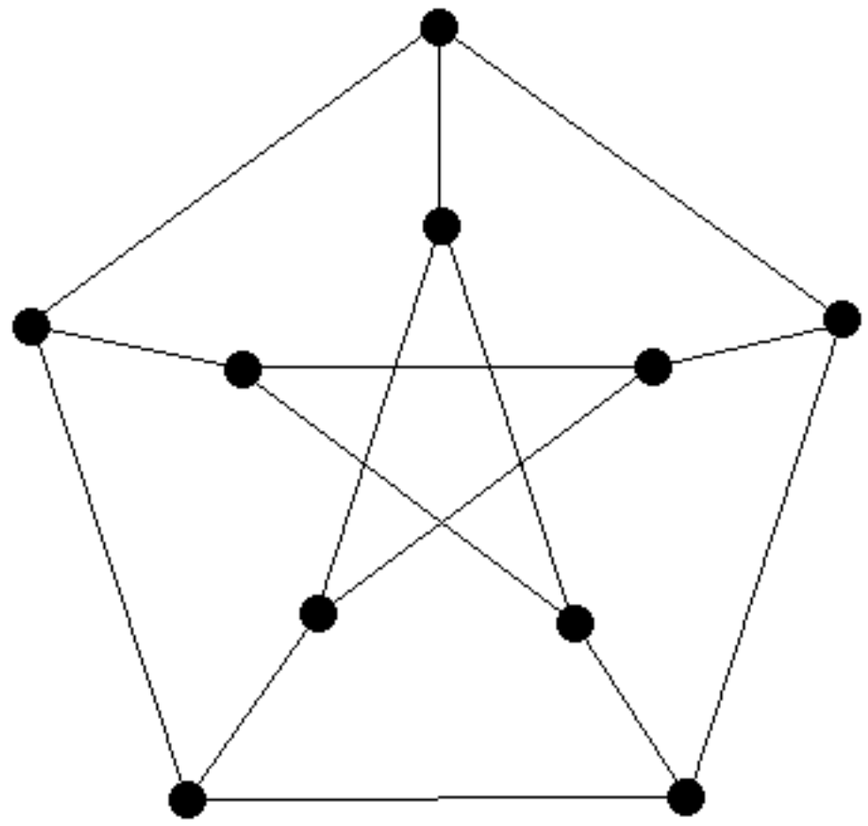




Degrees

- What's the relationship between **vertex degrees** and **edges**?

- **Exercise**: Give the relationship described above.
  (hint: think about the sum of all the degrees and the number of edges.)



Idea

- Enumerate ordered pairs (i,j) in the graph, where i and j is connected by an edge.

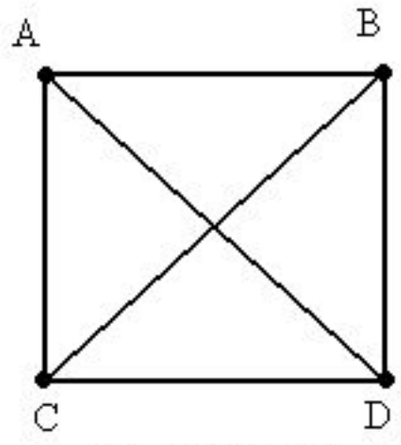
- Count by vertices:
  - If node i has degree di, then there are di ordered pairs of of (i,j)
  - The number of ordered pairs=sum of degrees of all nodes

- Count by edges:
  - If (i,j) is an edge, then both (i,j) and (j,i) are ordered pairs
  - The number of ordered pairs=2 times number of edges

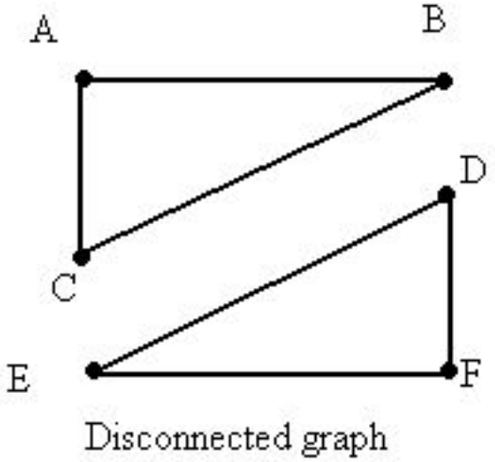
- Sum of degrees of all nodes=2 times number of edges

Connectivity

- A series of edges is called a “path.”
- If there exists a path between two nodes, the two nodes are “connected.”
- If every pair of nodes in a graph is connected, the entire graph is “connected”, otherwise it is “disconnected”.
  - A disconnected graph can be divided into multiple connected components
  - Connected graph has only 1 connected component

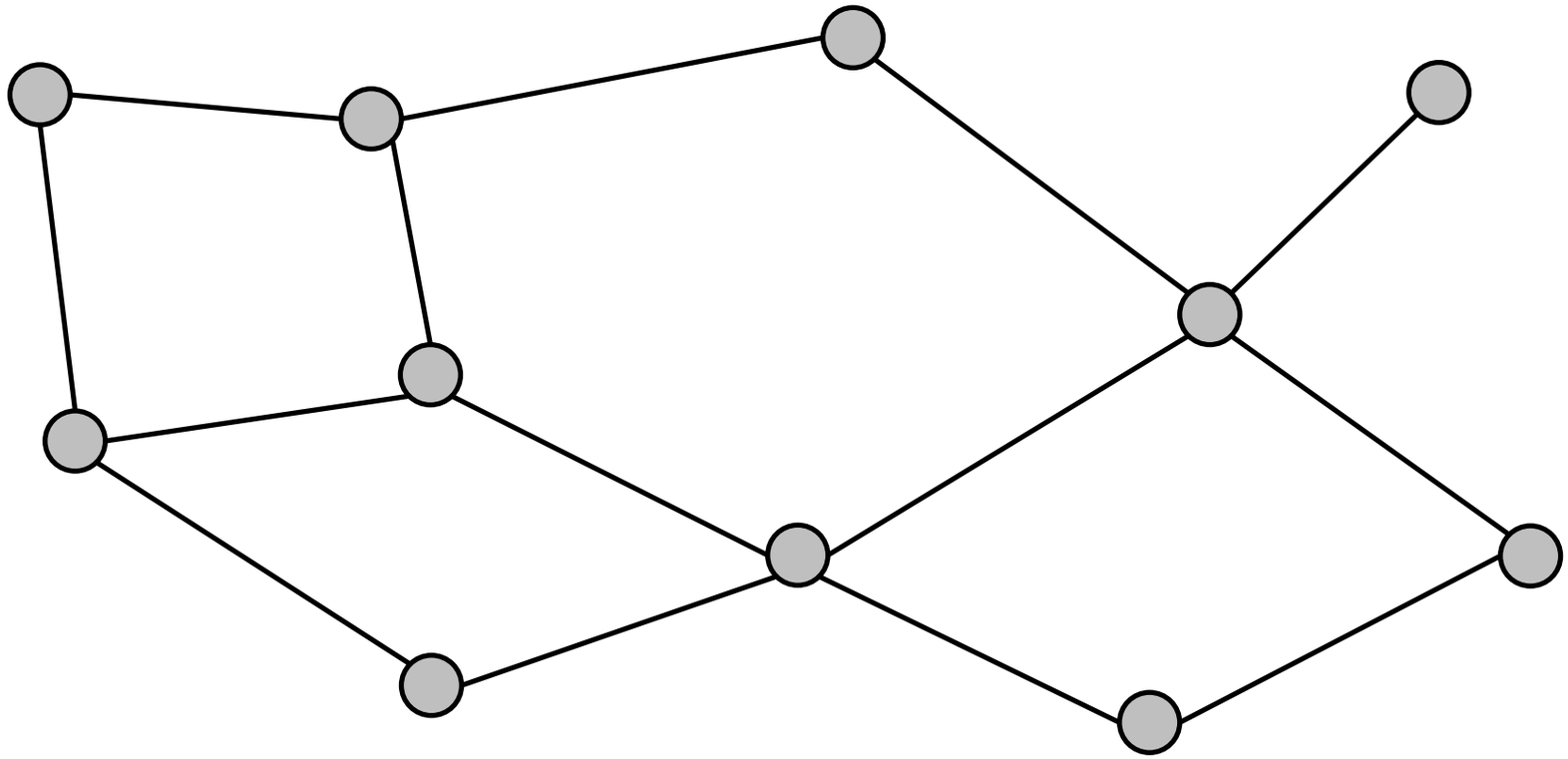


Connected graph



Disconnected graph

How to find the shortest path between two nodes?



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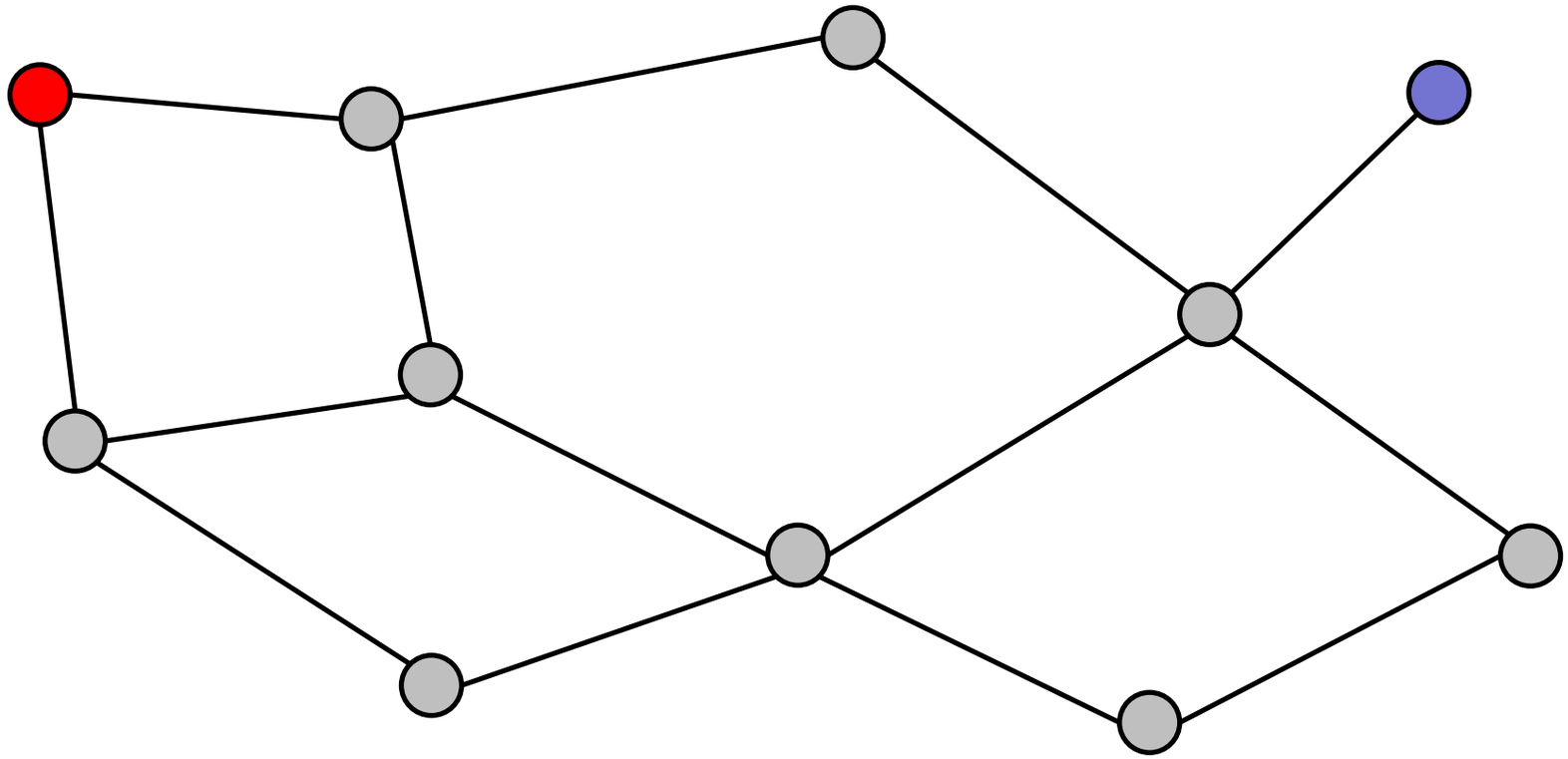

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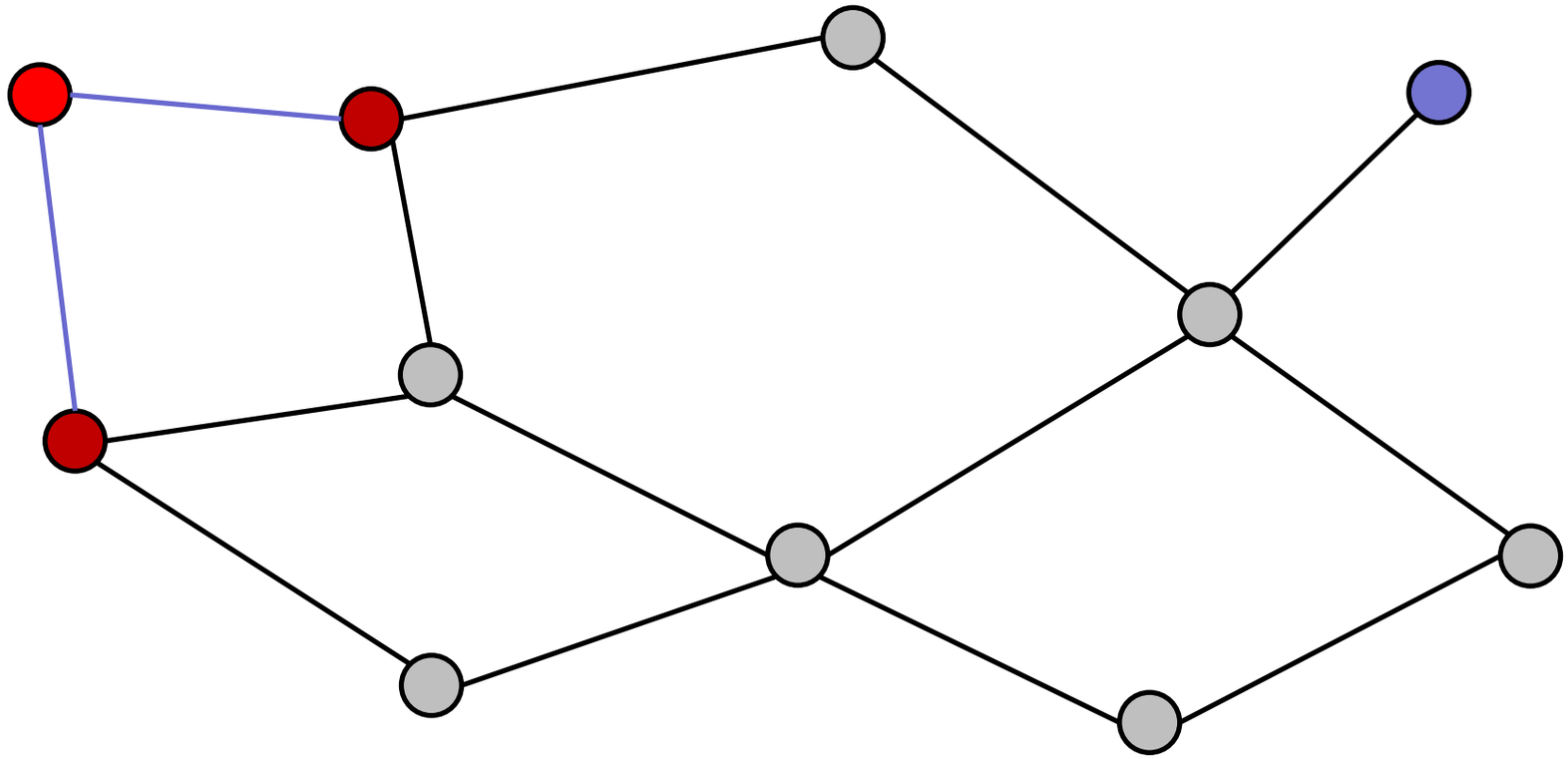

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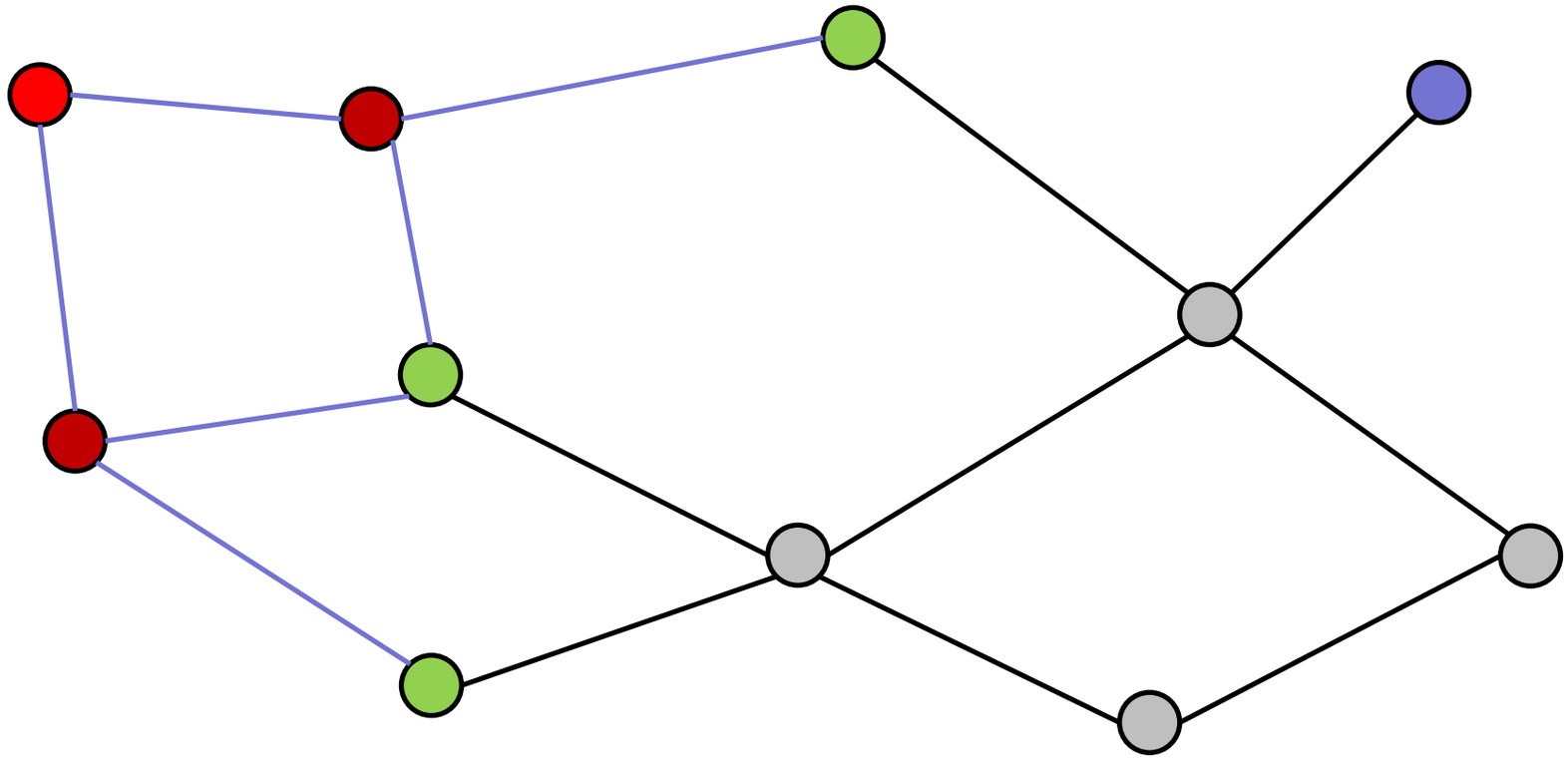

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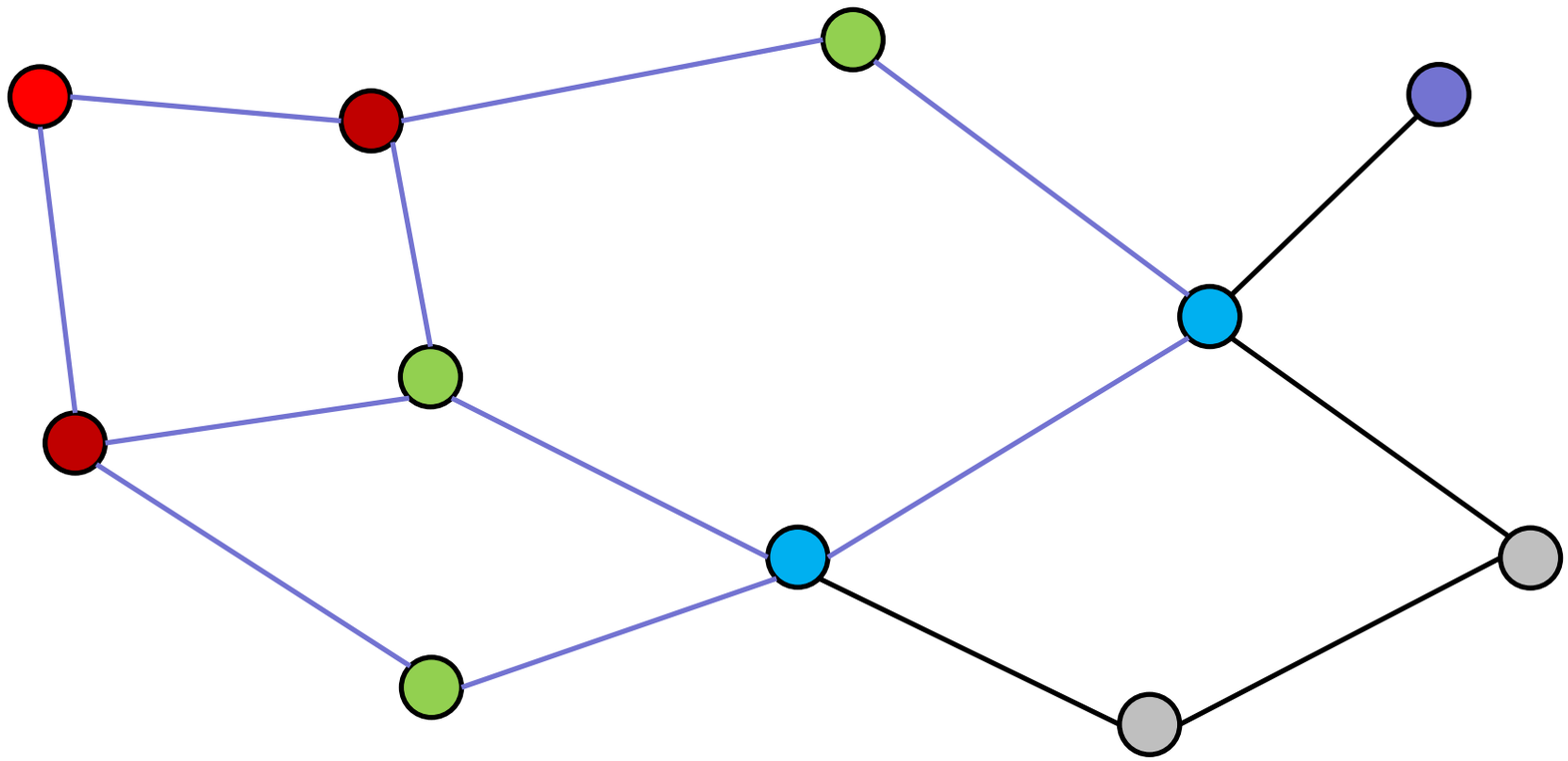

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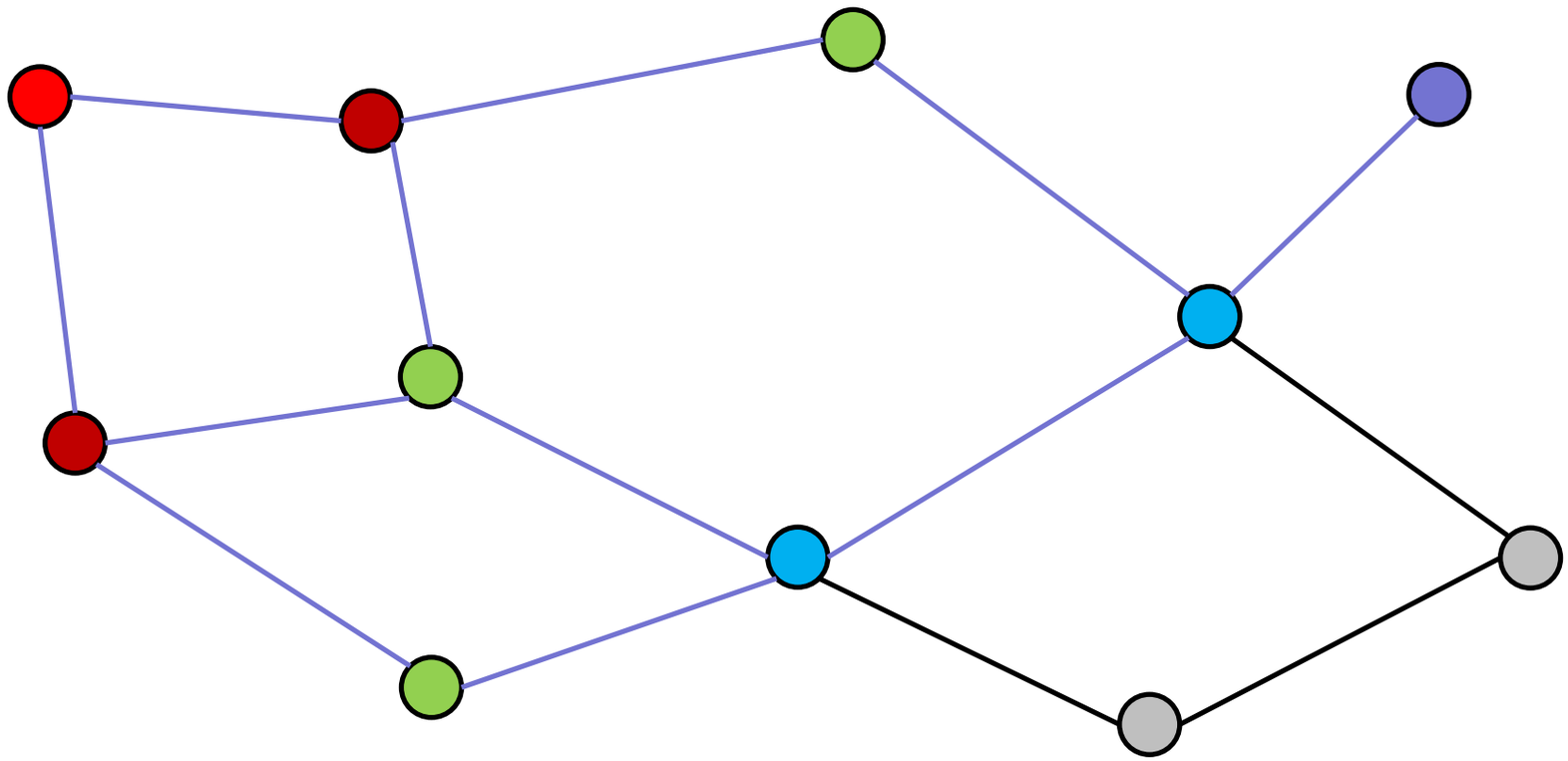

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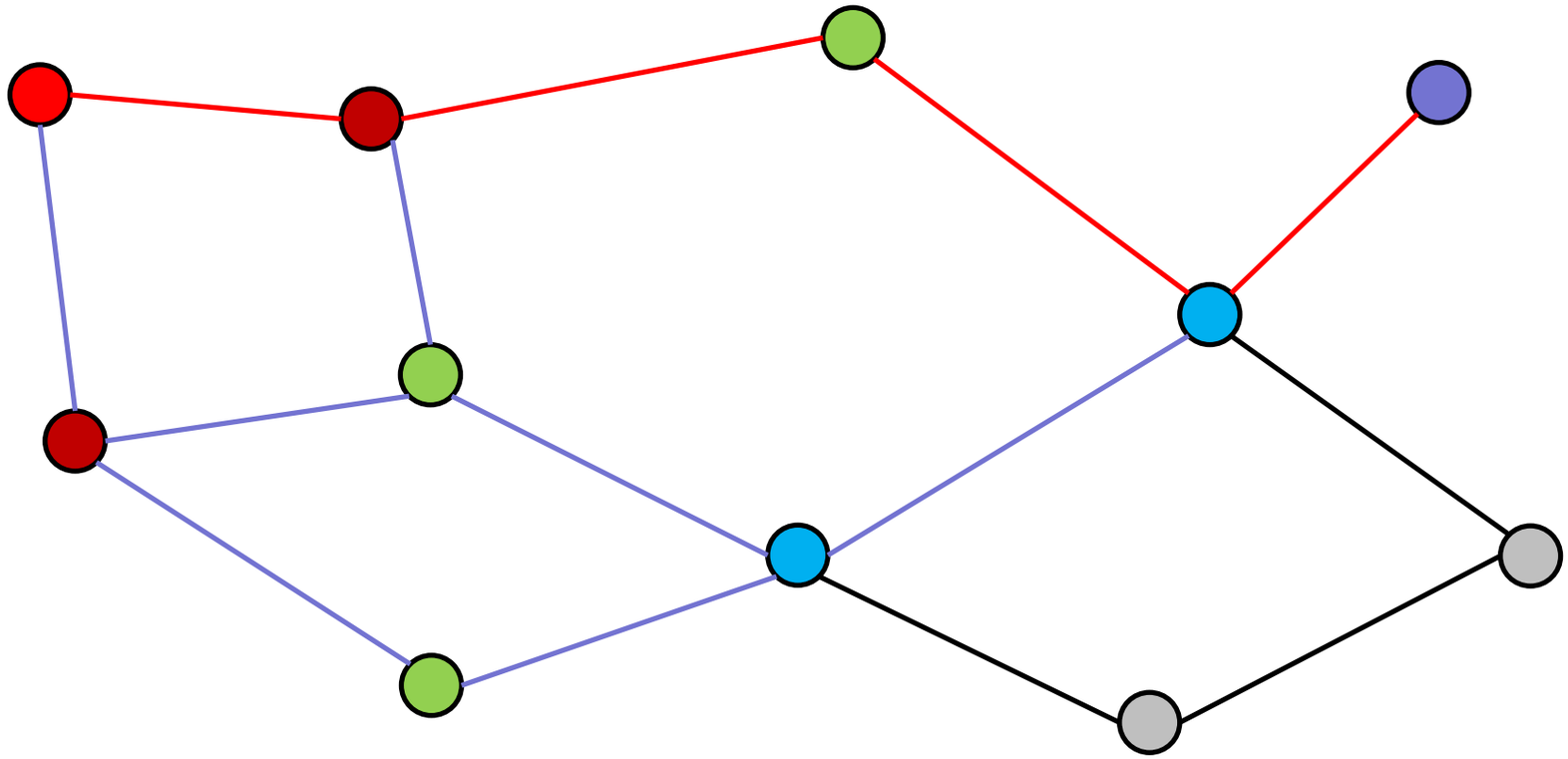
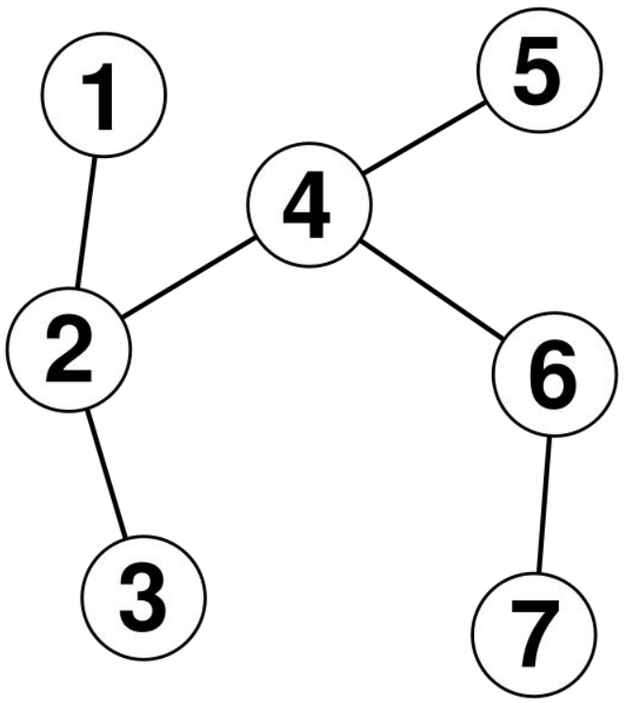
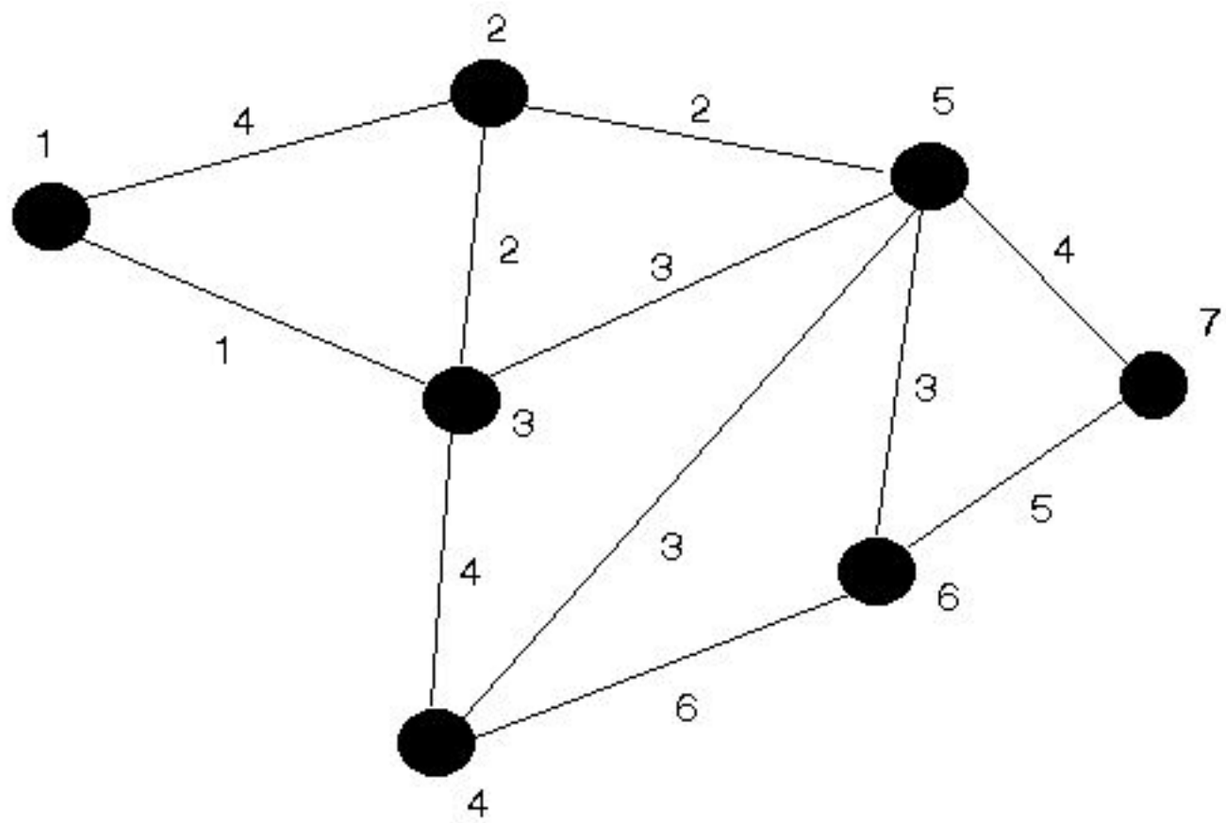


Cycles

- A series of edges (a path) starting and ending in the same node is called a “cycle.”
- Q: Which graphs have cycles? If so, how many cycles are there?

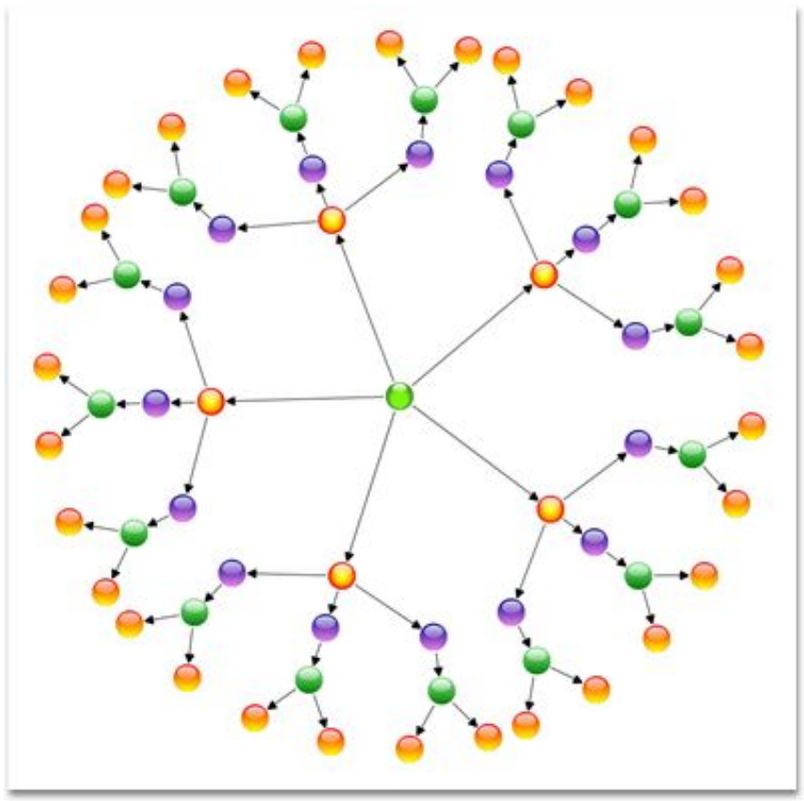




Trees

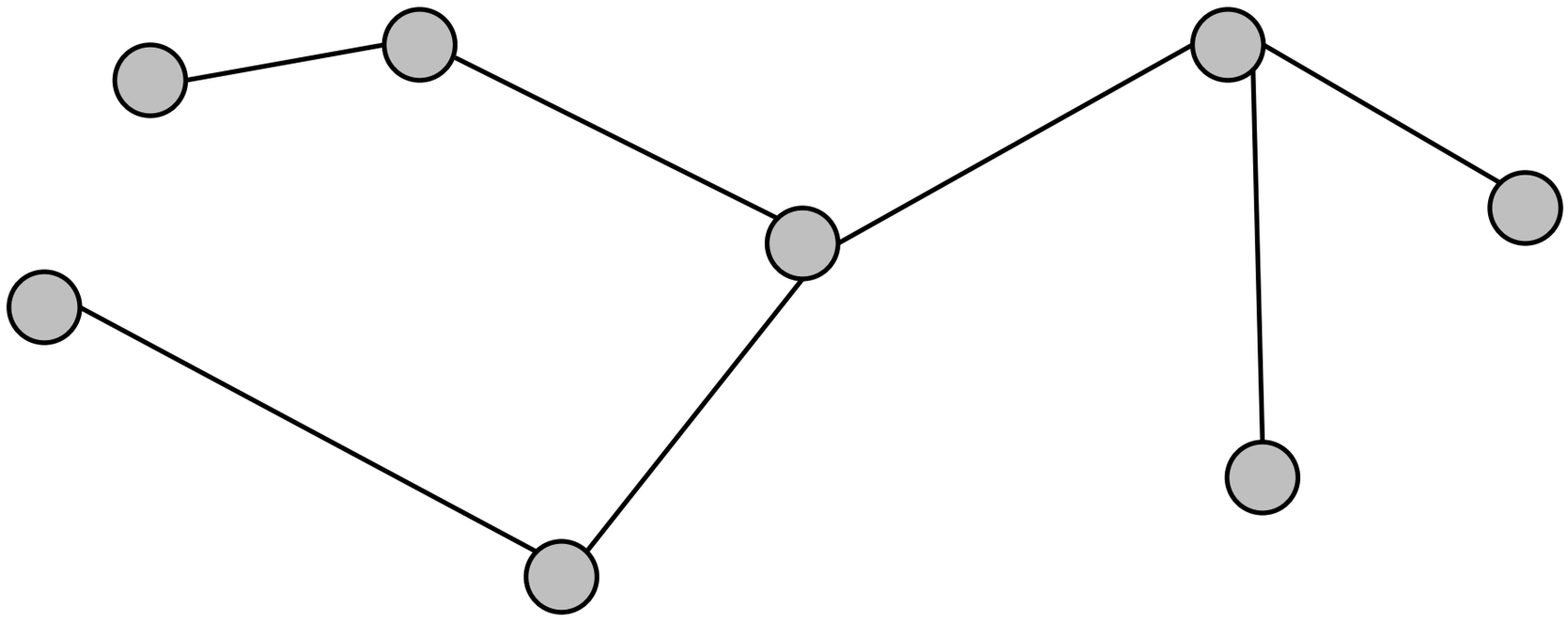
- A connected graph with no cycles is called a “tree”.
  - If we remove “connected” from this definition, the graph will be called a “forest”.

- **Exercise**: If we have a tree with n nodes, how many edges will it have?
  - (hint: draw some small examples of trees and see what you get.)



Idea

- Leaves in a tree: a node with degree 1 is called a leaf of the tree
  - Every tree has at least two leaves



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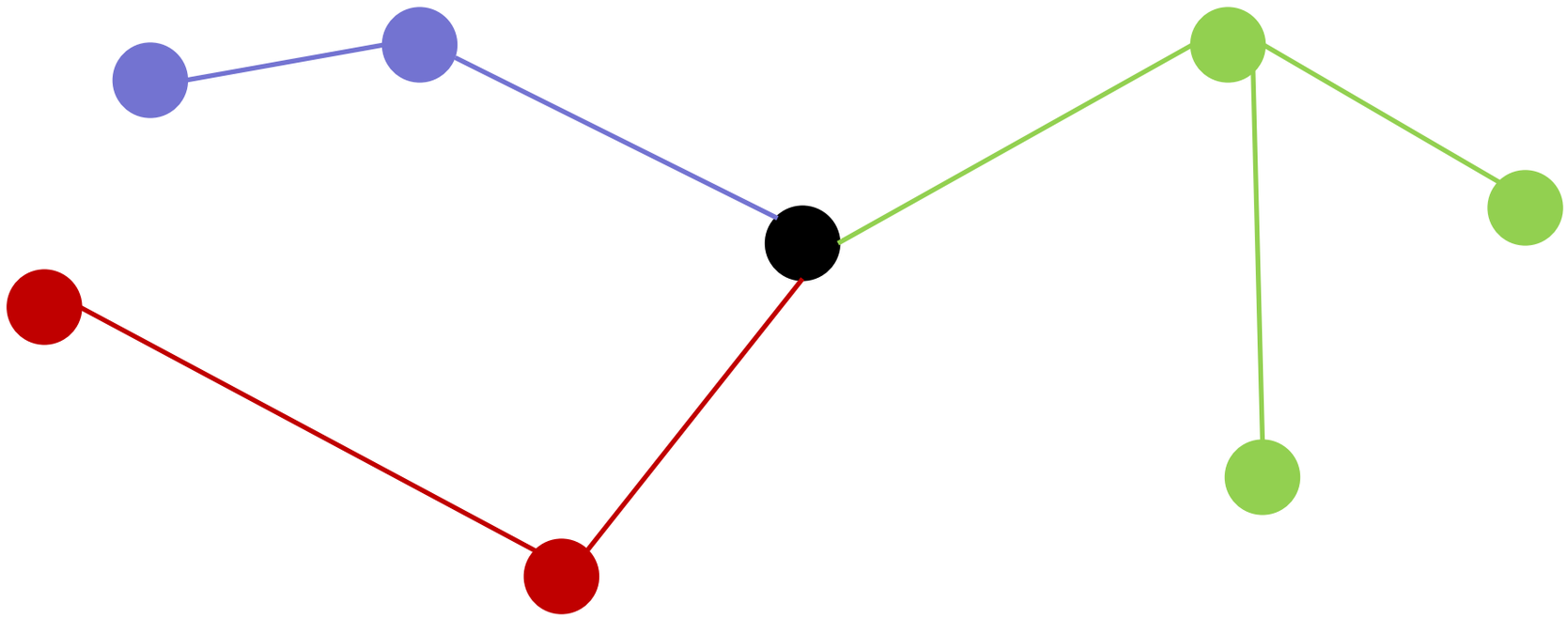

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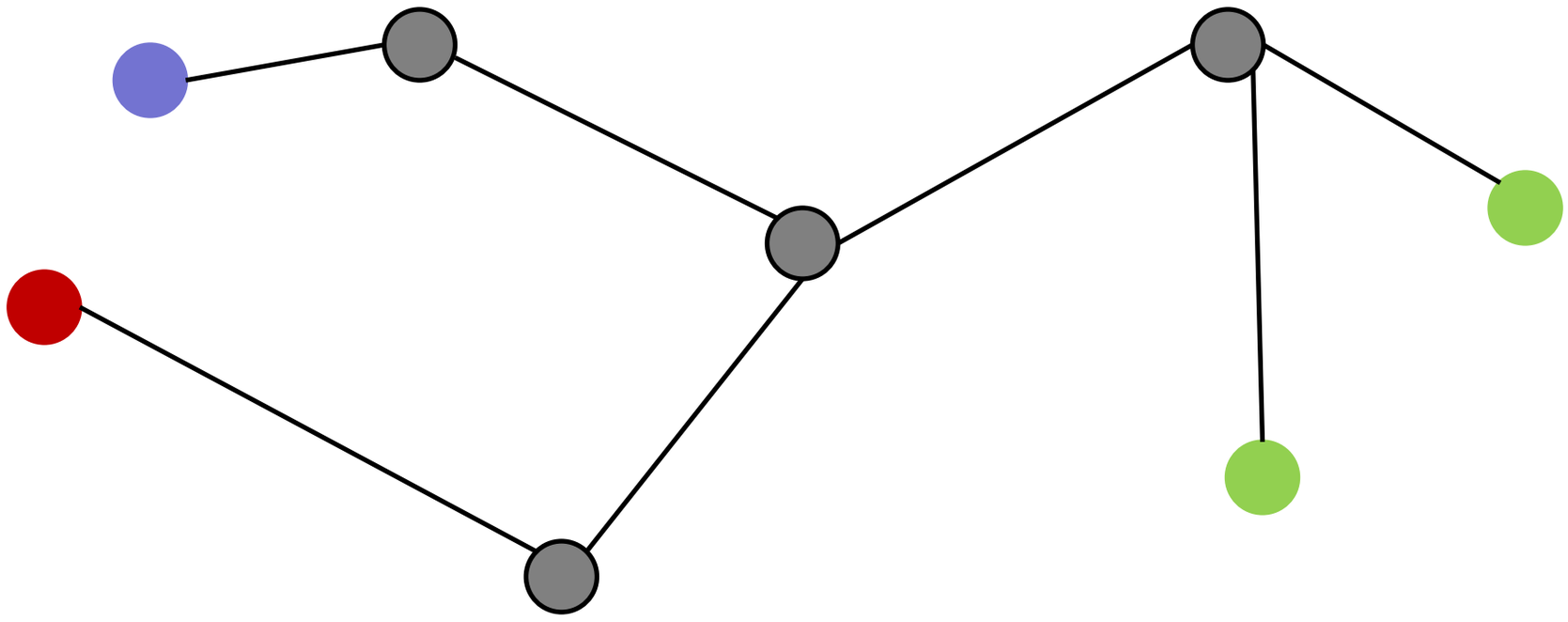

Idea

- Leaves in a tree: a node with degree 1 is called a leaf of the tree
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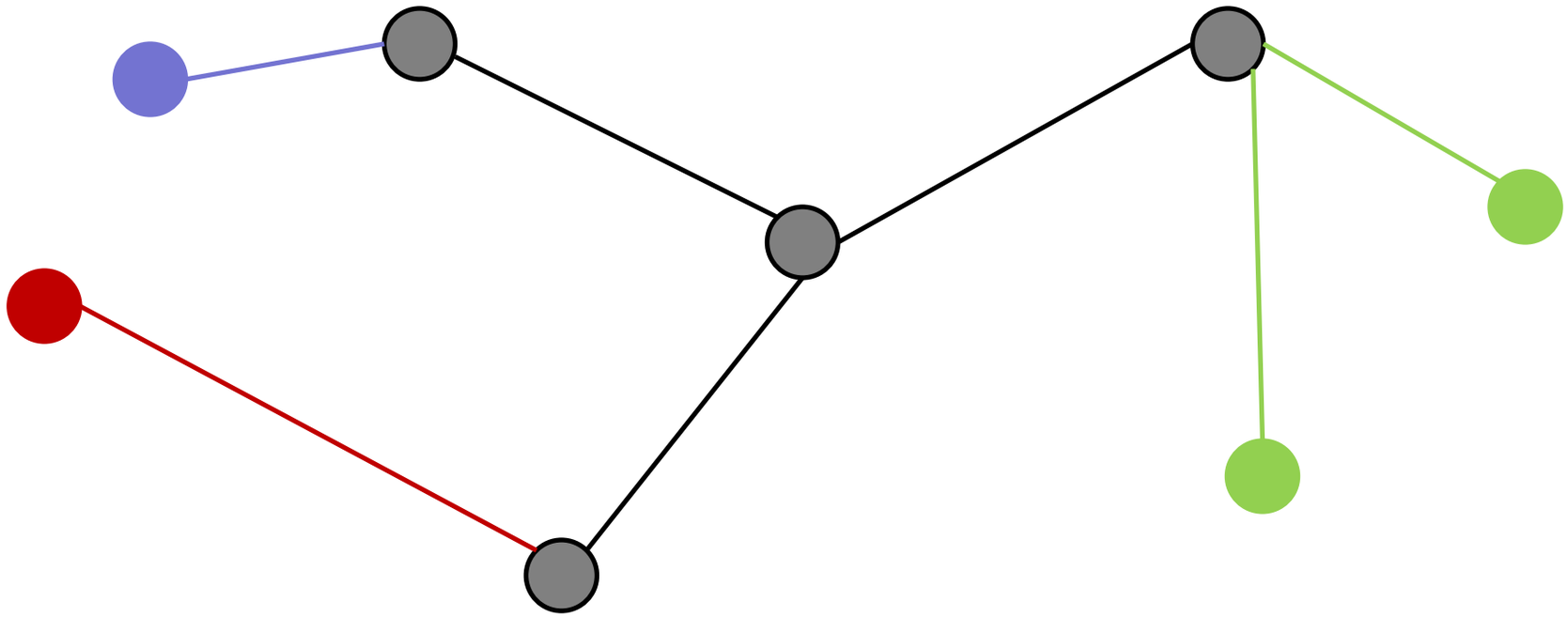
  - Remove leaves and its connected edges will not change the difference between the number of nodes and the number of edges



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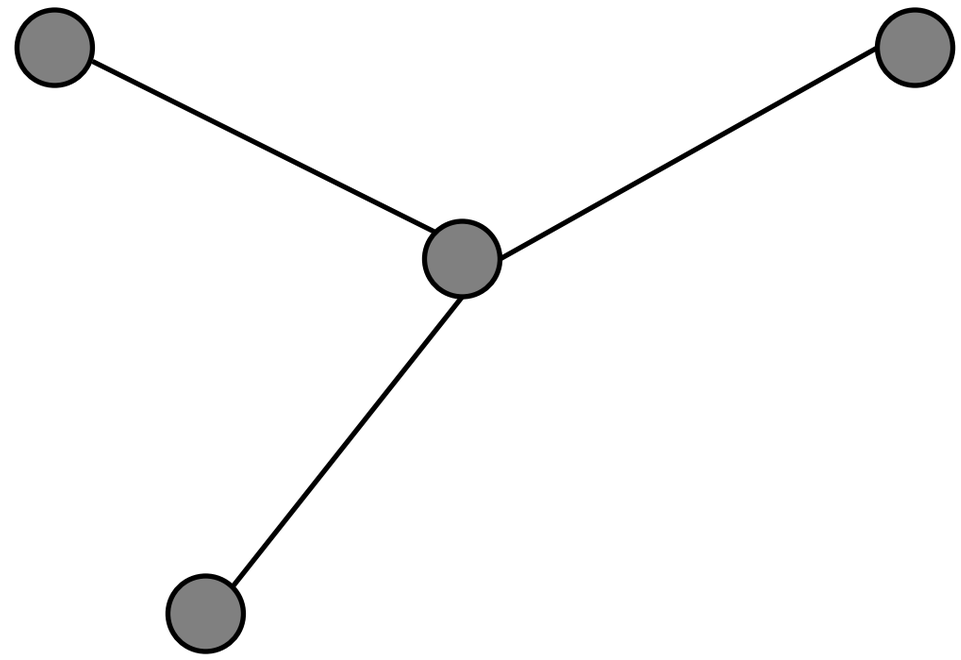
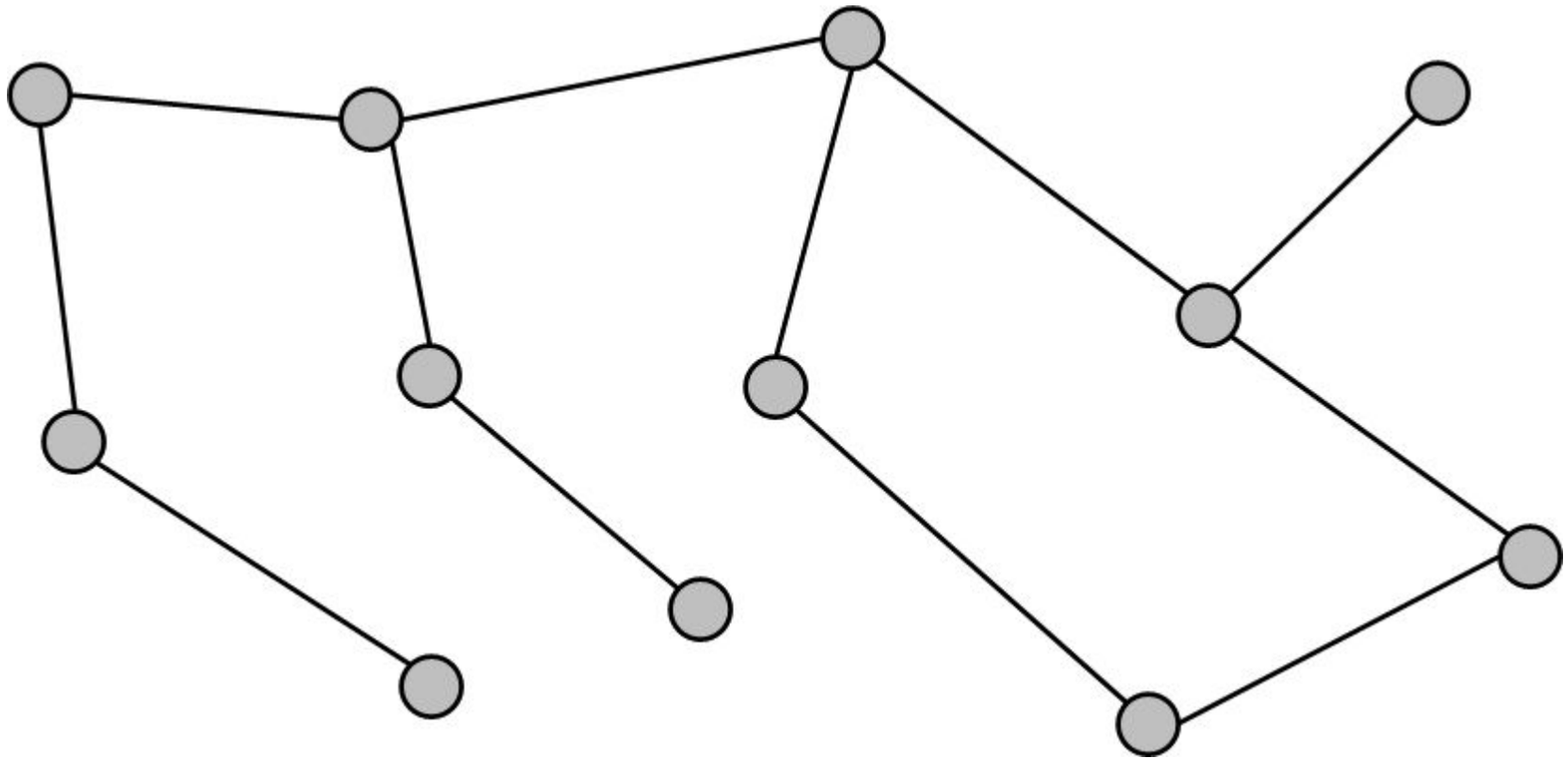


Idea

- Leaves in a tree: a node with degree 1 is called a leaf of the tree
  - Every tree has at least two leaves
  - Remove leaves and its connected edges will not change the difference between the number of nodes and the number of edges
  - Kept removing leaves from the tree, we finally obtain a graph of a single node
  - #nodes-#edges=1 in a tree

How to find a cycle in a graph?



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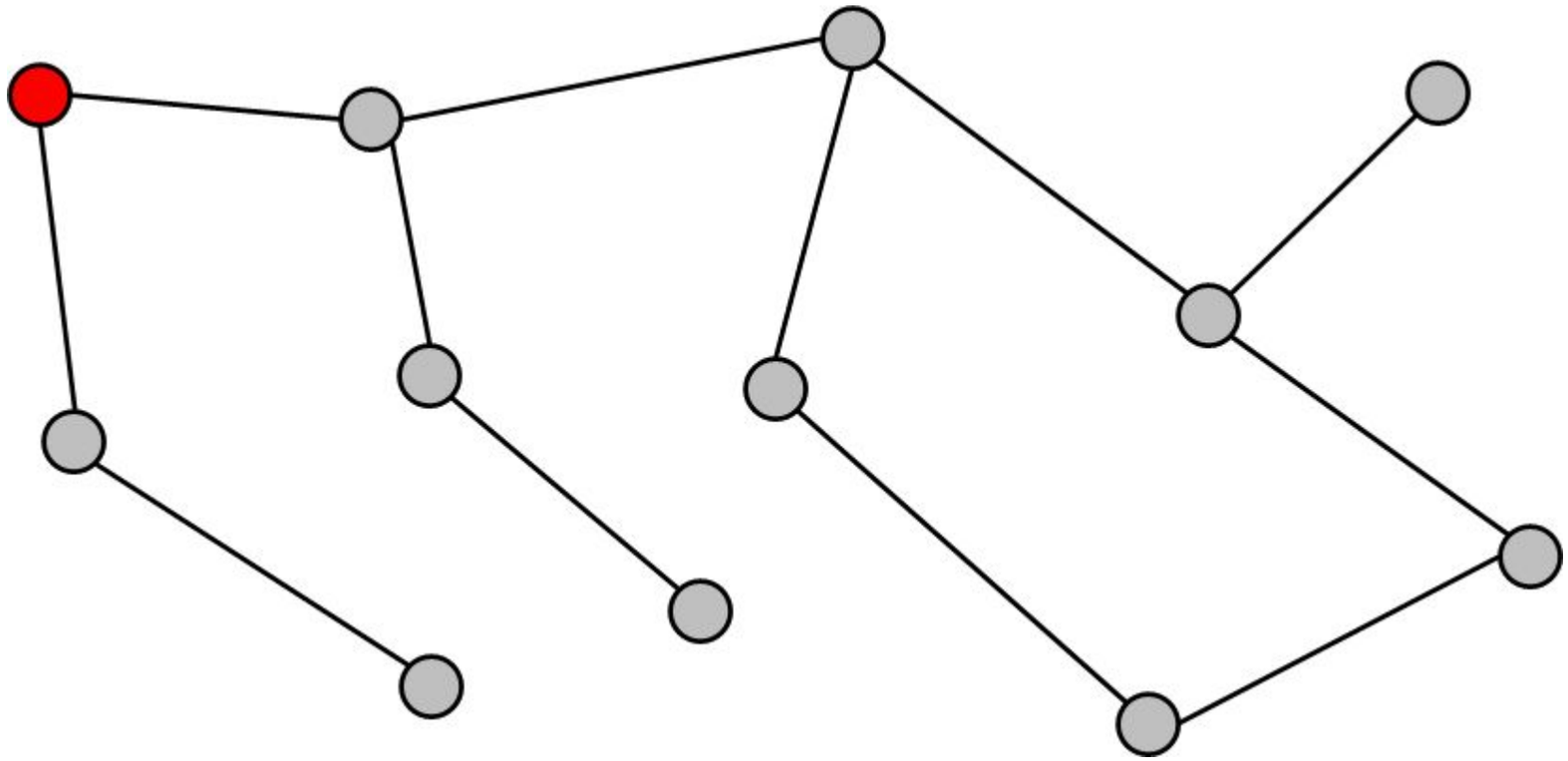

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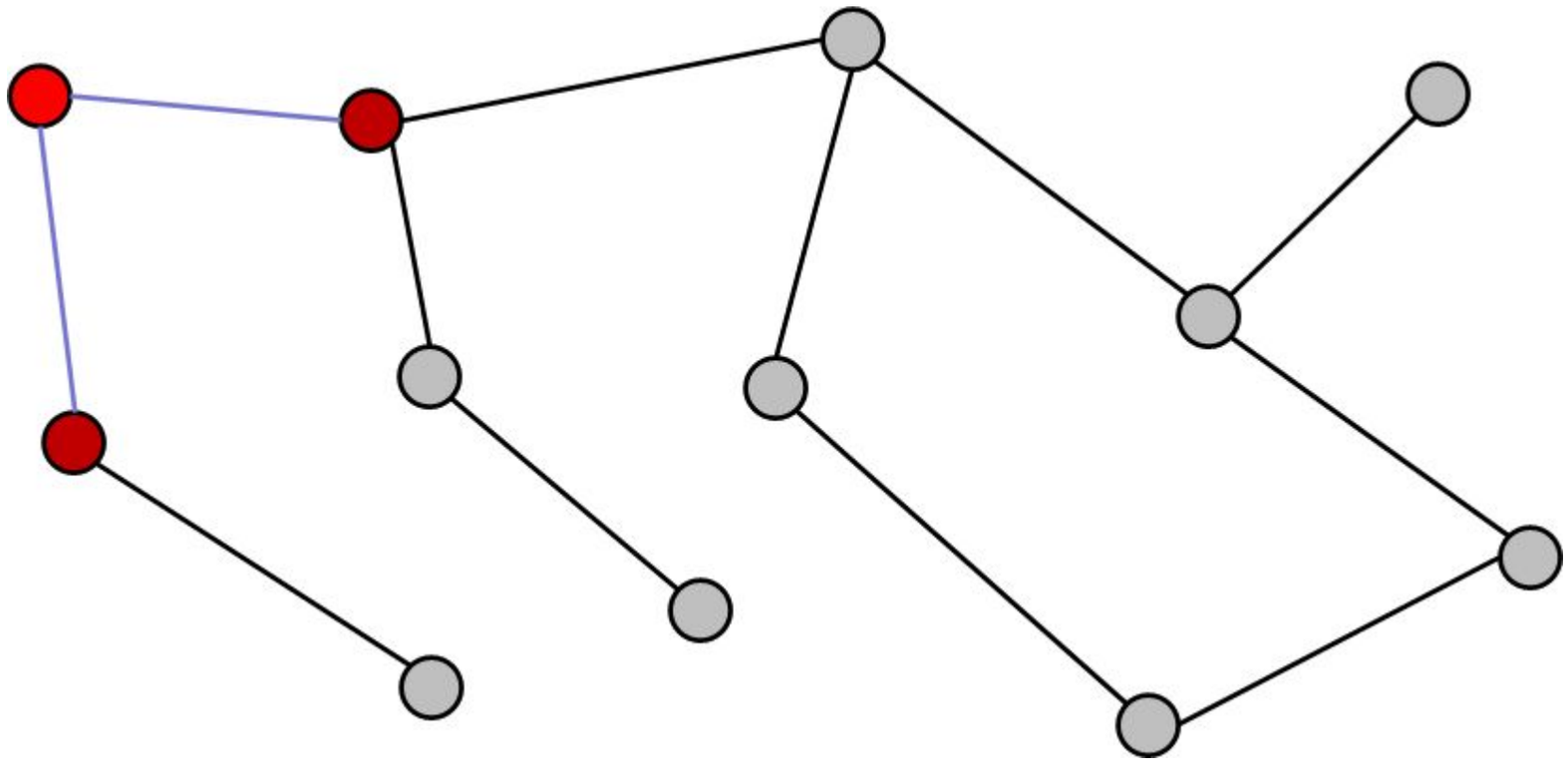

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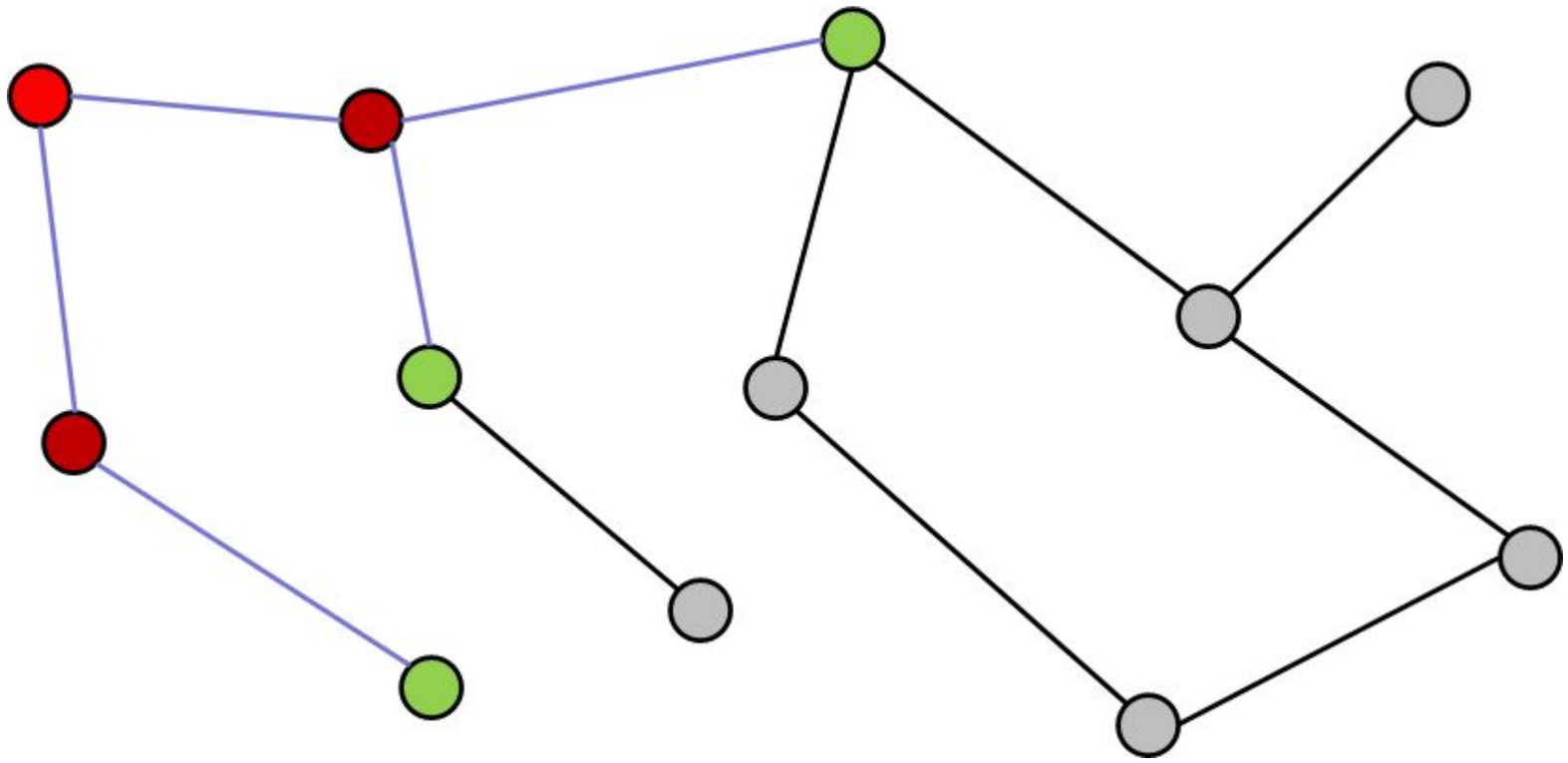

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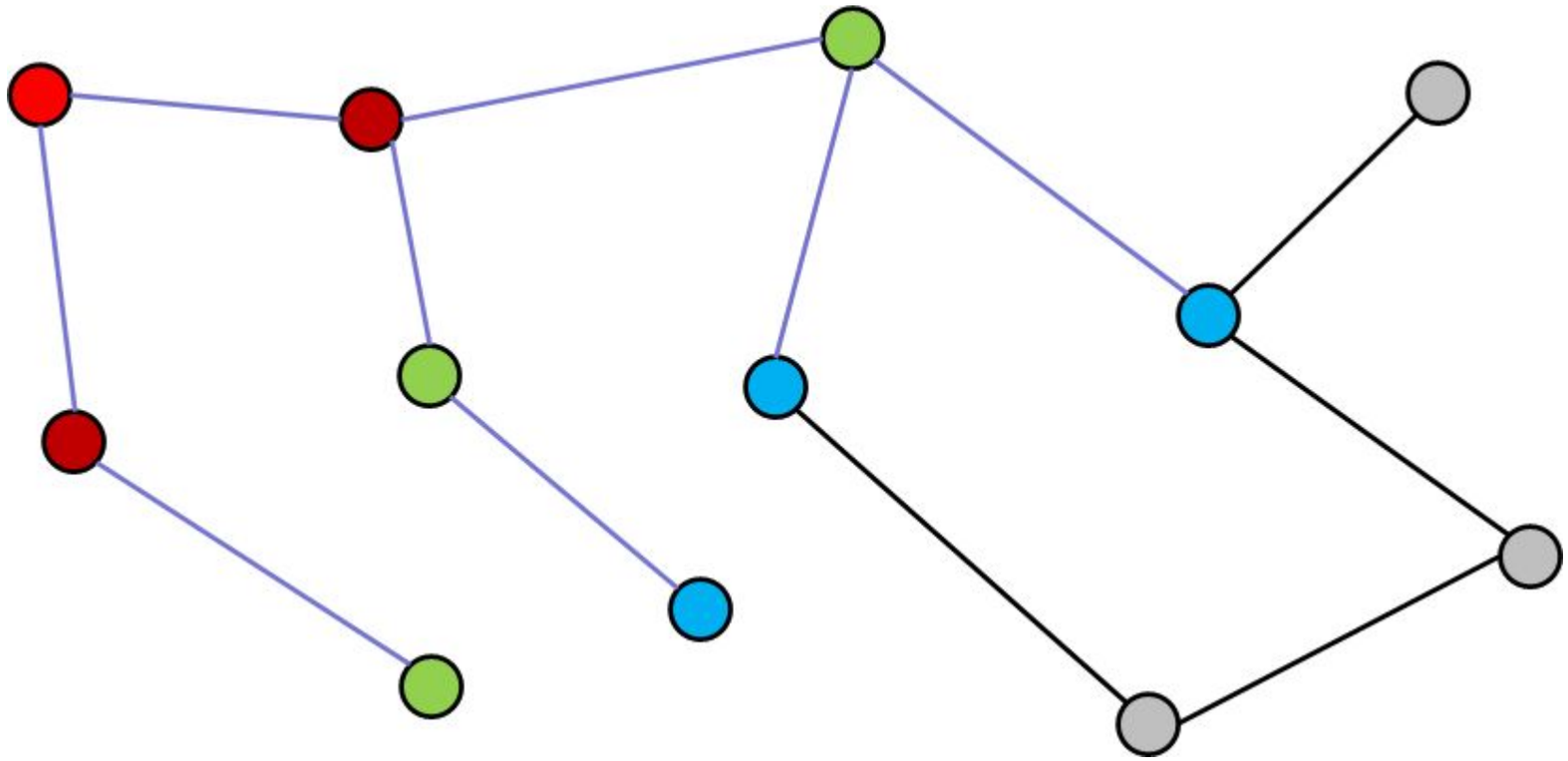

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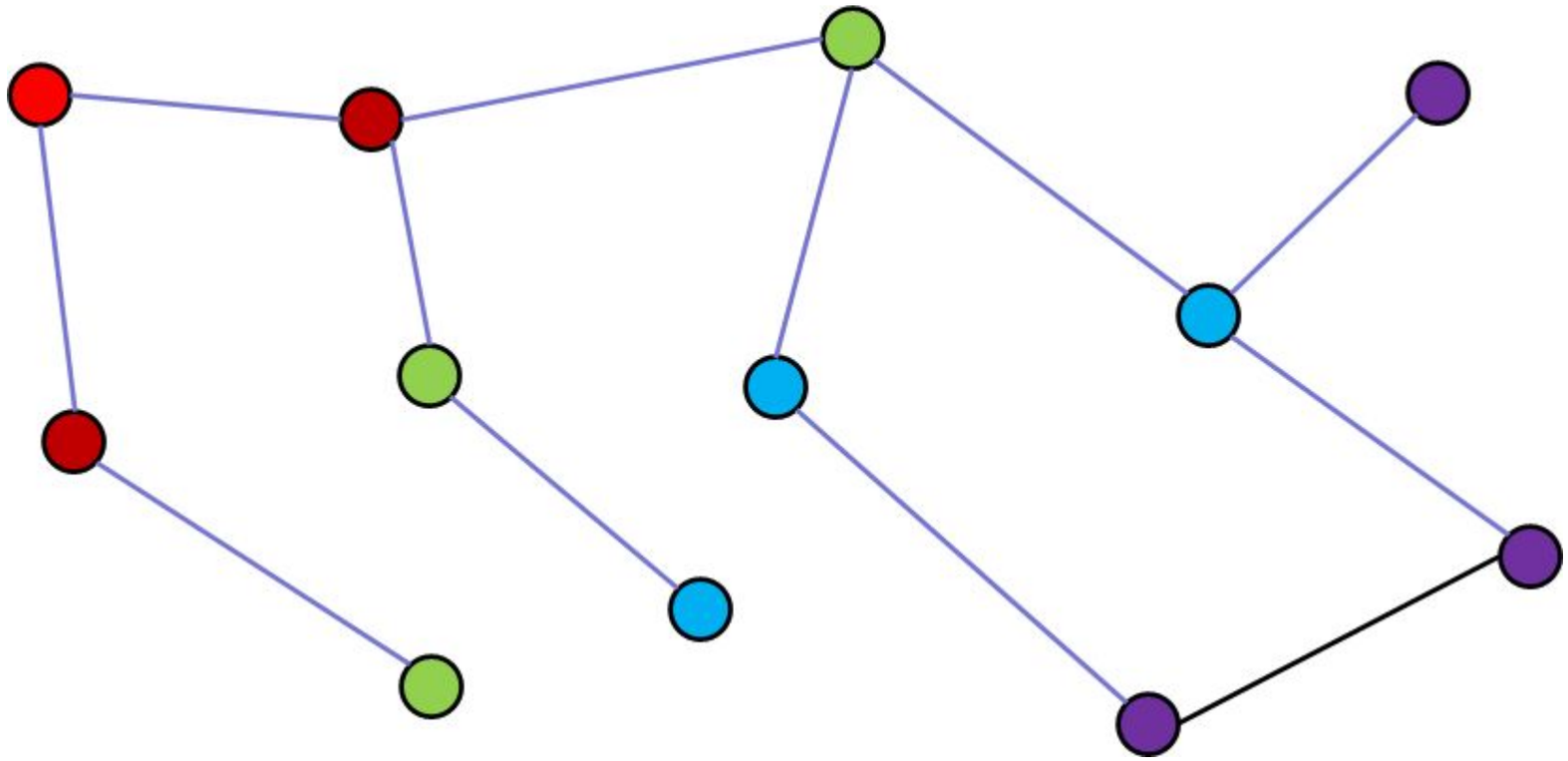

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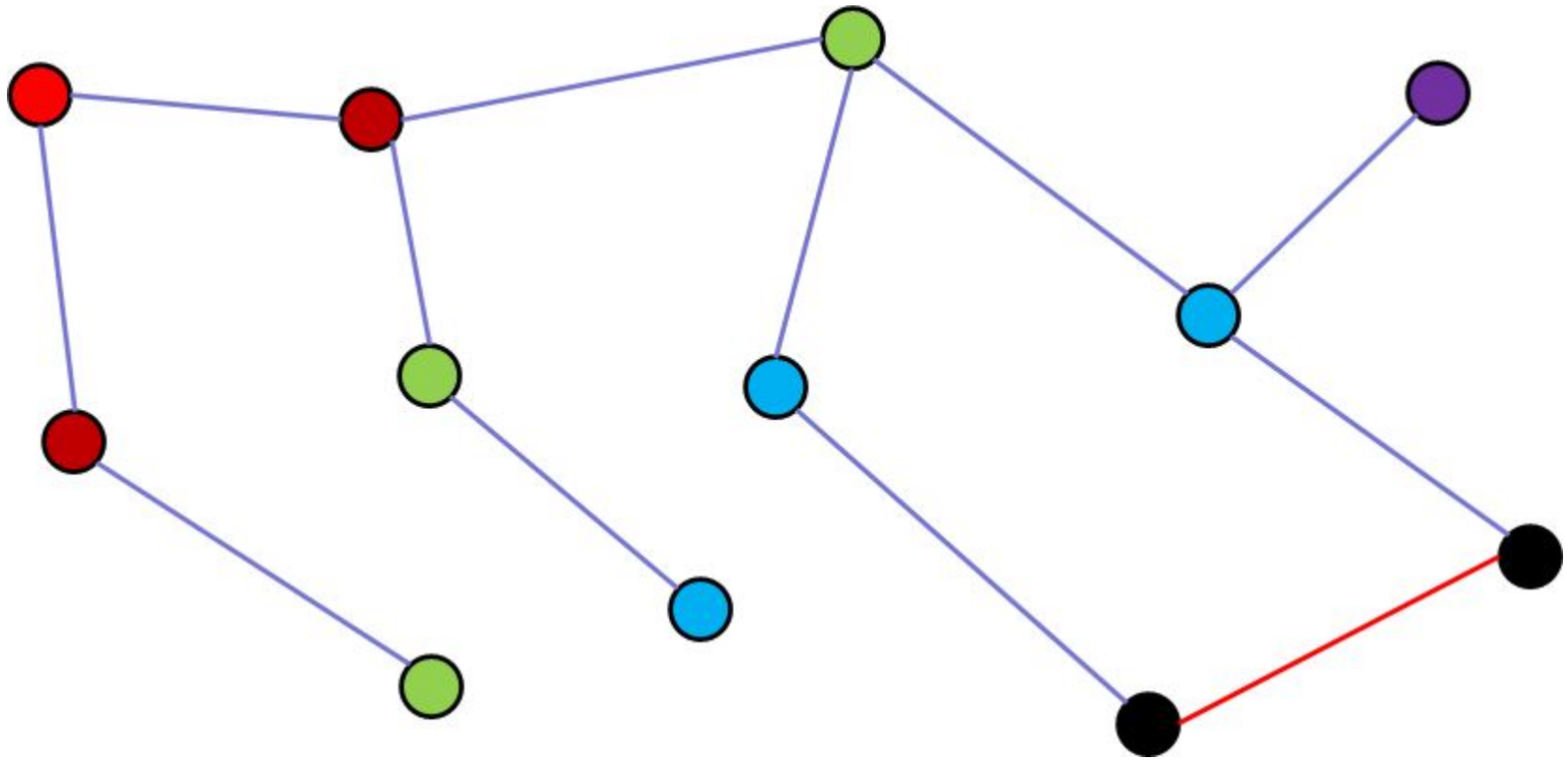

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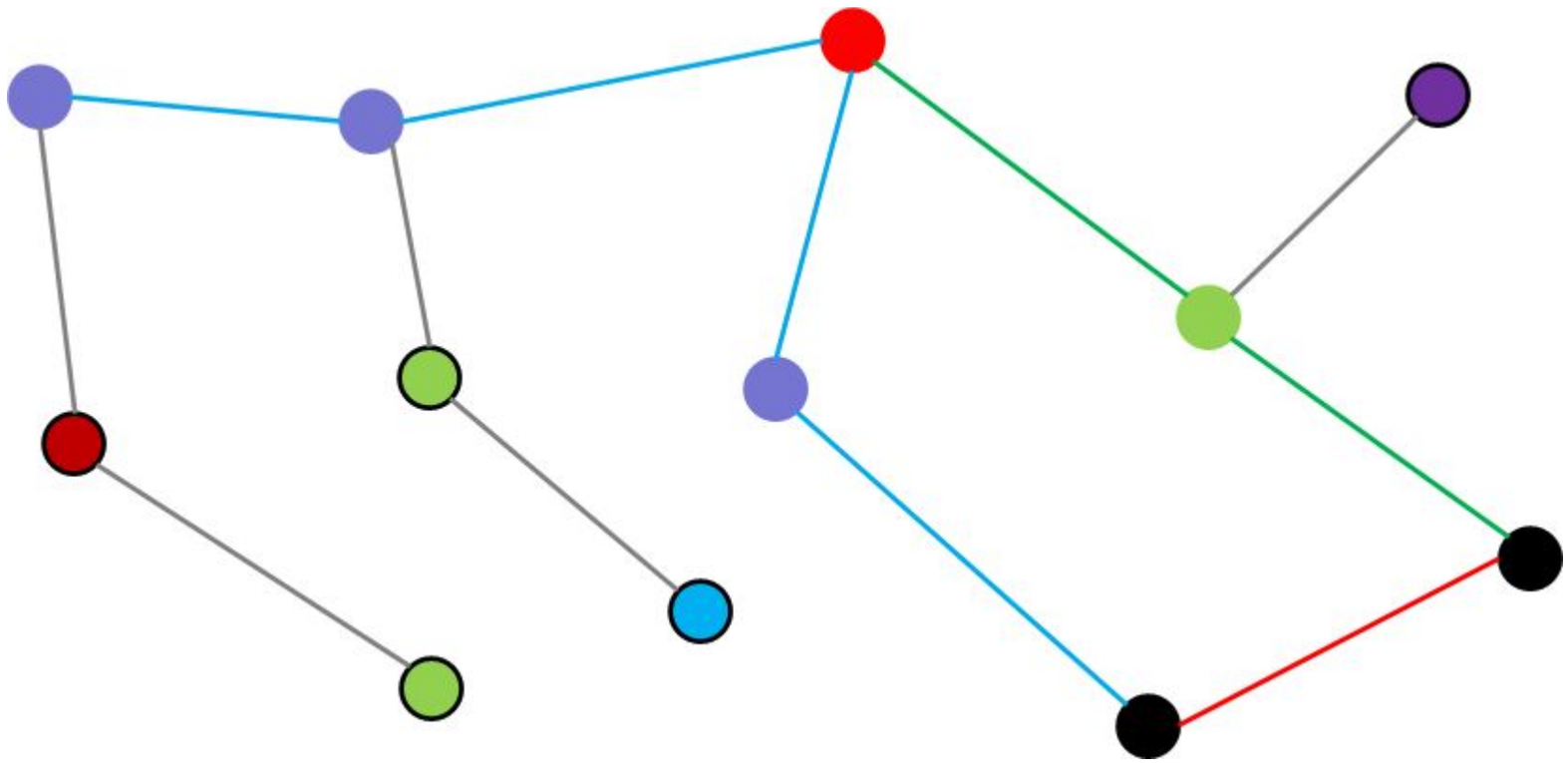

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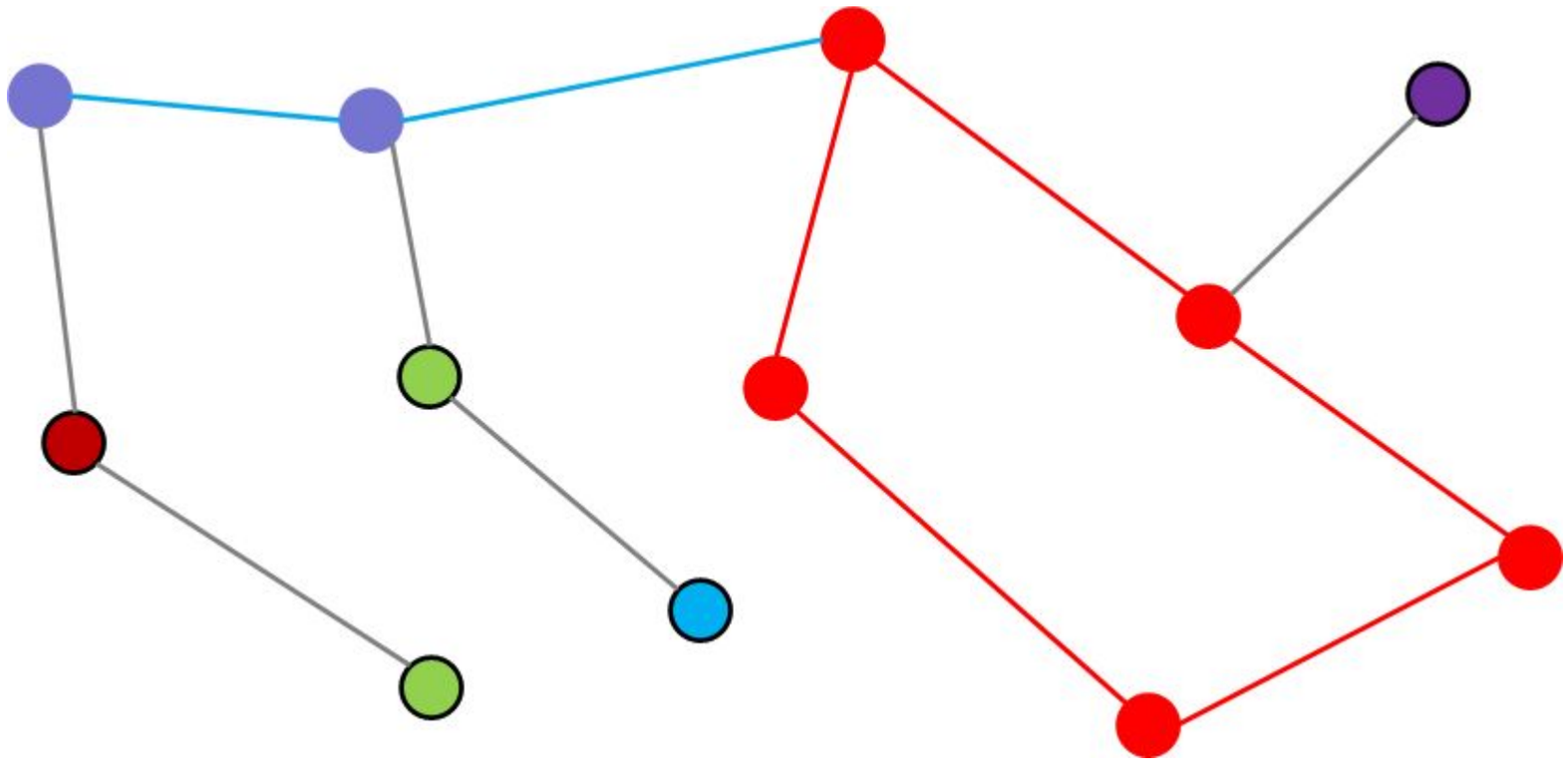

Adjacency Matrix

- When we can't draw a graph, we can represent it with a matrix.
- If row i, column j contains a 1, there is an edge between nodes i and j. If there's a 0, there's no edge.
- **Exercise**: Draw the graph for this adjacency matrix. What's the degree of each node? Is there something special about node 1? Are there any cycles? What are their lengths?

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

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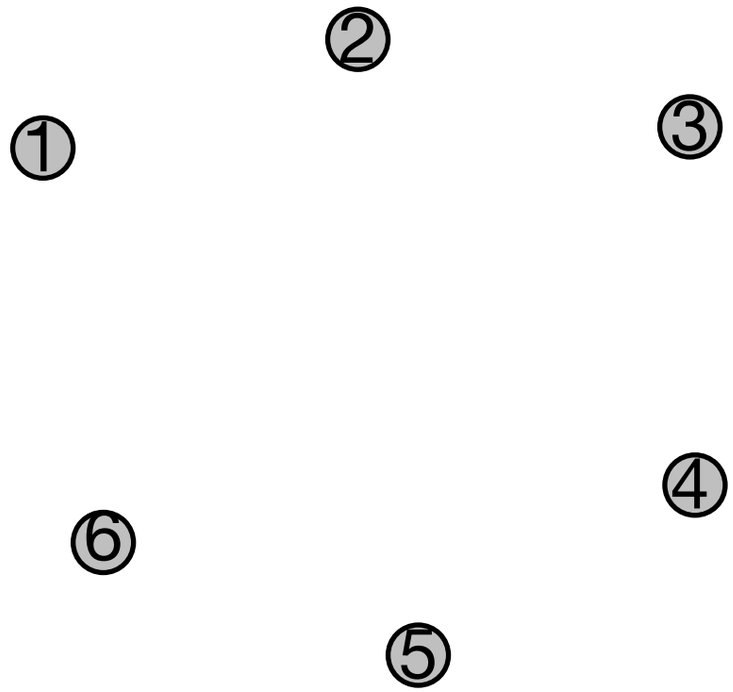

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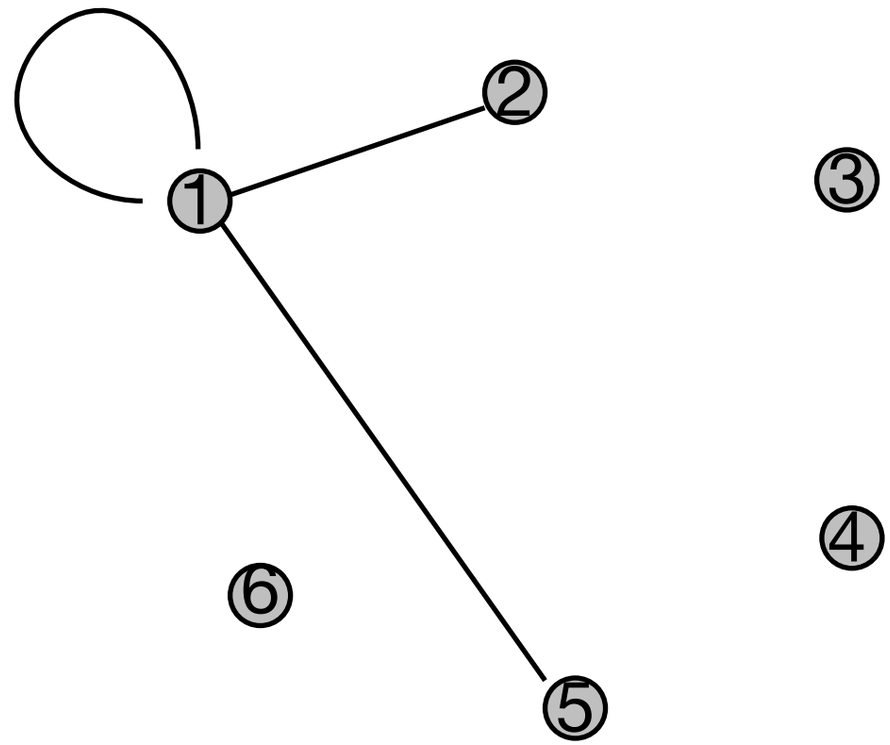

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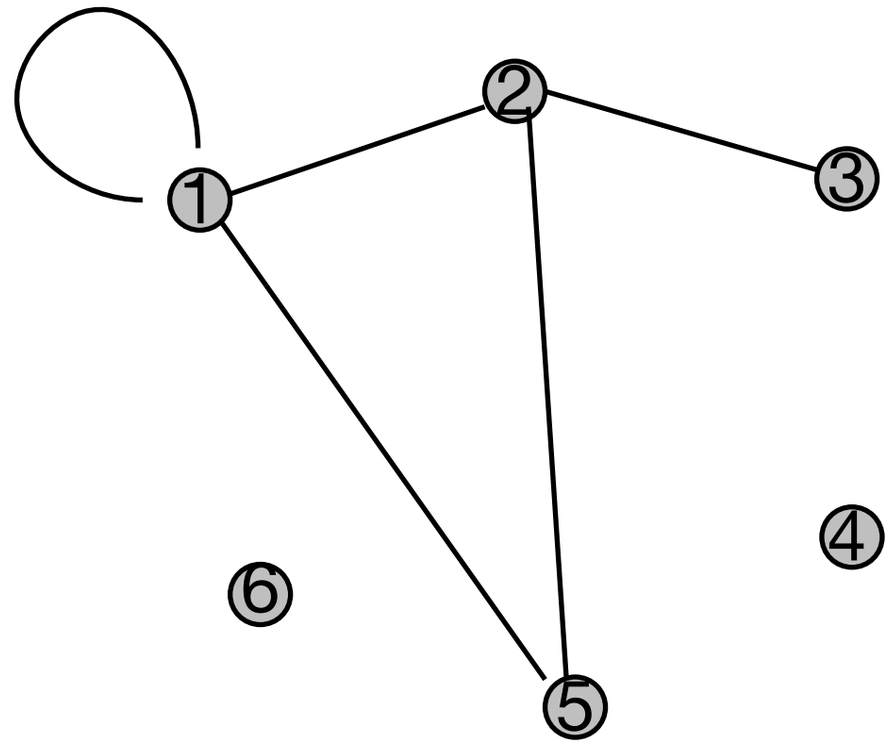

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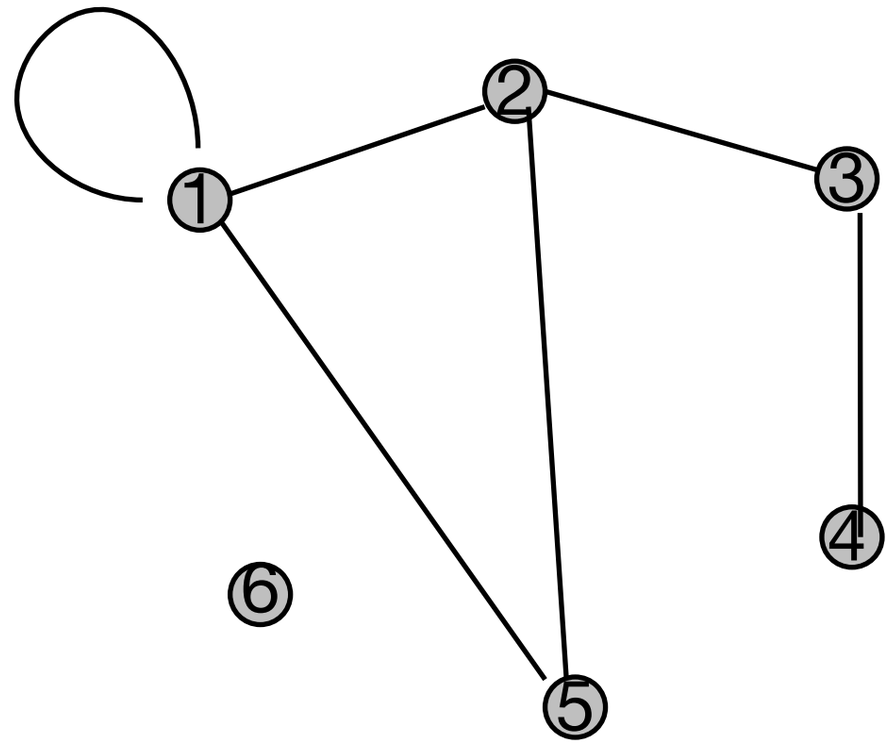

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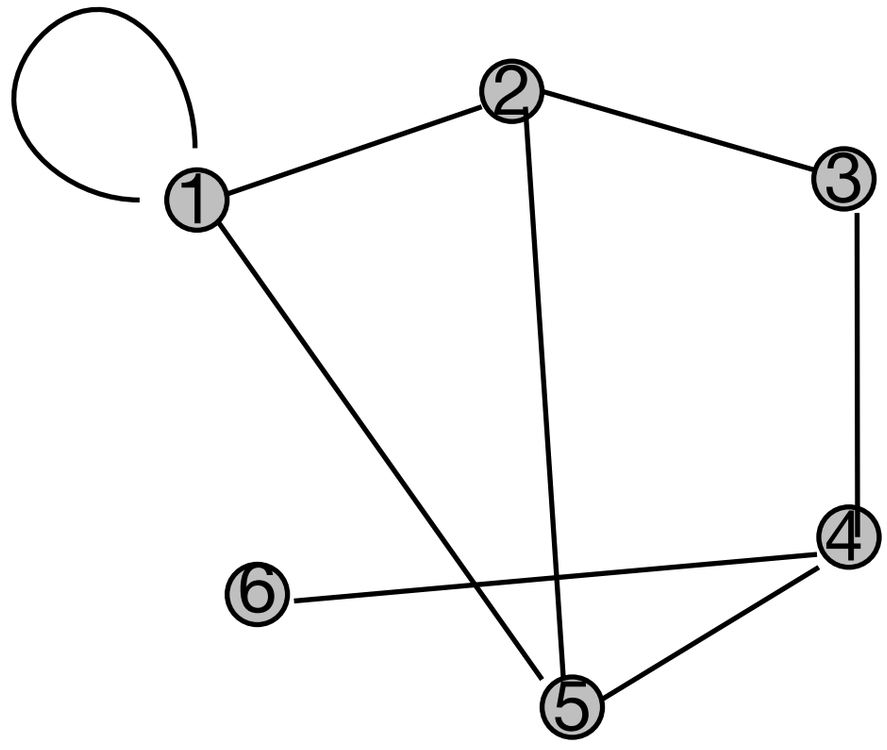

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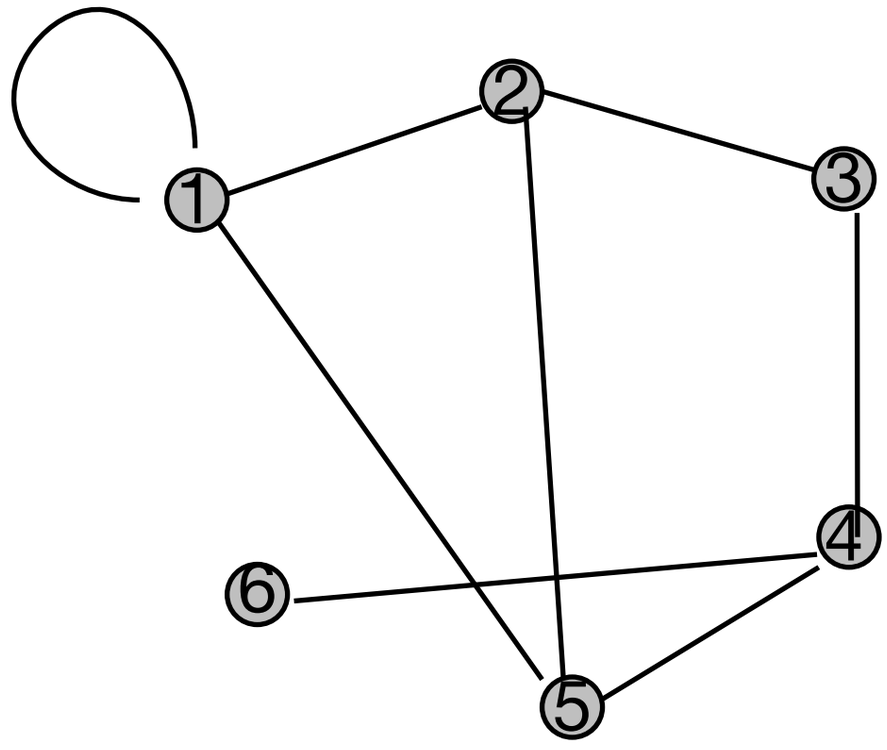


Exercise:

- What's the degree of each node? How is it related to the adjacency matrix?
- Is there something special about node 1?
- Are there any cycles? What are their lengths?
- Is the Adjacency matrix of a graph symmetric?

Further topics:

- Social Network:
  - Social Network Visualization

    http://www.science.smith.edu/dftwiki/index.php/Visualizations:_Social_Networks

- Classical Graph Theoretical Problems:
  - Seven Bridges of Konigsberg
  - Traveling Salesman Problem
  - Four Color Theorem
  - Algorithms on graphs

- What's more?
  - Algebraic graph theory: Complexity theory
  - Extremal graph theory: Cryptography, Coding theory
  - Random graph theory: Social networks, Network navigation
  - Machine learning

Exercise 1: Easy graph theory

- First, take a look at matrix0.txt in data folder
- It’s an adjacency matrix
- Now, look at LACC2021_graph_theory_day1_Exc.ipynb
  - All we’ll do is implement two simple functions:
  - matrix_load() reads the matrix.txt file and loads it into a matrix (an array of arrays)
  - print_degrees() prints the degrees of each node
  - Comments / hints are provided above each function.

- Optional: is_connected() returns 1 if the graph is connected, 0 otherwise.

Mini-Project: Shortest Path

- Next we'll do something harder.
  - Implement a function that finds the shortest path between arbitrarily two nodes:

    def shortest_path (mat,node1,node2)

    - Print out 'No such path' and return [] if node1 and node2 are not connected
    - Print out the shortest path by the edges on the path and return a list containing the nodes on the graph otherwise.

  - It's not obvious how to do this at first – try out some ideas on paper, then code it up only when you have a working algorithm that you can do manually.

  - Generate multiple random graphs to test on your program

Mini-Project: Cycle Detection

- Next we'll do something harder.
  - Implement a function that detects whether or not there is a cycle in the graph. The definition will be:

def detect_cycle(mat)

- Print out an arbitrary edge on the cycle and return a list containing all nodes on the cycle if there is one
- Print out 'No cycle in the graph' and return [] otherwise
- In the first step, assume the graph is connected and solve the problem.

- Generate multiple graphs to test on your program
- Question: Does it make any difference if the graph is directed?

Mini-Project: Cycle Detection for General Graphs

- Next we'll do something harder.
  - Bonus: solve it for more general graphs that contain multiple connected components
  - The definition will be:

def detect_cycle_gen(mat)

- The requirements are specified in CycleGen_E.py
- Generate multiple graphs to test on your program
- Question: Does it make any difference if the graph is directed?

Mini-Project: Preliminaries

- Things you might need in Python:
  - Lists
    - Suppose A is a list
    - A.append(x) appends x to A
    - The expression (x in A) returns True if A contains x, otherwise it returns False
    - A[n] returns the (n+1)-th element of A
    - The elements of a list could also be lists
      - A=[2,3,[4,5],6,[7,[8,9]]]
      - A[0]=2,A[1]=3,A[2]=[4,5],A[3]=6,A[4]=[7,[8,9]]
      - A[2][0]=4,A[2][1]=5
      - A[4][0]=7,A[4][1]=[8,9]
      - A[4][1][0]=8,A[4][1][1]=9
  - Reading files
    - Filename “matrix.txt”
    - Fh=open(“matrix.txt”)
    - A=Fh.readlines() will return a list containing all the lines of the file

Mini-Project: Preliminaries

- Things you might need in Python:
  - Reading files
    - Filename “B.txt”
    - Fh=open(“B.txt”)
    - A=Fh.readlines() will return a list containing all the lines of the file
    - Example
    - If the file “B.txt” contains the following text:

Hello world!
010001
Python

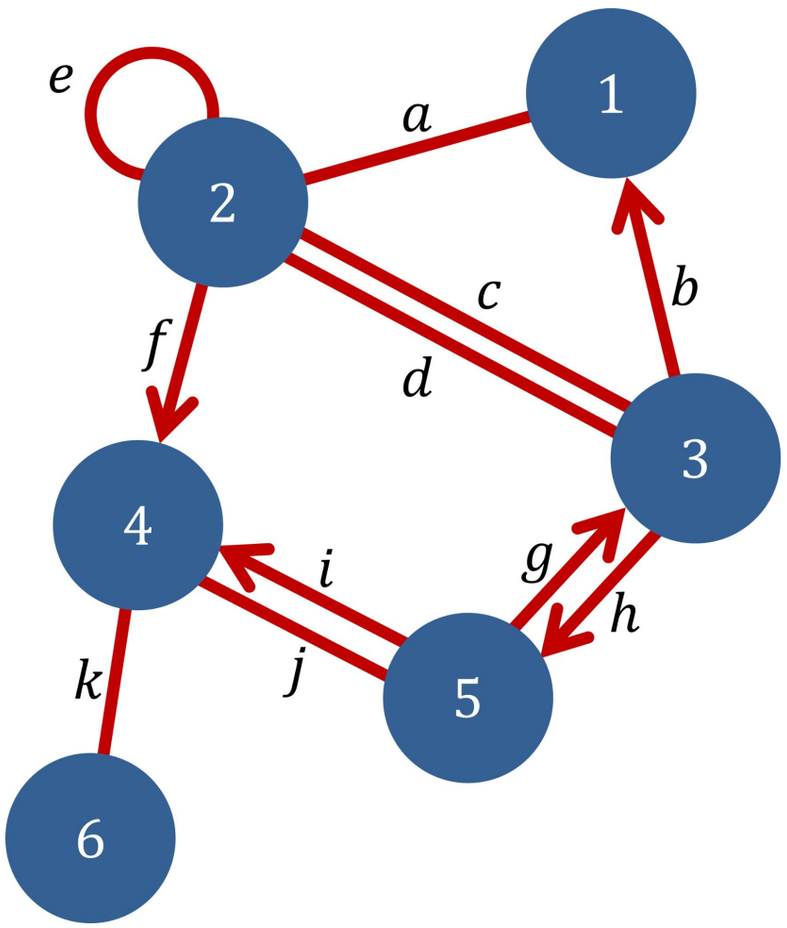
    - Then A=[‘Hello world!’, ‘010001’, ‘Python’]
      - Each component of A is a string

References

- Graphs: http://people.brunel.ac.uk/~mastjjb/jeb/or/graph.html
- Graph Theory: http://en.wikipedia.org/wiki/Graph_theory
- Kevin Bacon: http://oracleofbacon.org/

Quick Recap

- Last day, we learnt about:
  - Graphs: Vertices, Edges, Weights, Directivity
  - Degree, Connectivity
  - Shortest Path, Cycles, Tree
  - Adjacency Matrix



Contents

- Common random graphs:
  - Barabasi-Albert Networks with Preferential Attachment
  - Erdos-Renyi Networks

- Community Structures and Modularity

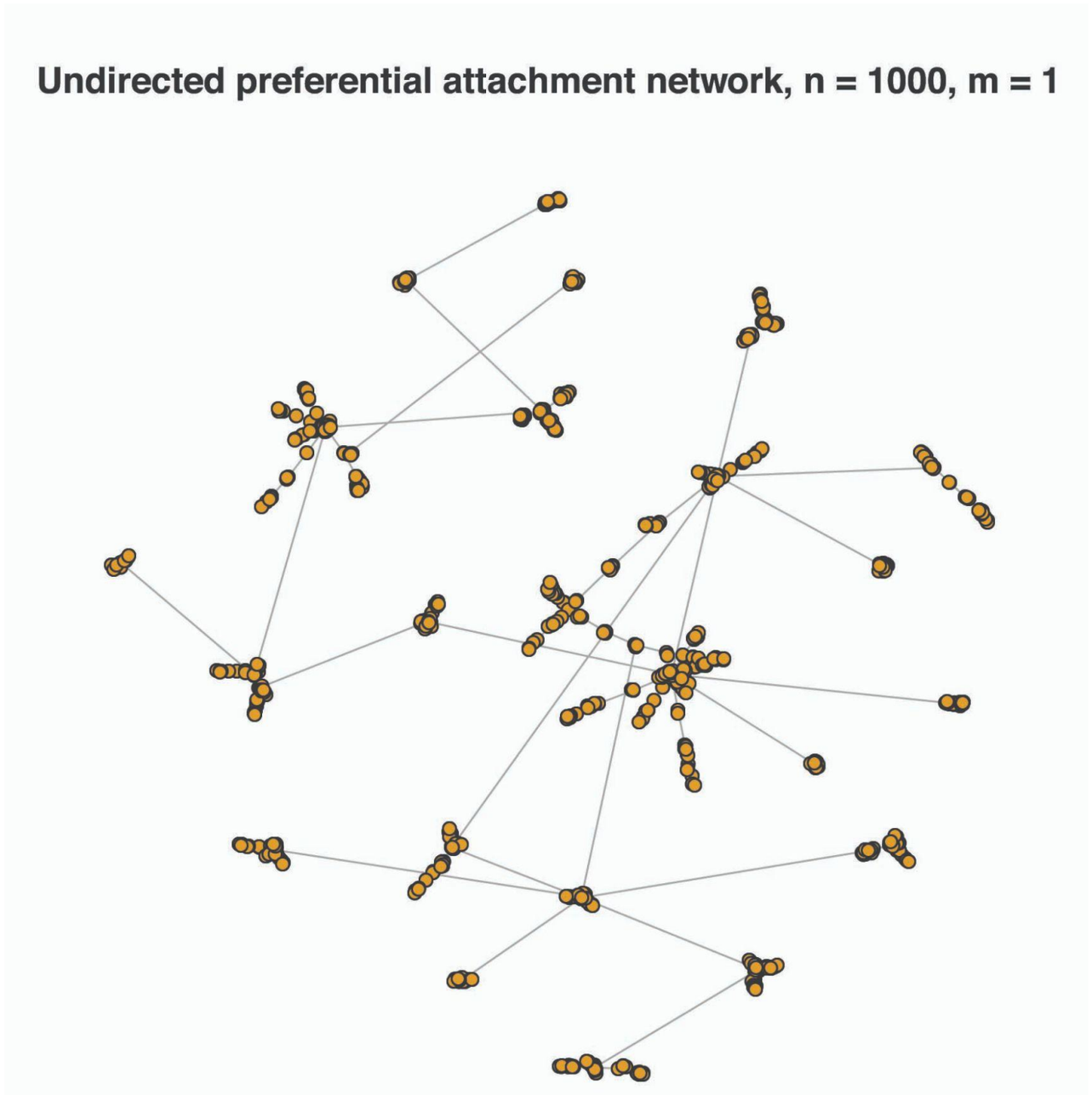
- Minimum Spanning Tree (MST)
- Shortest Path: Dijkstra's Algorithm

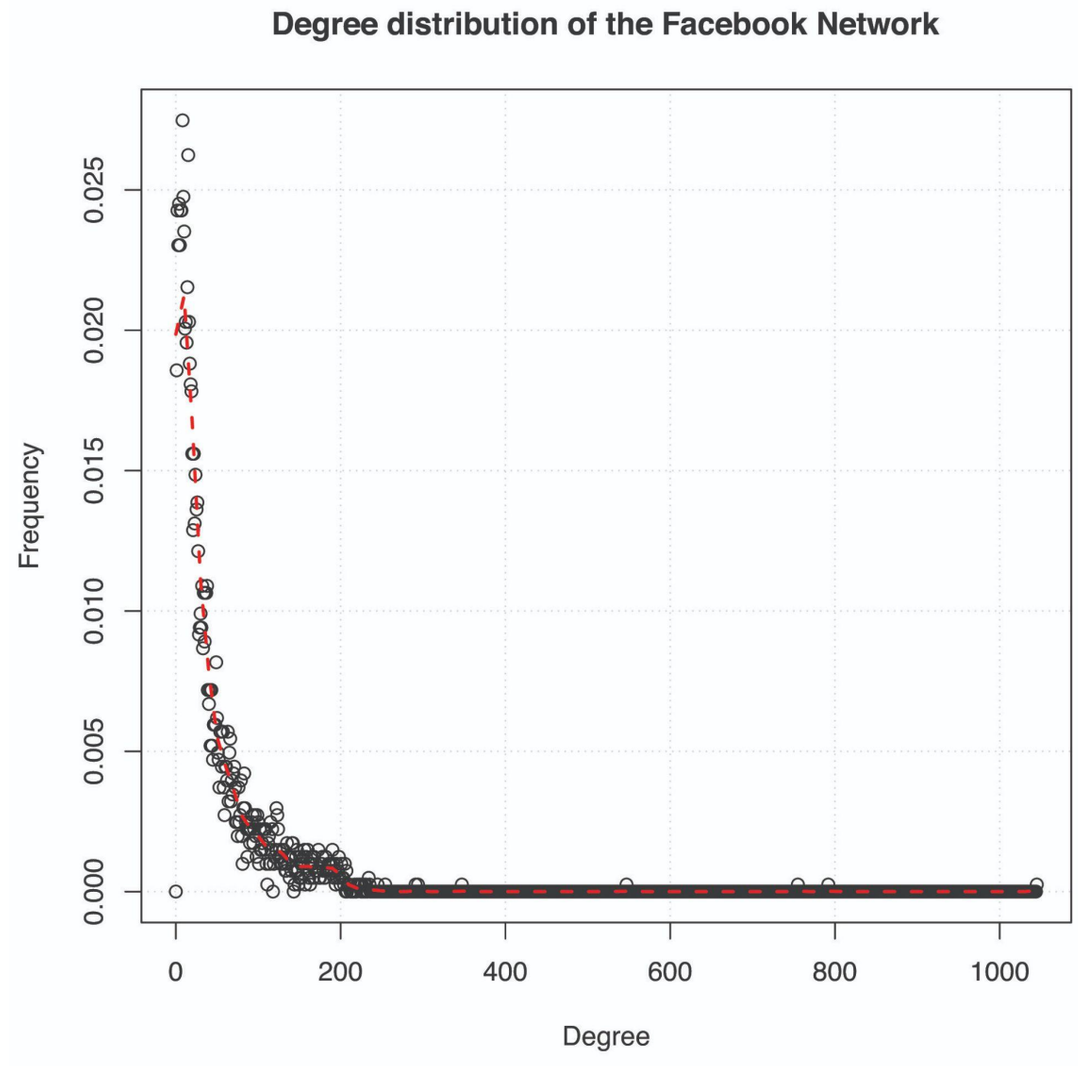
- Traveling Salesman Problem (TSP)

Preferential attachment

- Popular nodes act as bridge between unpopular nodes.
- The more popular a node, the higher is the probability for it to be connected to incoming nodes.
- Popularity is measured by the degree of a node.
- The Barabasi-Albert model creates **scale-free**, **evolving** and **connected** networks with preferential attachment.
- The degree distribution of such networks follow power-law.

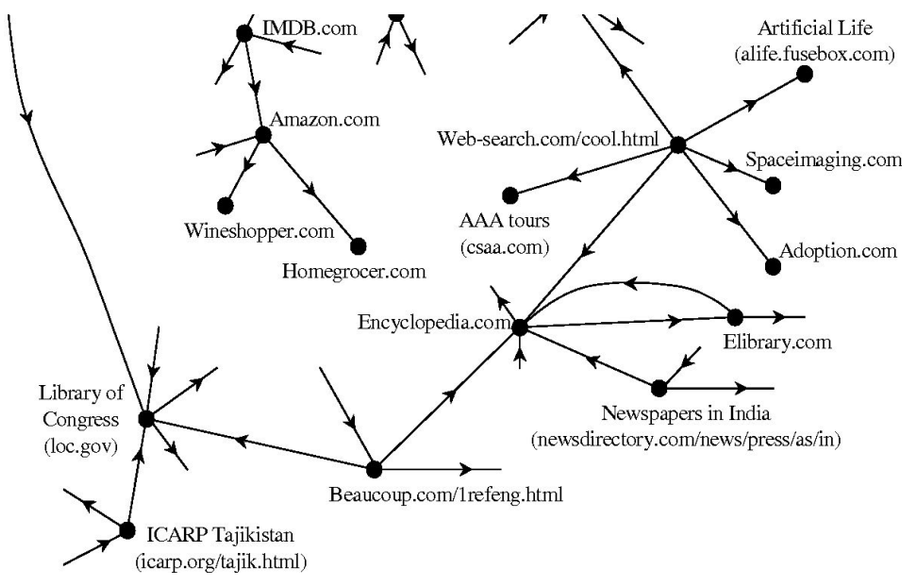
Preferential attachment

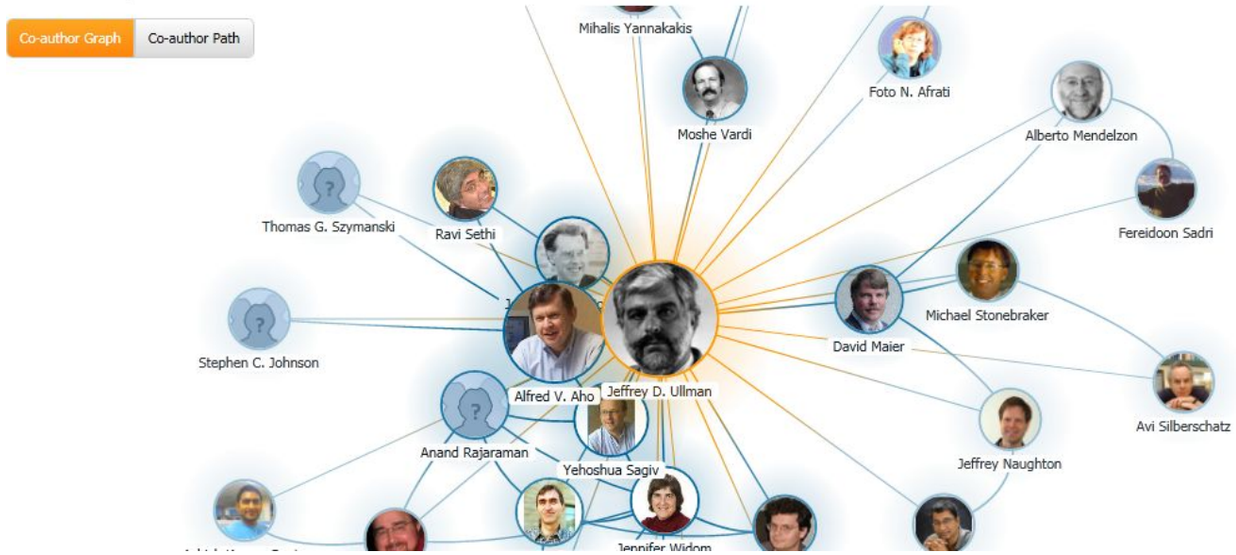




Applications

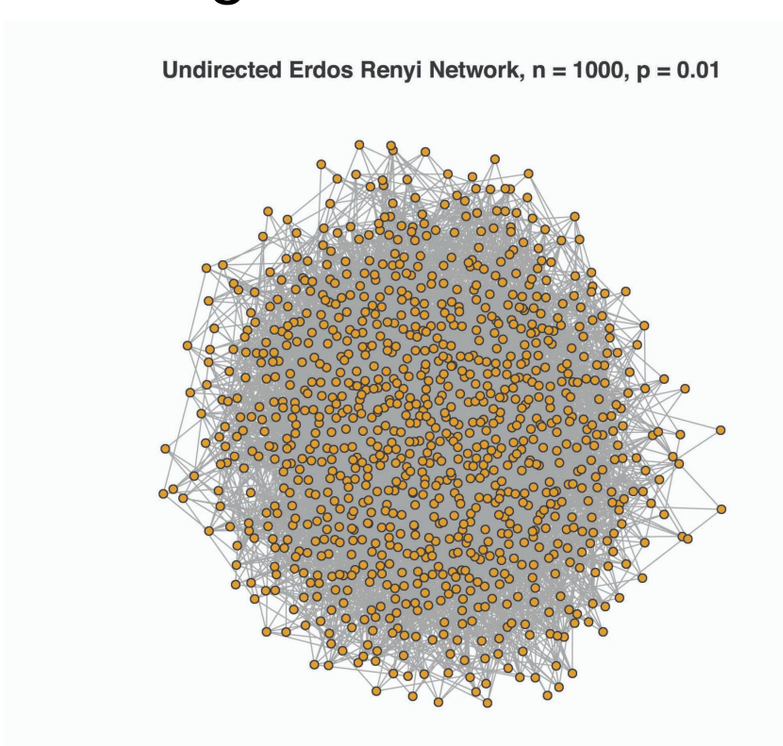
- Social Networks
- Search engines (PageRank)
- World Wide Web
- Citations among researchers
- Internet
- Recommendation Systems
- Family / Erdos Trees

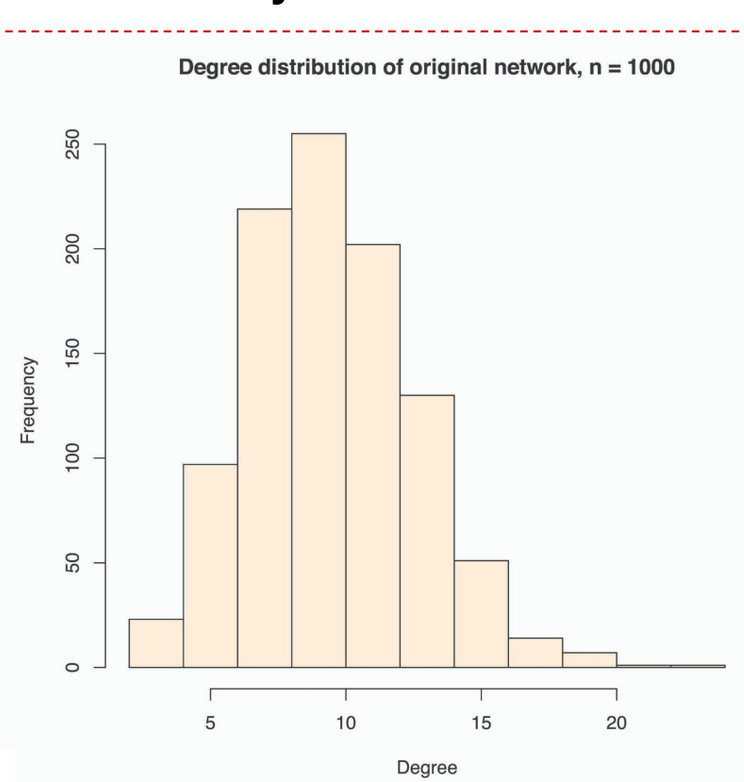


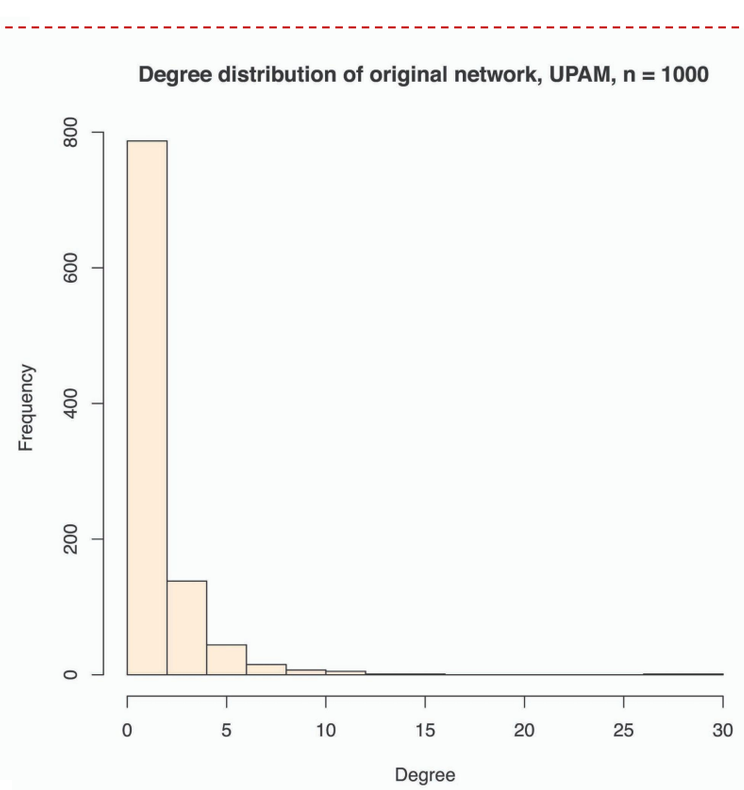


Erdos-Renyi Models

- All edges have a certain probability of being present or absent (probability of connectedness).
- Network may or may not be fully connected; concept of Giant Connected Component (GCC)
- Degree distribution of Erdos-Renyi networks are binomial or Poisson.



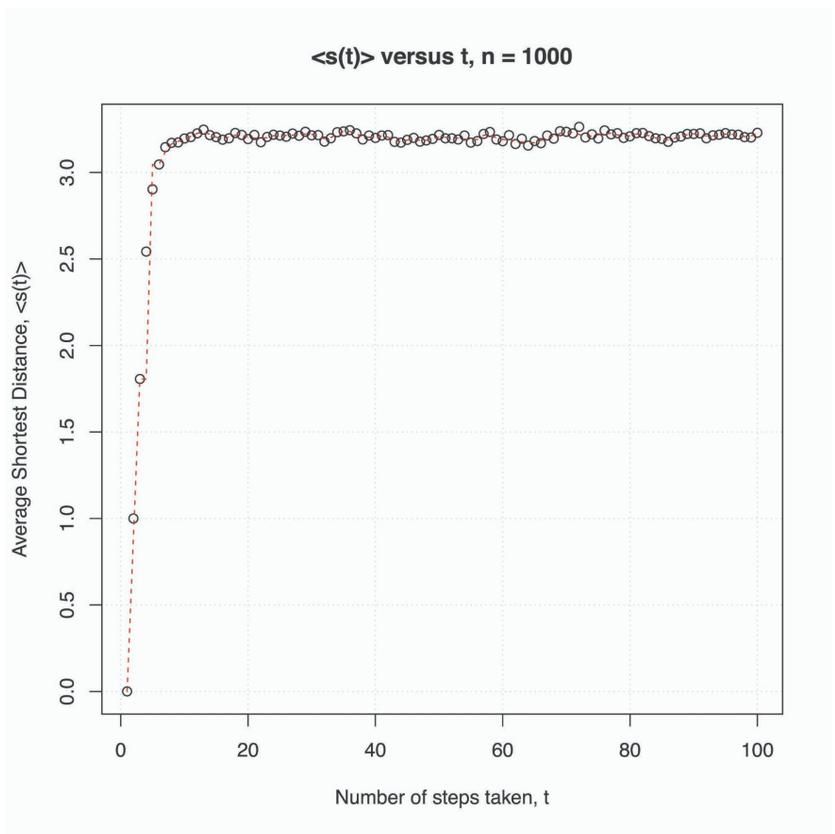


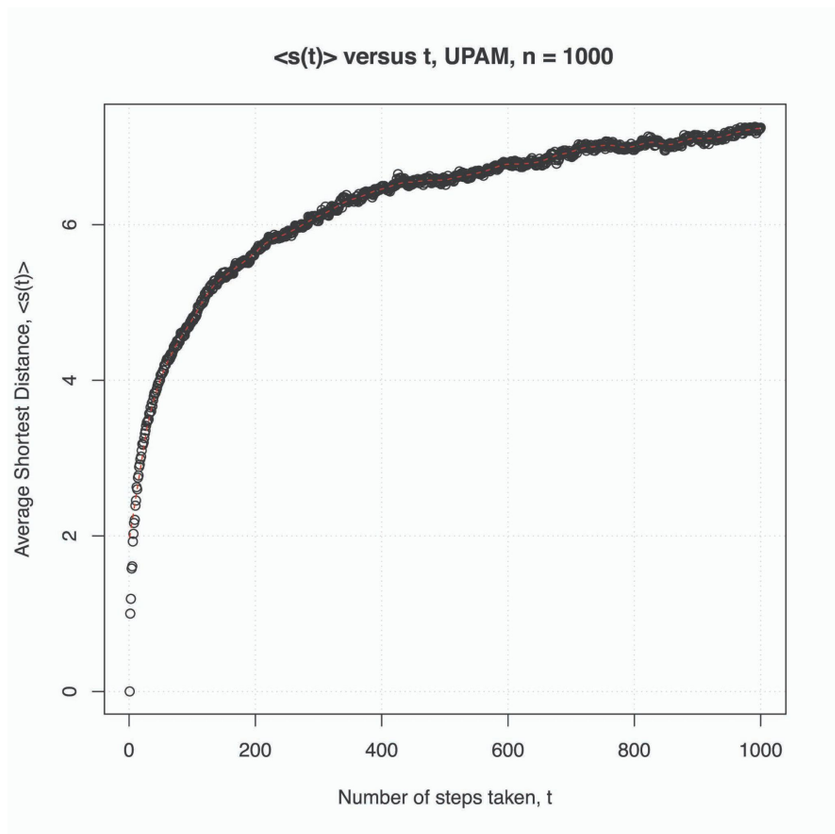


Random Walk on Random Graphs

- If you walk randomly on an Erdos-Renyi graph, you will likely end up within a “circular region” corresponding to the average shortest distance of the nodes in the entire network from an initial node.

- However, for Barabasi-Albert graphs, your average shortest distance will increase non-monotonically with more steps from the initial node.





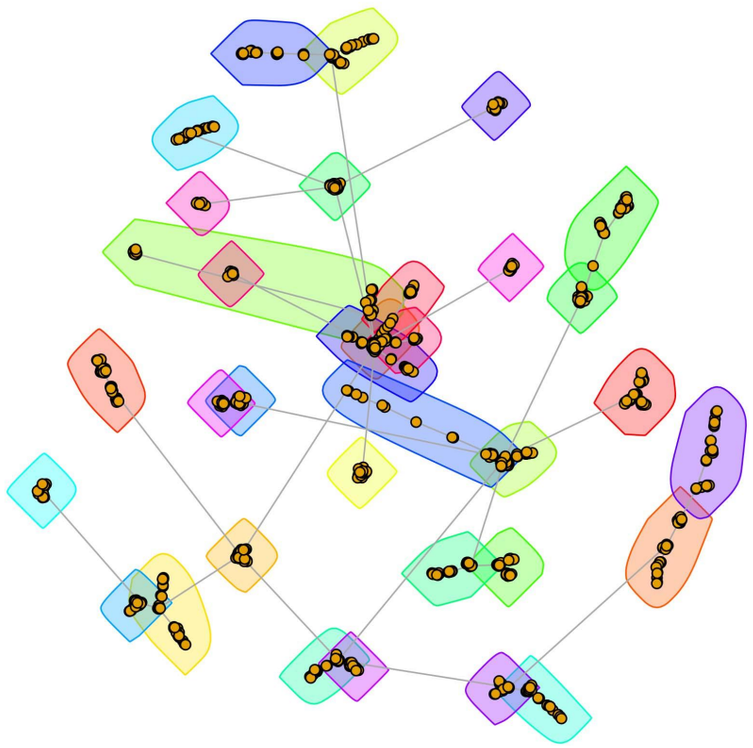
Community Structures and Modularity

- Community structures: clusters or regions of high connectedness in the graph
- We measure the strength of clustering using modularity score.

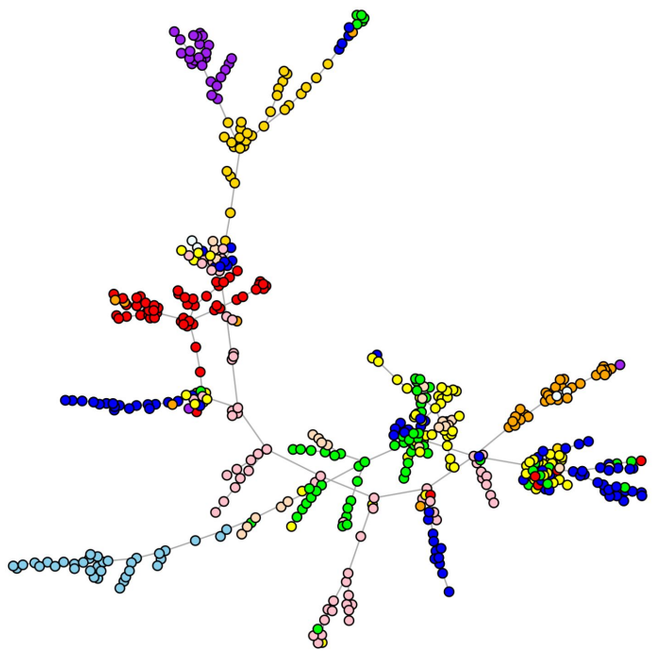
- Application example: Correlation graph of stocks

- Techniques: Fast-Greedy, InfoMap, Edge-Betweenness

Community Structure of UNPA, n = 1000, m = 1



Example of stock market correlation graph MST

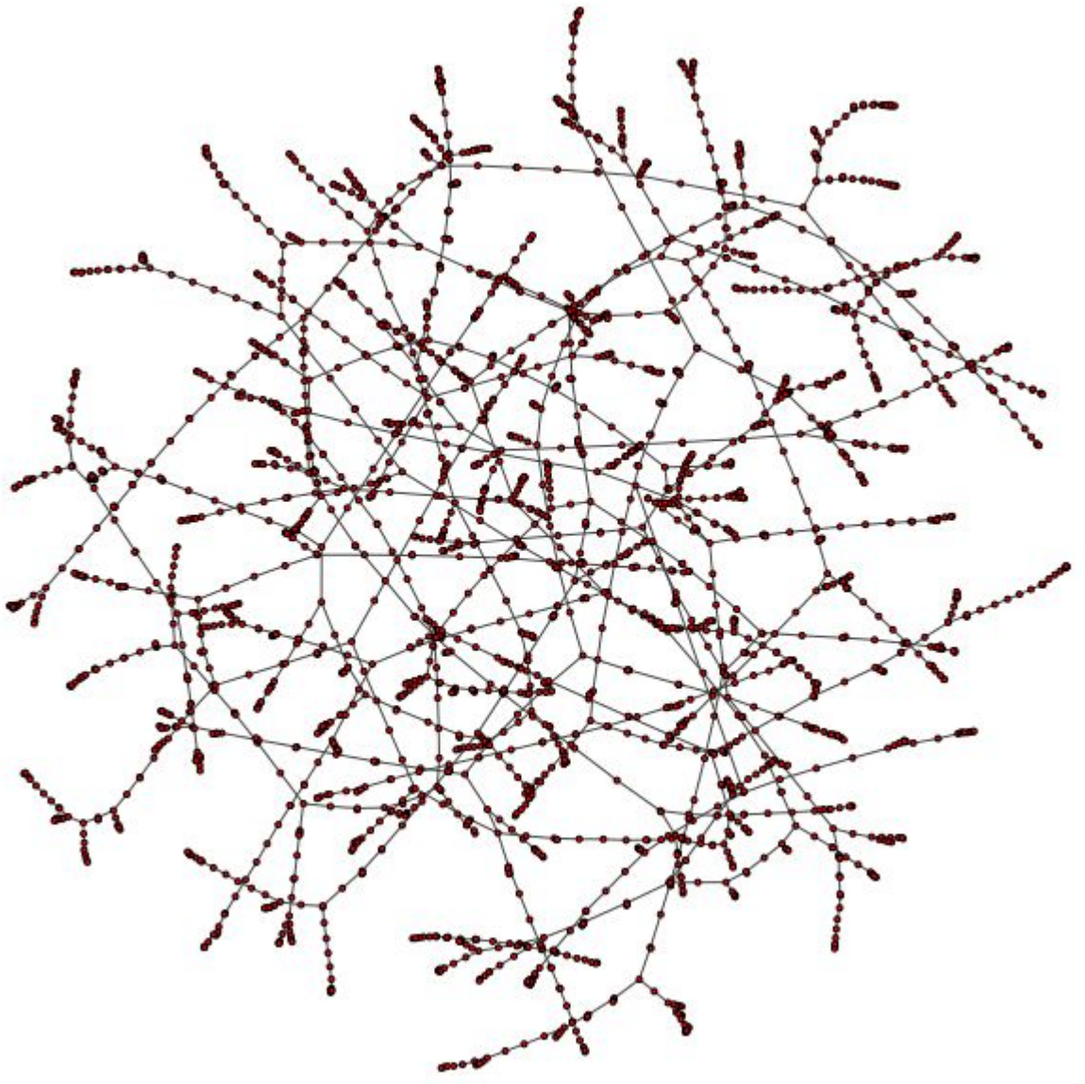


Minimum Spanning Tree (MST)

- MST: Connects all the nodes of a connected and edge-weighted undirected graph:
  - **without any cycles**
  - **with the lowest possible cumulative edge weights (lowest cost).**

- Techniques to find MST:
  - Prim’s Algorithm
  - Kruskal’s Algorithm

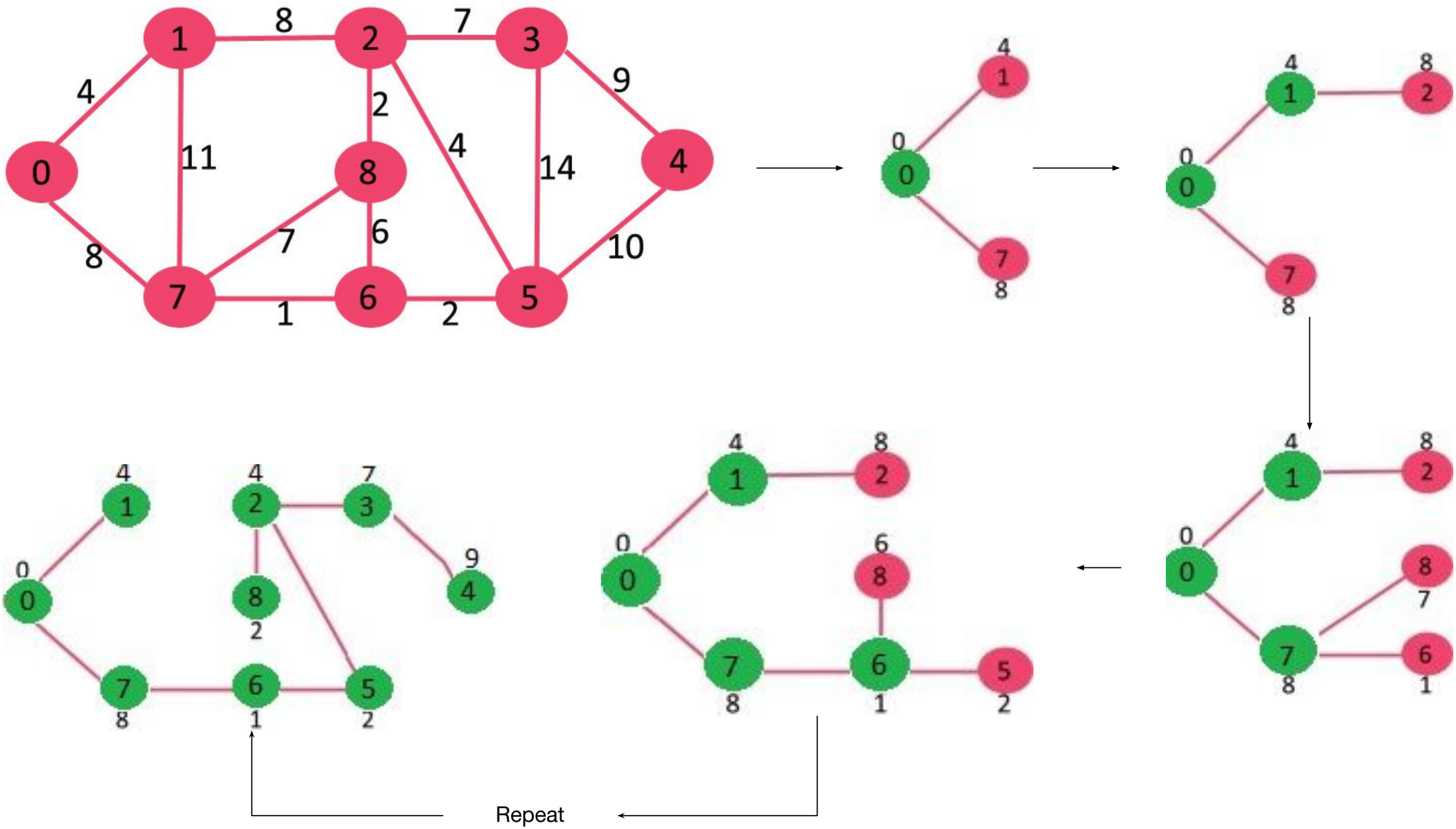
- Prim’s Algorithm (uses two sets):
  - Find a cut
  - Pick the edge with minimum weight.
  - Include the edge in the list of included vertices.



MST of Uber Dataset

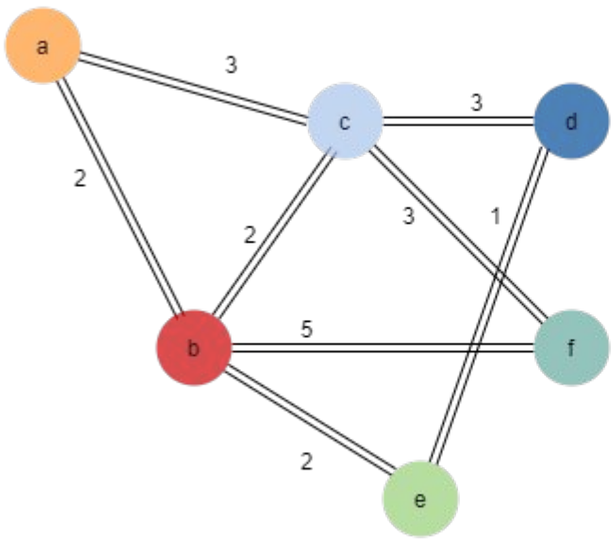
https://www.geeksforgeeks.org/prims-minimum-spanning-tree-mst-greedy-algo-5/

Prim's Algorithm

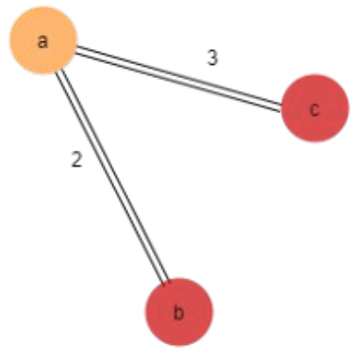


https://medium.com/star-gazers/graph-theory-finding-the-shortest-paths-afb35d28176b

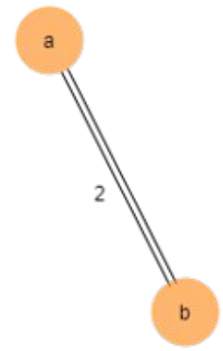
Dijkstra's Shortest Path Algorithm



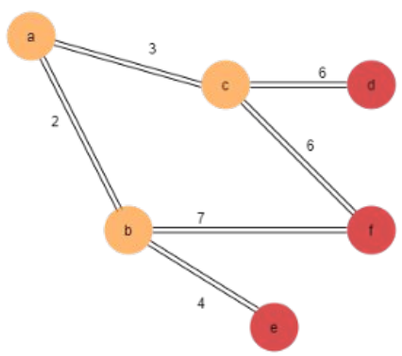
Edge		Weight
a	b	2
a	c	3
b	c	2
b	e	2
b	f	5
c	d	3
c	f	3
d	e	1



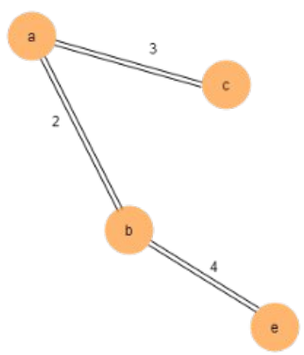
Edge		Weight
a	b	2
a	c	3



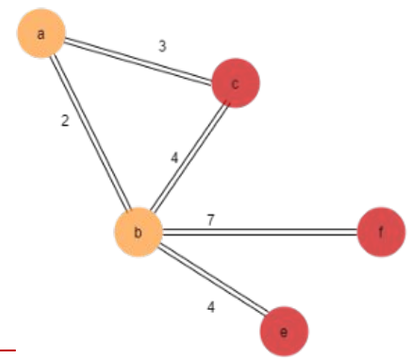
Edge		Weight
a	b	2



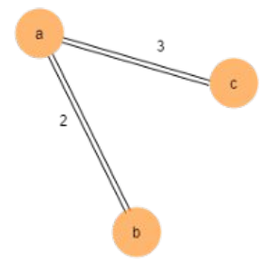
Edge		Weight
c	d	6
c	f	6
b	e	4
b	f	7



Edge		Weight
a	b	2
a	c	3
b	e	4



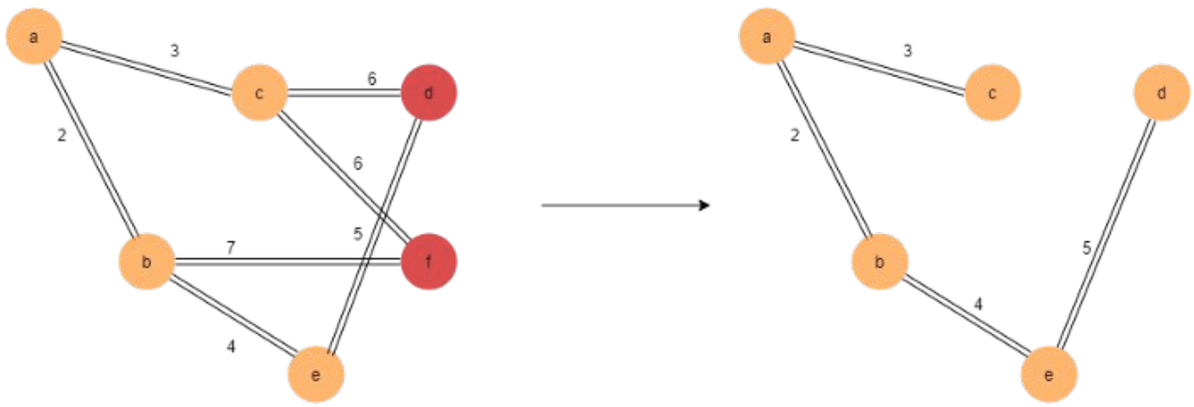
Edge		Weight
a	c	3
b	c	4
b	e	2
b	f	5



Edge		Weight
a	b	2
a	c	3

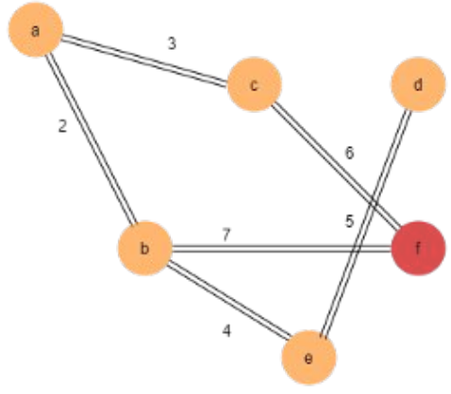
https://medium.com/star-gazers/graph-theory-finding-the-shortest-paths-afb35d28176b

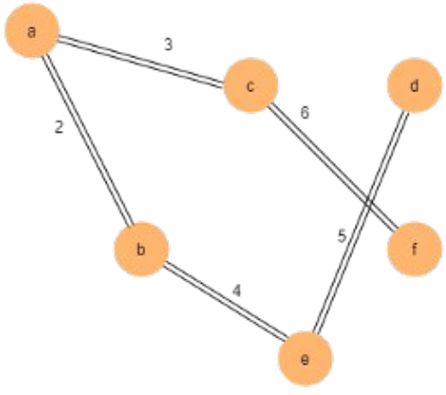
Dijkstra's Shortest Path Algorithm (contd.)



Edge		Weight
c	d	6
c	f	6
d	e	5
b	f	7

Edge		Weight
a	b	2
a	c	3
b	e	4
d	e	5





Edge		Weight
c	f	6
b	f	7

Edge		Weight
a	b	2
a	c	3
b	e	4
d	e	5
c	f	6

https://medium.com/star-gazers/graph-theory-finding-the-shortest-paths-afb35d28176b
https://www.baeldung.com/cs/prim-dijkstra-difference

Prim’s versus Dijkstra's

- Dijkstra’s algorithm finds the shortest path, but Prim’s algorithm finds the MST.
- Dijkstra’s algorithm can work on both directed and undirected graphs, but Prim’s algorithm only works on undirected graphs.
- Prim’s algorithm can handle negative edge weights, but Dijkstra’s algorithm may fail to accurately compute distances if at least one negative edge weight exists.
- Prim’s Algorithm adds the next vertex that is closest to the tree. Dijkstra’s Algorithm, on the other hand, adds the next vertex that is closest to the root.
- Intuitive example:
  - Dijkstra's: Find shortest and lowest cost path in existing road network.
  - Prim’s: How to create the road network itself efficiently.

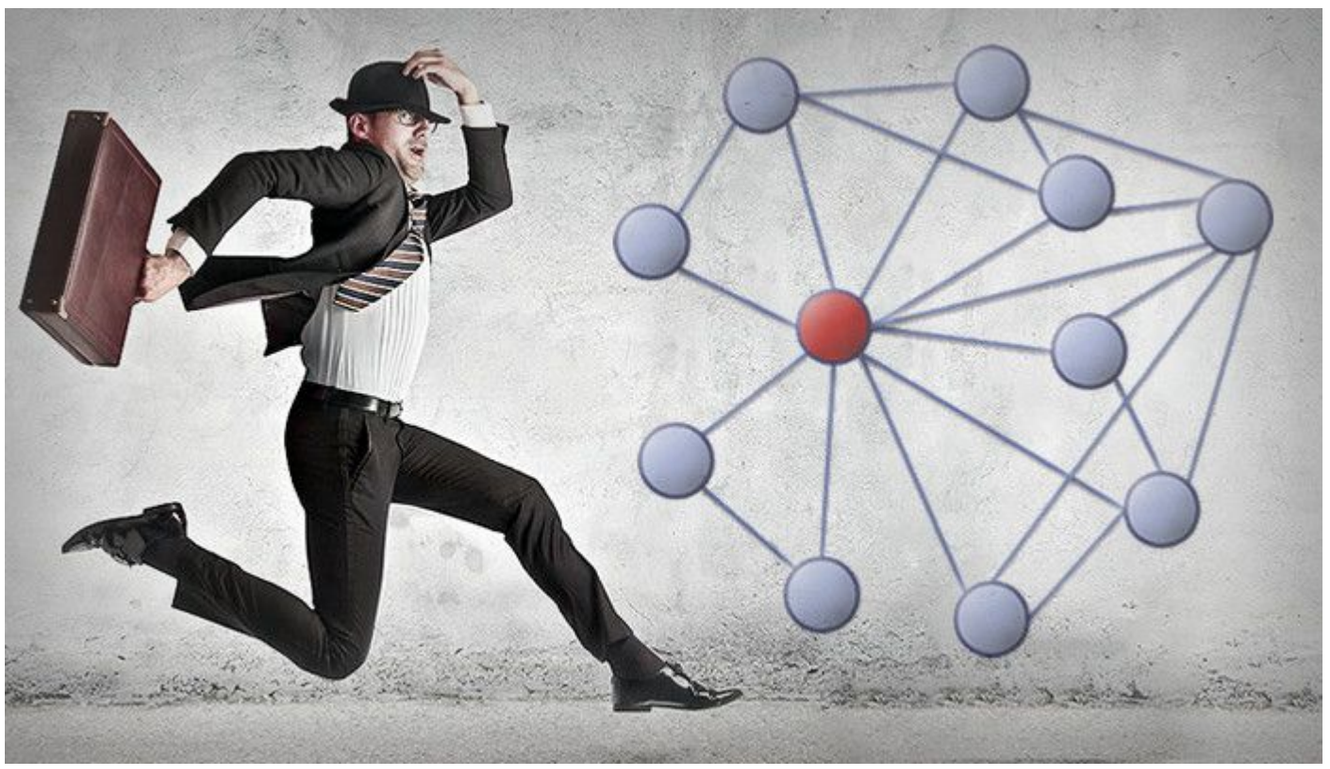
Traveling Salesman Problem (TSP)

- Problem formulation: Find the shortest route from a city such that all other cities are visited only once and the traveller returns to the original city.

- The triangle inequality: sum of any two sides of a triangle must always be greater than the third side: $a + b > c$, $a + c > b$, $b + c > a$.

- The route is also called a Hamiltonian cycle satisfying the triangle inequality

- Graph:
  - Nodes: cities
  - Edges: Roads
  - Weights: Distance, travel time, etc.



https://towardsdatascience.com/how-to-solve-the-traveling-salesman-problem-a-comparative-analysis-39056a916c9f

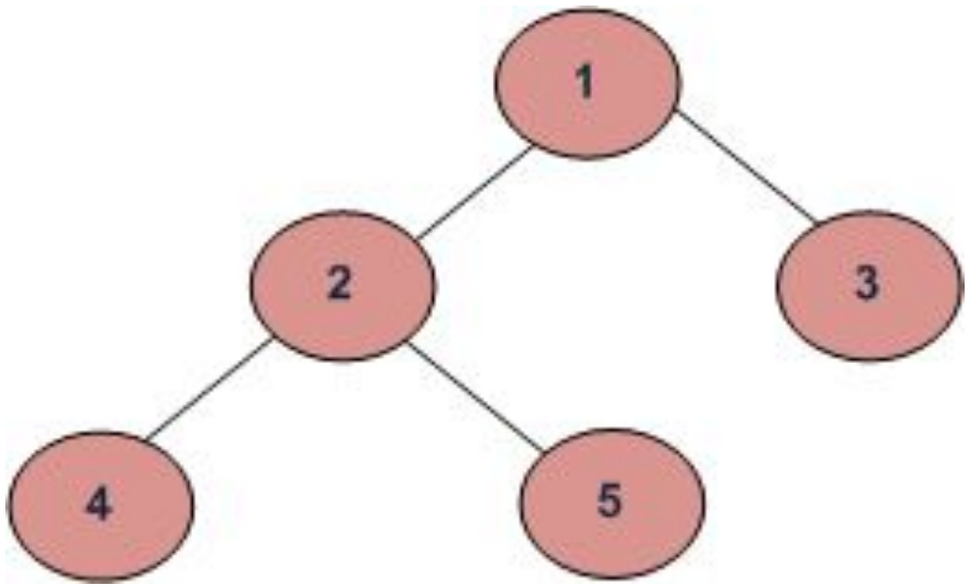
TSP Intuition

- TSP is NP-hard: It is not possible to guarantee you will find the shortest path within some predefined time-limit; n cities can have n! paths (nX(n-1)X(n-2)...1), e.g., 15 cities can result in trillion possible routes.

- We use approximation techniques to select pareto-optimal routes.

www.variability.org

Pre-order traversal

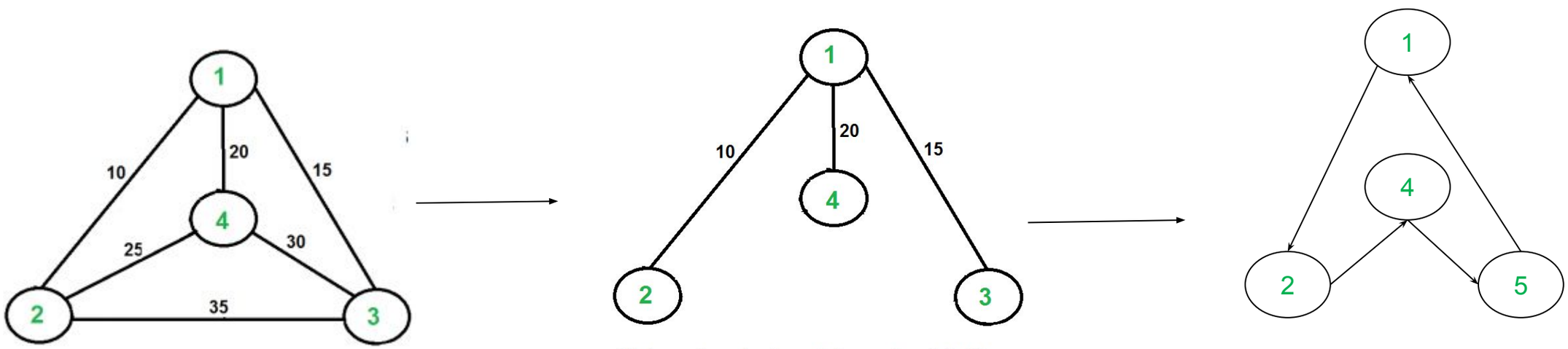


1 - 2 - 4 - 5 - 3

https://www.geeksforgeeks.org/travelling-salesman-problem-set-2-approximate-using-mst/
https://www.geeksforgeeks.org/tree-traversals-inorder-preorder-and-postorder/

2-approximate algorithm for TSP using MST

- The cost of the output is never more than twice the cost of best possible output.
- Algorithm:
  - Use Prim's algorithm to generate MST, using the starting/ending point as the root node.
  - Find depth-first pre-order traversal of the MST (root, left, right)
  - Add 1 at the end of the pre-order walk to get the Hamiltonian cycle.



Minimum Spanning Tree of the graph on left side with 1 as root. The Preorder Traversal of MST is 1-2-4-3. So the output is 1-2-4-3-1

Other TSP solutions

- Exact algorithms: Dynamic programming

- Approximate algorithms:
  - 1-approximate using Eulerian circuits and Dijkstra's algorithm
  - 2-approximate using MST
  - Simulated Annealing (guided local search)
  - Ant-colony Optimization

- Special cases
  - Independent set
  - 2-SAT