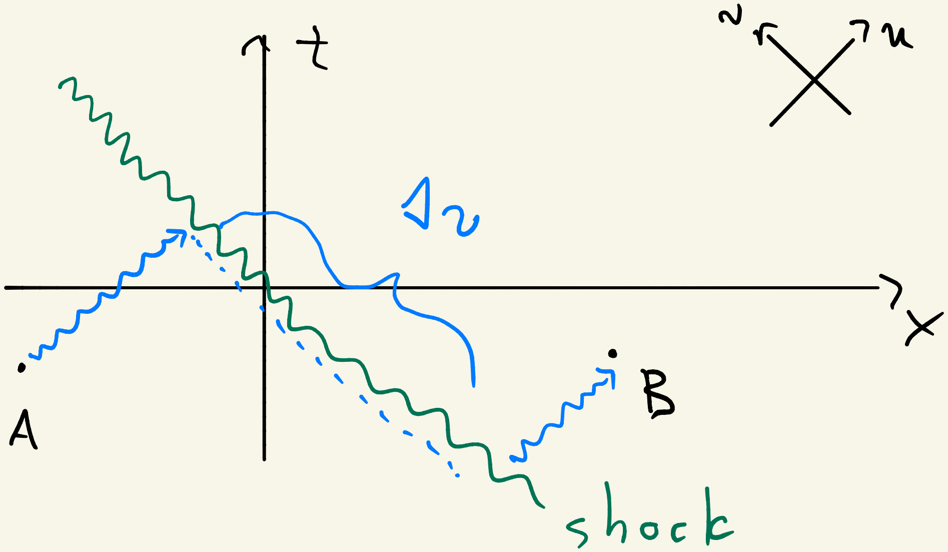
Causality in EFTs

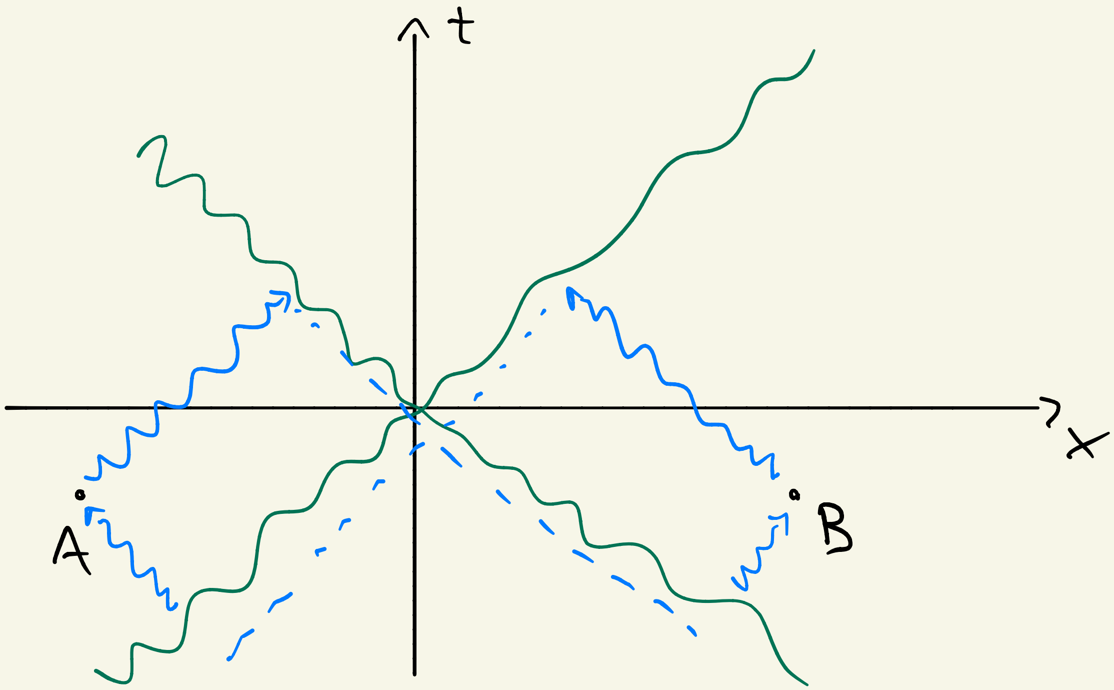
1407.5597 Camanho, Edelstein, Maldecena, Zhiboedov
1904.05905 Koloğlu, Kravchuk, Simmons-Duffin, Zhiboedov
today

0602178 Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazz further reference

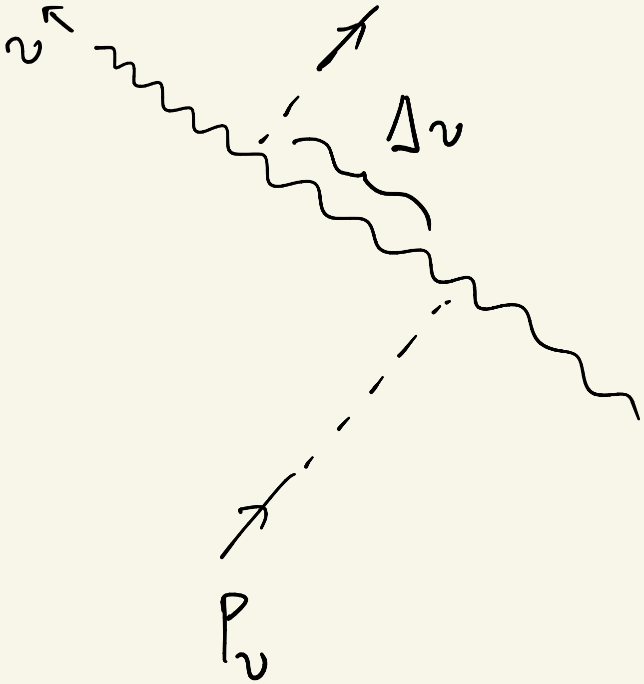


$$u = \frac{1}{\sqrt{2}} (t - x)$$

$$v = \frac{1}{\sqrt{2}} (t + x)$$



$\Delta v < 0 \Rightarrow$ Causality violation



Shock wave stress Tensor:

$$T_{uu} = -\,\underline{P_u}\,\delta(u)\,\delta^{D-2}(\vec{x})$$

$P_u < 0$: Momentum of particle

Shock wave metric ansatz:

$$ds^2 = -\,du\,dv + h(u,\vec{x})\,du^2 + \sum_{i=1}^{D-2}(dx_i)^2$$

Einstein eqt: $\partial_\perp^2 h(u,x_i) = -16\pi G\,|P_u|\,\delta(u)\,\delta^{D-2}(\vec{x})$

x_i

$$h(u,x_i) = \frac{4G|P_u|}{\pi^{\frac{D-4}{2}}}\,\Gamma\left(\frac{D-4}{2}\right)\frac{\delta(u)}{r^{D-4}}$$

(*) In $D=4$

$$h \sim \ln\left(\frac{r^2}{r_0^2}\right)$$

Dray, t' Hooft 1984

$r_0 = \ell_{pl} \rightsquigarrow$ choice of units

Change coords $v = v_{new} + \underbrace{\frac{4\Gamma\left(\frac{D-4}{2}\right)}{\pi^{\frac{D-4}{2}}} \frac{G\,|P_u|}{b^{D-4}}}\,\theta(u)$

No disc $\Rightarrow$ geodesic continuous

$$\Delta v = \frac{4\Gamma\left(\frac{D-4}{2}\right) G\,|P_u|}{\pi^{\frac{D-4}{2}}\; b^{D-4}} > 0$$

$$\Delta v \underset{4D}{\sim} \ln \frac{\ell_{pl}^2}{b^2}$$

QM - analysis

$$-i\partial_v \phi \simeq p_v \phi \quad (*)$$

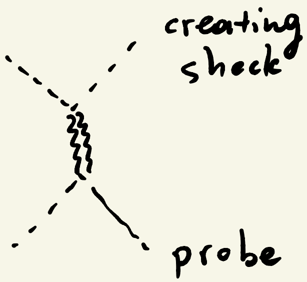
$$\nabla^2 \phi = 0 \Rightarrow \partial_u \partial_v \phi + h\partial_v^2 - \frac{1}{4}\partial_i \phi = 0$$

Around $u=0$: $\partial_u \partial_v \phi(\underbrace{0+\varepsilon, v, x^i}_{\text{light cone}}) + h\partial_v^2 \phi \simeq 0$

$$\Rightarrow \partial_u \phi \simeq -h\partial_v \phi \Rightarrow \phi(0^+, v, \underset{\substack{\uparrow \\ \text{impact par.}}}{b}) = e^{\int_{0^-}^{0^+} du\, \partial_u} \phi(0^-, v, b)$$

$$= e^{-\int_{0^-}^{0^+} du\, h\, \partial_v} \phi(0^-, v, b) \overset{(*)}{=} e^{-i\Delta v\, p_v} \phi(0^-, \ldots)$$

Amplitudes



$$= -8\pi G \frac{s^2}{t} = \mathcal{A}_{tree}(s,t)$$

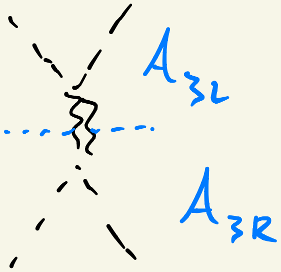
Eikonal appx ($t/s \to 0$)

$$i\mathcal{A}_{eik} = 2s \int d^{D-2}\vec{b}\, e^{i\vec{q}\cdot\vec{b}} \left[e^{i\delta(b,s)} - 1 \right]$$

w/ $$\delta(b,s) = \frac{1}{2s} \int \frac{d^{D-2}\vec{q}}{(2\pi)^{D-2}} e^{i\vec{q}\cdot\vec{b}} \mathcal{A}_{tree}(s, -\vec{q}^{\,2}) = \frac{\Gamma\left(\frac{D-4}{2}\right) G s}{\pi^{\frac{D-4}{2}} b^{D-4}}$$

Here $s = 4 P_u p_v \quad \rightsquigarrow \quad \delta(b,s) = -p_v \Delta v$

$$\delta(\vec{b}, s) = \frac{\Gamma\left(\frac{D-4}{2}\right)}{4\pi^{\frac{D-4}{2}}} \frac{1}{2s} A_{3L}^{13I}(-i\partial_{\vec{b}}) A_{3R}^{24I}(-i\partial_{\vec{b}}) \frac{1}{|\vec{b}|^{D-4}}$$



$\longrightarrow$ A is polynomially bounded

$\longrightarrow$ Only massless pole

Spin 1

e.g. integrate out e^-

$$S = \int d^D x \sqrt{-g}\, \frac{1}{4}\left[F^2 + \hat{\alpha}_2 R^{\mu\nu}{}_{\sigma\delta} F_{\mu\nu} F^{\sigma\delta} \right]$$

Spin 1

e.g. integrate out e^-

$$S = \int d^D x \sqrt{-g} \, \frac{1}{4} \left[F^2 + \hat{\alpha}_2 R^{\mu\nu}{}_{\sigma\delta} F_{\mu\nu} F^{\sigma\delta} \right]$$

$$A_{g\gamma\gamma} = \sqrt{32\pi G} \left(A_{F^2} + \hat{\alpha}_2 A_{RFF} \right)$$

where $A_{F^2}^{13I} = \varepsilon^I_{\mu\nu} \left[p_1^\mu p_3^\nu (\varepsilon_1 \cdot \varepsilon_2) - \varepsilon_1^\mu p_3^\nu (\varepsilon_3 \cdot p_1) - \varepsilon_3^\mu p_1 \varepsilon \cdot p \right]$

$$A_{FFR}^{13I} = \varepsilon^I_{\mu\nu} \, p_1^\mu p_3^\nu (\varepsilon_1 \cdot p_3)(\varepsilon_3 \cdot p_1)$$

$$p_1^\mu = \left(p_u, \frac{q^2}{16 p_u}, \frac{\vec{q}}{2} \right) \qquad \varepsilon_1^\mu = \left(-\frac{\vec{q} \cdot \vec{e}_1}{2 p_u}, 0, \vec{e}_1 \right) \qquad s = 4 p_u p_v$$

$$t = -\vec{q}^{\,2}$$

$$p_3^\mu = \left(-p_u, \frac{-q^2}{16 p_u}, \frac{\vec{q}}{2} \right) \qquad \varepsilon_3^\mu = \left(\frac{\vec{q} \cdot \vec{e}_3}{2 p_u}, 0, \vec{e}_3 \right)$$

$$\varepsilon_1 \cdot \varepsilon_2 = \vec{e}_1 \cdot \vec{e}_2, \quad \varepsilon_1 \cdot p_3 = \vec{q} \cdot \vec{e}_1, \quad \varepsilon_3 \cdot p_1 = \vec{q} \cdot \vec{e}_3$$

$$\mathcal{A}^{13\,\mathrm{I}}_{F^2} = \varepsilon^{\mathrm{I}}_{\mu\nu}\, p_1^{\mu} p_3^{\nu}\, \vec{e}_1 \cdot \vec{e}_3 \quad ; \quad \mathcal{A}^{13\,\mathrm{I}}_{RFF} = \varepsilon^{\mathrm{I}}_{\mu\nu}\, p_1^{\mu} p_3^{\nu}\, \vec{e}_1 \cdot \vec{q}\;\; \vec{e}_3 \cdot \vec{q}$$

$$\varepsilon^{\mathrm{I}}_{\mu\nu}\, p_1^{\mu} p_3^{\nu} = \varepsilon^{\mathrm{I}}_{vv}\, p_n^2 + \mathcal{O}(q)$$

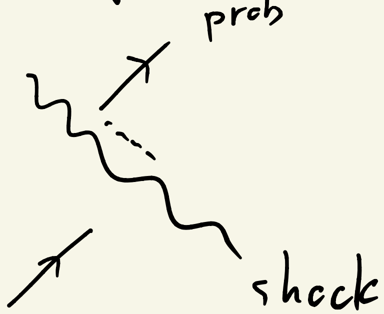
$\mathcal{A}^{13\,\mathrm{I}}(-i\partial_{\vec{b}})$: To get set $\vec{q} \to -i\partial_{\vec{b}}$

$$\frac{1}{m^2}$$

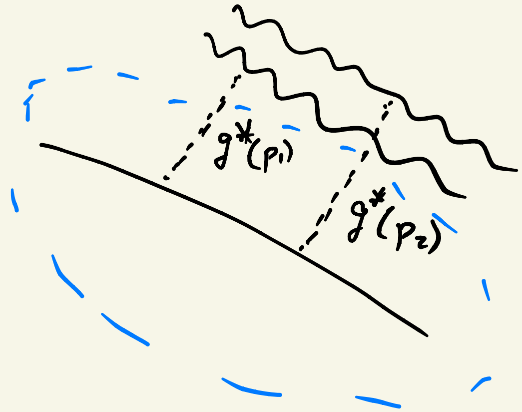
$$\Delta v = \frac{4\,\Gamma\!\left(\frac{D-4}{2}\right)}{\pi^{\frac{D-4}{2}}}\, \frac{G\,|p_n|}{b^{D-4}}\, \varepsilon\cdot\varepsilon \left[\, \underset{\mathcal{A}_{F^2}}{1} + \frac{\hat{\alpha}_2}{b^2}\,(D-4)(D-2) \left[\frac{(\varepsilon\cdot n)^2}{\varepsilon\cdot\varepsilon} - \frac{1}{D-2} \right] \right]$$

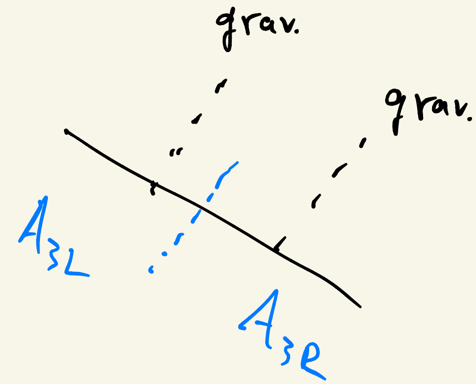
$\mathcal{A}_{RFF}$ $\qquad \frac{1}{E^2}$

So far



Now





Causality :

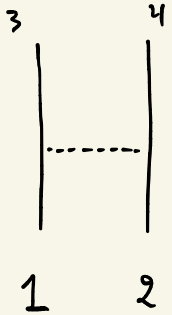
$$\sum_{X} \Big[\mathcal{A}(2,3,-X)\,\mathcal{A}(X,1,4) - \mathcal{A}(1,3,-X)\,\mathcal{A}(X,2,4) \Big] = 0$$

For our RFF theory:

$$\sum \left(\mathcal{A}\mathcal{A} - \mathcal{A}\mathcal{A} \right) =$$

$$\hat{\alpha}_2^2 \; \underset{\text{shock1}}{\vec{q}_1} \cdot \underset{\text{shock2}}{\vec{q}_2} \; \underset{\text{ph}}{e_{3,i}} \left(\underbrace{q_{1,i}\, q_{2,j} - q_{2,i}\, q_{1,j}}_{\text{commutator}} \right) \underset{\text{photons}}{e_{4,j}} \neq 0$$

δ from 3pt On Shell Amplitudes

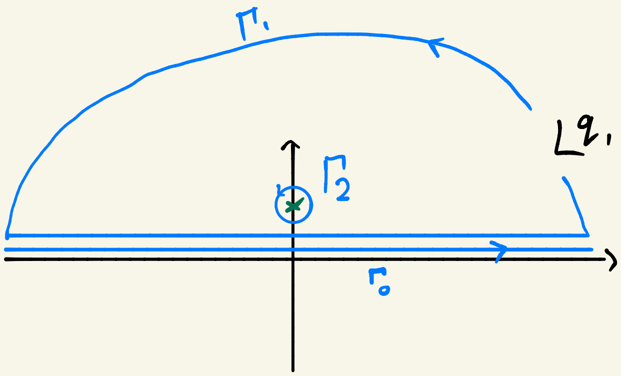


$$s = -(p_1+p_2)^2$$
$$t = -(p_1+p_3)^2$$
$$\frac{t}{s} \ll 1$$

lightcone coords

$$p_1 \sim \Big(\underbrace{p_u, 0}, \frac{\vec{q}}{2}\Big) \qquad b = (0, 0, |b|, 0, \dots)$$
$$p_2 \sim \Big(0, p_v, -\frac{\vec{q}}{2}\Big) \qquad \vec{q} = (0, 0, q_1, \vec{q}_{rest})$$

$$\delta(\vec{b}, s) = \frac{1}{2s}\int \frac{d^{D-2}\vec{q}}{(2\pi)^{D-2}}\, e^{i\vec{q}\cdot\vec{b}} A_4(\vec{q}) = \frac{1}{2s}\int \frac{d^{D-3}\vec{q}_{rest}}{(2\pi)^{D-3}} \underbrace{\int_{\Gamma_0} \frac{dq_1}{2\pi}\, e^{i\vec{q}\cdot\vec{b}} A_4(\vec{q})}_{I_1}$$



1) Polynomially bounded : $\Gamma_0 \to \Gamma_1$

2) Massless pole $q_*^2 = 0 \Rightarrow q_{1*}^2 = -q_{rest*}^2 \Rightarrow q_1^* = \pm i|q_{rest}|$

$\Gamma_1 \to \Gamma_2$

$$I_1 = \oint_{\Gamma_2} \frac{dq_1}{2\pi} \frac{A_{3L}^{13I}(\vec{q})\, A_{3R}^{24I}(\vec{q})}{\vec{q}^{\,2}}\, e^{i\vec{q}\cdot\vec{b}} = A_{3L}^{13I}(-i\partial_{\vec{b}})\, A_{3R}^{24I}(-i\partial_{\vec{b}}) \oint_{\Gamma_2} \frac{dq_1}{2\pi} \frac{e^{i\vec{q}\cdot\vec{b}}}{\vec{q}^{\,2}}$$

$$\overset{\Gamma_2 \to \Gamma_1 \to \Gamma_0}{=} A_{3L}^{13I}(-i\partial_{\vec{b}})\, A_{3R}^{24I}(-i\partial_{\vec{b}}) \int_{\Gamma_0} \frac{dq_1}{2\pi} \frac{e^{i\vec{q}\cdot\vec{b}}}{\vec{q}^{\,2}}$$

$$\delta(\vec{b}, s) = \frac{1}{2s} A_{3L}^{13I}(-i\partial_{\vec{b}})\, A_{3R}^{24I}(-i\partial_{\vec{b}}) \int \frac{d^{D-2}\vec{q}}{(2\pi)^{D-2}} \frac{e^{i\vec{q}\cdot\vec{b}}}{\vec{q}^{2}}$$

$$= \frac{\Gamma\left(\frac{D-4}{2}\right)}{4\pi^{\frac{D-4}{2}}} \frac{1}{2s} A_{3L}^{13I}(-i\partial_{\vec{b}})\, A_{3R}^{24I}(-i\partial_{\vec{b}}) \frac{1}{|\vec{b}|^{D-4}}$$