HQET Power Corrections

$$\mathcal{L}^{HQET} = \bar{Q}_v \, i v\cdot D \, Q_v \leftarrow (\text{lowest order } \mathcal{L}^{(0)})$$
$$- \bar{B}_v (i v\cdot D + 2M_Q) B_v$$
$$+ \bar{Q}_v \, i \not{D}_T B_v + \bar{B}_v \, i \not{D}_T Q_v + \mathcal{L}^{QCD}_{light}$$

with $\not{v} Q_v = Q_v$, $\not{v} B_v = - B_v$

Integrate out B_v (anti-particle):

$$\mathcal{L}^{(1)} = \frac{-1}{2M_Q} \bar{Q}_v \not{D} \not{D} Q_v = - \frac{\bar{Q}_v D_T^2 Q_v}{2M_Q} - g \frac{\bar{Q}_v \sigma_{\mu\nu} G^{\mu\nu} Q_v}{4 M_Q}$$

Reparametrization Invariance (part a)

$$P_Q^\mu = M_Q v^\mu + k^\mu$$

Invariance under $\boxed{v^\mu \to v^\mu + \frac{\varepsilon^\mu}{M_Q}}$

$$k^\mu \to k^\mu - \varepsilon^\mu$$

$v^2 = 1$, so $v\cdot\varepsilon = 0$

We still want $\not{v} Q_v(0) = Q_v(0)$

$$\to \left(\not{v} + \frac{\not{\varepsilon}}{M_Q}\right)(Q_v + \delta Q_v) = Q_v + \delta Q_v$$

solved by $\delta Q_v = \frac{\not{\varepsilon}}{2M_Q} Q_v \qquad (x=0)$

so $\boxed{Q_v(x) \to e^{i\varepsilon x}\left(1 + \frac{\not{\varepsilon}}{2M_Q}\right) Q_v}$

q doesn't depend on v, so $\boxed{q \to q}$

(part b)

$$J^{(0)}(x) = C_1 \bar{q} \gamma^\mu Q_v + C_2 \bar{q} v^\mu Q$$

$$J^{(1)}(x) = \frac{1}{2M_Q} \sum_i B_i O_i^{(1)}(x)$$

$$O_1^{(1)} = \bar{q} \gamma^\mu i \not{D} Q_v \qquad O_4^{(1)} = \bar{q}(-iv\cdot\overleftarrow{D})\gamma^\mu Q_v$$

$$O_2^{(1)} = \bar{q} v^\mu i \not{D} Q_v \qquad O_5^{(1)} = \bar{q}(-iv\cdot\overleftarrow{D}) v^\mu Q_v$$

$$O_3^{(1)} = \bar{q}\, i D^\mu Q_v \qquad O_6^{(1)} = \bar{q}(-i\overleftarrow{D}^\mu) Q_v$$

under transformation

$$J^{(0)} + \delta J^{(0)} = C_1 \bar{q} \gamma^\mu e^{i\varepsilon x}\left(1 + \frac{\not{\varepsilon}}{2M_Q}\right) Q_v$$

$$+ C_2 \bar{q}\left(v^\mu + \frac{\varepsilon^\mu}{M_Q}\right) e^{i\varepsilon x}\left(1 + \frac{\not{\varepsilon}}{2M_Q}\right) Q_v$$

$$\to \delta J^{(0)} = C_1 \bar{q} \gamma^\mu e^{i\varepsilon x} \frac{\not{\varepsilon}}{2M_Q} Q_v$$

$$+ C_2 \bar{q} v^\mu e^{i\varepsilon x} \frac{\not{\varepsilon}}{2M_Q} Q_v + C_2 \bar{q} \frac{\varepsilon^\mu}{M_Q} e^{i\varepsilon x} Q_v$$

$$J^{(1)} + \delta J^{(1)} = \frac{1}{2M_Q} B_1 \bar{q} \gamma^\mu i \not{D} e^{i\varepsilon x}\left(1 + \frac{\not{\varepsilon}}{2M_Q}\right) Q_v$$

$$+ \frac{1}{2M_Q} B_2 \bar{q}\left(v^\mu + \frac{\varepsilon^\mu}{M_Q}\right) i\not{D} e^{i\varepsilon x}\left(1 + \frac{\not{\varepsilon}}{2M_Q}\right) Q_v$$

$$+ \frac{1}{2M_Q} B_3 \bar{q}\, i D^\mu e^{i\varepsilon x}\left(1 + \frac{\not{\varepsilon}}{2M_Q}\right) Q_v$$

$+ \to$

$$+\frac{1}{2M_Q} B_4\, \bar{q}\left(-i\left(v+\frac{\varepsilon}{M_Q}\right)\cdot \overleftarrow{D}\right)\gamma^\mu e^{i\varepsilon x}\left(1-\frac{\not{\varepsilon}}{2M_Q}\right) Q_v$$

$$+\frac{1}{2M_Q} B_5\, \bar{q}\left(-i\left(v+\frac{\varepsilon}{M_Q}\right)\cdot \overleftarrow{D}\right) v^\mu e^{i\varepsilon x}\left(1-\frac{\not{\varepsilon}}{2M_Q}\right) Q_v$$

$$+\frac{1}{2M_Q} B_6\, \bar{q}\left(-i\, \overleftarrow{D}^\mu\right) e^{i\varepsilon x}\left(1-\frac{\not{\varepsilon}}{2M_Q}\right) Q_v$$

$$\rightarrow \delta J^{(1)} = \frac{1}{2M_Q} B_1\, \bar{q}\, \gamma^\mu\, i\,(i\not{\varepsilon})\, e^{i\varepsilon x} Q_v$$

$$+\frac{1}{2M_Q} B_2\, \bar{q}\, v^\mu\, i\,(i\not{\varepsilon})\, e^{i\varepsilon x} Q_v$$

$$+\frac{1}{2M_Q} B_3\, \bar{q}\;\, i\,(i\,\varepsilon^\mu)\, e^{i\varepsilon x} Q_v \quad + \mathcal{O}\left(\frac{1}{M_Q^2}\right)$$

for RP1 want $\quad \delta J^{(0)} + \delta J^{(1)} = 0 + \mathcal{O}\left(\frac{1}{M_Q^2}\right)$

$$\therefore \boxed{\begin{array}{l} B_1 = C_1\,, \quad B_2 = C_2\,, \quad B_3 = 2C_2 \\[1em] B_4,\ B_5,\ B_6 \ \text{unconstrained} \end{array}}$$