



Graphs and Social Networks

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*Note: Slides adapted from original LACC slides from Dr. Luis Garcia & Saptadeep Pal
Some materials adapted from UCLA ECE 232E course by Prof. Vwani Roychowdhury*



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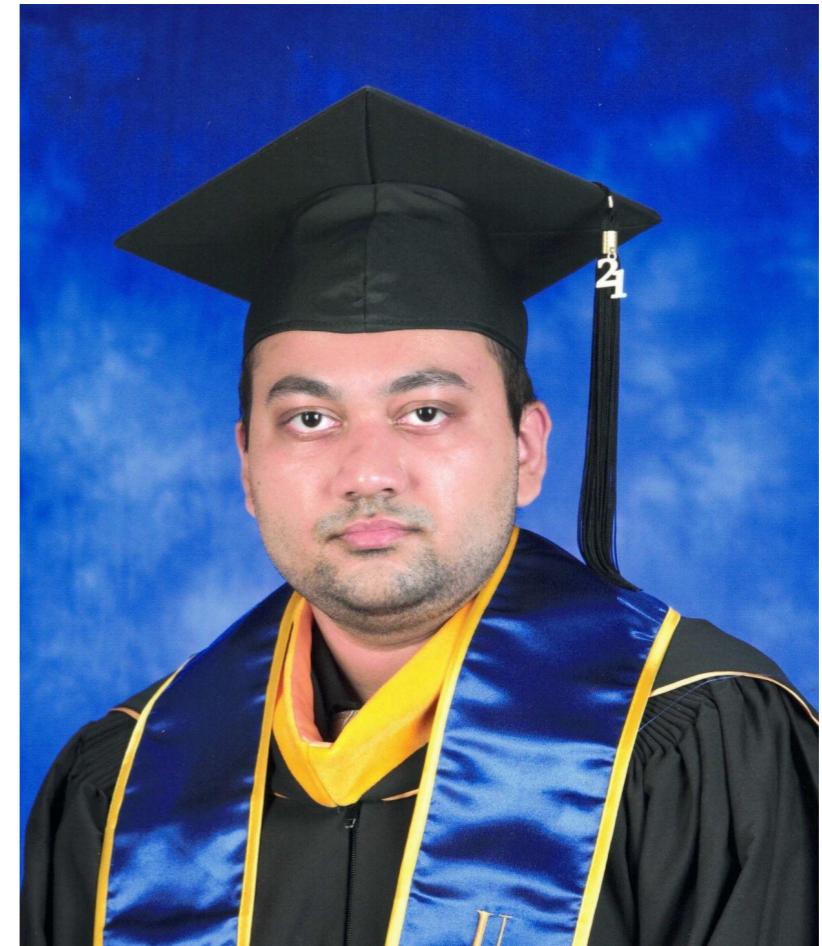
- Vice President, CS-GSA
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- Research Interests:
Graph neural networks, knowledge graphs
for various applications



Instructors

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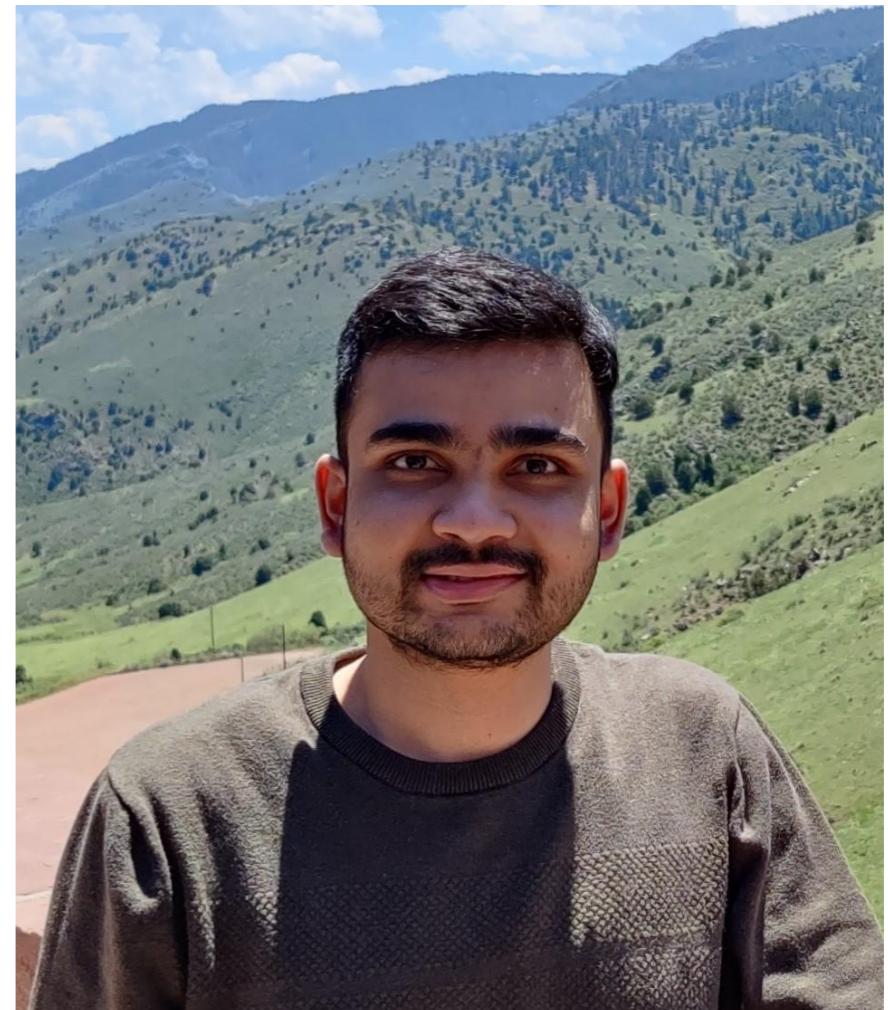
- ECE PhD @ UCLA,
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- Entering Event's Director, EGSA
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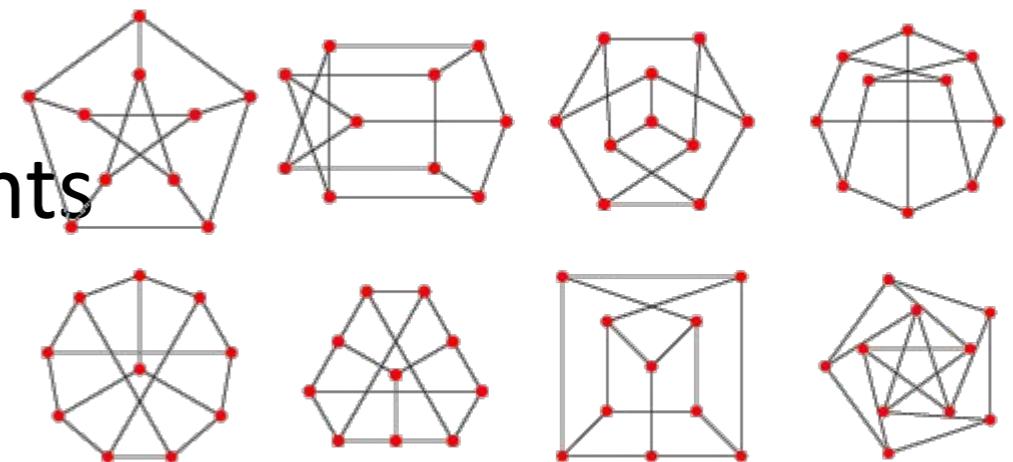
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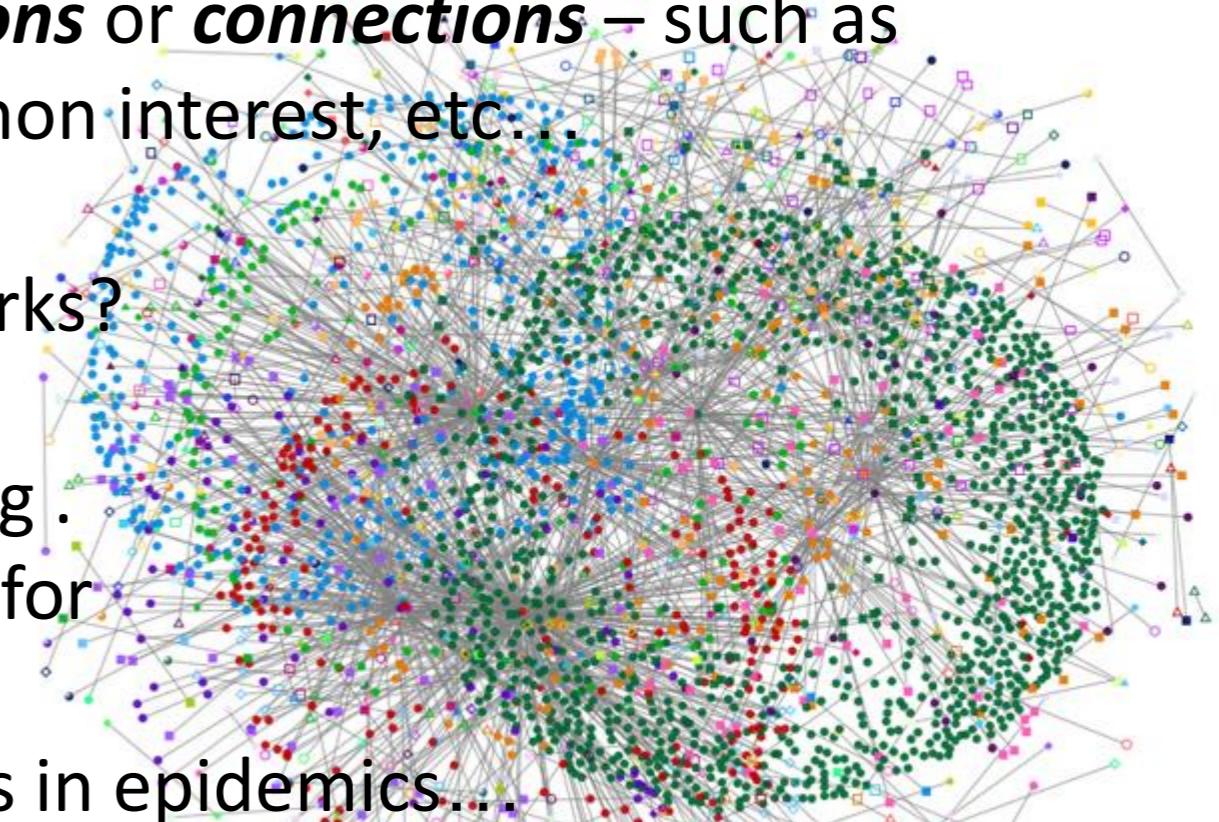
In This Lecture...

- We are going to:
 - Introduce the concepts of a social network and graph theory
 - Play ‘Six Degrees of Kevin Bacon’
 - Learn some basic graph theory
- Then we will do programming exercises:
 - **Easier:** some simple graph theory
 - **Harder:** path and cycle detection
find connected components



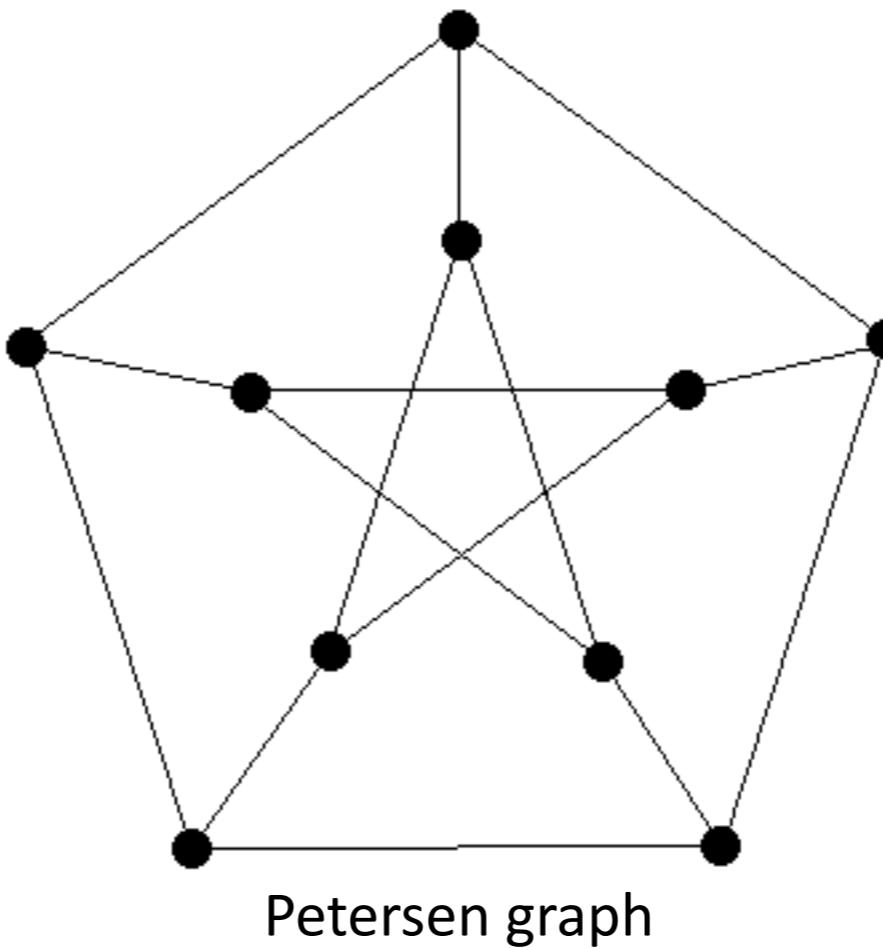
Social Networks

- Q: What's a **social network**?
- A: It is a structure made up of **individuals** (or organizations, or agents, or anything else) tied together by **relations** or **connections** – such as friendship, alliance, financial or common interest, etc...
- Q: Why should we study social networks?
 - It's useful! Tons of applications:
 - Finding friends and matchmaking.
 - Locating appropriate customers for companies (marketing).
 - Preventing spreading of diseases in epidemics...



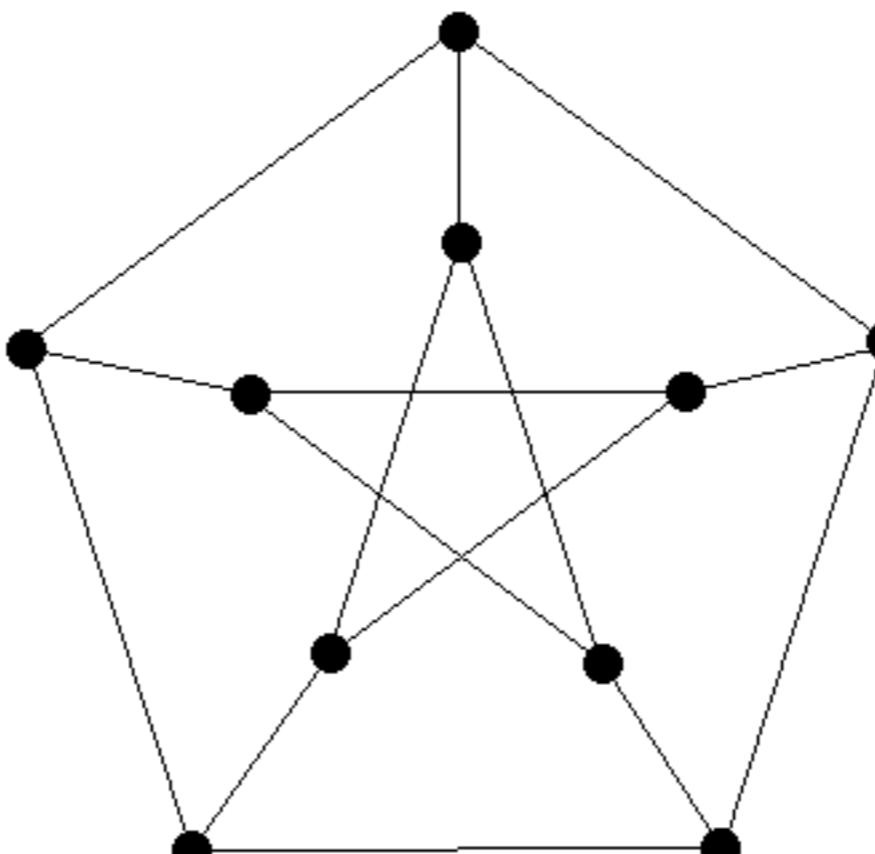
Graph Theory

- The main tool used to understand social networks is graph theory.
- An example of a graph:



Graph Theory

- A graph is made up of nodes (also called vertices) and edges:
- Which are the nodes and which are the edges in the graph below?

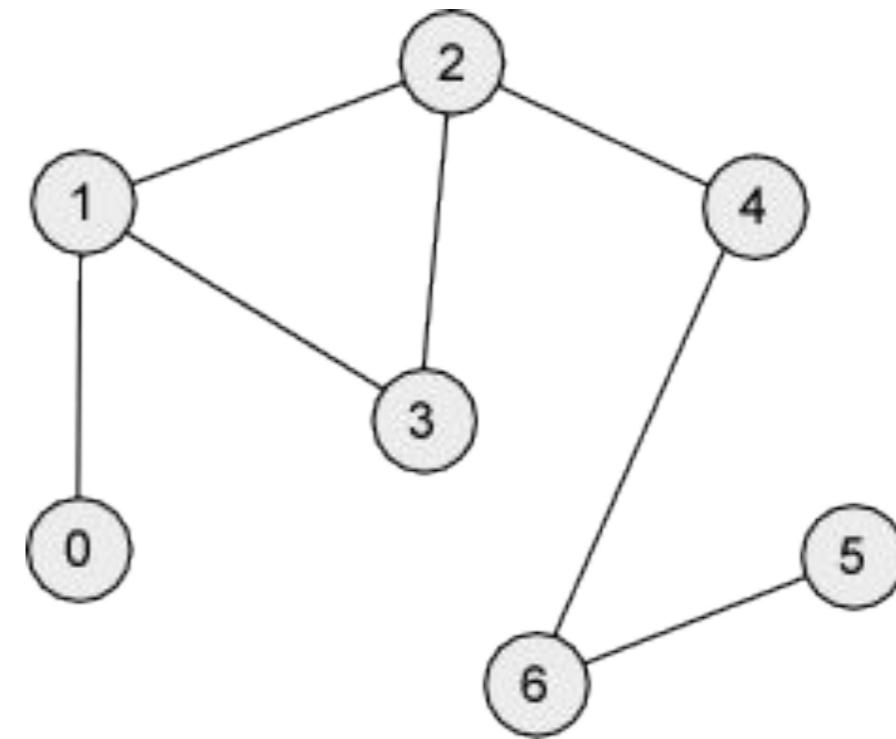
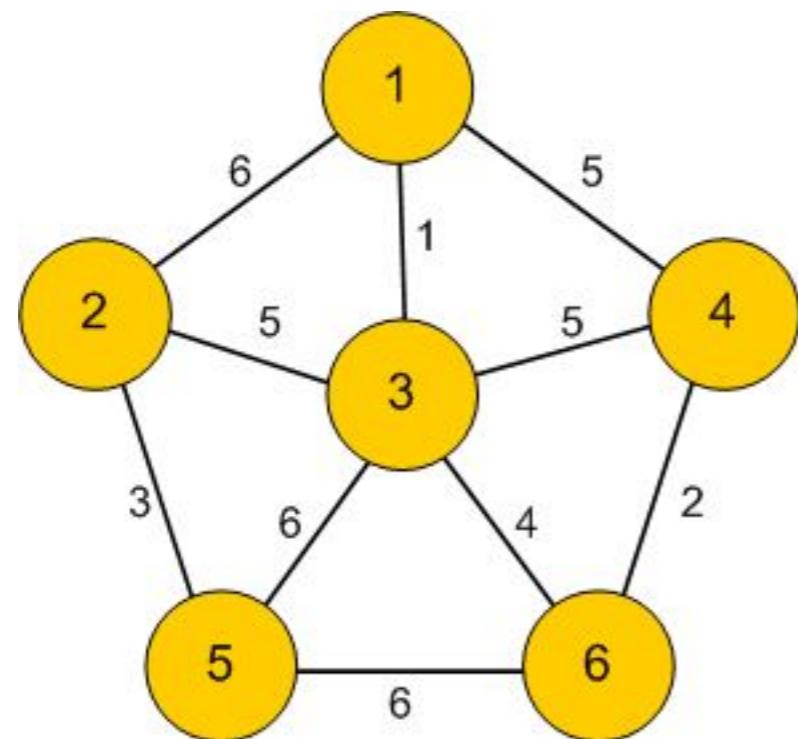


Petersen graph

Graph Theory

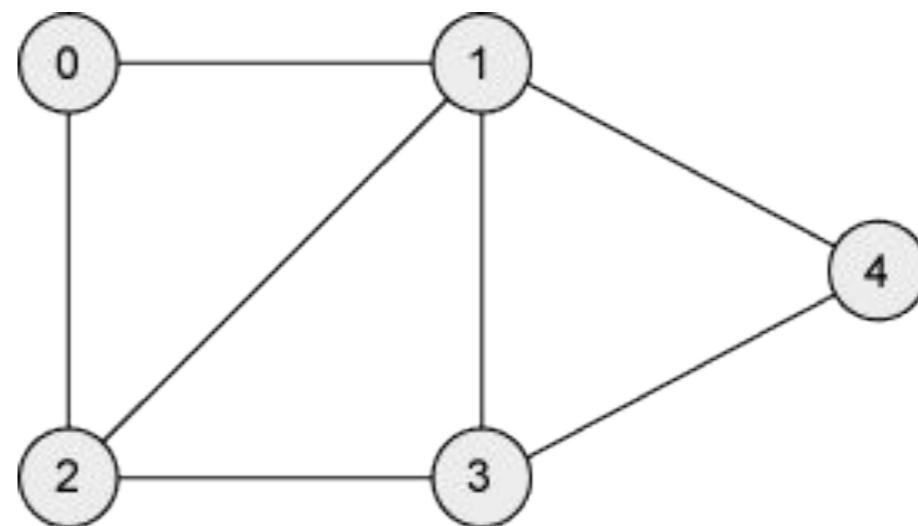
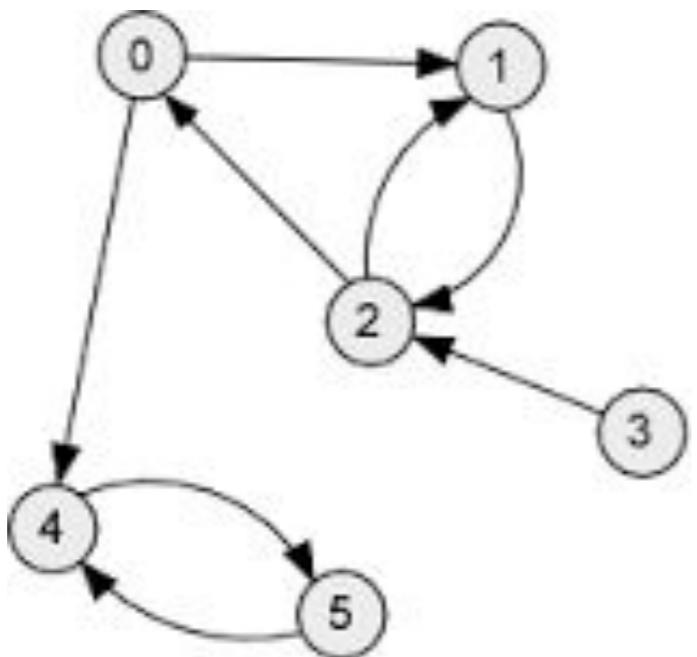
- Edges can be **weighted** or **unweighted** (= uniformly weighted graph).
- Weights: allow us to express the importance of a connection.

Q: Can you think of some applications where we need edge weights?



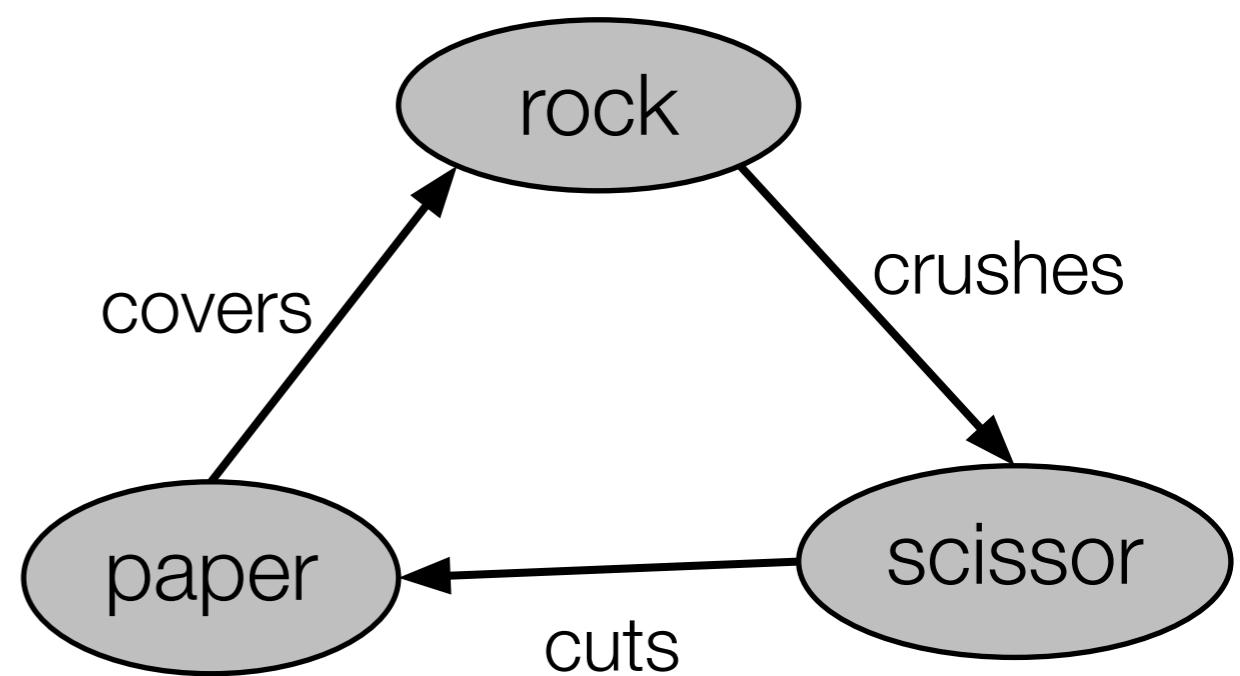
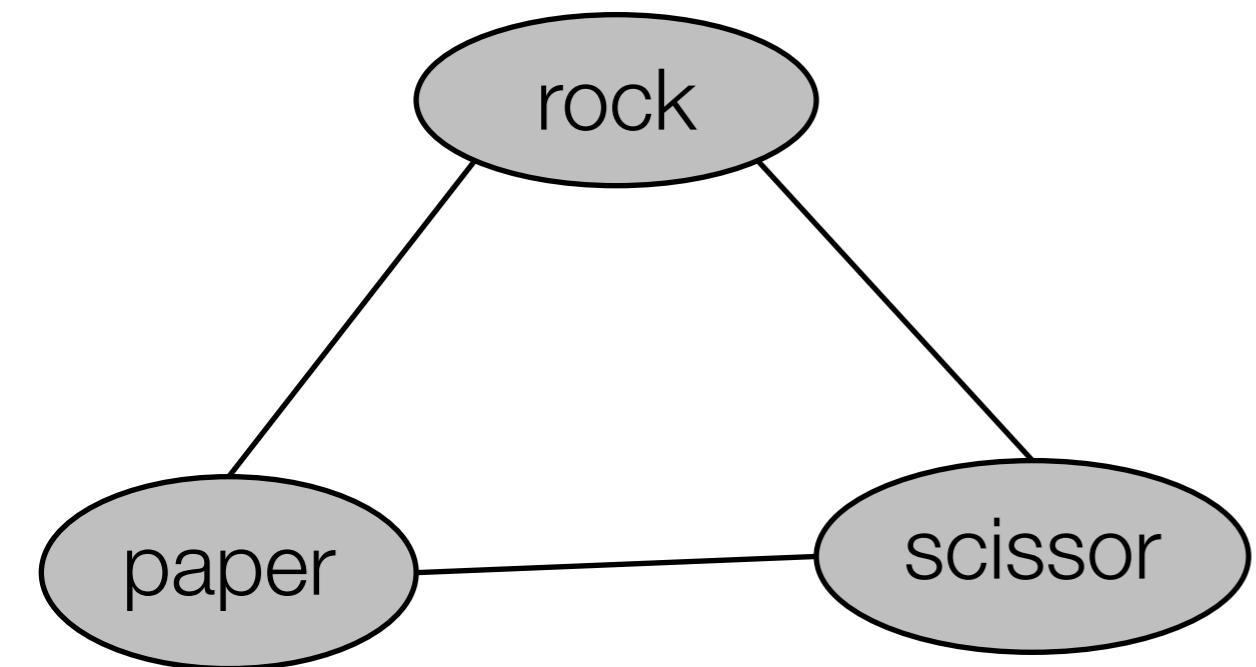
Graph Theory

- Edges can be **directed** or **undirected**.
- Directions allow us to have influence go one way (rather than mutual).
- The type of graph we use is dependent on



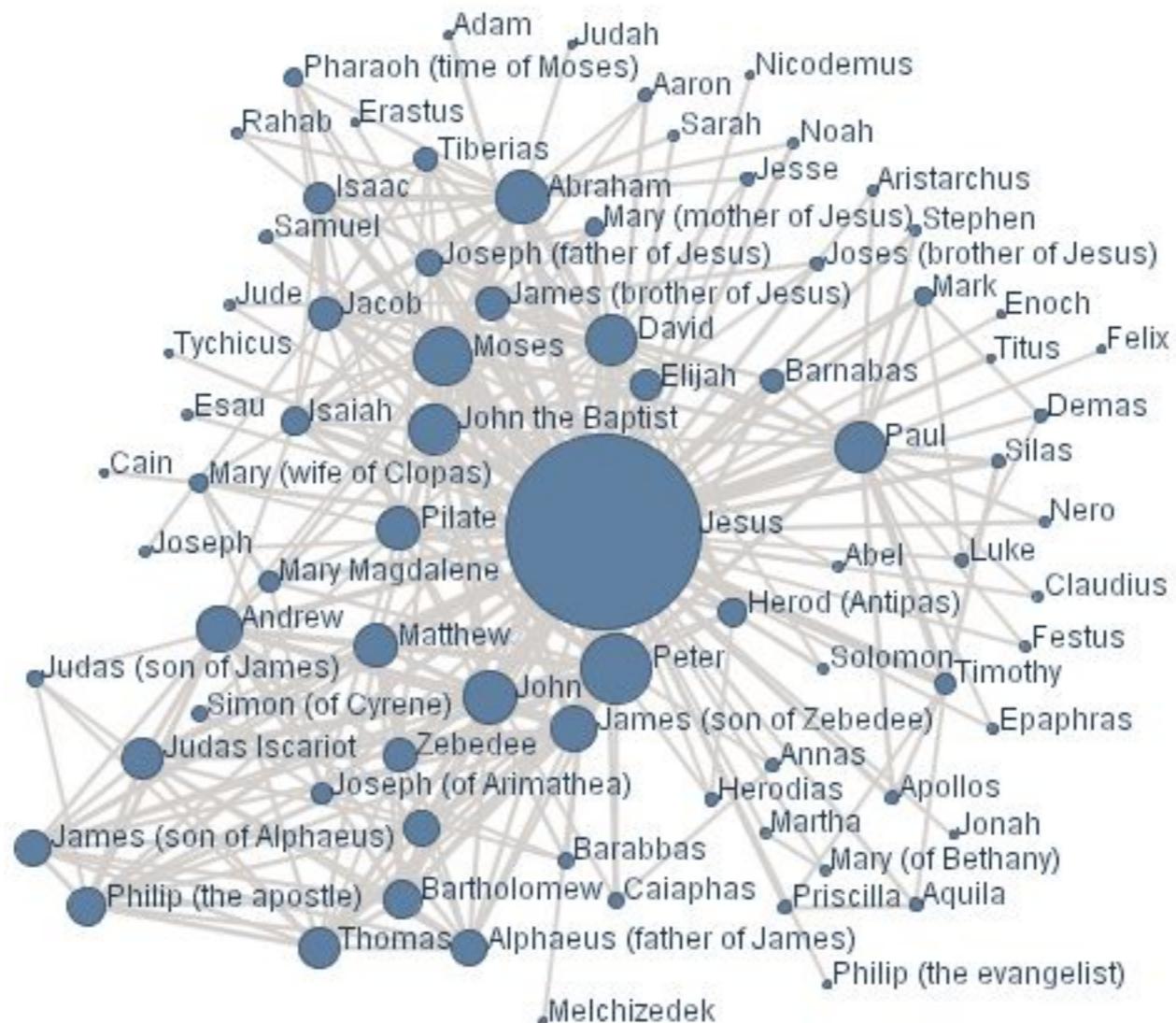
Example of directed edges in a graph

- Rock – Paper – Scissor Game



Graph Theory

- Q: Why do we care about graphs?
- A: We can use them to represent a social network:



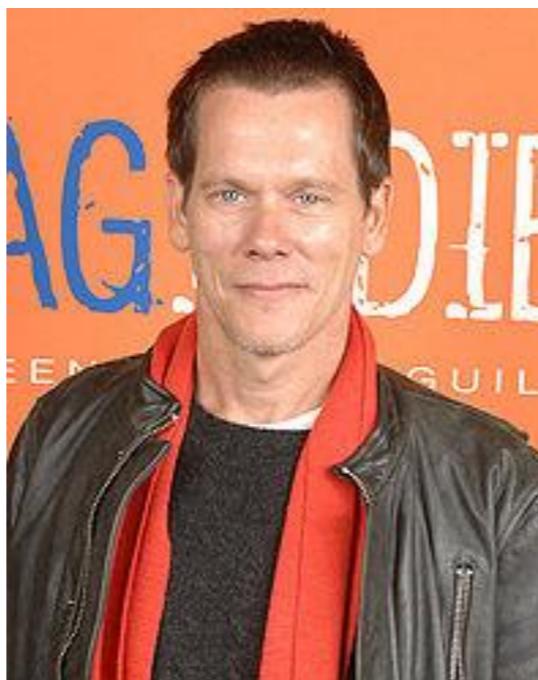
Acting Networks

- A good example of a network is formed by our coworkers, their coworkers, etc...
- We'll look at Hollywood actors.
- We'll say that two actors are connected if they played in the same film.



Kevin Bacon

- Q: Who is the “center” of the movie universe?
- A: Kevin Bacon. Have you heard of him? If not – it’s because he’s known for playing a supporting role in a huge number of films:



Kevin Bacon

Kevin Bacon

- We can play a game called ‘Six Degrees of Kevin Bacon.’
- The rules:
 - Pick an actor or an actress.
 - If they have played in a film with Kevin Bacon, their Bacon number is 1.
 - If they have played in a film with someone with a Bacon number of 1, they have a Bacon number of 2.
 - Etc...
- Example: Bradley Cooper:
 - Bradley Cooper was in “Silver Linings Playbook” (2012) with Jennifer Lawrence.
 - Jennifer Lawrence was in “X-Men: First Class” (2011) with Kevin Bacon.
- So Bradley Cooper has a Bacon number of 2.

“Six Degrees” Game

- **Exercise:**
- Using IMDB, find the Bacon number (and path) for:
 - Julia Roberts
 - Selena Gomez
 - Christian Bale



“Six Degrees” Game

- It's a lot easier with an automated tool that searches IMDB:

<http://oracleofbacon.org/cgi-bin/movielinks>

- For example, it produces this output for Christian Bale:



“Six Degrees” Game

- **Exercise:** Try out the Oracle tool with your favorite actors.
- What's the largest Bacon number you can get?



“Six Degrees” Game

- We saw a lot of actors with Bacon numbers 1 and 2.
- What about the extremes?
 - What's the smallest possible Bacon number? Who has it?
 - What's the largest possible Bacon number?
 - What if someone is not connected to Kevin Bacon at all?
 - Among those who are, what's the largest Bacon number?



“Six Degrees” Theorem

- The “Six Degrees” idea states that any two people selected at random are connected by at most 6 degrees.
- Of course, there are some counterexamples, but they’re rare.
- Same applies to Hollywood – which is why the game is called “Six Degrees of Kevin Bacon.”



Back to Kevin Bacon

- Here's a statistic about Kevin Bacon numbers for other actors:

Bacon Number	# of People
0	1
1	3434
2	398768
3	1478688
4	382332
5	33473
6	4101
7	679
8	144
9	16
10	7



Back to Kevin Bacon

- What can we see from the table?
- Most people have a Bacon number of 3.
- Slightly less have a Bacon number of 2 and 4.
- Bacon numbers higher than 4 are rare.
- The maximum is 10.



Back to Kevin Bacon

- You can replace Bacon with other actors to see who makes a good “center” for connections.
- Try it out:

<http://oracleofbacon.org/cgi-bin/center-cgi>

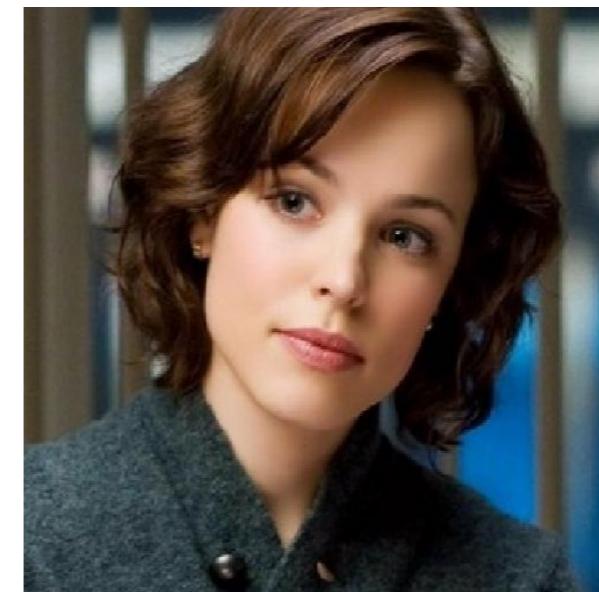


Back to Kevin Bacon

- What does it mean to be a good “center?”
- For example, we see that Tom Cruise has an average number of 2.976 while Bacon has an average of 3.026. Who’s better?
- **Exercise:** Try out your favorite actor. Who picked the best center?

Rachel McAdams Number	# of People
0	1
1	1897
2	300058
3	1505279
4	451047
5	38285
6	4249
7	641
8	163
9	16
10	7

Total number of linkable actors: 2301643
Weighted total of linkable actors: 7144962
Average Rachel McAdams number: 3.104



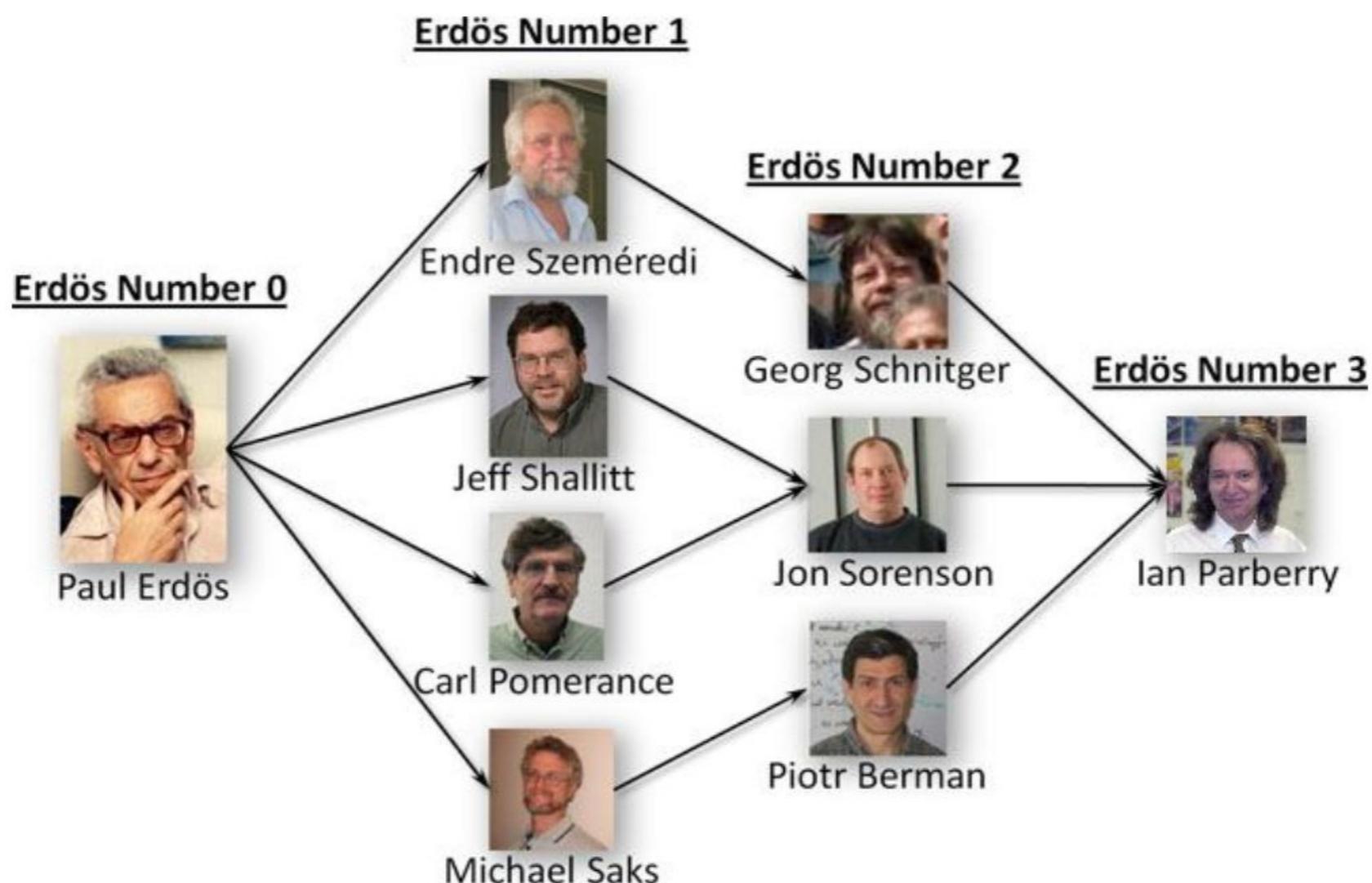
What about the world of science?

- Erdős numbers:



Erdős numbers

- Describes the "collaborative distance" between mathematician Paul Erdős and another person, as measured by authorship of mathematical papers.



What's your Erdos number?

- Roshni Iyer (instructor of this module): 2

Paul Erdős

Satish Rao

Roshni Iyer

Compute:

<https://oakland.edu/enp/compute/>



www.variability.org



Larger networks

- We saw, from the Kevin Bacon game, that
 - 1. Hollywood has about 1 million actors.
 - 2. Most of them can be related to each other within 4 connections.
- What about all the people in the US?
 - Population is about 300 million.
- Q: About how many connections on average between two random people?
 - (Note: define connection here as “being acquainted with.”)



Larger Networks

- “Six Degrees” Theorem still applies!
- So, if we pick two random people in the US, using a chain of “friends of friends”, we should be able to connect them within 6 connections.
- Studied by the famous psychologist Stanley Milgram in his “Small world experiment” in the 1960’s.
- Easily recreated today using Facebook!



dw 2010 www.variability.org



Stanley Milgram

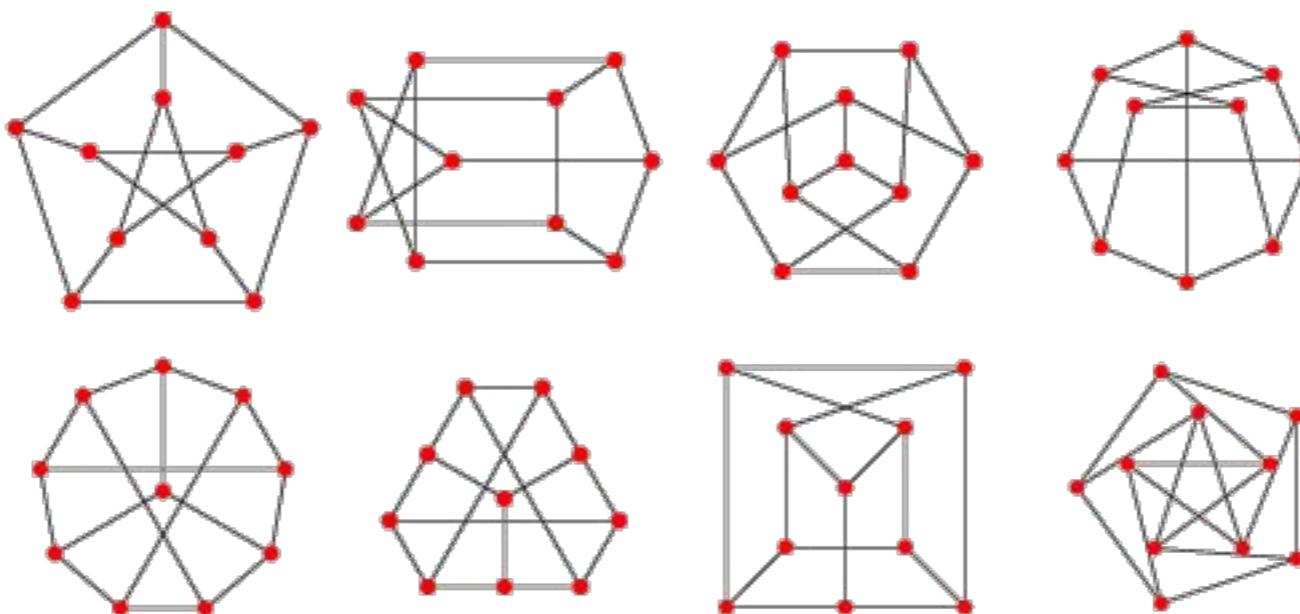
“Six degrees” Theorem

- So what's useful about it? Applications?
- One example: Insurance companies request customers to provide 5 referrals in order to expand their business.
- **Exercise:** Come up with another example.



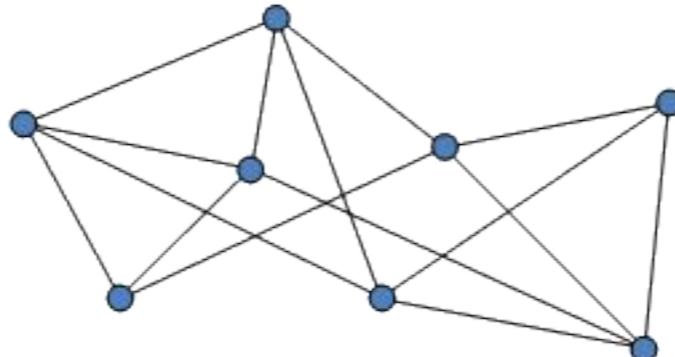
Graph Theory

- We've talked about networks, connections, "distances", etc...
- Now it's time to describe the math behind this.
- This domain is called "graph theory."



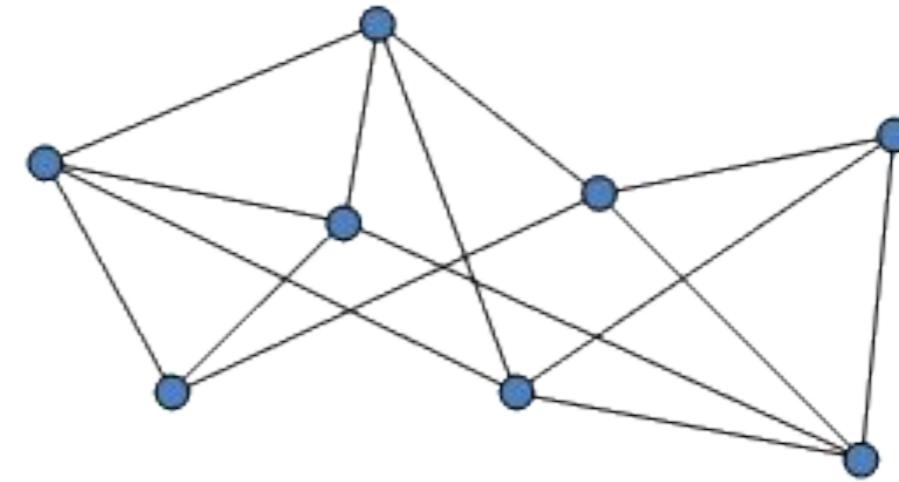
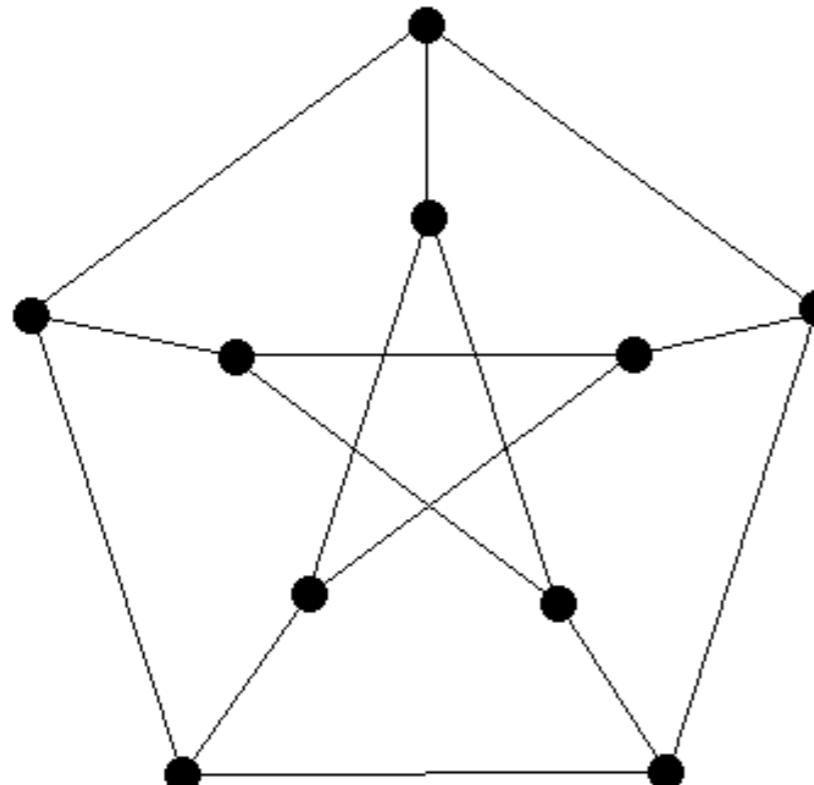
Graphs

- A graph is made up of “nodes” and “edges.”
- Nodes are denoted by a dot/vertex.
- Edges connect two nodes.
- If two nodes have an edge between them, they are “adjacent” or “neighbors.”



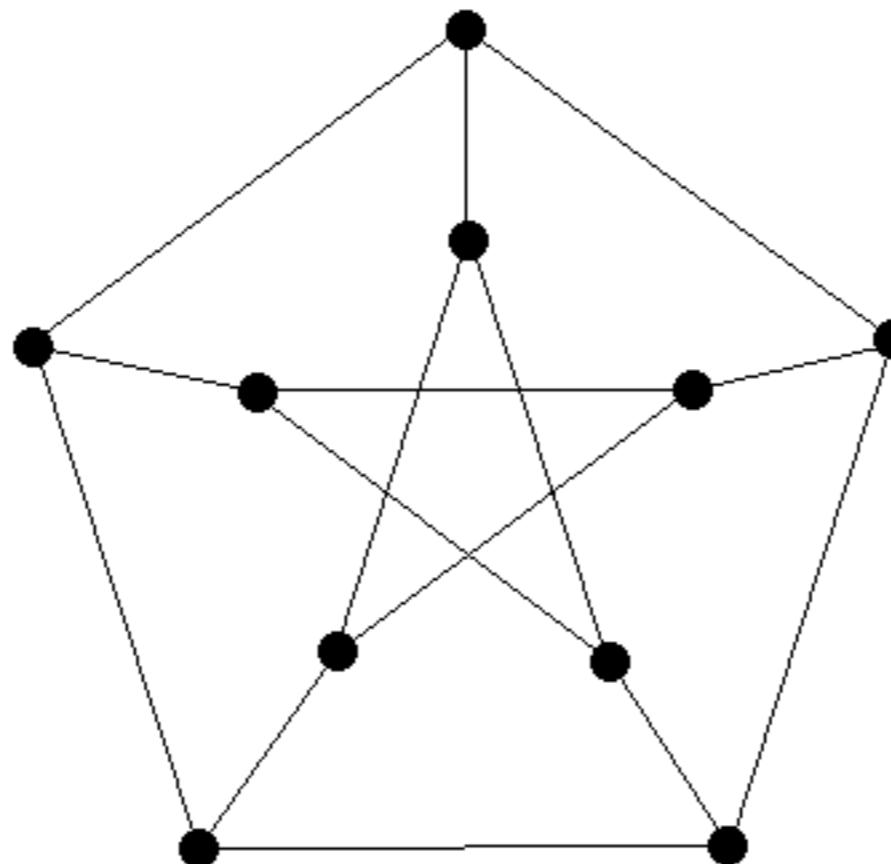
Degrees

- The number of edges coming out of a node is called its “degree.”
 - Sometimes degree is specified to be ***out-degree*** (outgoing edges) or ***in-degree*** (incoming edges).
- What are the in-degrees & out-degrees of the nodes in the graph below?
- All nodes have same degree: **regular** graph. If not, **irregular**.
- Q: Are complete graphs (maximally connected graphs) regular or irregular?



Degrees

- What's the relationship between **vertex degrees** and **edges**?
- **Exercise:** Give the relationship described above.
(hint: think about the sum of all the degrees and the number of edges.)



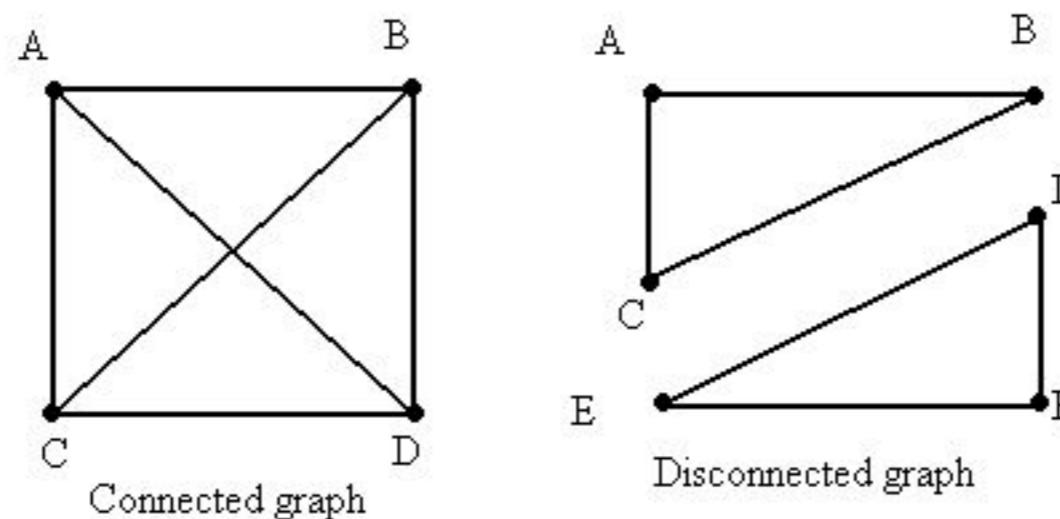
Idea

- Enumerate ordered pairs (i,j) in the graph, where i and j is connected by an edge.
- Count by vertices:
 - If node i has degree d_i , then there are d_i ordered pairs of (i,j)
 - The number of ordered pairs = sum of degrees of all nodes
- Count by edges:
 - If (i,j) is an edge, then both (i,j) and (j,i) are ordered pairs
 - The number of ordered pairs = 2 times number of edges
- Sum of degrees of all nodes = 2 times number of edges

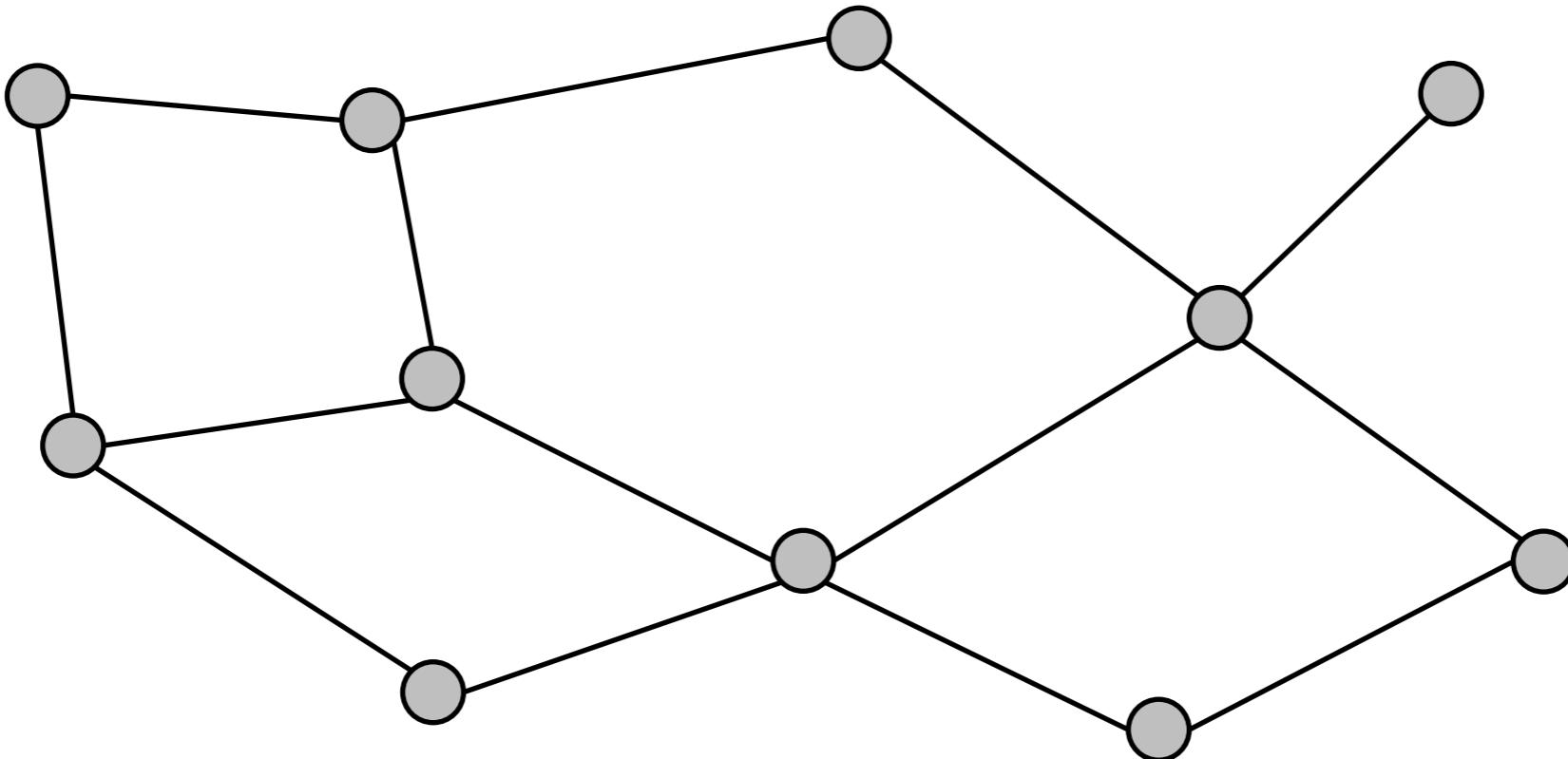


Connectivity

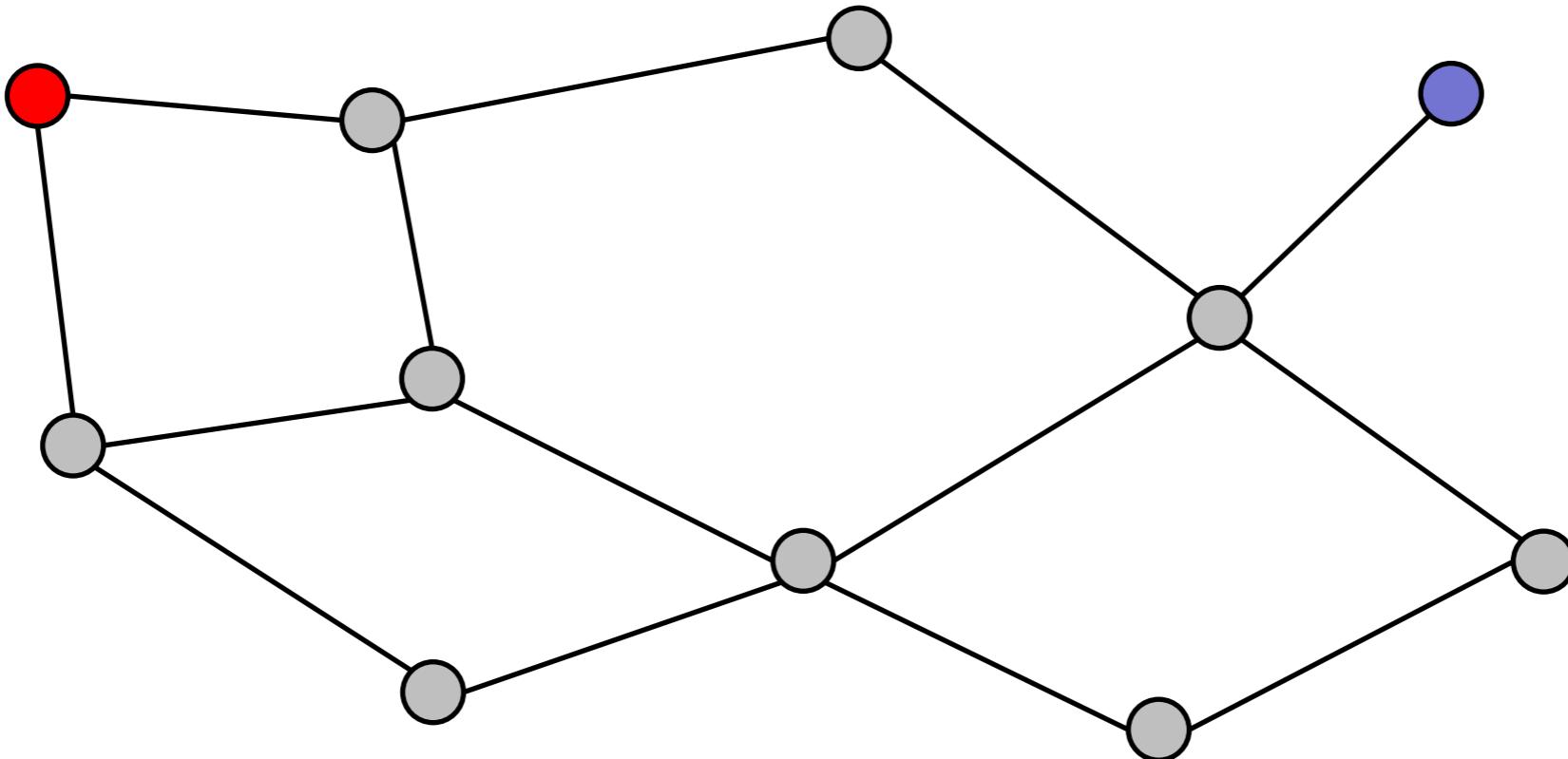
- A series of edges is called a “path.”
- If there exists a path between two nodes, the two nodes are “connected.”
- If every pair of nodes in a graph is connected, the entire graph is “connected”, otherwise it is “disconnected”.
 - A disconnected graph can be divided into multiple connected components
 - Connected graph has only 1 connected component



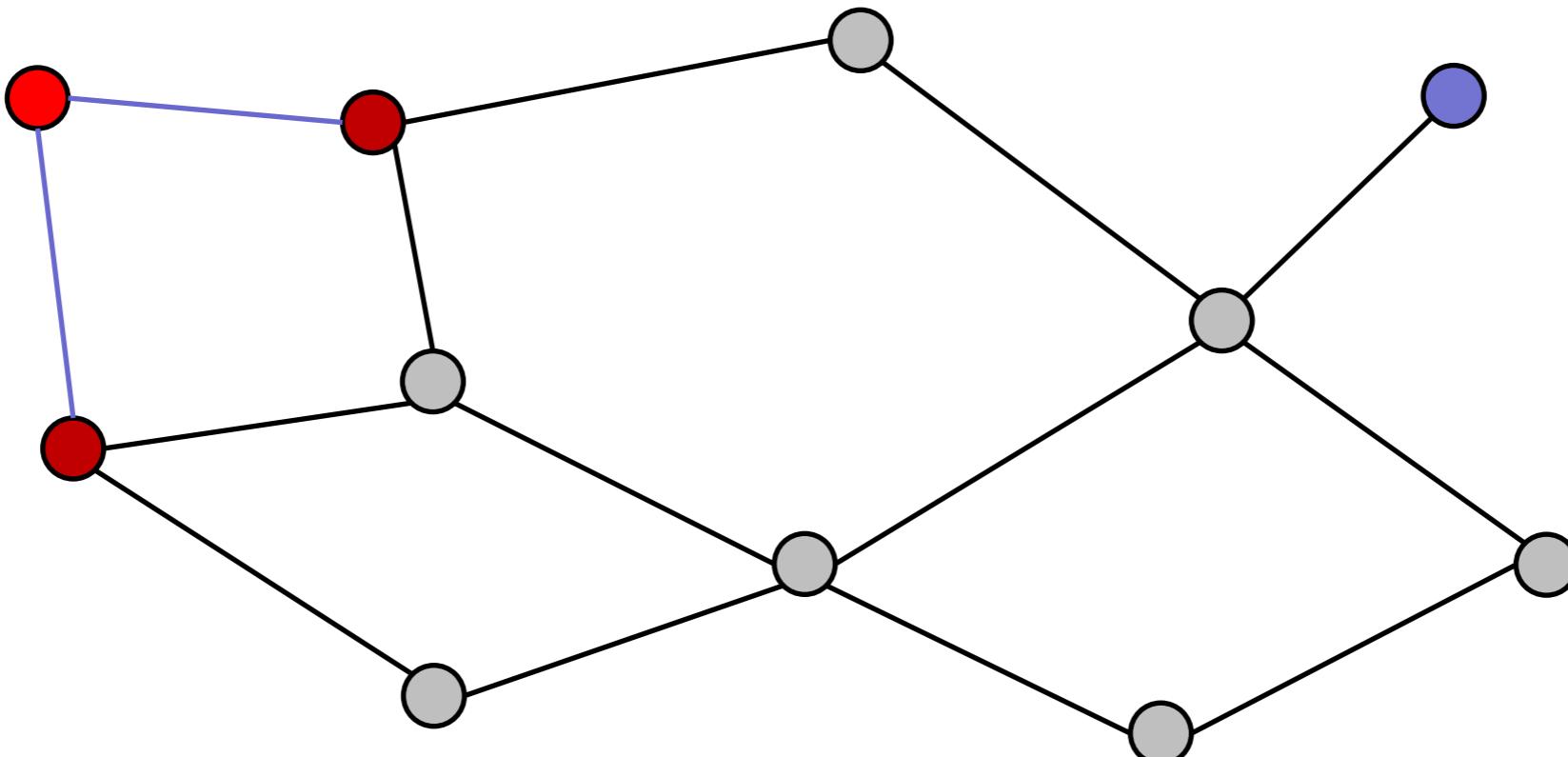
How to find the shortest path between two nodes?



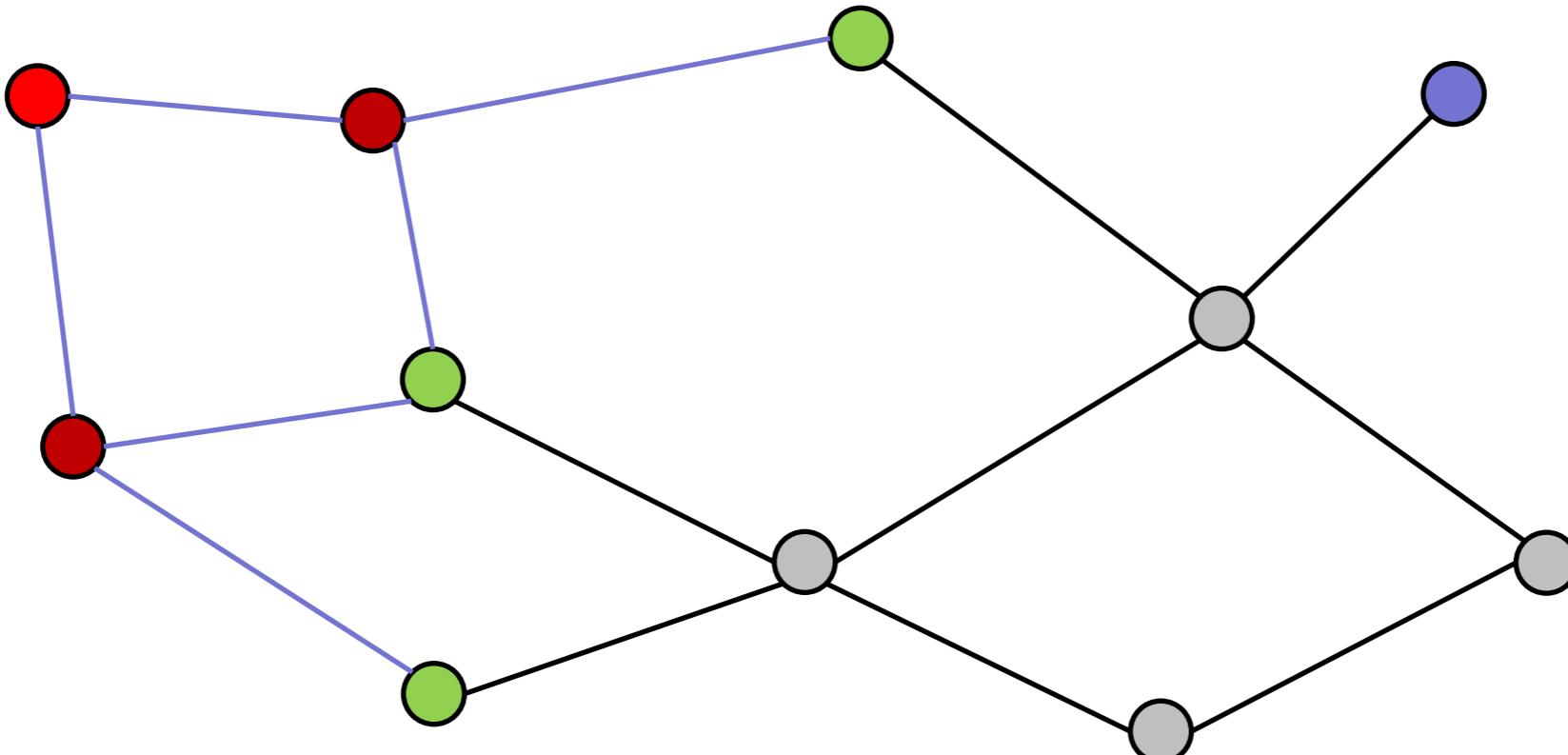
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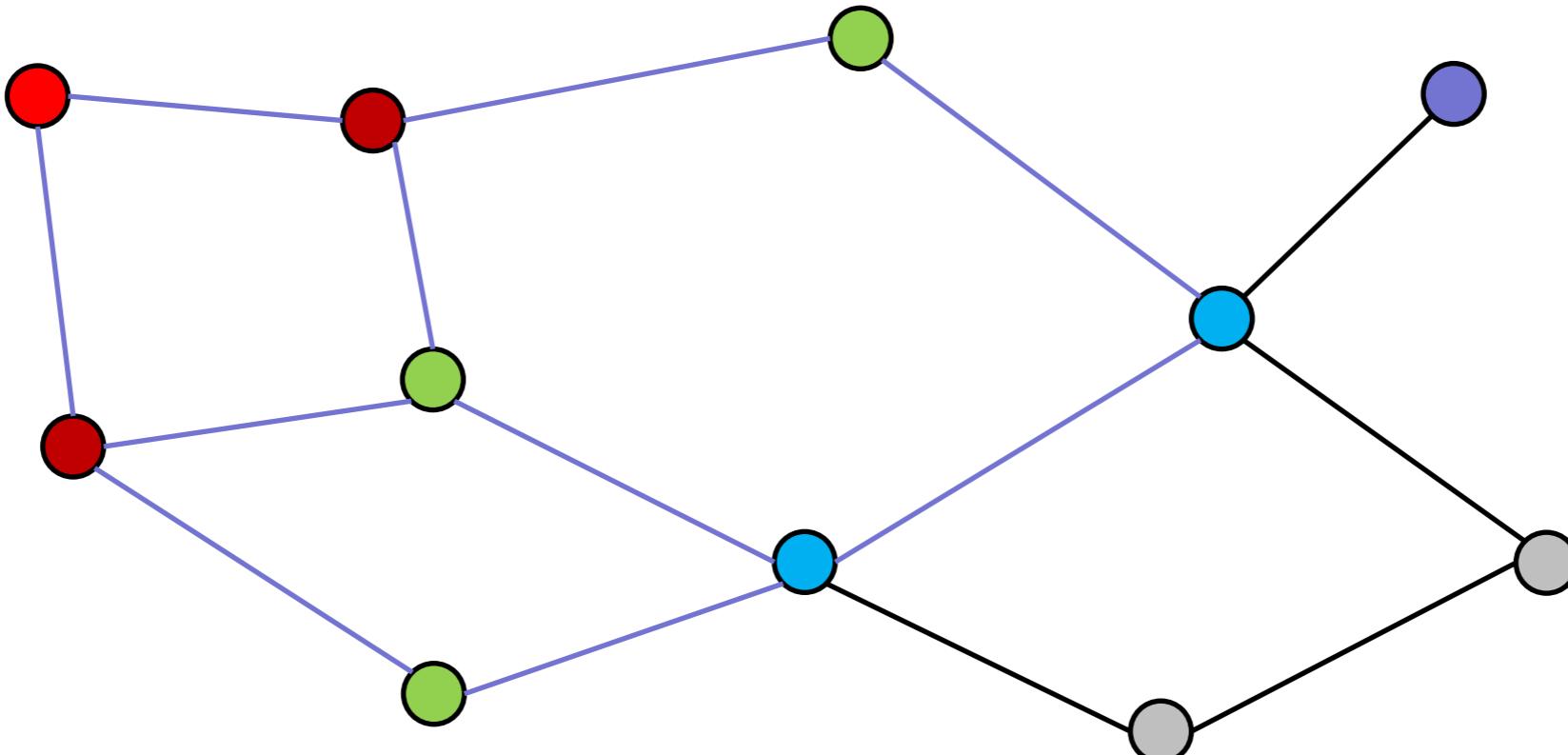
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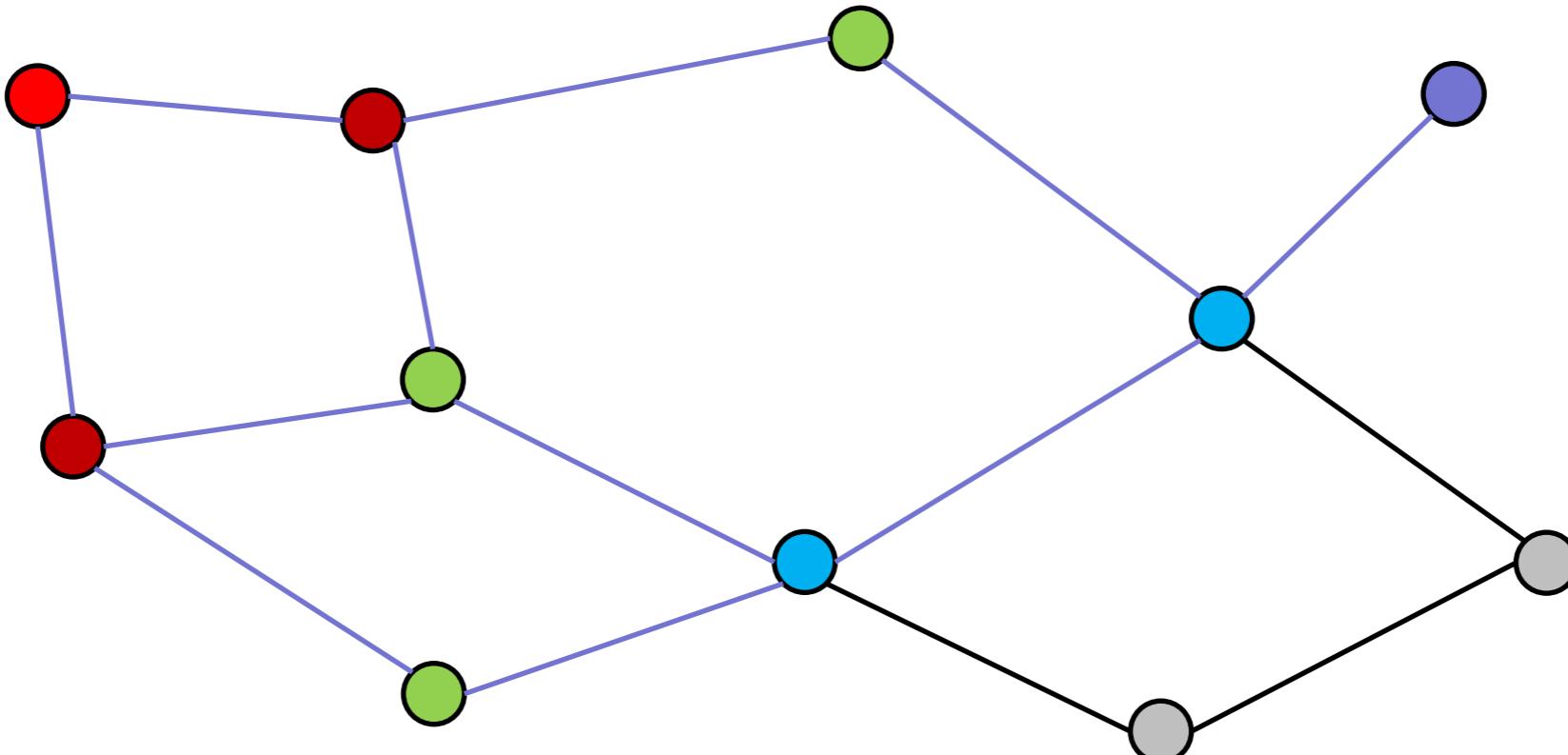
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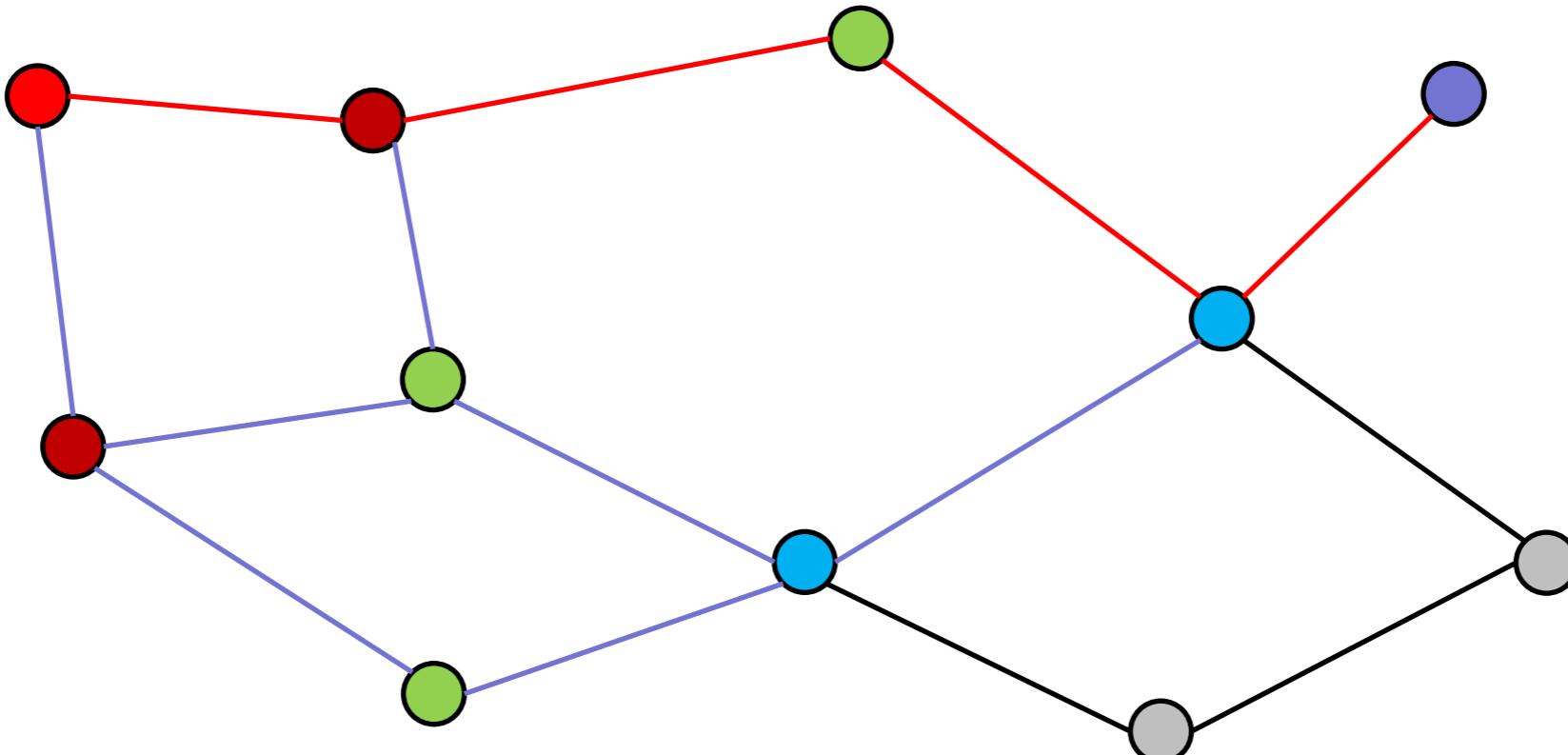
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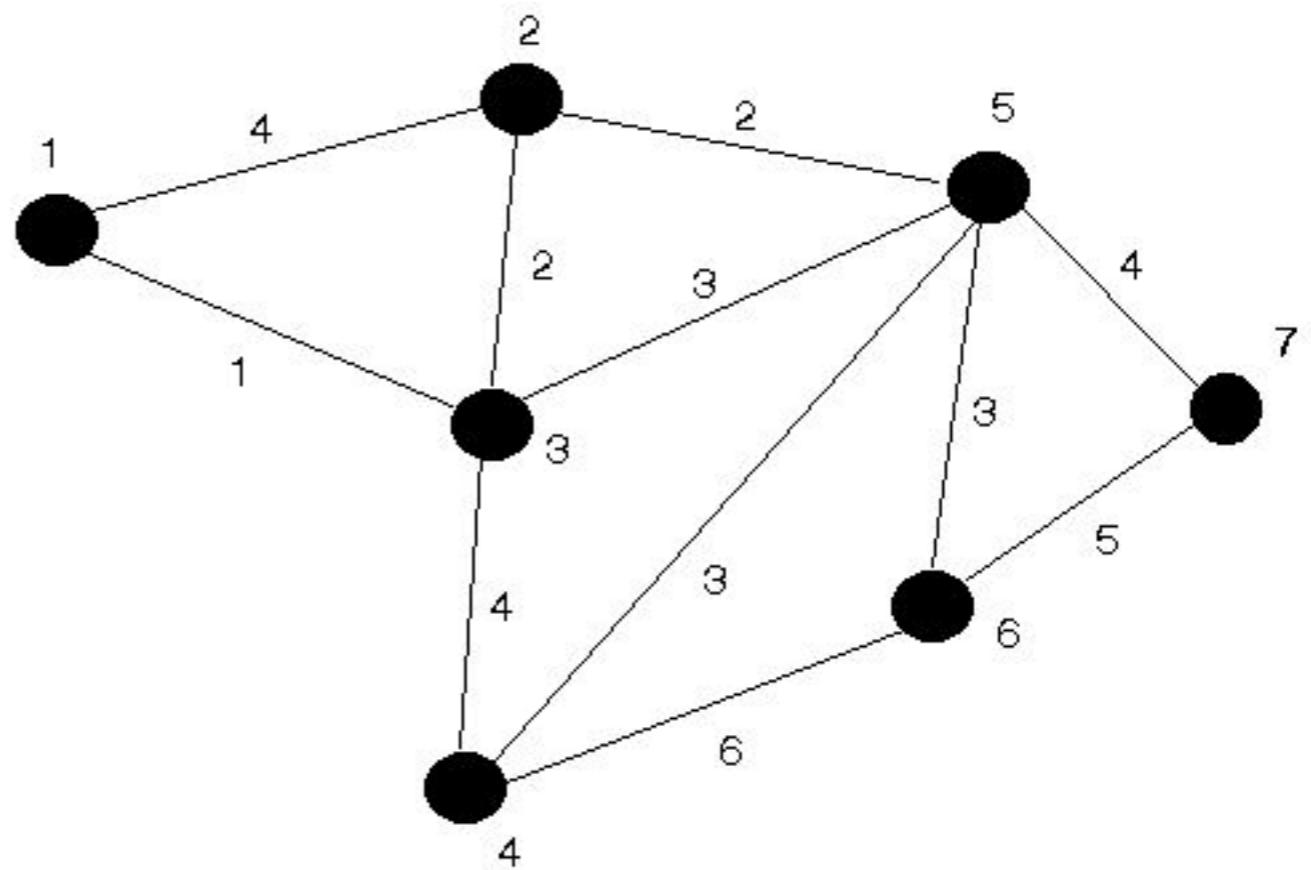
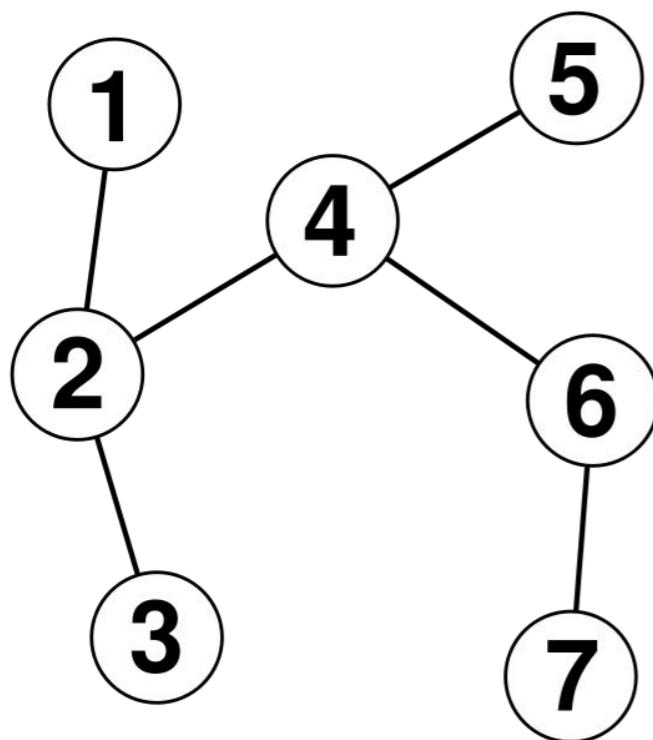


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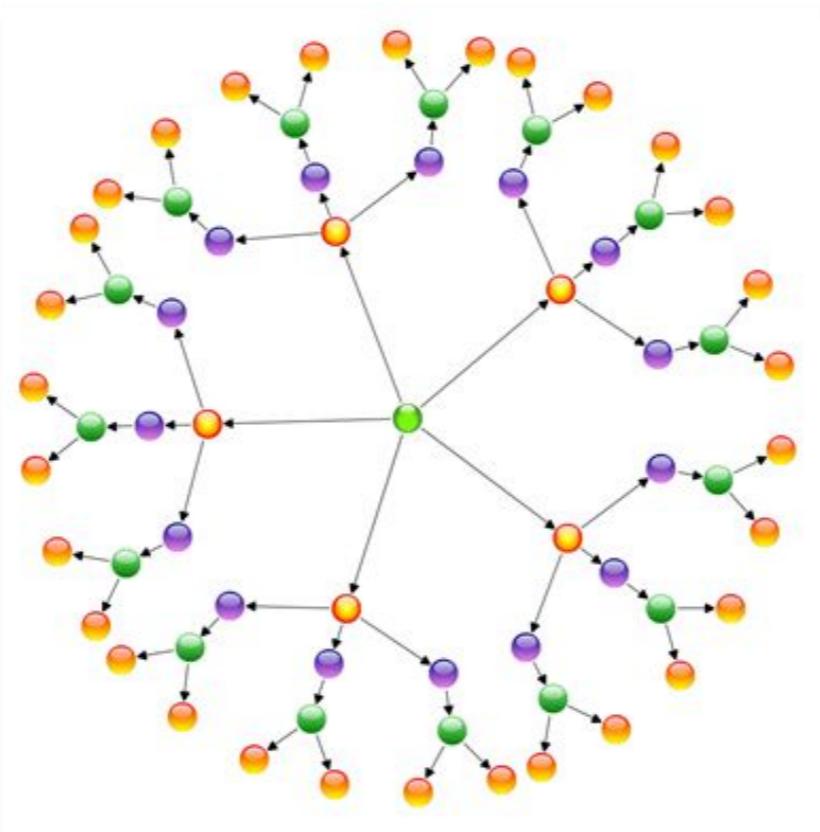
Cycles

- A series of edges (a path) starting and ending in the same node is called a “cycle.”
- Q: Which graphs have cycles? If so, how many cycles are there?



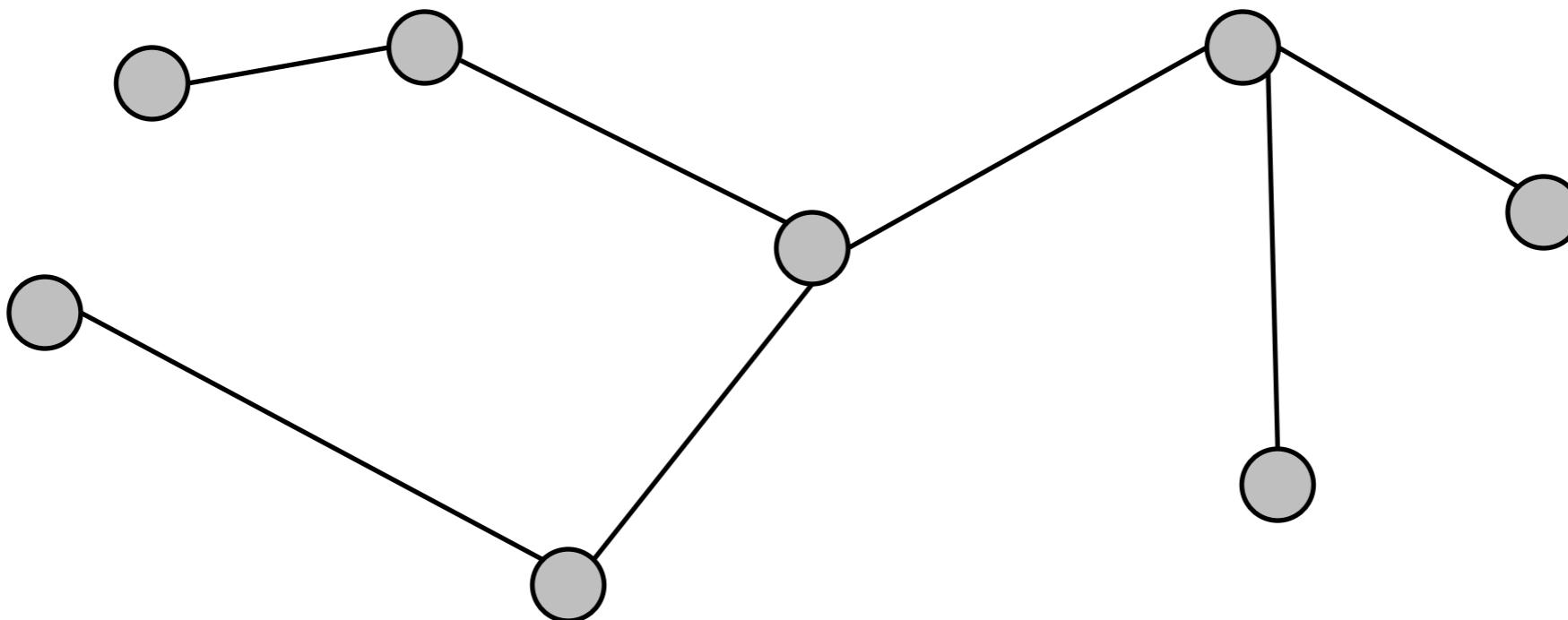
Trees

- A connected graph with no cycles is called a “tree”.
 - If we remove “connected” from this definition, the graph will be called a “forest”.
- **Exercise:** If we have a tree with n nodes, how many edges will it have?
 - (hint: draw some small examples of trees and see what you get.)



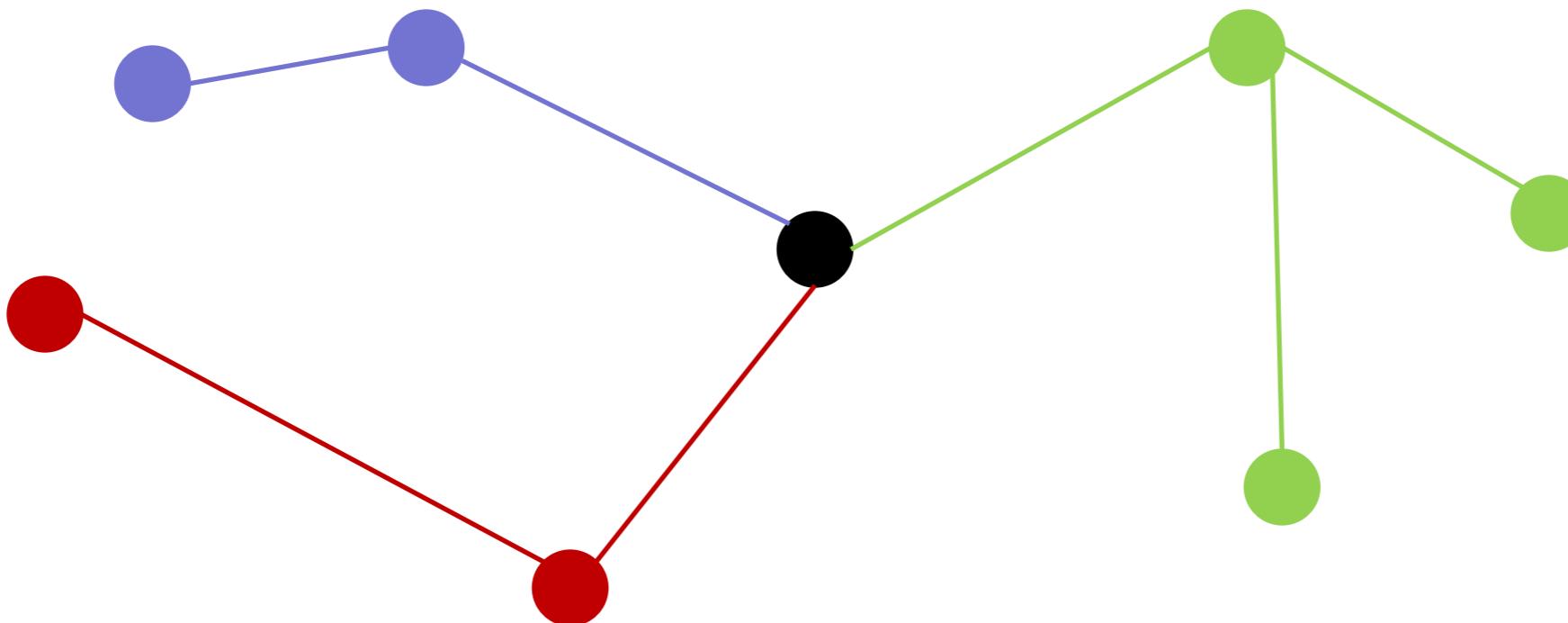
Idea

- Leaves in a tree: a node with degree 1 is called a leaf of the tree
 - Every tree has at least two leaves



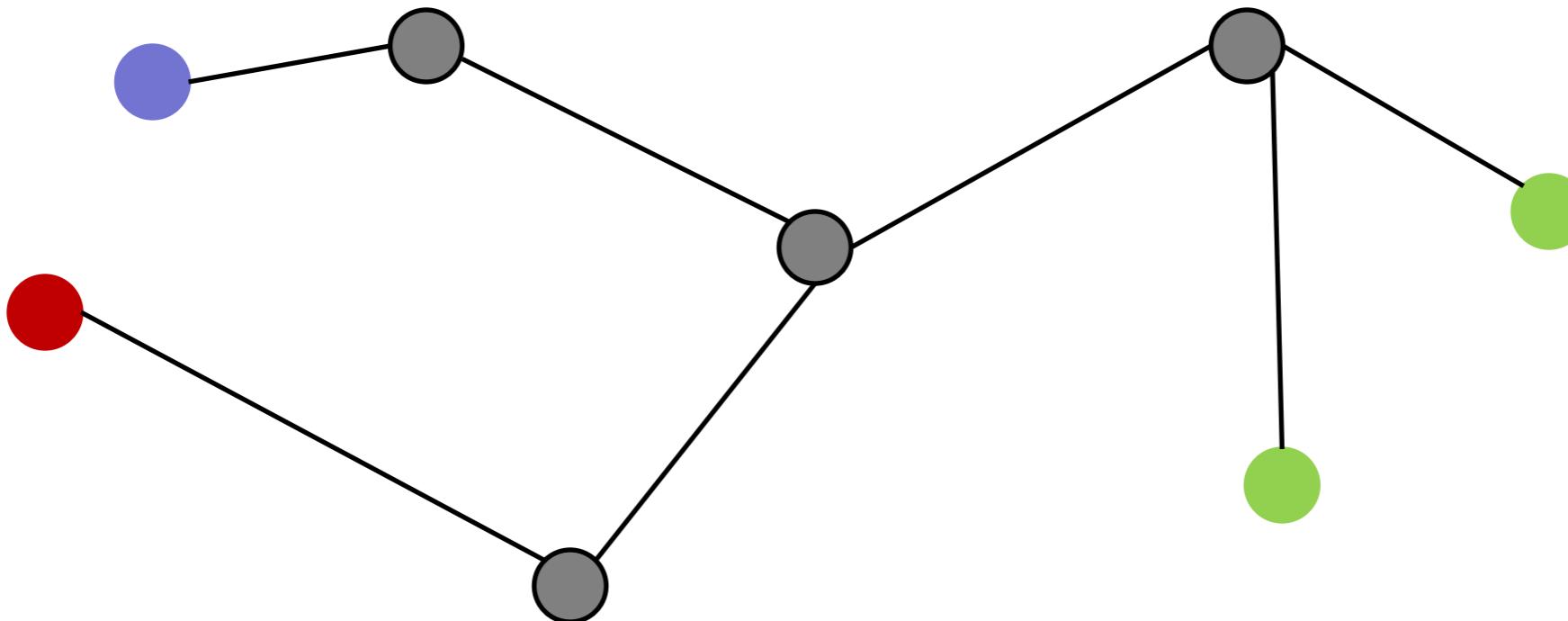
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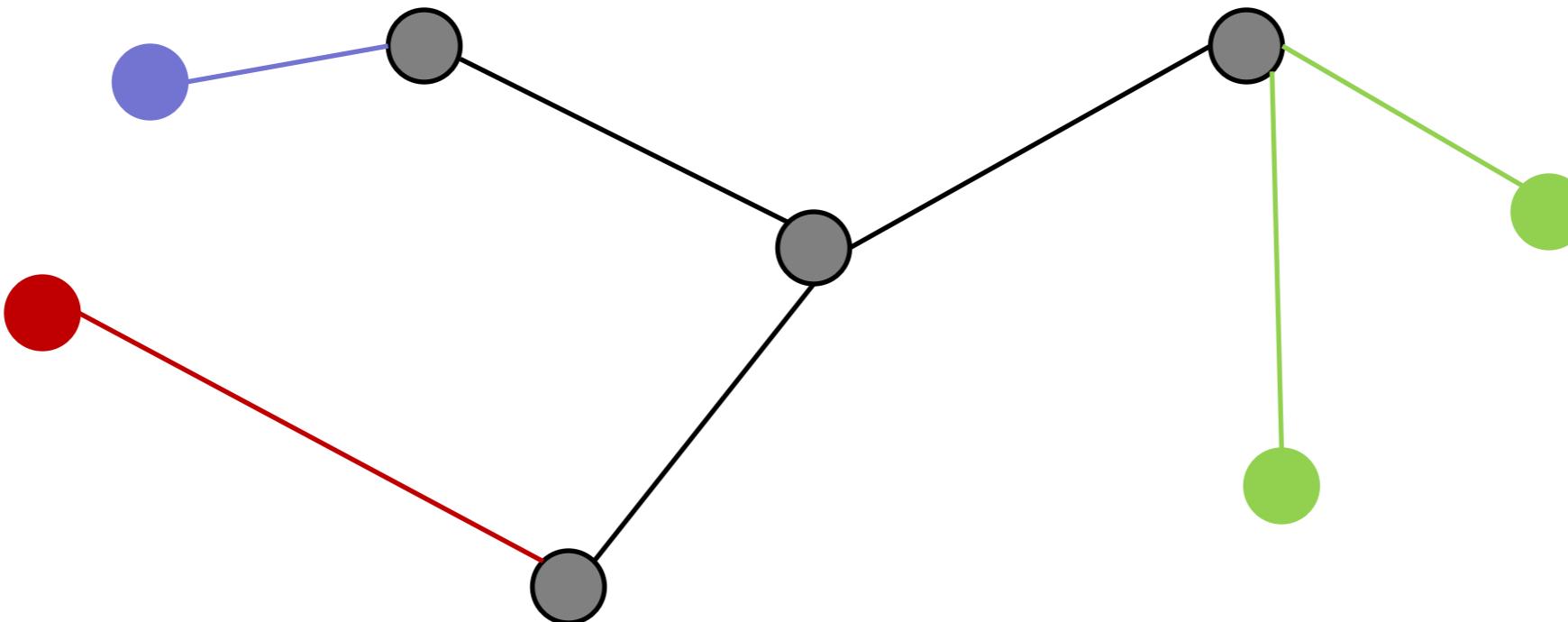
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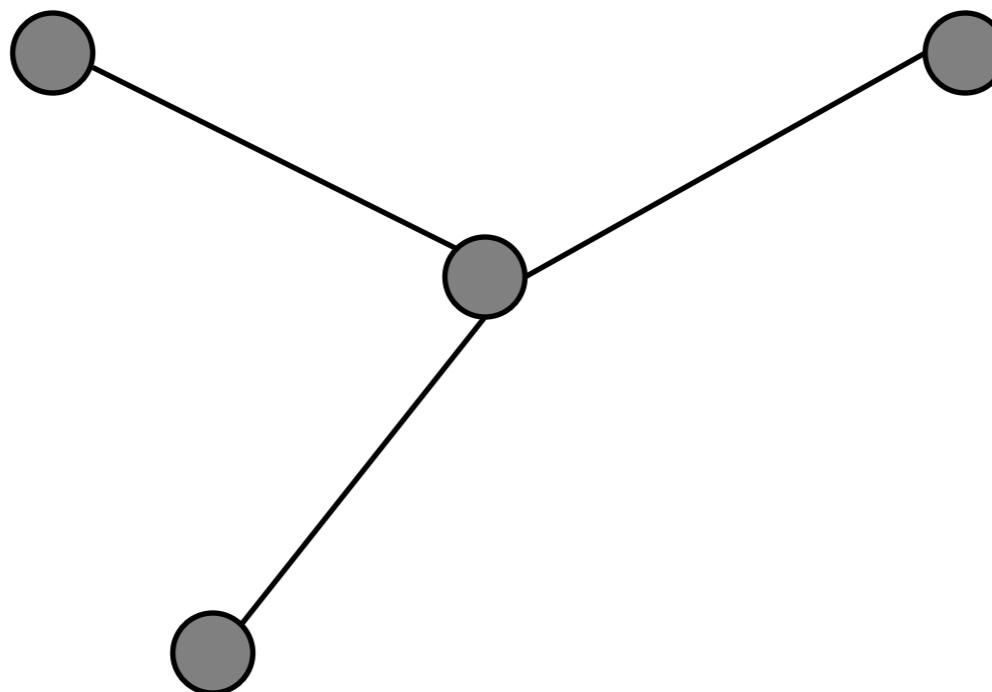
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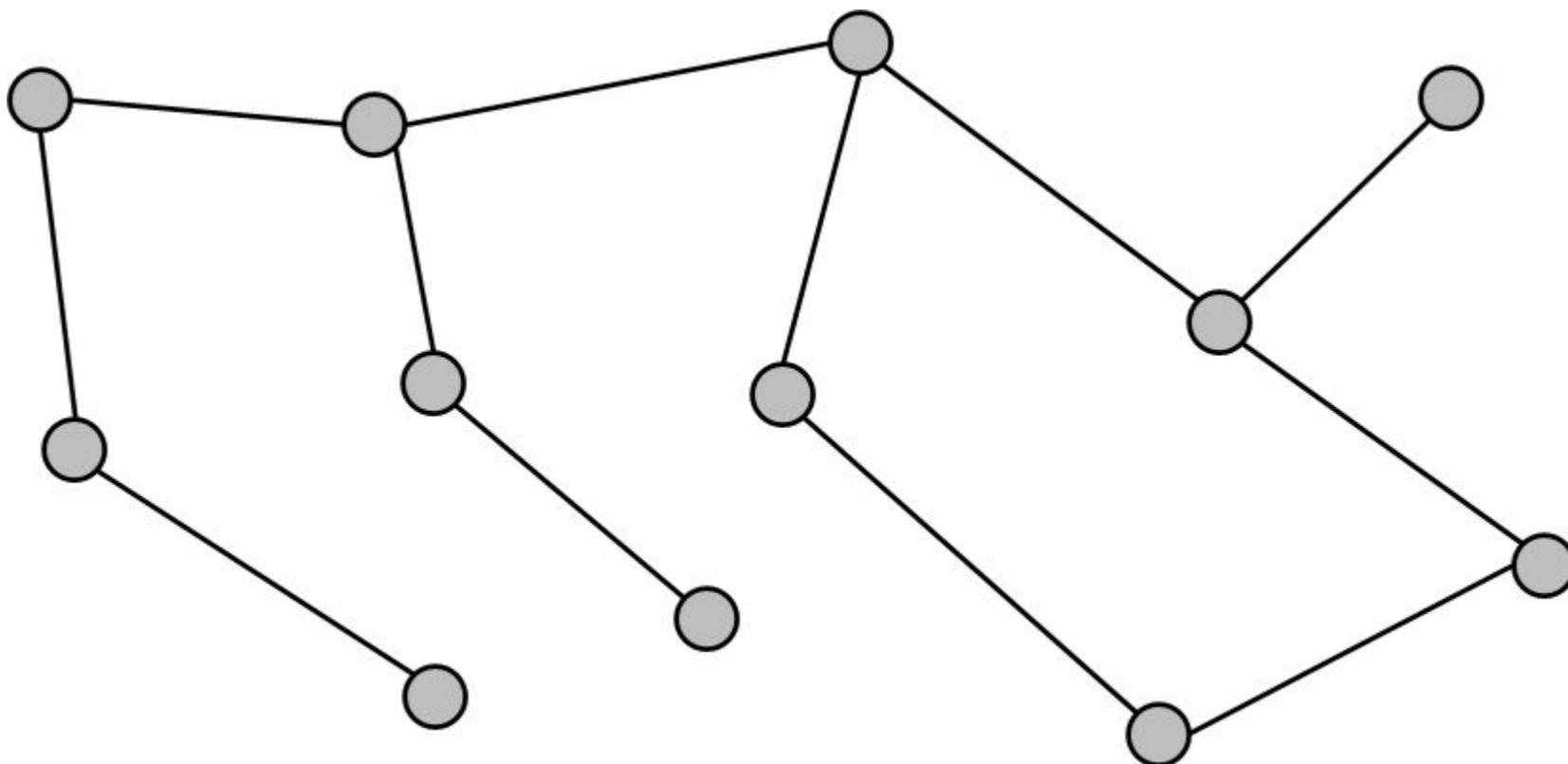


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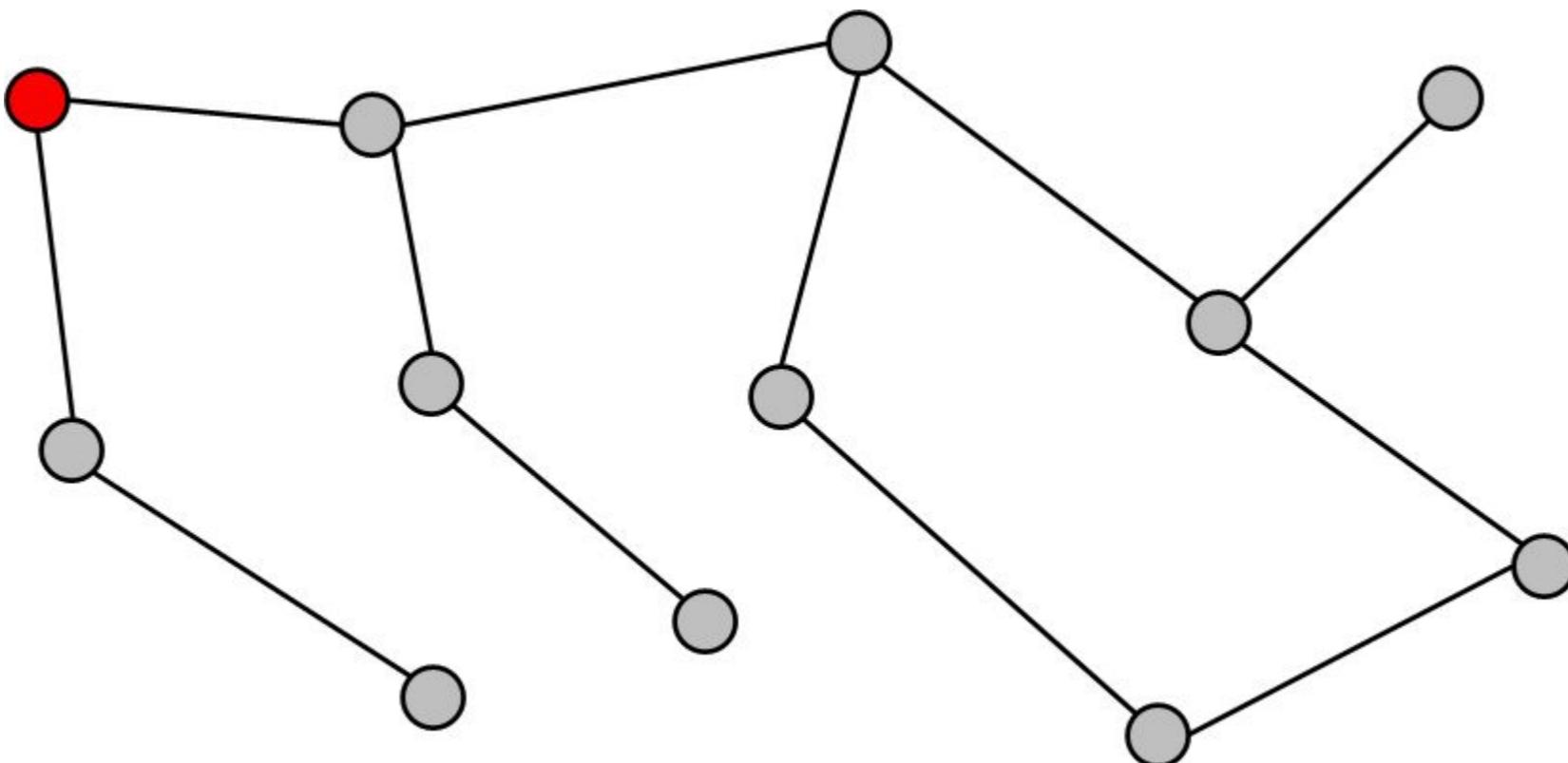
- Leaves in a tree: a node with degree 1 is called a leaf of the tree
 - Every tree has at least two leaves
 - Remove leaves and its connected edges will not change the difference between the number of nodes and the number of edges
 - Kept removing leaves from the tree, we finally obtain a graph of a single node
 - $\# \text{nodes} - \# \text{edges} = 1$ in a tree



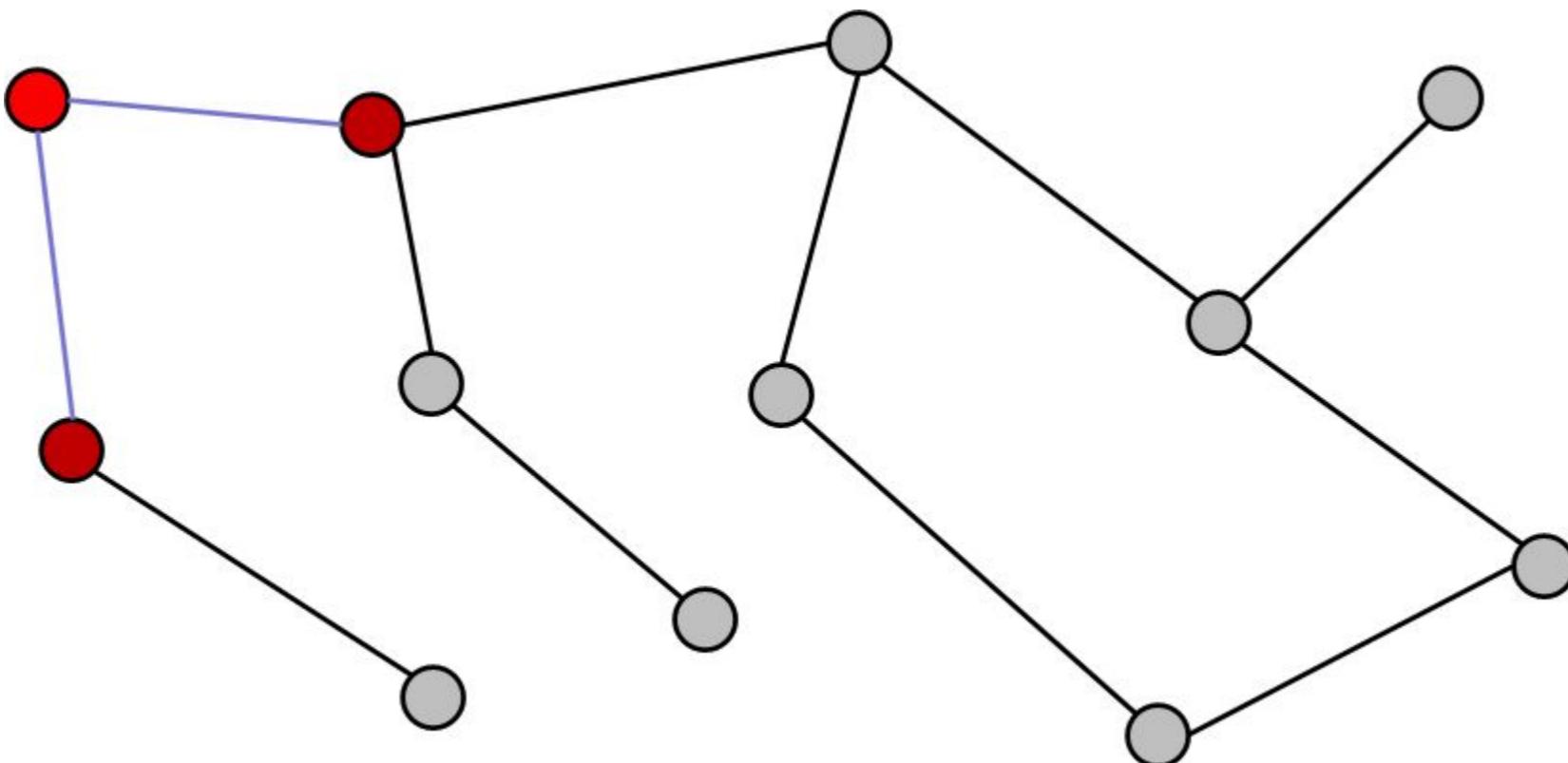
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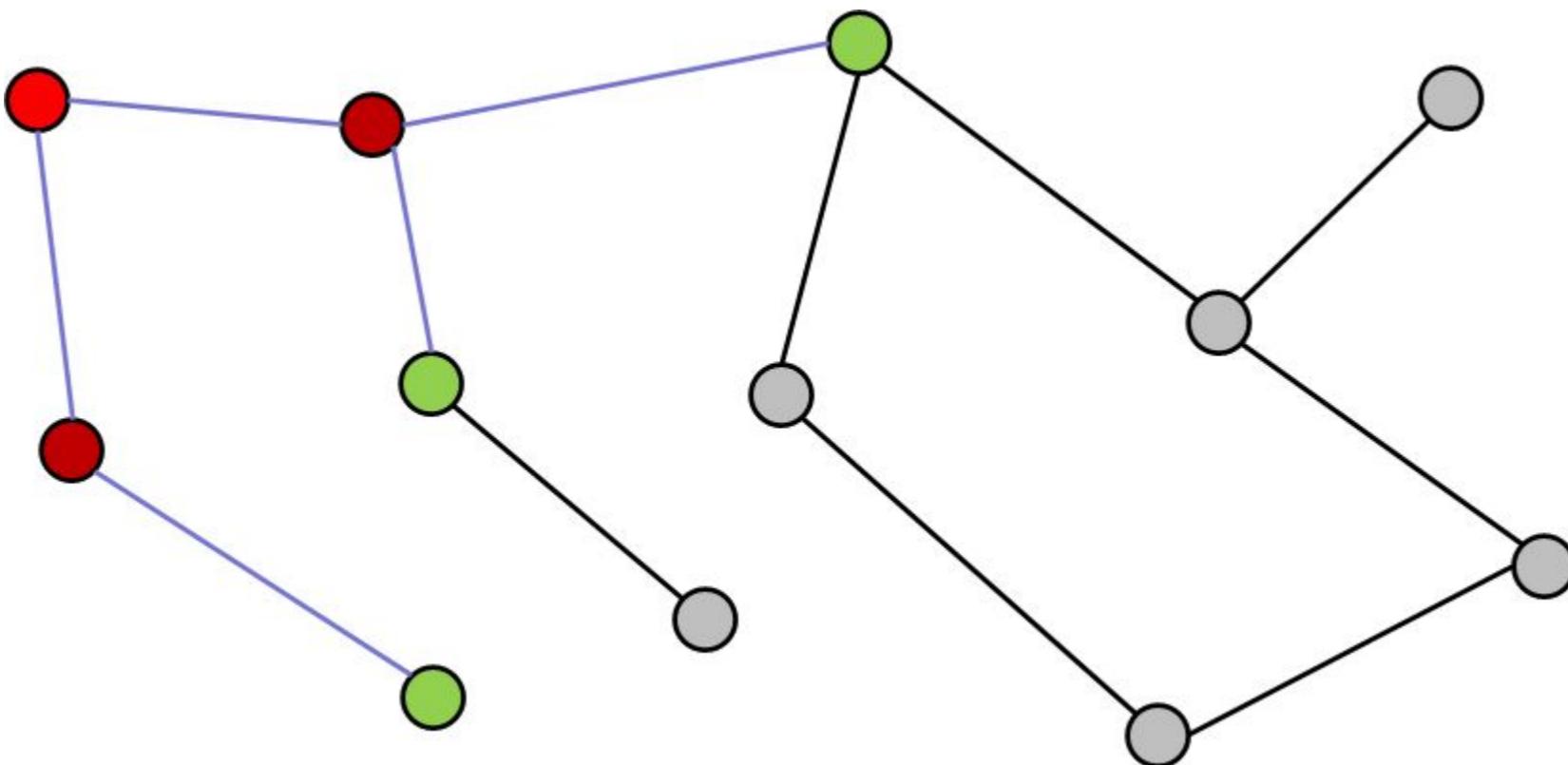
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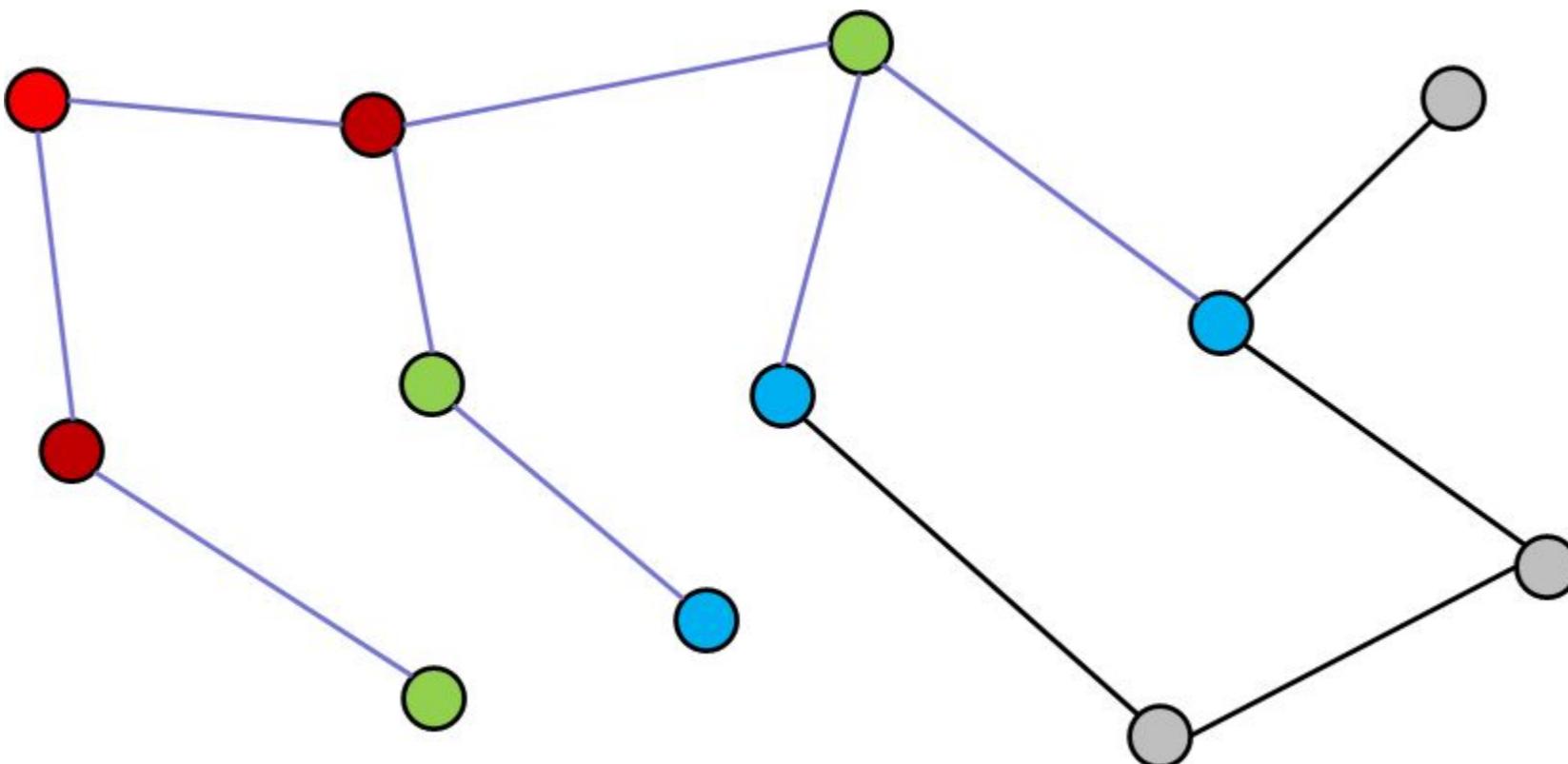
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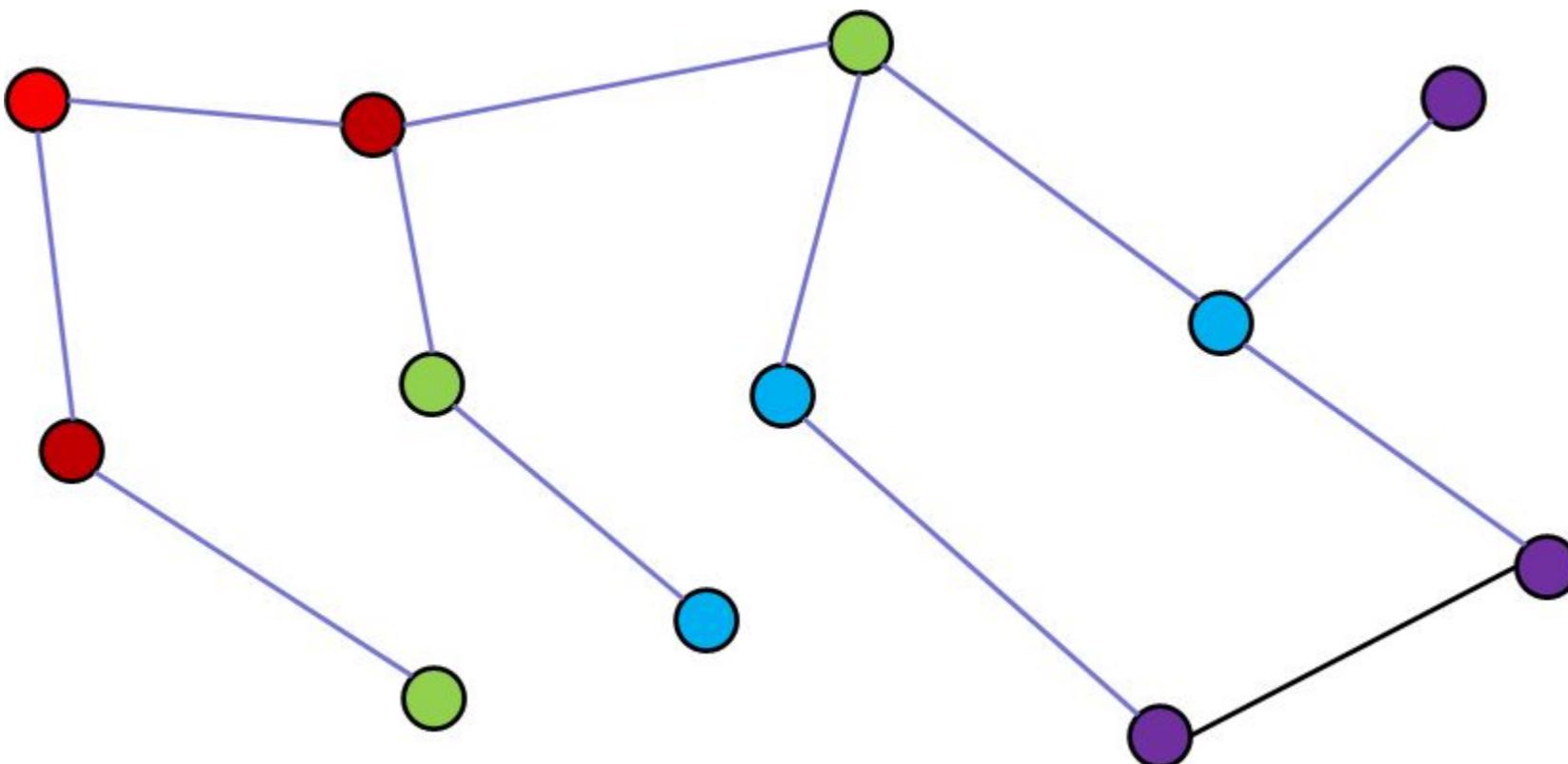
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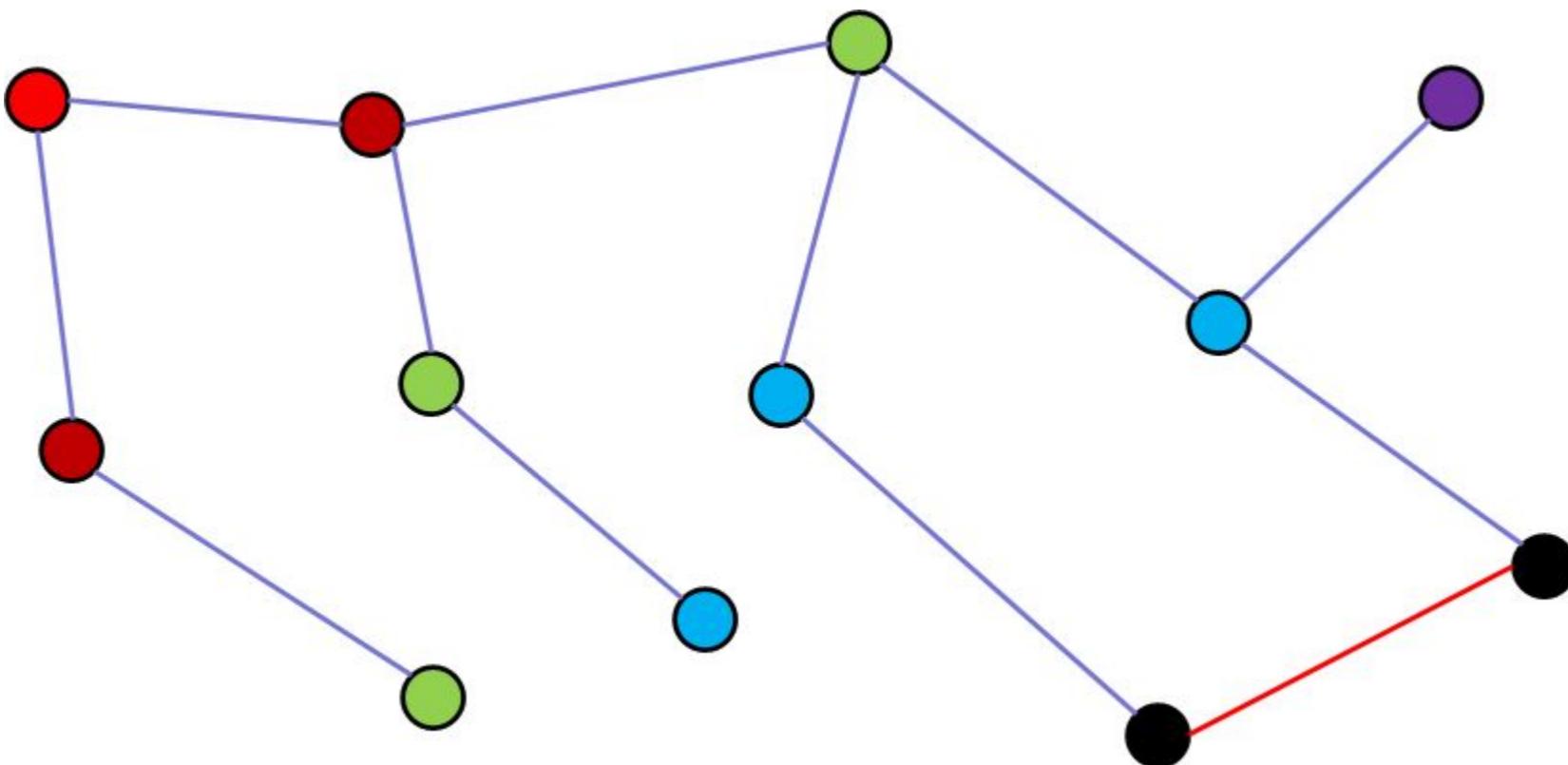
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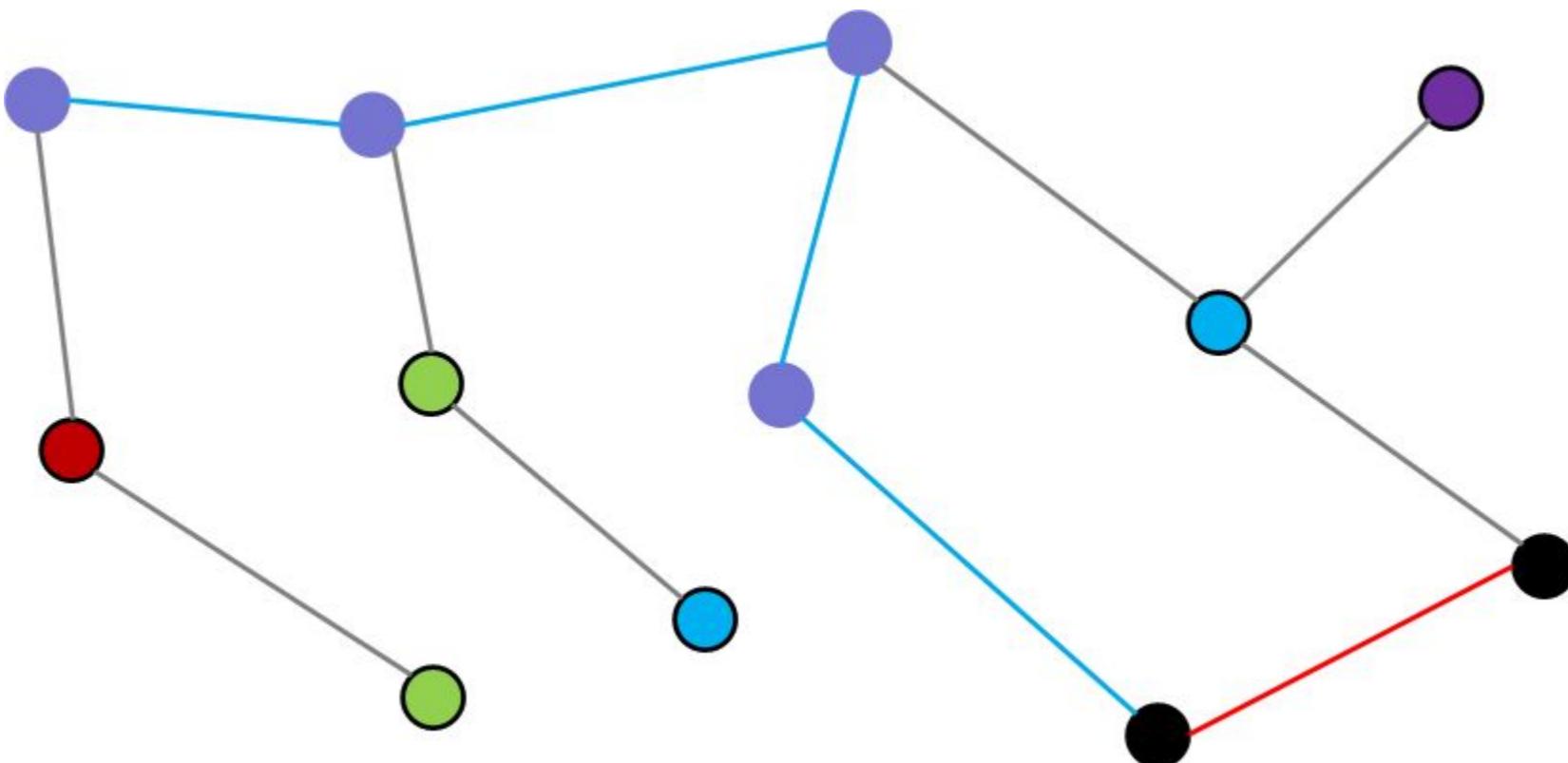
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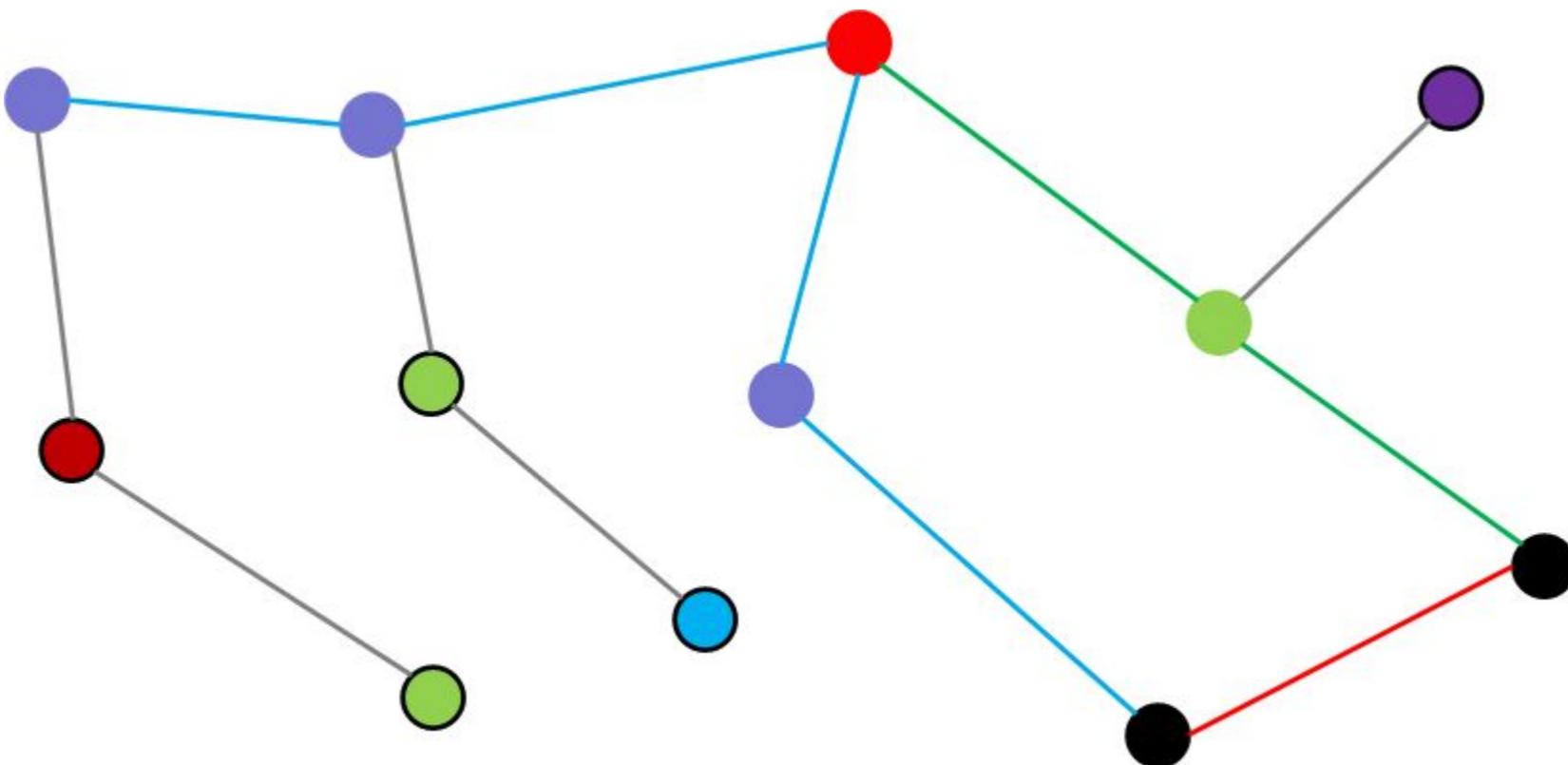
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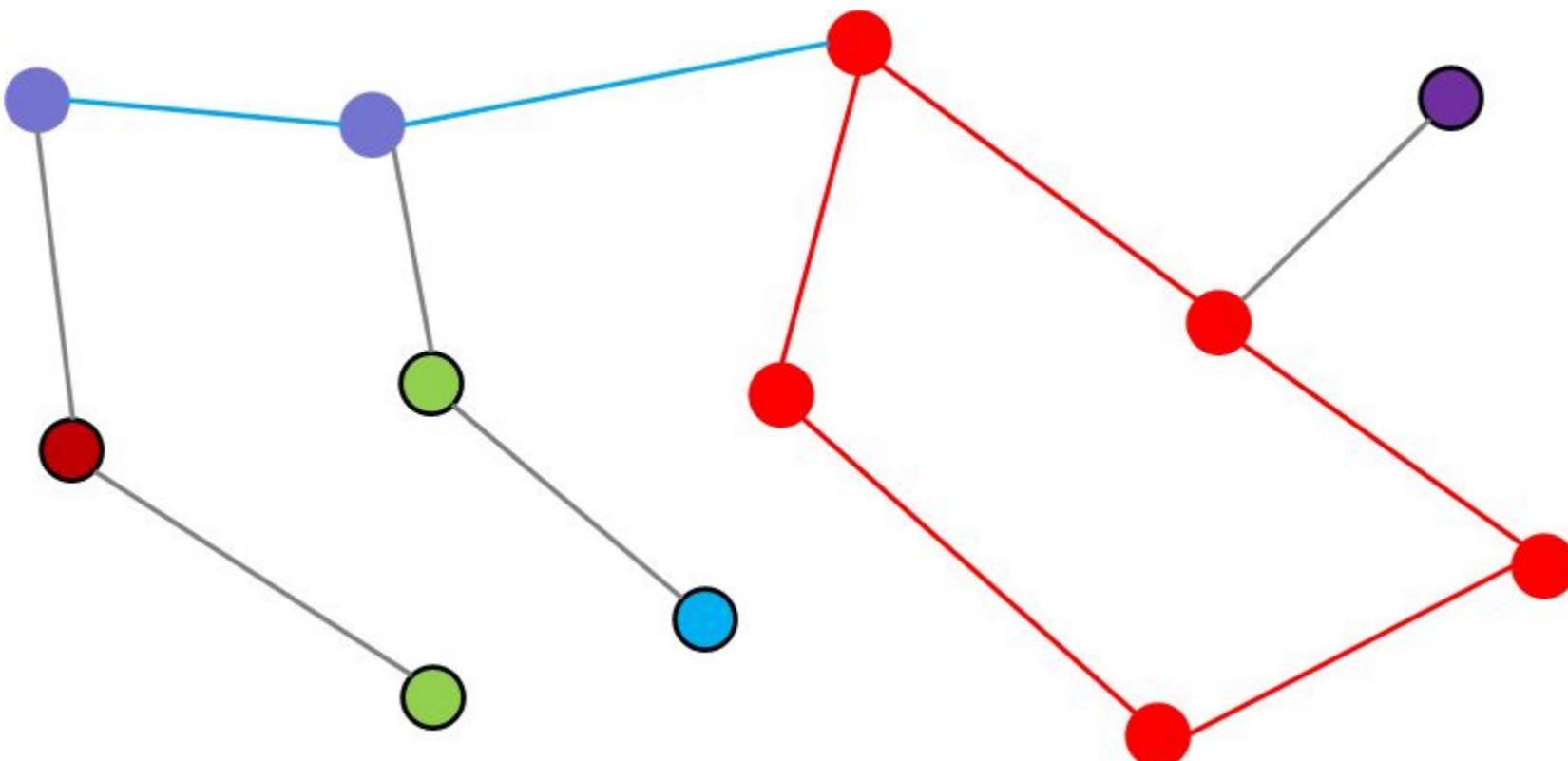
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How to find a cycle in a graph?



How to find a cycle in a graph?



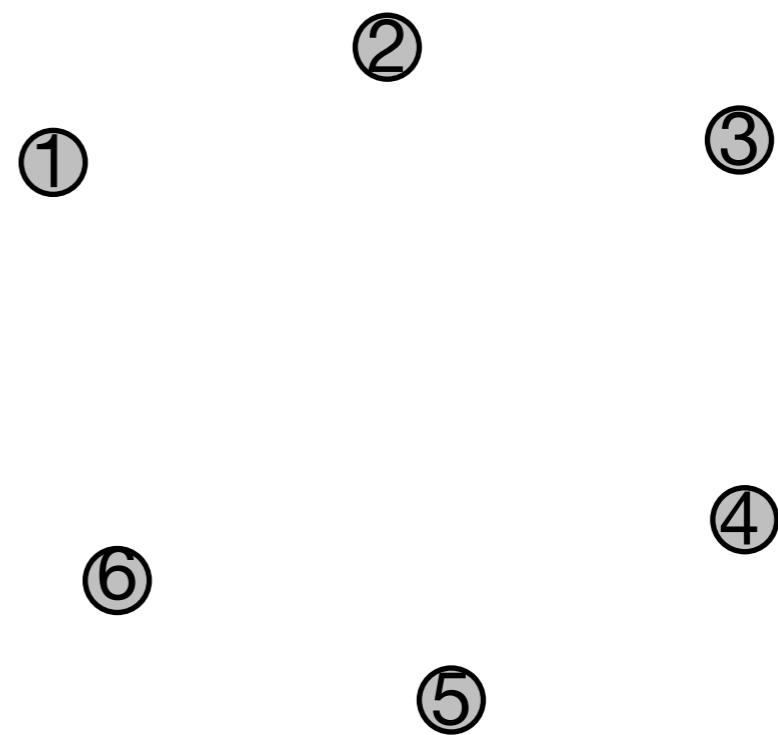
Adjacency Matrix

- When we can't draw a graph, we can represent it with a matrix.
- If row i , column j contains a 1, there is an edge between nodes i and j . If there's a 0, there's no edge.
- **Exercise:** Draw the graph for this adjacency matrix. What's the degree of each node? Is there something special about node 1? Are there any cycles? What are their lengths?

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

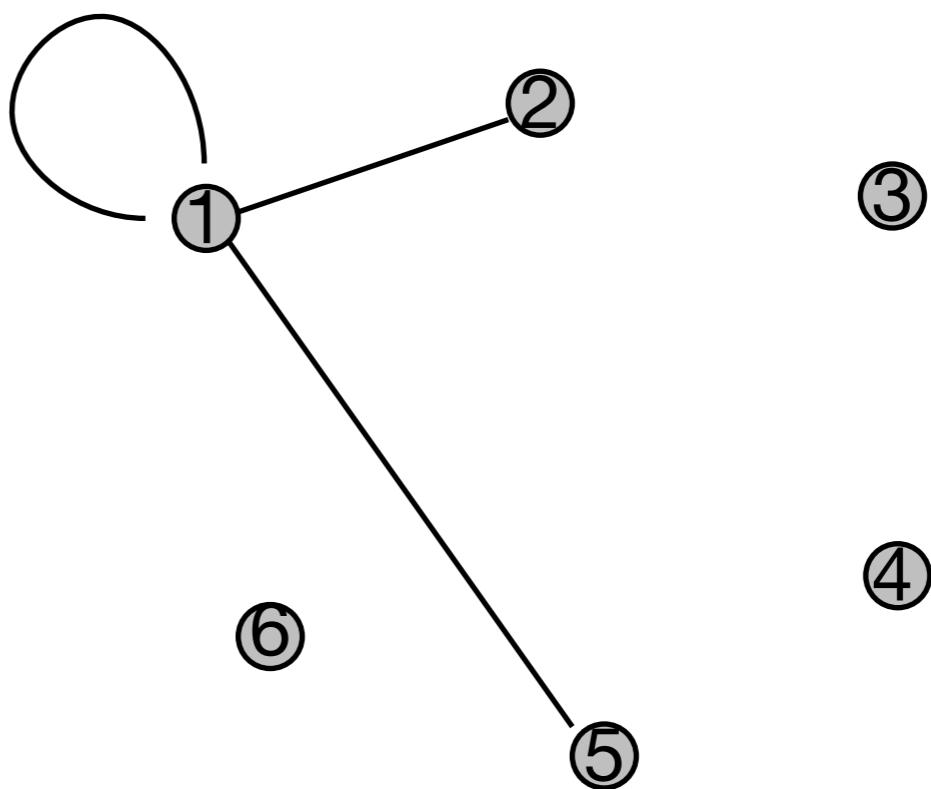
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$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$



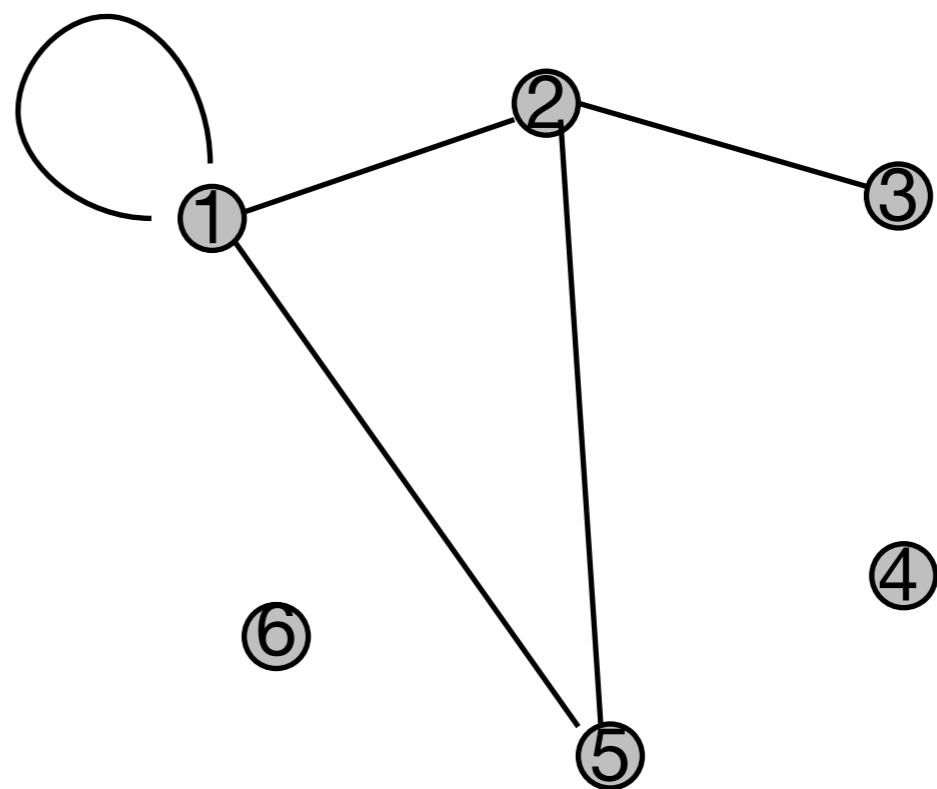
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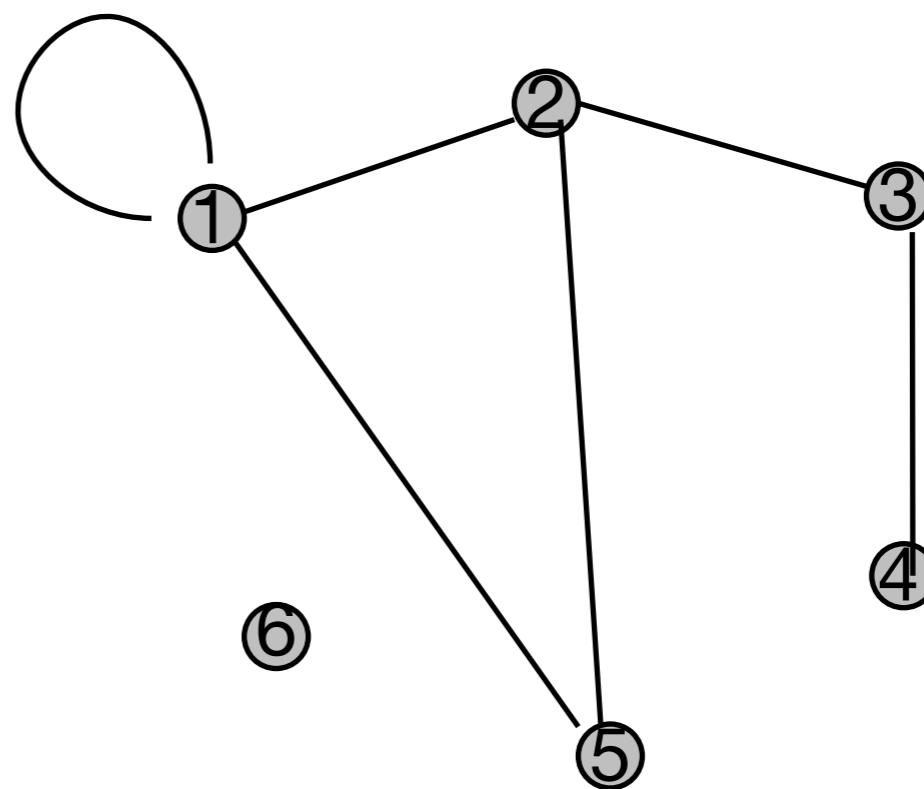
Adjacency Matrix

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$



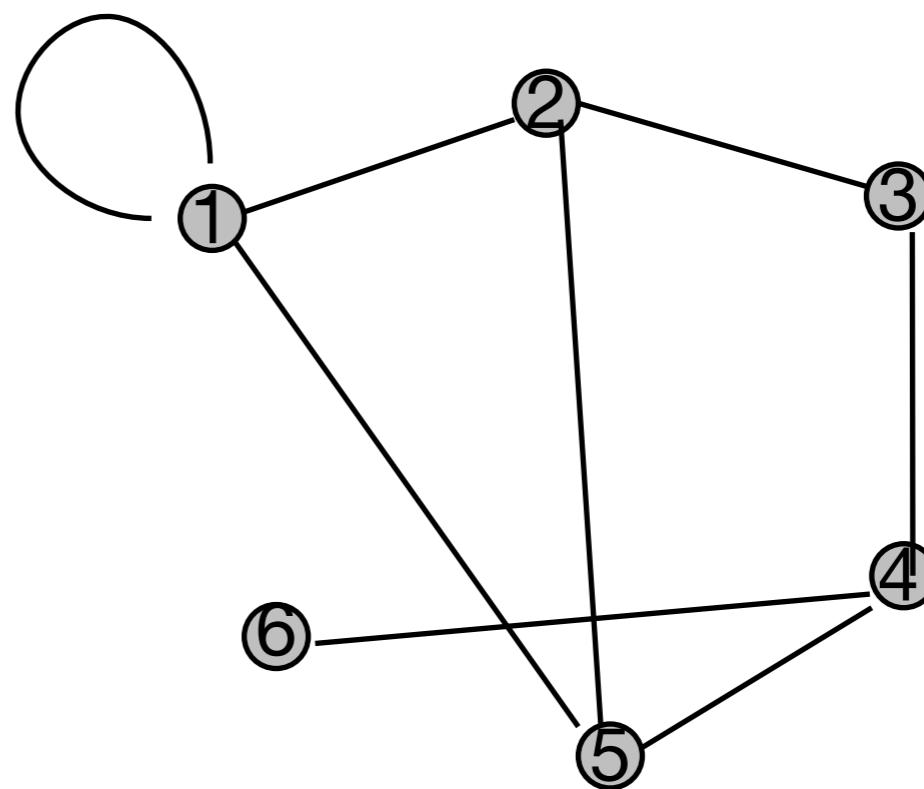
Adjacency Matrix

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$



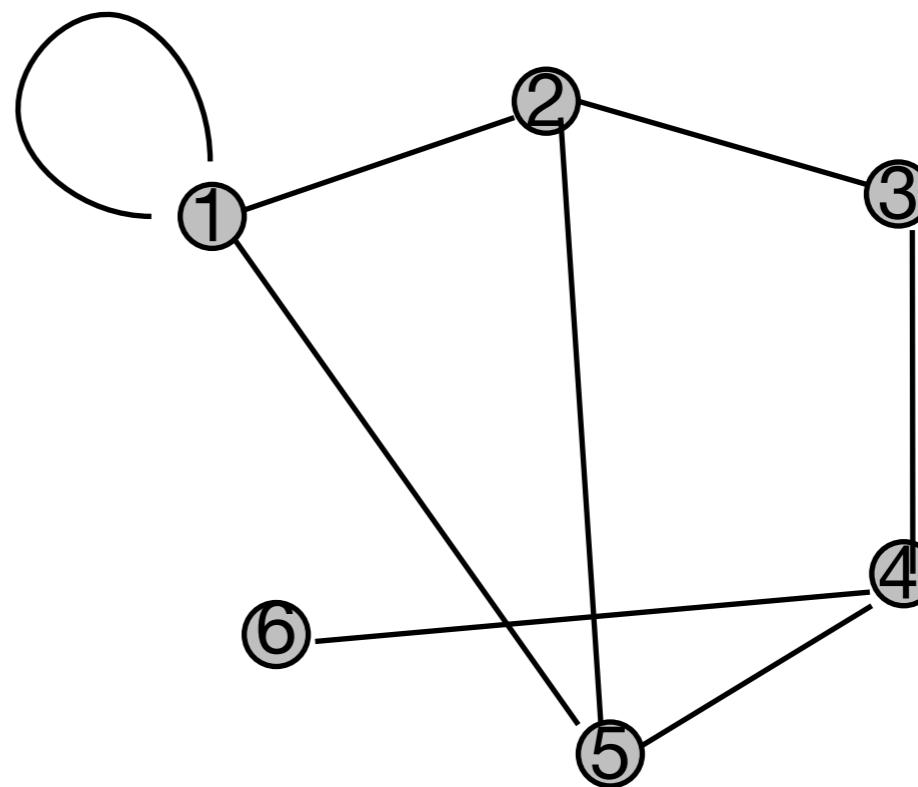
Adjacency Matrix

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$



Adjacency Matrix

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$



Exercise:

- What's the degree of each node? How is it related to the adjacency matrix?
- Is there something special about node 1?
- Are there any cycles? What are their lengths?
- Is the Adjacency matrix of a graph symmetric?

Further topics:

- Social Network:
 - Social Network Visualization
http://www.science.smith.edu/dftwiki/index.php/Visualizations:_Social_Networks
- Classical Graph Theoretical Problems:
 - Seven Bridges of Konigsberg
 - Traveling Salesman Problem
 - Four Color Theorem
 - Algorithms on graphs
- What's more?
 - Algebraic graph theory: Complexity theory
 - Extremal graph theory: Cryptography, Coding theory
 - Random graph theory: Social networks, Network navigation
 - Machine learning

Exercise 1: Easy graph theory

- First, take a look at matrix.txt
- It's an adjacency matrix
- Now, look at graphs.py
 - All we'll do is implement two simple functions:
 - `matrix_load()` reads the matrix.txt file and loads it into a matrix (an array of arrays)
 - `print_degrees()` prints the degrees of each node
 - Comments / hints are provided above each function.
 - **Optional:** `is_connected()` returns 1 if the graph is connected, 0 otherwise.



Mini-Project: Shortest Path

- Next we'll do something harder.
 - Implement a function that finds the shortest path between arbitrarily two nodes:

```
def shortest_path (mat,node1,node2)
```
 - Print out 'No such path' and return [] if node1 and node2 are not connected
 - Print out the shortest path by the edges on the path and return a list containing the nodes on the graph otherwise.
 - It's not obvious how to do this at first – try out some ideas on paper, then code it up only when you have a working algorithm that you can do manually.
 - Generate multiple random graphs to test on your program



Mini-Project: Cycle Detection

- Next we'll do something harder.
 - Implement a function that detects whether or not there is a cycle in the graph. The definition will be:

```
def detect_cycle(mat)
```
 - Print out an arbitrary edge on the cycle and return a list containing all nodes on the cycle if there is one
 - Print out 'No cycle in the graph' and return [] otherwise
 - In the first step, assume the graph is connected and solve the problem.
 - Generate multiple graphs to test on your program
 - **Question: Does it make any difference if the graph is directed?**



Mini-Project: Cycle Detection for General Graphs

- Next we'll do something harder.
 - **Bonus:** solve it for more general graphs that contain multiple connected components
 - The definition will be:

```
def detect_cycle_gen(mat)
```

- The requirements are specified in CycleGen_E.py
- Generate multiple graphs to test on your program
- **Question:** Does it make any difference if the graph is directed?



Mini-Project: Preliminaries

- Things you might need in Python:
 - Lists
 - Suppose A is a list
 - A.append(x) appends x to A
 - The expression (x in A) returns True if A contains x, otherwise it returns False
 - A[n] returns the (n+1)-th element of A
 - The elements of a list could also be lists
 - A=[2,3,[4,5],6,[7,[8,9]]]
 - A[0]=2,A[1]=3,A[2]=[4,5],A[3]=6,A[4]=[7,[8,9]]
 - A[2][0]=4,A[2][1]=5
 - A[4][0]=7,A[4][1]=[8,9]
 - A[4][1][0]=8,A[4][1][1]=9
 - Reading files
 - Filename “matrix.txt”
 - Fh=open(“matrix.txt”)
 - A=Fh.readlines() will return a list containing all the lines of the file



Mini-Project: Preliminaries

- Things you might need in Python:
 - Reading files
 - Filename “B.txt”
 - Fh=open(“B.txt”)
 - A=Fh.readlines() will return a list containing all the lines of the file
 - Example
 - If the file “B.txt” contains the following text:
Hello world!
010001
Python
 - Then A=['Hello world!', '010001', 'Python']
 - Each component of A is a string



References

- Graphs: <http://people.brunel.ac.uk/~mastjeb/jeb/or/graph.html>
- Graph Theory: http://en.wikipedia.org/wiki/Graph_theory
- Kevin Bacon: <http://oracleofbacon.org/>

Quick Recap

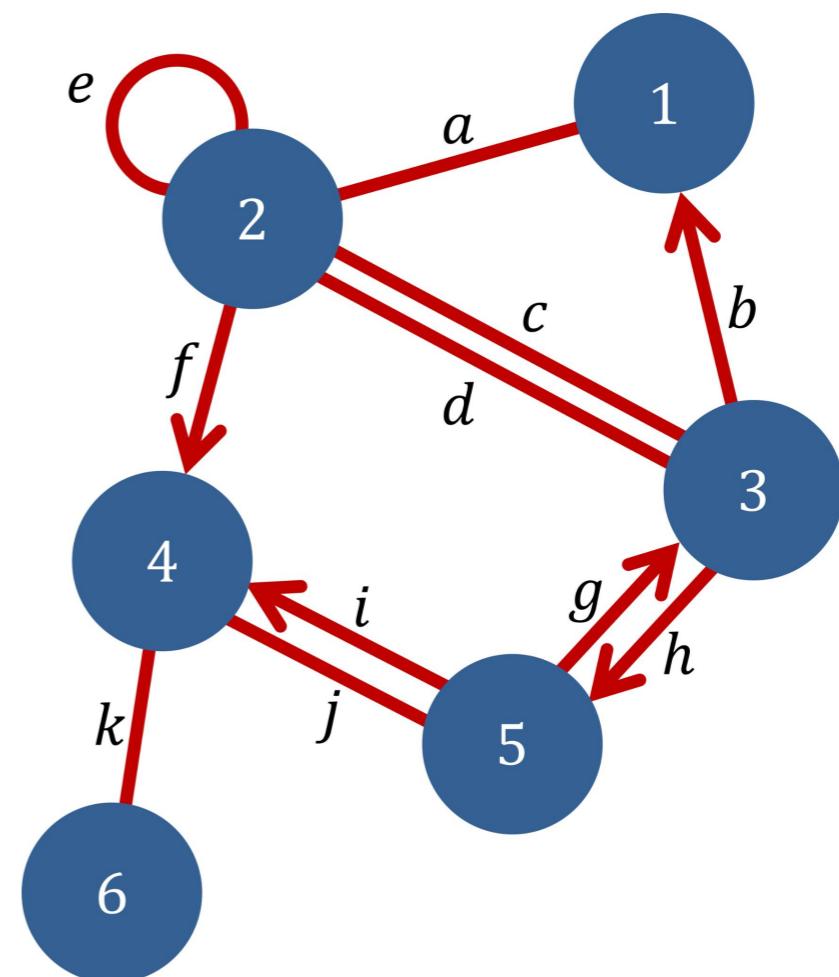
- Last day, we learnt about:

- Graphs: Vertices, Edges, Weights, Directivity

- Degree, Connectivity

- Shortest Path, Cycles, Tree

- Adjacency Matrix



Contents

- Common random graphs:
 - Barabasi-Albert Networks with Preferential Attachment
 - Erdos-Renyi Networks
- Community Structures and Modularity
- Minimum Spanning Tree (MST)
- Shortest Path: Dijkstra's Algorithm
- Traveling Salesman Problem (TSP)



Preferential attachment

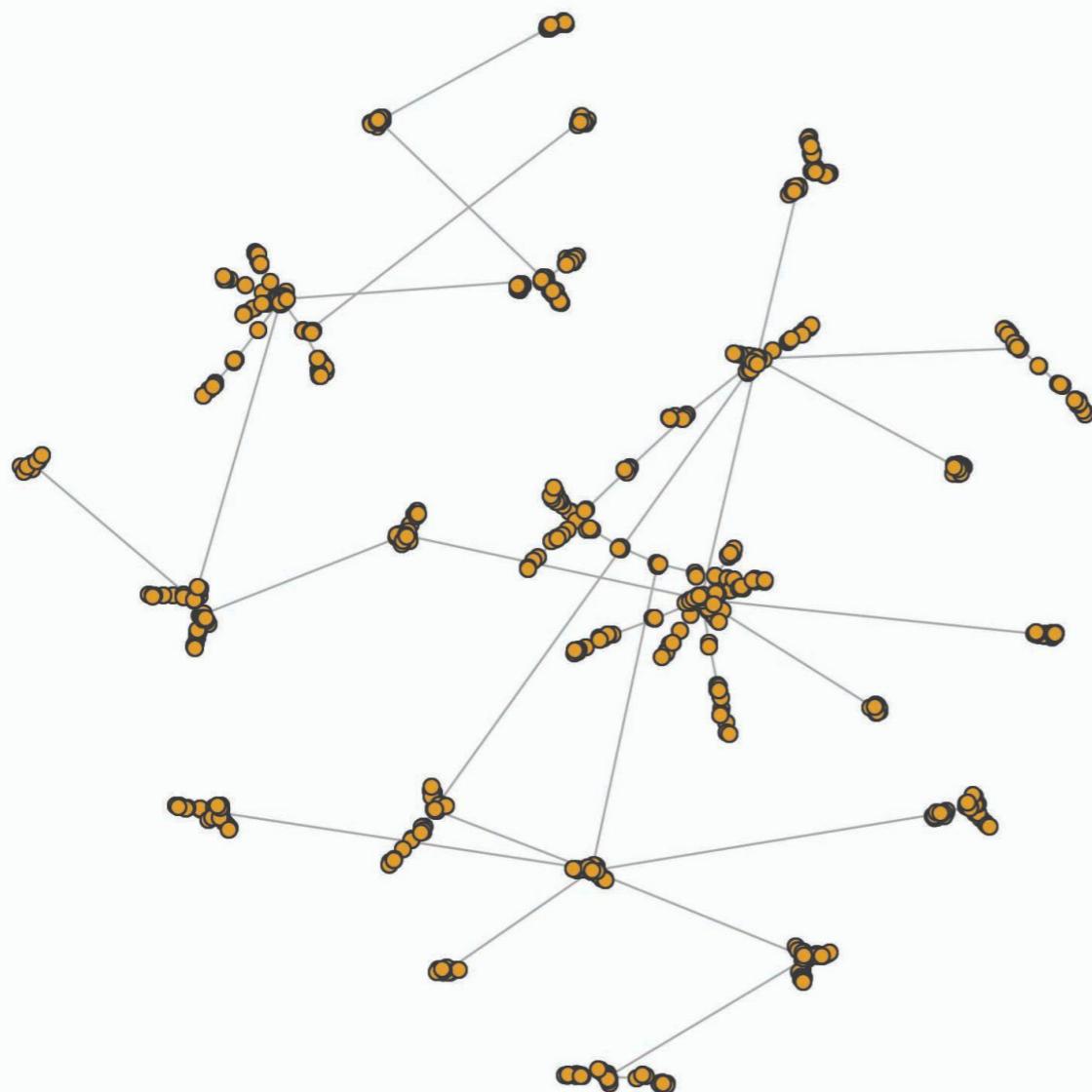
- Popular nodes act as bridge between unpopular nodes.
- The more popular a node, the higher is the probability for it to be connected to incoming nodes.
- Popularity is measured by the degree of a node.
- The Barabasi-Albert model creates **scale-free, evolving and connected** networks with preferential attachment.
- The degree distribution of such networks follow power-law.



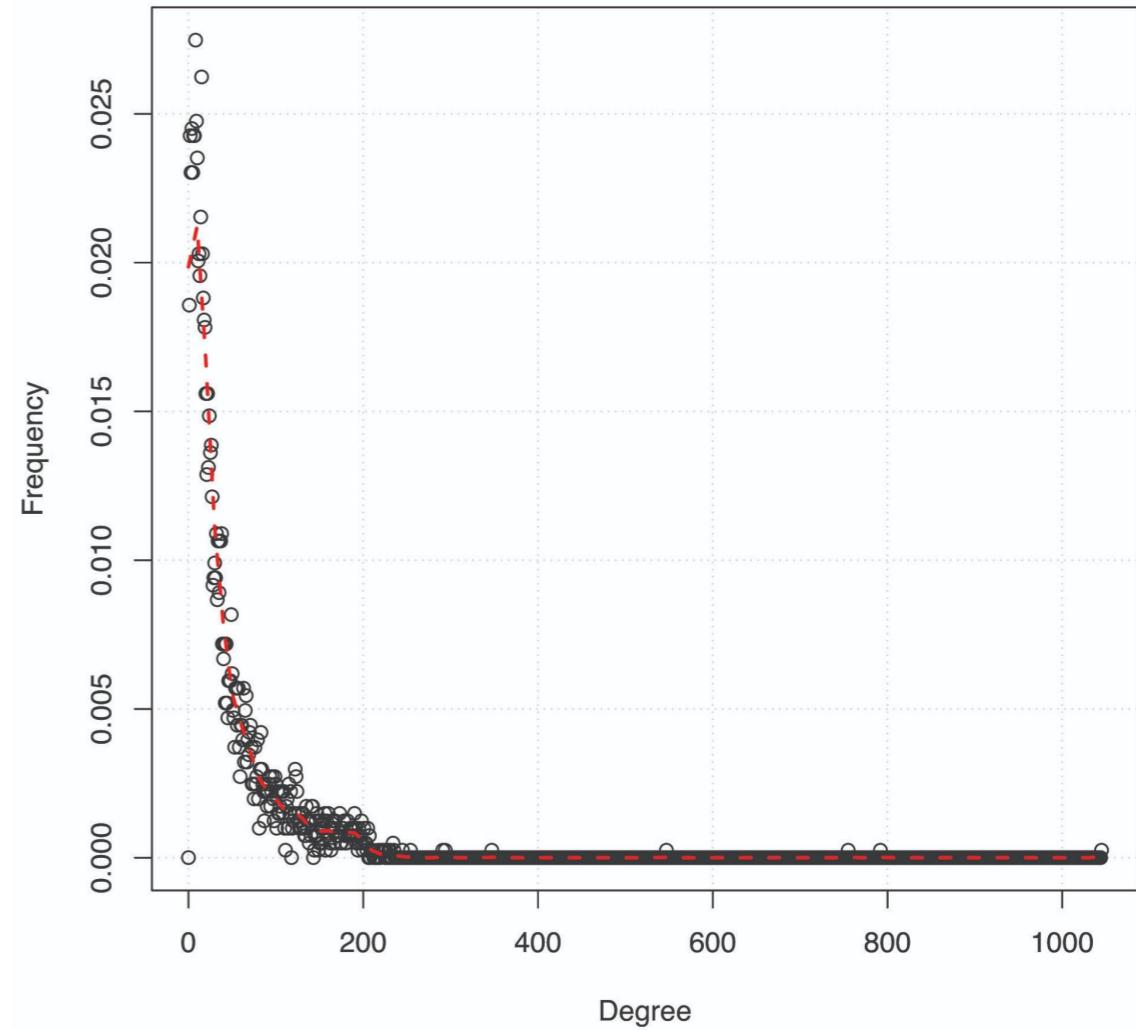
freegifmaker.me

Preferential attachment

Undirected preferential attachment network, $n = 1000$, $m = 1$



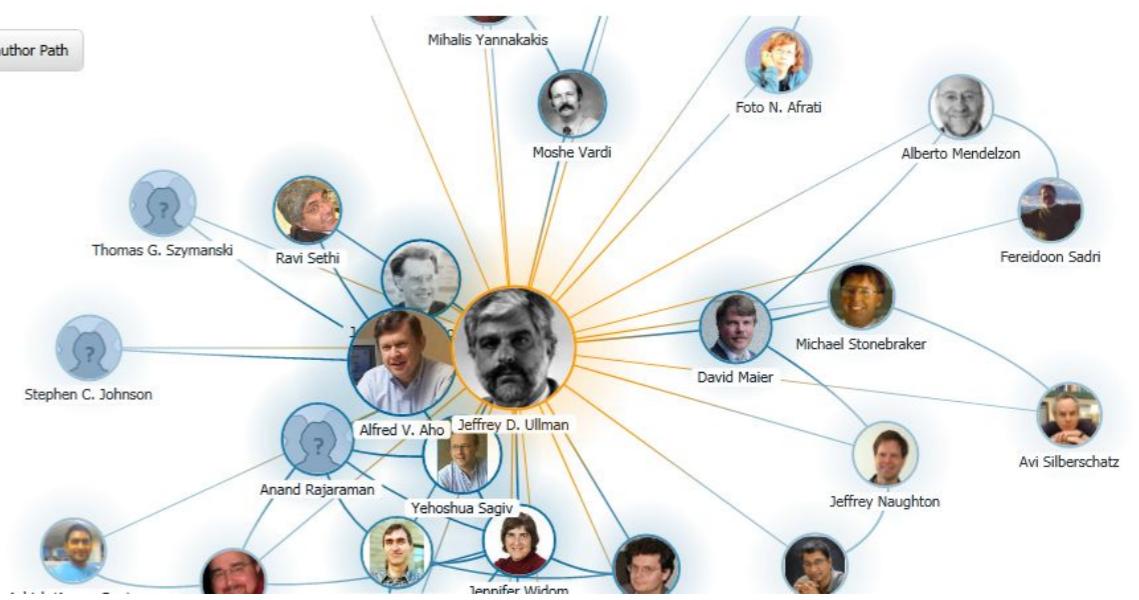
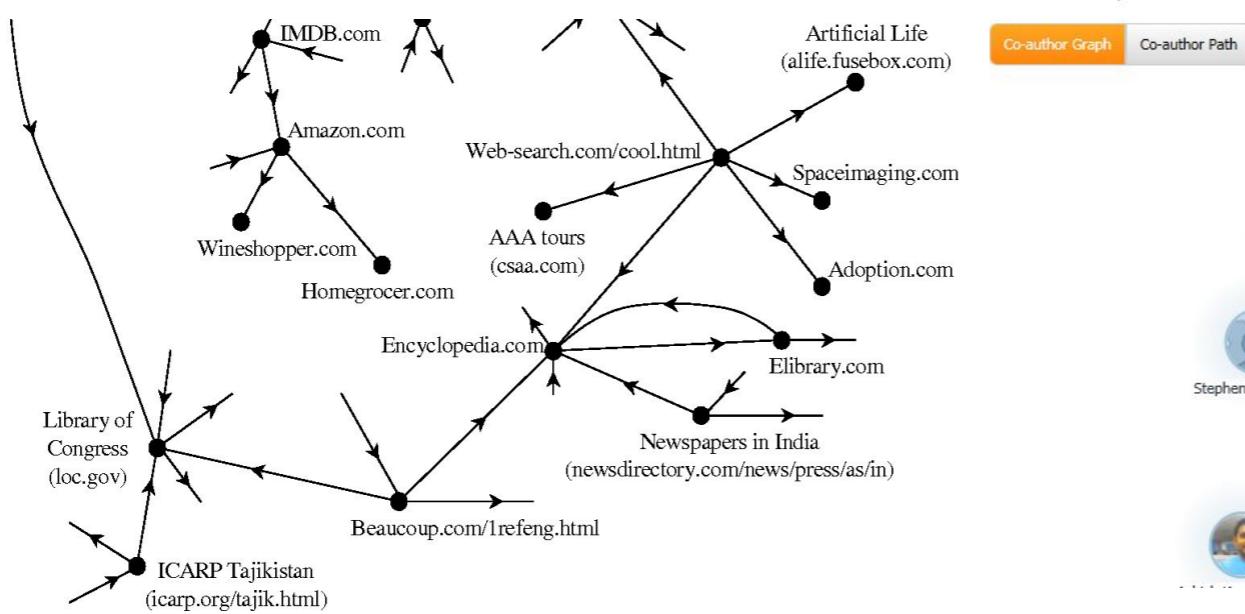
Degree distribution of the Facebook Network



Applications

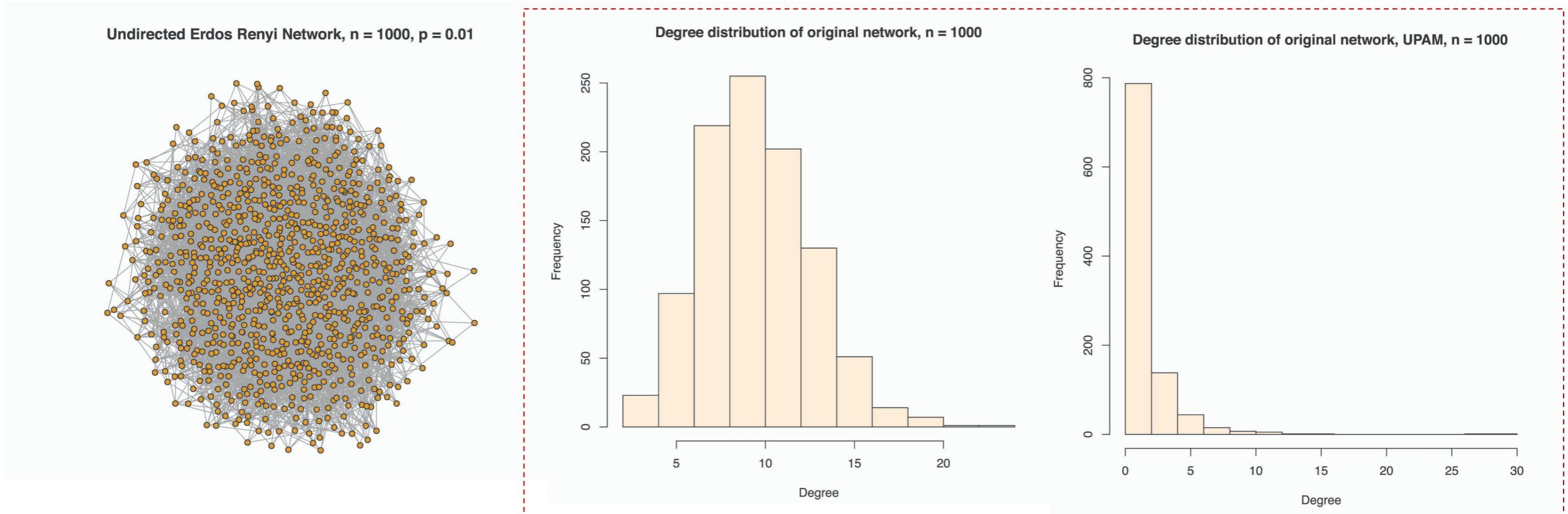


- Social Networks
- Search engines (PageRank)
- World Wide Web
- Citations among researchers
- Internet
- Recommendation Systems
- Family / Erdos Trees



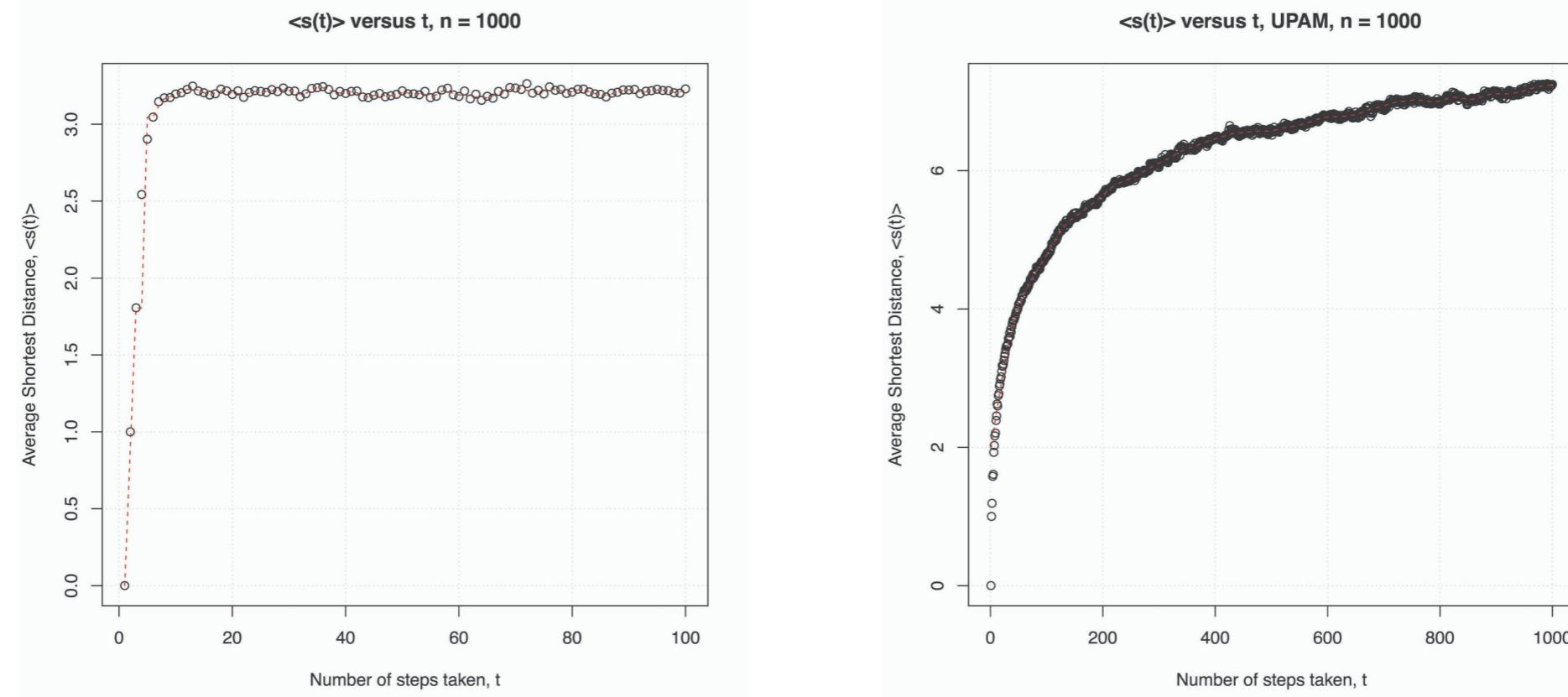
Erdos-Renyi Models

- All edges have a certain probability of being present or absent (probability of connectedness).
- Network may or may not be fully connected; concept of Giant Connected Component (GCC)
- Degree distribution of Erdos-Renyi networks are binomial or Poisson.



Random Walk on Random Graphs

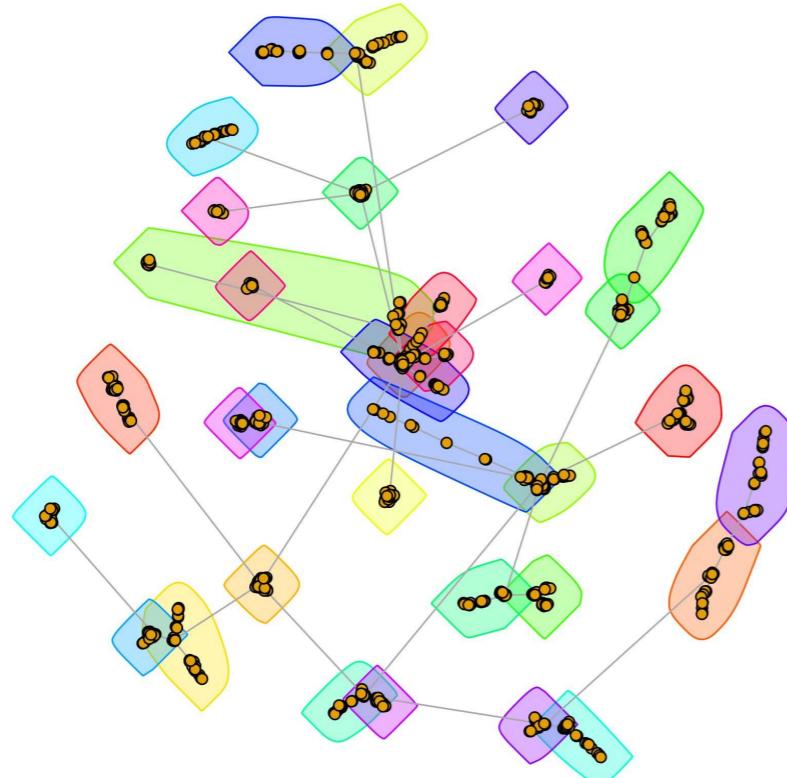
- If you walk randomly on an Erdos-Renyi graph, you will likely end up within a “circular region” corresponding to the average shortest distance of the nodes in the entire network from an initial node.
- However, for Barabasi-Albert graphs, your average shortest distance will increase non-monotonically with more steps from the initial node.



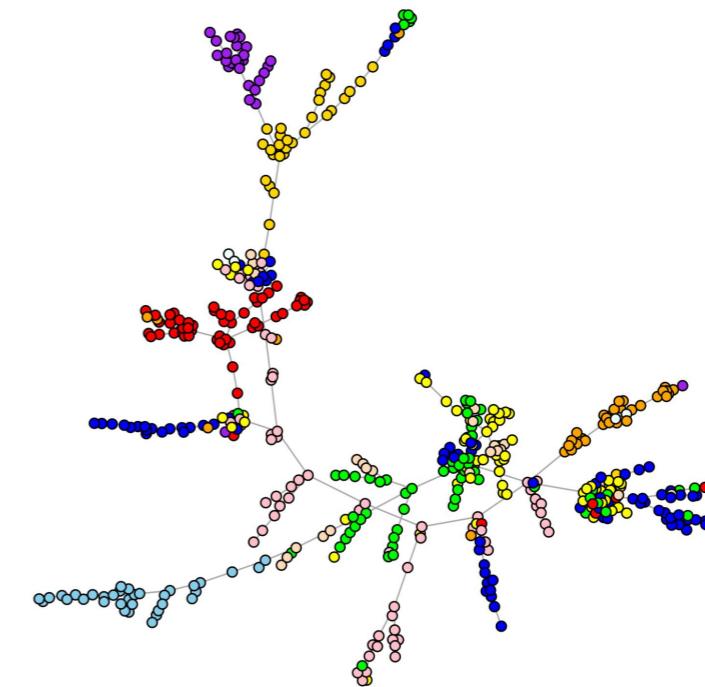
Community Structures and Modularity

- Community structures: clusters or regions of high connectedness in the graph
- We measure the strength of clustering using modularity score.
- Application example: Correlation graph of stocks
- Techniques: Fast-Greedy, InfoMap, Edge-Betweenness

Community Structure of UNPA, $n = 1000, m = 1$

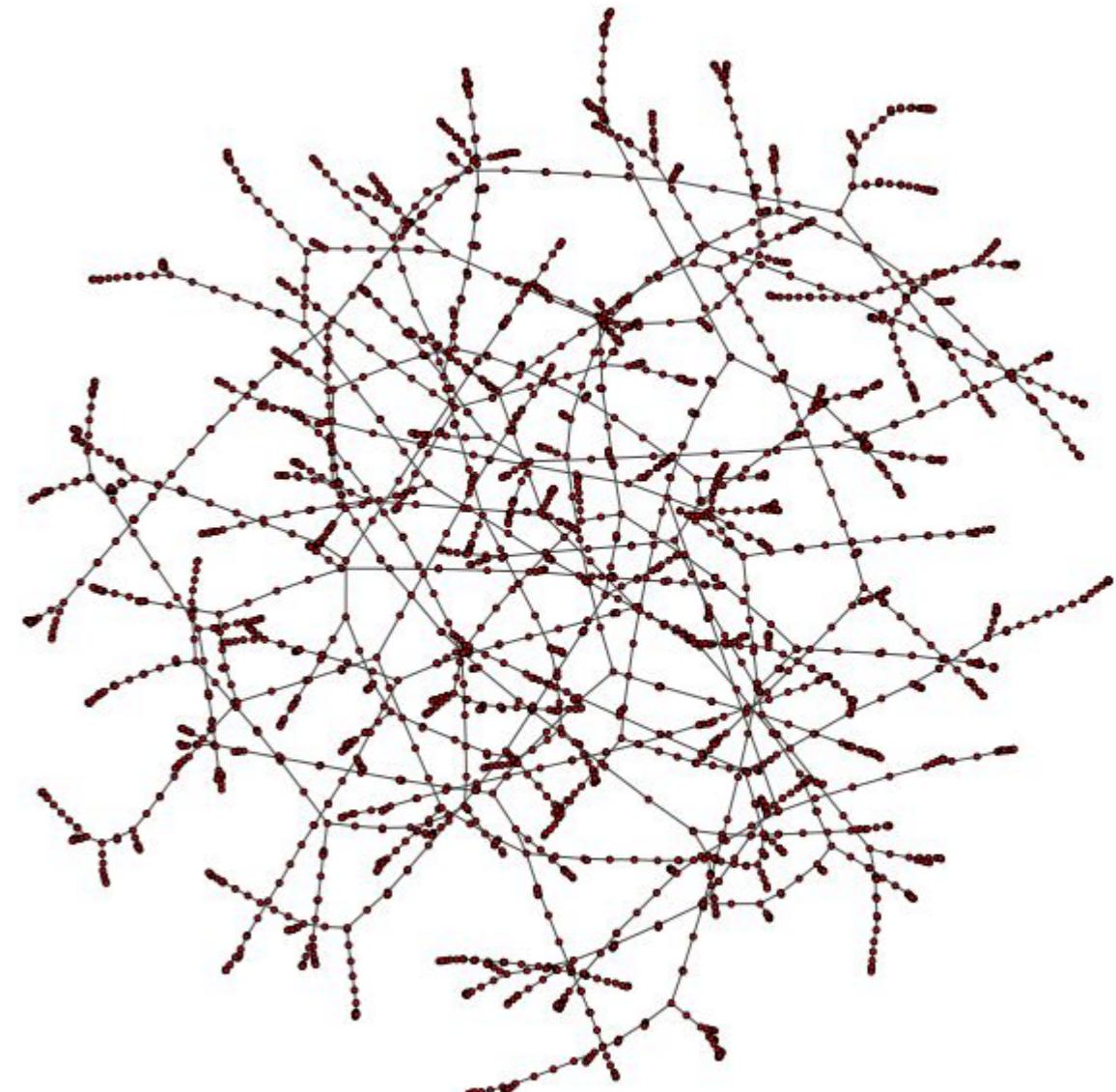


Example of stock market correlation graph MST



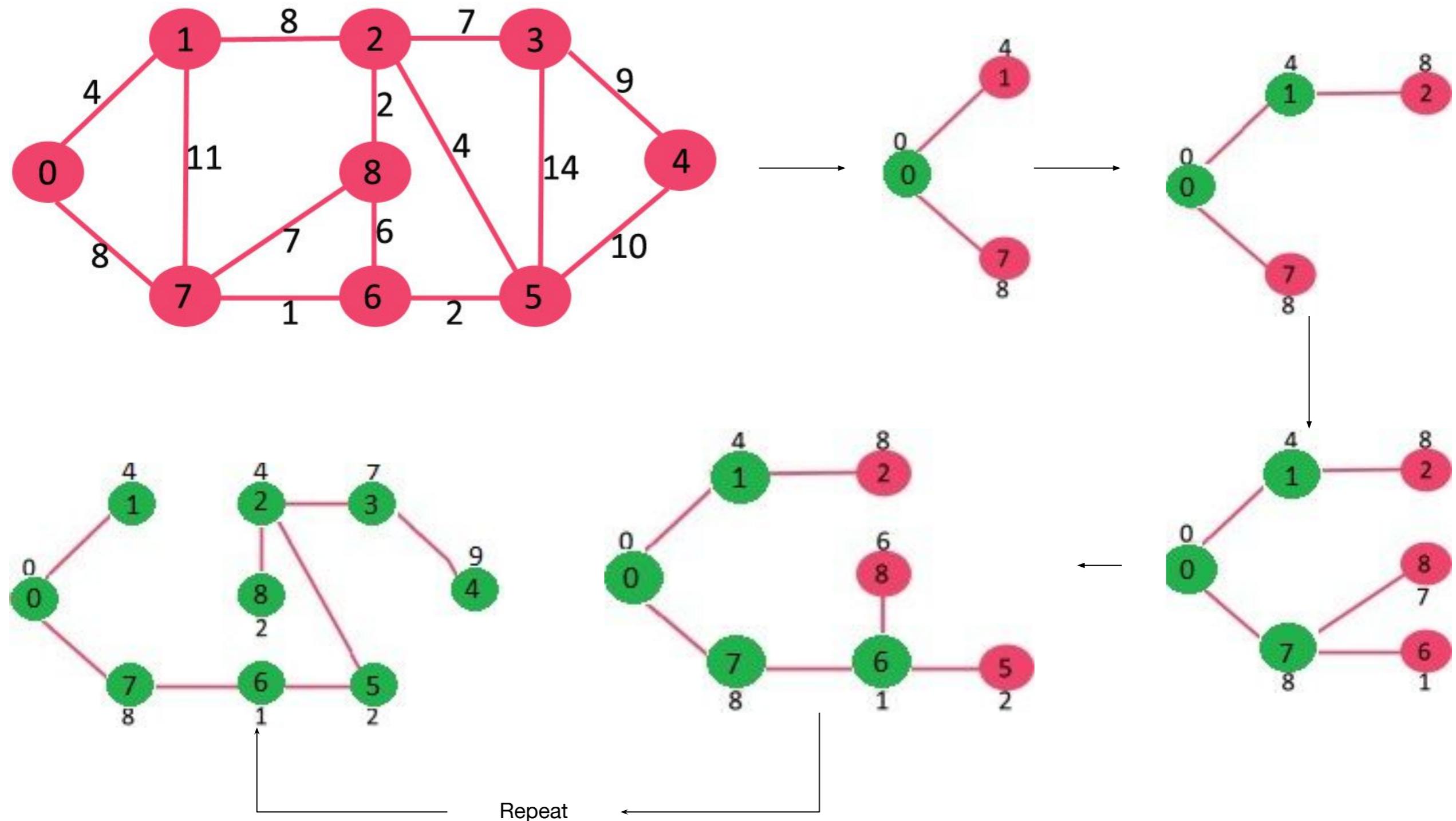
Minimum Spanning Tree (MST)

- MST: Connects all the nodes of a connected and edge-weighted undirected graph:
 - **without any cycles**
 - **with the lowest possible cumulative edge weights (lowest cost).**
- Techniques to find MST:
 - Prim's Algorithm
 - Kruskal's Algorithm
- Prim's Algorithm (uses two sets):
 - Find a cut
 - Pick the edge with minimum weight.
 - Include the edge in the list of included vertices.

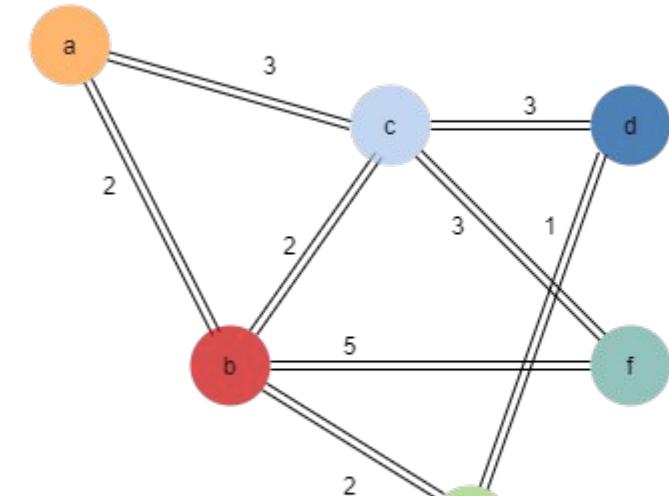


MST of Uber Dataset

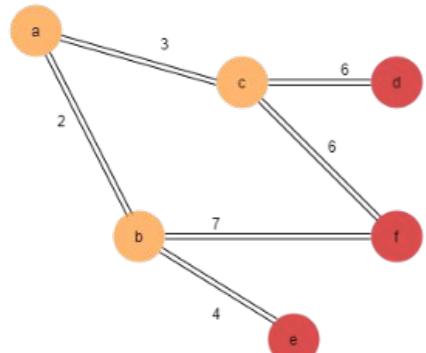
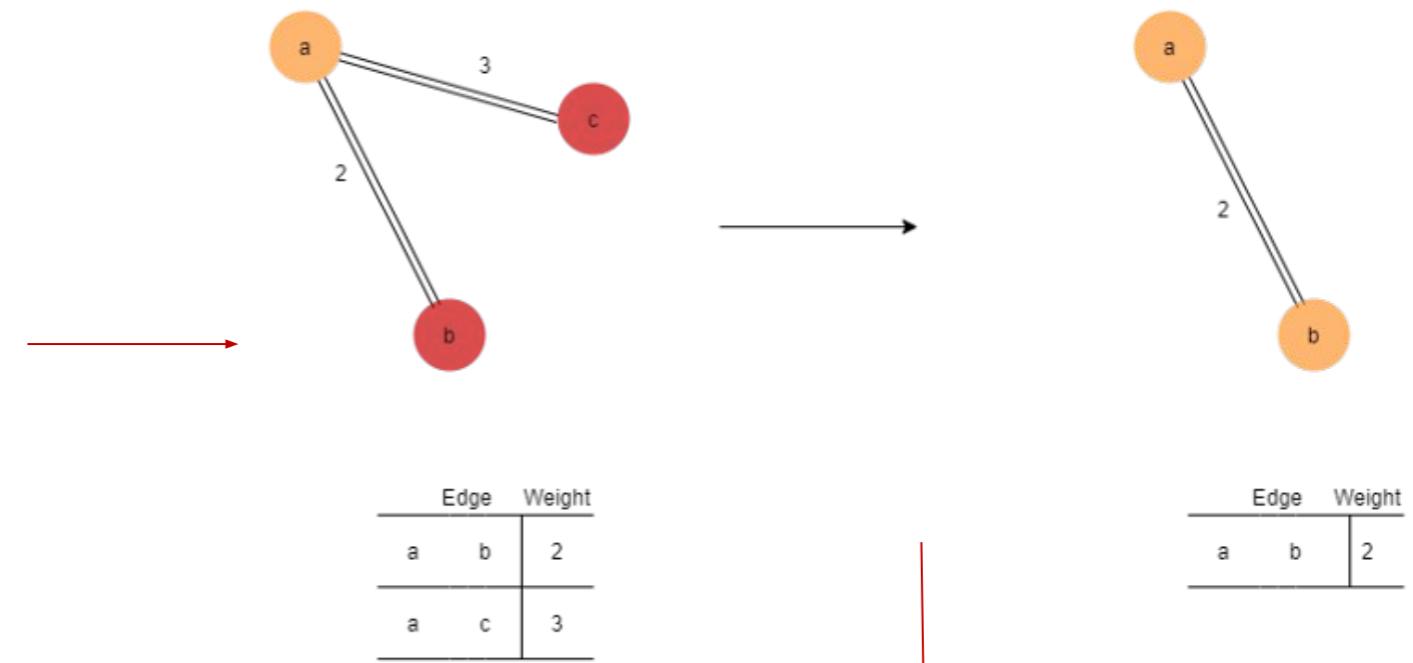
Prim's Algorithm



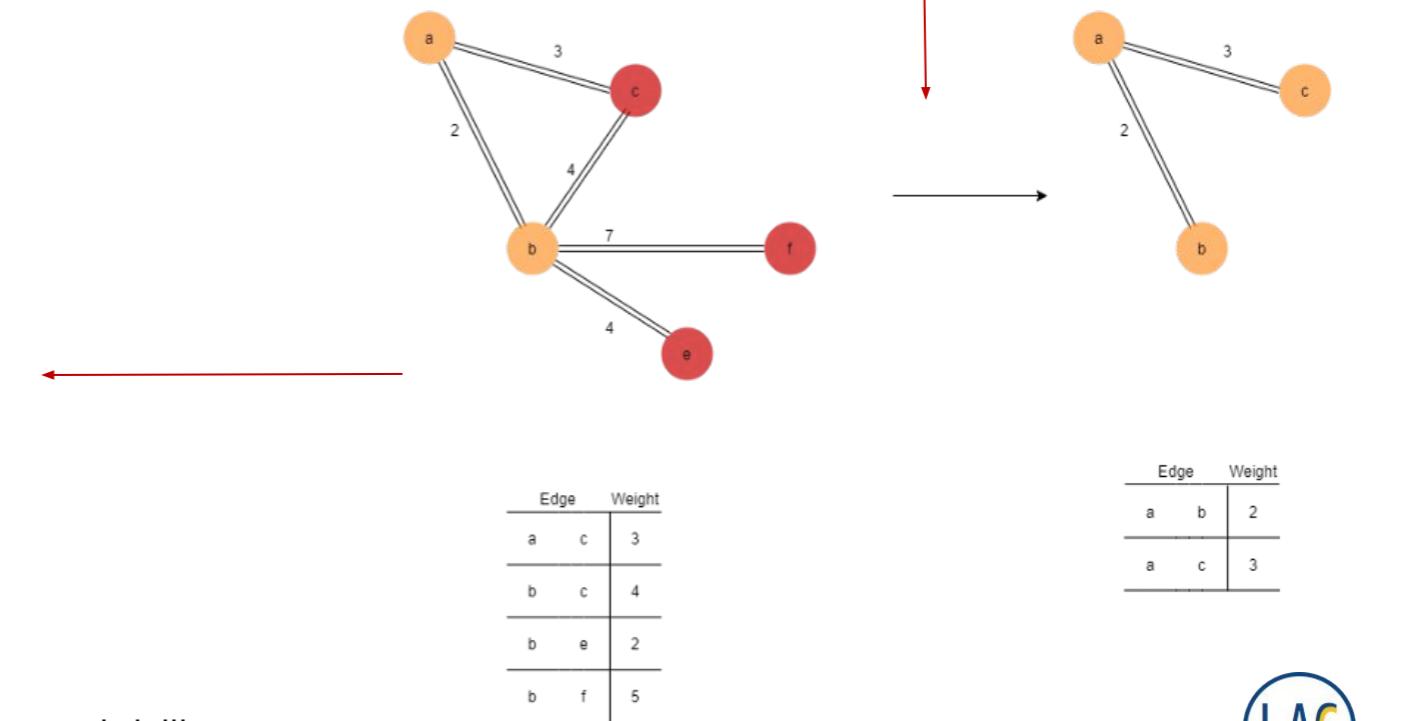
Dijkstra's Shortest Path Algorithm



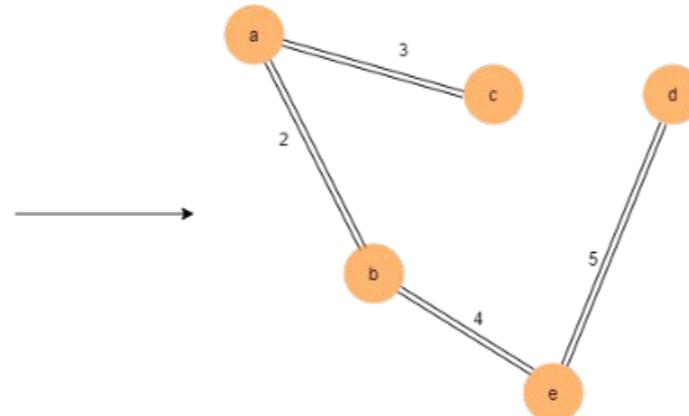
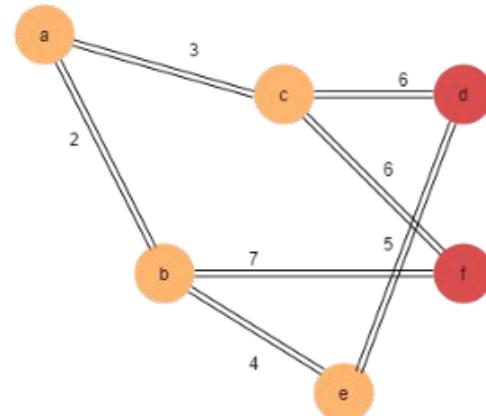
Edge	Weight
a b	2
a c	3
b c	2
b e	2
b f	5
c d	3
c f	3
d e	1



Edge	Weight
c d	6
c f	6
b e	4
b f	7

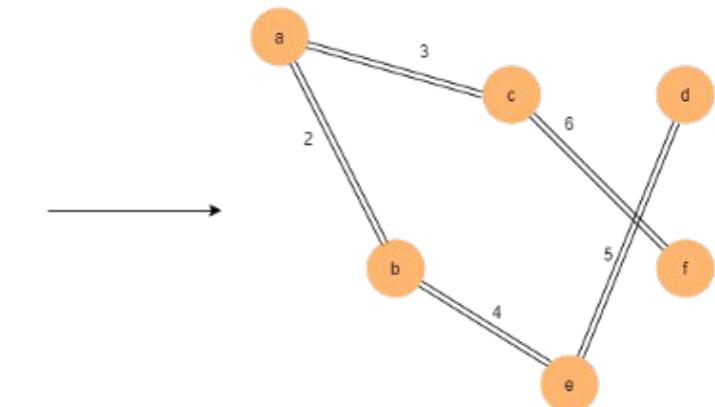
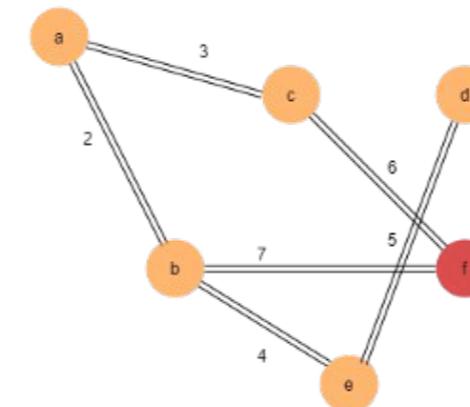


Dijkstra's Shortest Path Algorithm (contd.)



Edge	Weight
c - d	6
c - f	6
d - e	5
b - f	7

Edge	Weight
a - b	2
a - c	3
b - e	4
d - e	5



Edge	Weight
c - f	6
b - f	7

Edge	Weight
a - b	2
a - c	3
b - e	4
d - e	5
c - f	6

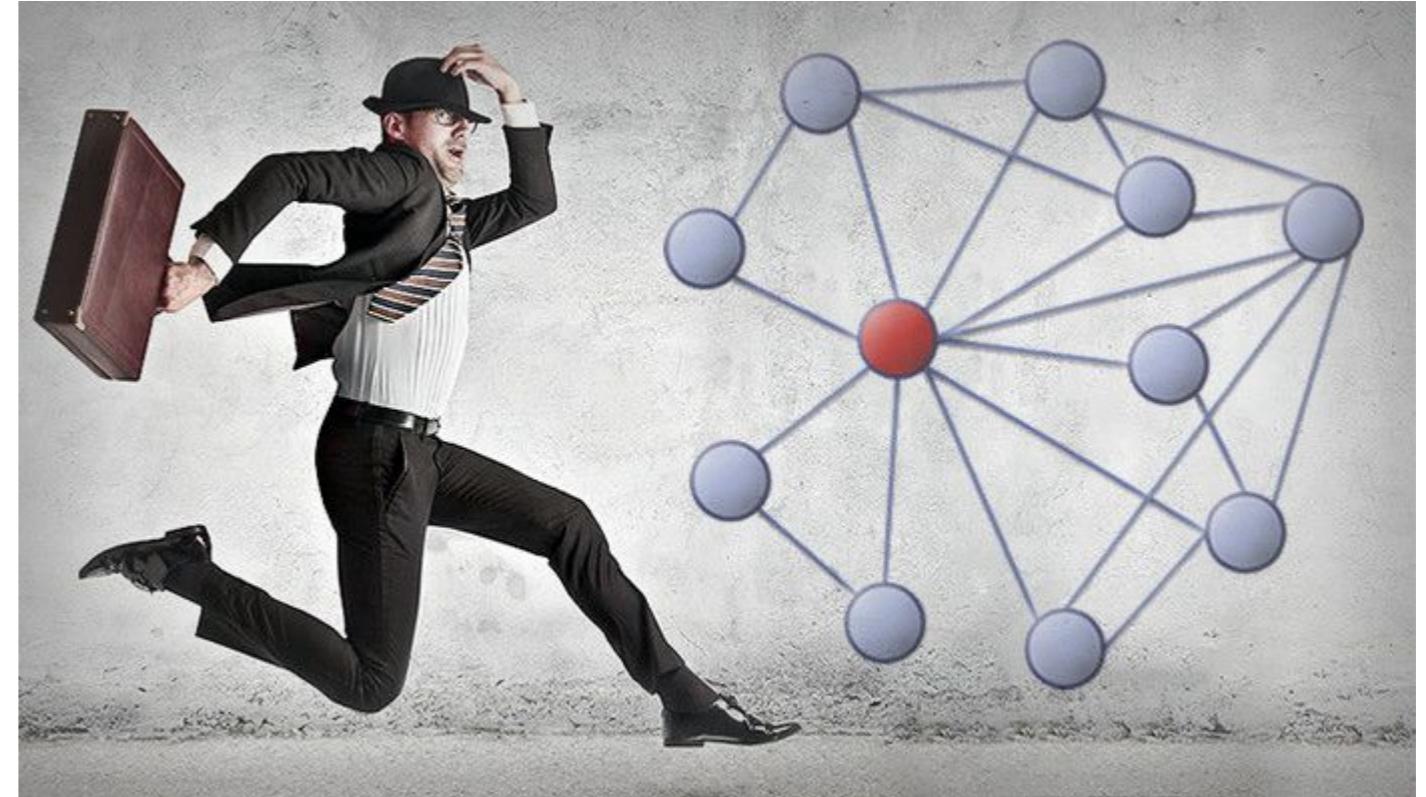
Prim's versus Dijkstra's

- Dijkstra's algorithm finds the shortest path, but Prim's algorithm finds the MST.
- Dijkstra's algorithm can work on both directed and undirected graphs, but Prim's algorithm only works on undirected graphs.
- Prim's algorithm can handle negative edge weights, but Dijkstra's algorithm may fail to accurately compute distances if at least one negative edge weight exists.
- Prim's Algorithm adds the next vertex that is closest to the tree. Dijkstra's Algorithm, on the other hand, adds the next vertex that is closest to the root.
- Intuitive example:
 - Dijkstra's: Find shortest and lowest cost path in existing road network.
 - Prim's: How to create the road network itself efficiently.



Traveling Salesman Problem (TSP)

- Problem formulation: Find the shortest route from a city such that all other cities are visited only once and the traveller returns to the original city.
- The triangle inequality: sum of any two sides of a triangle must always be greater than the third side: $a + b > c$, $a + c > b$, $b + c > a$.
- The route is also called a Hamiltonian cycle satisfying the triangle inequality
- Graph:
 - Nodes: cities
 - Edges: Roads
 - Weights: Distance, travel time, etc.

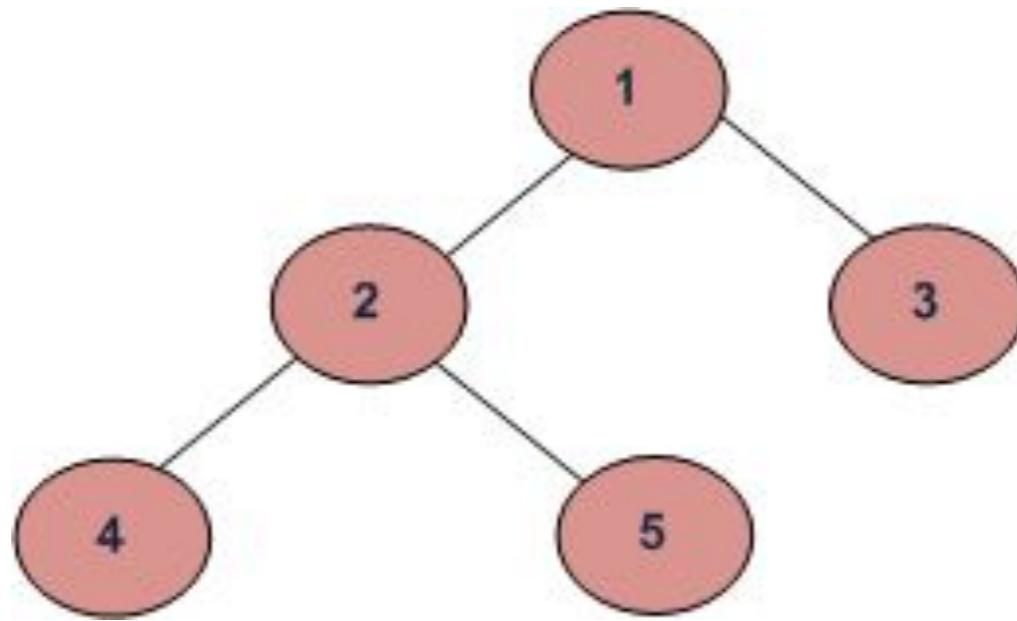


TSP Intuition

- TSP is NP-hard: It is not possible to guarantee you will find the shortest path within some predefined time-limit; n cities can have $n!$ paths ($n \times (n-1) \times (n-2) \dots 1$), e.g., 15 cities can result in trillion possible routes.
- We use approximation techniques to select pareto-optimal routes.



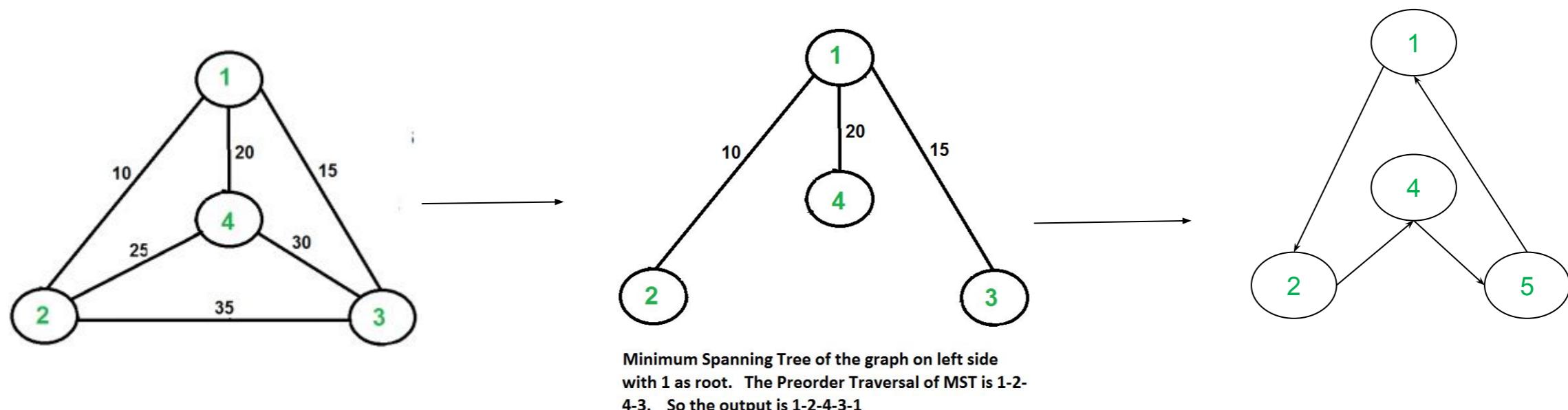
Pre-order traversal



1 - 2 - 4 - 5 - 3

2-approximate algorithm for TSP using MST

- The cost of the output is never more than twice the cost of best possible output.
- Algorithm:
 - Use Prim's algorithm to generate MST, using the starting/ending point as the root node.
 - Find depth-first pre-order traversal of the MST (root, left, right)
 - Add 1 at the end of the pre-order walk to get the Hamiltonian cycle.



Other TSP solutions

- Exact algorithms: Dynamic programming
- Approximate algorithms:
 - 1-approximate using Eulerian circuits and Dijkstra's algorithm
 - 2-approximate using MST
 - Simulated Annealing (guided local search)
 - Ant-colony Optimization
- Special cases
 - Independent set
 - 2-SAT