findBestHyperparam

July 15, 2022

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0.1 1. Datasets insight

We used a corpus made of 16 datasets: 2 public (pX datasets), 4 synthetic sampled for agile projects (aX datasets), and 10 synthetic sampled for plan-driven projects (cX and dX datasets): - pX datasets: 2 small datasets which have been used in several works, as in 'Ant Colony Optimization for the Next Release Problem'. P1 has very few requirements (20) and also very few dependencies (7). P2 makes a bit more effort in providing more realistic project datasets, althouth dependencies are requirements are not too large.

- aX datasets: commonly agile projects have a lower number of stakeholders actively managed, althouth their buy-in in the development of the product if more constant. Thus, aX datasets present a lower number of stakeholders. Since requirements are not decided a priori with a long elicitation process, requirements are dependencies among them are not usually large for a given minor o functional release. Thus, we produced datasets with not a ver high number of dependencies and requirements. For the sake of completenees, two aX dataset have a large number of requirements. Effort estimations are computed using a fibonacci scale, similar to some agile estimation techniques.
- cX datasets: classic or plan-driven datsets tend to have a large number of requirements and, due to a long and expensive planning, algo a large number of dependencies. Also due to usual processes which deal with management of stakeholders interests, it is also common to identify more stakeholders than in agile datasets. Effort values were simulated by using Function Points values extracted from the 2015 version of the International Software Benchmarking Standards Group (ISBSG) dataset, using {A,B} values for "Unadjusted Function Points rating", "New development" for "Development type" and "IFPUG 4+" for "Count approach". This procedure is used to generate percentile 25,50,75 of total FPs of a classic project, in order to generate a realistic sample of classic estimation of requirements, done by selecting randomly, for a given number of requirements, a list of costs that sums up to the percentile value.
- dX datasets: following the same procedure than for cX datasets, here we simulate the most complex classical projects, with very large number of requirements (column #PBI) and dependencies (column #(PBI-->[PBI])). In fact, this is the case in which the MONRP might be of greater help for the decision maker.
- eX|datasets: again, derived from classical estimation of effort with Function Points from SBSG 2015, these datasets not only contain large number of requirements, and requirements which imply dependencies, but also the cardinality of these dependencies is also large (column Avg_len[PBI]); that is, when a requirement has a dependency X --> [list of requirements], this list is larger in the eX datasets compared to the others. ---> aún no añadidos en la experimentación, pendiente de que termine MIMIC.

	Dataset	#Stakeholders	#PBI	#(PBI>[PBI])	%(PBI>[PBI])	Avg_len[PBI]
0	p1	5	20	7	0.3500	1.8571
1	p2	5	100	29	0.2900	2.6897
2	a1	5	50	18	0.3600	2.2222
3	a2	15	50	18	0.3600	2.7222
4	a3	5	200	74	0.3700	1.9459
5	a4	15	200	75	0.3750	2.2533
6	c1	15	50	20	0.4000	2.4000

7	c2	100	50	17	0.3400	3.5294
8	с3	15	200	69	0.3450	1.9420
9	c4	100	200	75	0.3750	2.0933
10	d1	15	100	45	0.4500	2.8444
11	d2	50	100	39	0.3900	2.1026
12	d3	15	200	88	0.4400	3.3523
13	d4	50	200	88	0.4400	4.8523
14	d5	50	200	88	0.4400	3.4773
15	d6	15	300	131	0.4367	3.7710
16	d7	50	300	145	0.4833	3.6966

0.2 2. FEDA description:

Given an initial set of requirements dependencies in the form of X1-->X2, FEDA uses this knowledge as a prefixed structure. e.g: we can have an acyclic graph like this: $G=\{0-->2, 1-->2, 3, 2-->4\}$, where requirements 0,1,3 do not have parents, parents(2)= $\{0,1\}$ and parents(4)= $\{2\}$.

Thus, learning is not structural and only applies to data. Sampling is always performed following a topological (ancestral) order ([3,0,1,2,4] in the example above).

Algorithm is as follows:

- 1. Sampling of First generation:
- -- If X does not have any parents, then sample using P(X)=1/num_requirements
- -- If any Y in parents(X) is set to 1, then X=1, else use P(X)=1/num_requirements

do

2. Learning

- $\mbox{--}$ If X does not have any parents in graph structure, then learn its marginal probability
- $\mbox{--}$ If X does have parents in graph structure, learn Conditional:
- P(X | all Y in parents(X)==0) In the example above, P(2 | 0==0,1==0).

Thus, we only need to learn P(X|parents(X)) from requirements whose parents Y are not selected.

That is, we do not need $P(X \mid any parents(X)==1)$, just the all parents(X)==0 case.

This means that conditional probability can be stored in a unidimensional array,

using the same array to store either marginal or conditional probability for each ${\tt X}$.

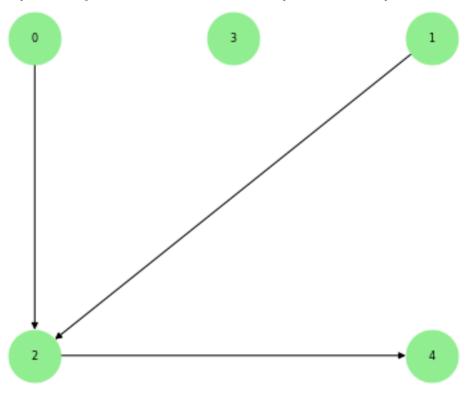
3. Sampling

-- In the case of requirements without parents in graph structure, use learned marginal probability

-- In any Y in parents(X) is set to 1, then X=1, else use P(X|parents(X)==0)

while(!stop_criterion)

Graph with prefixed structure for requriments dependencies



Ancestral order: [3,0,1,2,4]

a) Sampling of first generation: In the dependencies structure shown above, when sampling the first set of solutions, the requirement in each solution is selected given the following probabilities:

$$P(3) = 1/5$$

$$P(0) = 1/5$$

$$P(1) = 1/5$$

P(2) = 1 if requirement 0 or 1 has been selected in current solution; 1/5 otherwise.

P(4) = 1 if requirement 2 has been selected in current soluction; 1/5 otherwise.

Let us assume that we sample 6 individuals with the following result:

solutions =

[[0 0 1 0 1]

```
[1 0 1 1 1]
[0 1 1 0 1]
[0 0 0 0 0]
[0 1 1 0 1]
[0 1 0 1 1]
[0 0 0 1 0]
]
```

An impossible sampled individual would be, for example: $[0\ 1\ 0\ 0\ 1]$ because requirement 2 should be selected since requirement 1 is. Thus, the dependencies graph structured is respected.

The whole population is evaluated, and the local NDS in current iteration set is identified. Let us assume this NDS:

```
nds_local =

[
        [1 0 1 1 1]
        [0 0 1 0 1]

        [0 1 0 1 1]

        [0 0 0 1 0]
]
```

nds global = nds local

b) Learning This step consists in updating the sampling probability of each requirement from the nds_local population.

$$P(0) = 1/4$$

$$P(1) = 1/4$$

$$P(2) = P_nds_local(2|requirement_0=0 \text{ and } requirement_1=0) = 1/2$$

$$P(3) = 2/4$$

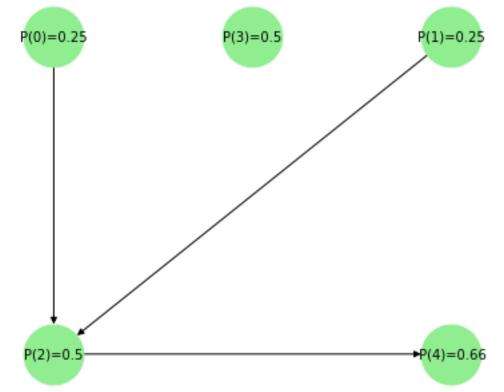
$$P(4) = P_nds_local(4|requirement_0=0) = 2/3$$
Thus, our probabilities vector is now:
$$probabilities = [0.25, 0.25, 0.5, 0.5, 0.66]$$

Sidenote about learning

we could have special cases in which a requirement is never selected. For instance, let us imagine this local NDS:

P(1) = 0, so sampling of new individuals would not select requirement 1 anymore. In this case, we keep P(1) as its previous stored probability. Another possible alleaviations to this cases could be learning P(1) from the global NDS, or using some smooth.

Marginal/conditioned probability for each requirement



c) Sampling Each new individual is sampled given the Ancestral order: [3,0,1,2,4], and using the following probabilities:

$$P(3) = 0.5$$

```
P(0) = 0.25
```

$$P(1) = 0.25$$

P(2) = 1 if requirement 0 or 1 is selected. 0.5 otherwise.

P(4) = 1 if requirement 2 is selected. 0.66 otherwise.

Let us assume that we sample 6 individuals with the following result:

```
{\it new\_solutions} =
```

```
[ [0 0 0 1 1]
  [0 0 1 1 1]
  [1 1 1 0 1]
  [0 0 1 1 1]
  [0 0 0 1 0]
  [0 0 0 0 1]
]
```

We evaluate the solutions to: - update the global NDS given the new_solution population - set local NDS to the nds found in new_solution population

Repeat Learning+Sampling until stop criterion.

0.3 3. Search of the best hyperparameters configuration for each algorithm.

3.1 Best configuration for: FEDA

These are the different values used to set hyperparameters in FEDA, for each dataset:

```
population_length: [ 100 200 500 700 1000]
```

max_generations: [50 100 200 300 400]

max_evaluations: [0]
selection_scheme: ['nds']

In total, 25 configuration per dataset.

Counts of best configurations found in 17 datasets. Please notice that those with less than the maximum possible #iterations or #solutions_per_iteration have converged sooner, and they obtain the same HV with higher configurations, thus the least configuration possible is shown. In conclusion: 'best configuration' can be interpreted as 'minimum configuration to converge'.

	population_length	max_generations	max_evaluations	selection_scheme	\
0	1000	300	0	nds	
1	1000	50	0	nds	
2	1000	400	0	nds	
3	1000	200	0	nds	

```
4
               1000
                                  100
                                                    0
                                                                    nds
                                                                             HV \
                    datasets
   [a1, a2, c2, c3, d2, d4]
                               [0.8728, 0.9435, 0.932, 0.9061, 0.8702, 0.808]
0
       [a3, a4, d1, d5, d7]
                                       [0.8244, 0.8047, 0.9126, 0.812, 0.781]
1
2
                    [c1, c4]
                                                              [0.8999, 0.8521]
3
                    [d3, p1]
                                                              [0.8099, 0.8789]
4
                    [d6, p2]
                                                              [0.8011, 0.7891]
    wins
0 6.0000
1 5.0000
2 2.0000
3 2.0000
4 2.0000
Best hyperparameter configuration for FEDA is:
population_length:1000
```

3.2 Best configuration for: GRASP It takes too long. In the few datasets in which results are available, GRASP's HV is better than all algorithms. However, in one week experiments did not finish for datasets with a large number of PBIs.

3.3 Best configuration for: GeneticNDS

These are the different values used to set hyperparameters in GeneticNDS, for each dataset:

population_length: [100 200 500 700 1000]

max_generations: [50 100 200 300 400]

selection_candidates: [2]
crossover_prob: [0.8]
mutation_prob: [0.1 0.3]
mutation: ['flip1bit']
replacement: ['elitismnds']

max_generations:300
max_evaluations:0
selection_scheme:nds

In total, 50 configuration per dataset.

Counts of best configurations found in 17 datasets. Please notice that those with less than the maximum possible #iterations or #solutions_per_iteration have converged sooner, and they obtain the same HV with higher configurations, thus the least configuration possible is shown. In conclusion: 'best configuration' can be interpreted as 'minimum configuration to converge'.

```
2
               1000
                                 400
                                                        2
                                                                      0.8
3
                                 100
                                                        2
                                                                      0.8
               1000
4
               1000
                                 200
                                                        2
                                                                      0.8
5
                                 400
                                                        2
                                                                      0.8
               1000
 mutation_prob mutation replacement
                                                        datasets
0
            0.3 flip1bit elitismnds
                                                         [a1, d1]
1
            0.3 flip1bit elitismnds
                                                     [a2, c4, d7]
2
                                       [a3, a4, c3, d4, d5, d6]
            0.3 flip1bit elitismnds
3
            0.3 flip1bit elitismnds
                                                         [c1, p2]
4
                                                     [c2, d2, d3]
            0.3 flip1bit elitismnds
5
            0.1 flip1bit
                           elitismnds
                                                             [p1]
                                                 HV
                                                      wins
                                   [0.8163, 0.7937] 2.0000
0
                            [0.8916, 0.724, 0.6546] 3.0000
1
2
   [0.7022, 0.6978, 0.808, 0.6969, 0.7019, 0.6686] 6.0000
3
                                    [0.8493, 0.685] 2.0000
4
                           [0.8285, 0.7663, 0.7074] 3.0000
5
                                           [0.8837] 1.0000
```

Best hyperparameter configuration for GeneticNDS is:

population_length:1000 max_generations:400 selection_candidates:2 crossover_prob:0.8 mutation_prob:0.3 mutation:flip1bit replacement:elitismnds

3.4 Best configuration for: UMDA

These are the different values used to set hyperparameters in UMDA, for each dataset:

population_length: [100 200 500 700 1000]

max_generations: [50 100 200 300 400]

selection_scheme: ['nds']

replacement_scheme: ['elitism']

In total, 25 configuration per dataset.

Counts of best configurations found in 17 datasets. Please notice that those with less than the maximum possible #iterations or #solutions_per_iteration have converged sooner, and they obtain the same HV with higher configurations, thus the least configuration possible is shown. In conclusion: 'best configuration' can be interpreted as 'minimum configuration to converge'.

```
population_length max_generations selection_scheme replacement_scheme \setminus 0 1000 50 nds elitism
```

```
1000
                                  400
                                                    nds
                                                                     elitism
1
2
                1000
                                  300
                                                    nds
                                                                     elitism
3
                1000
                                  200
                                                    nds
                                                                     elitism
4
                1000
                                  100
                                                    nds
                                                                     elitism
                                                                                HV
                    datasets
   [a1, a3, c3, c4, d3, p1]
                               [0.8639, 0.8031, 0.8779, 0.8204, 0.7945, 0.8783]
                                                          [0.9303, 0.831, 0.7516]
1
                [a2, d2, d7]
2
                [a4, c1, d1]
                                                          [0.7942, 0.8819, 0.865]
                    [c2, d4]
                                                                   [0.8631, 0.802]
3
4
                [d5, d6, p2]
                                                         [0.7844, 0.7648, 0.7713]
    wins
0 6.0000
1 3.0000
2 3.0000
3 2.0000
```

Best hyperparameter configuration for UMDA is:

population_length:1000 max_generations:50 selection_scheme:nds

replacement scheme:elitism

3.5 Best configuration for: PBIL

These are the different values used to set hyperparameters in PBIL, for each

dataset:

4 3.0000

population length: [100 200 500 700 1000]

max_generations: [50 100 200 300 400]

max evaluations: [0] learning_rate: [0.1] mutation_prob: [0.1] mutation_shift: [0.1]

In total, 25 configuration per dataset.

Counts of best configurations found in 17 datasets. Please notice that those with less than the maximum possible #iterations or #solutions_per_iteration have converged sooner, and they obtain the same HV with higher configurations, thus the least configuration possible is shown. In conclusion: 'best configuration' can be interpreted as 'minimum configuration to converge'.

```
population_length max_generations max_evaluations learning rate
0
          1000.0000
                            400.0000
                                               0.0000
                                                              0.1000
           700.0000
                            400.0000
                                               0.0000
                                                              0.1000
1
2
           200.0000
                            300.0000
                                               0.0000
                                                              0.1000
```

```
mutation_prob mutation_shift \
0
         0.1000
                         0.1000
1
         0.1000
                         0.1000
2
         0.1000
                         0.1000
                                              datasets \
   [a1, c1, c2, c3, c4, d1, d2, d3, d4, d5, d6, d...
1
                                          [a2, a3, a4]
2
                                                   [p1]
                                                     HV
                                                           wins
   [0.8968, 0.903, 0.9421, 0.7414, 0.7336, 0.9033... 13.0000
                              [0.9736, 0.6568, 0.6583]
1
2
                                               [0.8948]
                                                         1.0000
```

Best hyperparameter configuration for PBIL is:

population_length:1000.0000 max_generations:400.0000 max_evaluations:0.0000 learning_rate:0.1000 mutation_prob:0.1000 mutation_shift:0.1000

3.6 Best configuracion for: MIMIC ---> pendiente de terminar los experimentos.

These are the different values used to set hyperparameters in MIMIC, for each dataset:

population_length: [100 200 500 700 1000]

max_generations: [50 100 200 300 400]

max evaluations: [0] selection_scheme: ['nds'] selected individuals: [50 100]

In total, 50 configuration per dataset.

Counts of best configurations found in 23 datasets. Please notice that those with less than the maximum possible #iterations or #solutions_per_iteration have converged sooner, and they obtain the same HV with higher configurations, thus the least configuration possible is shown. In conclusion: 'best configuration' can be interpreted as 'minimum configuration to converge'.

	population_length	${\tt max_generations}$	${\tt max_evaluations}$	${\tt selection_scheme}$	\
0	1000	200	0	nds	
1	1000	300	0	nds	
2	1000	400	0	nds	
3	1000	100	0	nds	
4	1000	200	0	nds	
5	1000	50	0	nds	

```
datasets
  selected_individuals
0
                     50
                                                         [a1, e2, e5, e6]
                     50
                                                         [a2, c1, d3, d4]
1
2
                     50
                          [a3, a4, c2, c3, c4, d1, d2, d5, d6, d7, p2]
3
                                                                  [e1, e4]
                     50
4
                     100
                                                                      [e3]
5
                     50
                                                                      [p1]
                                                      HV
                                                             wins
                      [0.9106, 0.8031, 0.7809, 0.7826]
0
                                                           4.0000
                      [0.9701, 0.9313, 0.8293, 0.8286]
                                                           4.0000
1
   [0.8458, 0.8355, 0.9412, 0.9203, 0.8679, 0.912... 11.0000
2
                                       [0.7979, 0.8093]
3
                                                           2.0000
4
                                                [0.7831]
                                                           1.0000
5
                                                [0.9134]
                                                           1.0000
```

Best hyperparameter configuration for MIMIC is:

population_length:1000
max_generations:400
max_evaluations:0
selection_scheme:nds
selected individuals:50

All algorithms find their best results when using a Population Size = 1000, the maximum value among the 5 given for this hyperparameter.

Respect to the number of generations, all algorithms but UMDA are slow to converge, needing the maximum (400) almost the maximum (300 in FEDA) value among the possible values. UMDA seems to converge really soon, since in 6 out of 17 datasets it finds its best results with just 50 generations, in both agile and classic projects.

0.4 4. Pareto plots for each dataset, setting each algorithm with its best configuration found (wins among all datasets).

Given the most frequently best configuration (over all datasets), we plot the pareto for each dataset given that configuration. That is, the configuration for a given algorithm is the same across all datasets, concretely the one which performed the best more times (more wins).

We show a plot for each dataset. In each plot, for each algorithm, we show the pareto front found in all the executions (commonly 30). Since the solutions subset size is commonly 10, thus for each algorithm we plot 300 points. Since each algorithm has 30 paretos, please note that such paretos are not non-dominated among them, which can be seen in the shapes they create.

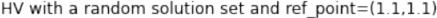
4.1 Metrics We show the average results over 30 executions for each algorithm. Each execution produces a NDS, from wich we keep the subset of 10 solutions which maximize HV, as suggested in 'Difficulties in Fair Performance Comparison of Multi-Objective Evolutionary Algorithms'. Such a subset is constructed by following an incremental forward greedy search.

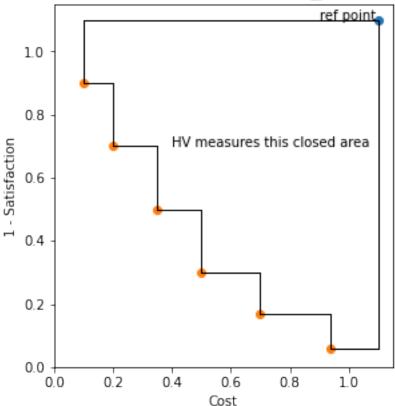
Metrics shown are: 4 Quality Indiciators (HV, gd+, UNFR, spread), time and cardinality of the

global NDS found by the algorithm during execution.

• HV (Hypervolume): this is the most widely used metric to assess paretos solutions for multiobjective problems and, concretly, in SBSE. It summarizes the four aspects of a solution set (convergence, spread, uniformity and cardinality); that is, this metric can be used as a compliance metric representing how good a Pareto front is. HV tends to be greater as knee points in the pareto are nearer the optimal point (0,0), thus it is preferable when Decision Makers prefer balanced solutions. In order to compute it, a reference point is needed, and this should be the same for all algorithms under comparison. Following the results and suggestions in ("How to evaluate solutions in pareto-based search-bases software engin."), we set the following reference point in our bi-objective problem: $ref_x = nadir_x + range_x/10$ $ref_y = nadir_y + range_y/10$ The nadir point is the worst point found by algorithms during search. Since we normalize both cost and satisfaction, our worst point is 1 for both metrics (satisfaction is plotted as 1-satisfaction). Range is the difference between the best and worst point found. Best point is 0, so clearly the value of the reference point for both goals is 1+(1-0)/10=1.1. HV is pare to compliance, so $HV_a>HV_b$ means that, visually, the pareto front of algorithm a dominates algorithm b. A great advantage of HV is that it does not need an ideal Pareto Reference, so its computation and fair comparison with other algorithm only needs a shared reference point which, thanks to goals normalization, is known a priori.

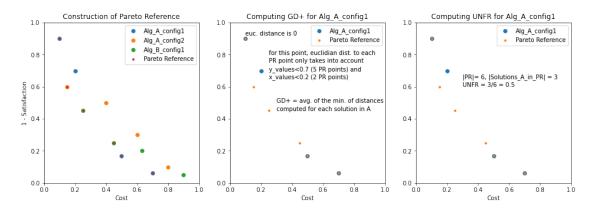
Text(0.4, 0.7, 'HV measures this closed area')





- GD+: General Distance (GD) covers the convergence aspect of the quality of solution set, measuring the Euclidian distance of such solution set to the ideal Pareto Reference. For each solution, its GD is the minimum of the distances to each point in the PF. In order to become GD compliant with Pareto dominance, GD+ enhances GD by measuring distances between points using only the goal coordinates which are superior in the Pareto Reference than those from the solutions set being measured. This metric is to be minimized, and a key point is the computation of the Pareto Reference, which needs to be done after execution of search algorithms. In our experiments, Pareto Reference is constructed by finding the Non Dominated Solutions set among all solutions sets found by all algorithm, under all hyperparameters configurations explained in Section 2.
- UNFR: Unique Non Dominated Front Ratio. It measures the ratio of solutio points in the PR which belong to the solution set of the evaluated algorithm. That is, it measures the contribution (from to 0 to 1) of an algorithm to the PR. Of curse, a point in the PR might be present in the solution sets of several algorithms. In our case, since the PR is constructed from such a large number of algorithms and configuration combinations, it presents a high cardinality of solutions. Furthermore, since each algorithm is evaluated using only using a selected subset of 10 points from its solution set, the UNFR value for each algorithm tends to be quite low, and the maximum possible is never 1, since the PR contais further more than 10 points. Anyway, greater UNFR values is desiderable.

Text(0.3, 0.6, 'UNFR = 3/6 = 0.5')

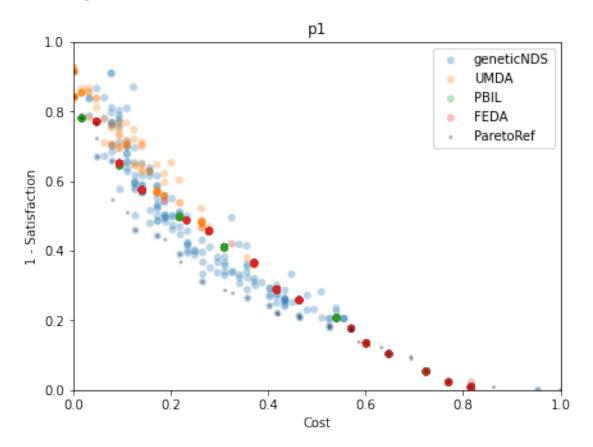


4.2 Results Here we show one plot pero datasets, containing the result of all algorithms with their best configuration. As mentioned before, each algorithm is run 30 times; afeter each execution, we keep a subset of the NDS constructed. This subset contains the 10 solutions from NDS which maximize HV in a forward greedy search over the NDS. Thus, for each algorithm, we plot 10*30=300 points. This way, it is easy to identify the common pareto shape relative to the algorithm. After each plot, all metrics are tabbed to show the average over the 30 executions in the dataset.

It is worth to mention that half of the execution time in algorithms is due to updating NDS after each iteration. Thus, although after each execution we select a subset of 10 solutions in the final

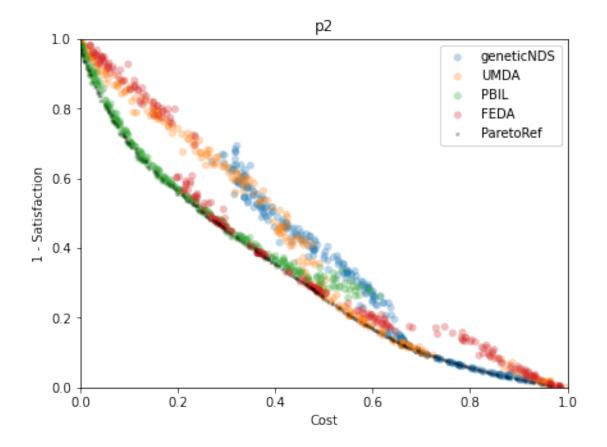
NDS, more time consuming algorithms are usually those which tend to develop a NDS with higher cardinality suring their search.

Pareto Reference has 40 points Maximum UNFR possible is 10/40=0.2500



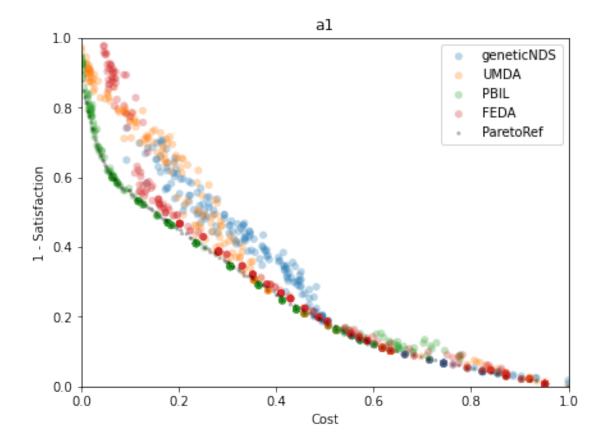
```
Method
                  ΗV
                       UNFR
                                    spread time(s)
                                                       |NDS|
                               gd+
  geneticNDS 0.8793 0.1142 0.0277
                                    0.6453 800.8662 47.0667
         UMDA 0.8783 0.0967 0.0374
                                    0.6236
                                            84.4473 34.8667
1
2
         PBIL 0.8948 0.1000 0.0316
                                    0.6038 452.2194 42.8333
3
        FEDA 0.8787 0.0733 0.0373
                                    0.6795 554.5325 31.9667
```

Pareto Reference has 283 points Maximum UNFR possible is 10/283=0.0353



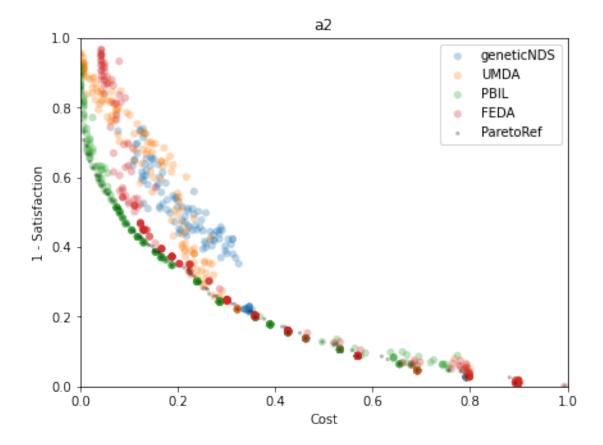
```
Method
                 HV
                      UNFR
                               gd+
                                   spread
                                            time(s)
                                                        |NDS|
  geneticNDS 0.6844 0.0068 0.0561
                                   0.6608 2213.7621 114.0000
        UMDA 0.7712 0.0033 0.0458
                                   0.5759 335.5445 151.1667
2
        PBIL 0.7390 0.0045 0.0113
                                   0.5571 1024.6246 107.2667
        FEDA 0.7886 0.0025 0.0299
                                   0.6018 3037.3493 217.2667
```

Pareto Reference has 142 points Maximum UNFR possible is 10/142=0.0704



```
time(s)
      Method
                 HV
                      UNFR
                               gd+
                                    spread
                                                        |NDS|
  geneticNDS 0.8150 0.0279 0.0369
                                    0.6649 1193.1049
                                                      64.5667
        UMDA 0.8639 0.0279 0.0299
                                   0.5756
                                           174.7565
                                                      67.0000
2
        PBIL 0.8968 0.0491 0.0031
                                    0.6078 698.1781 110.7667
        FEDA 0.8728 0.0120 0.0179
                                   0.5812 1427.4162 94.9333
```

Pareto Reference has 86 points Maximum UNFR possible is 10/86=0.1163



```
        Method
        HV
        UNFR
        gd+
        spread
        time(s)
        |NDS|

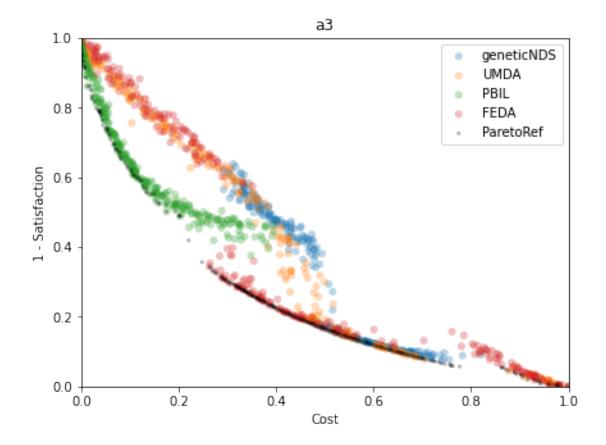
        0
        geneticNDS
        0.8877
        0.0717
        0.0338
        0.6420
        806.1419
        50.5000

        1
        UMDA
        0.9290
        0.0488
        0.0296
        0.5923
        136.8646
        47.1667

        2
        PBIL
        0.9736
        0.0795
        0.0029
        0.6097
        548.6406
        73.6000

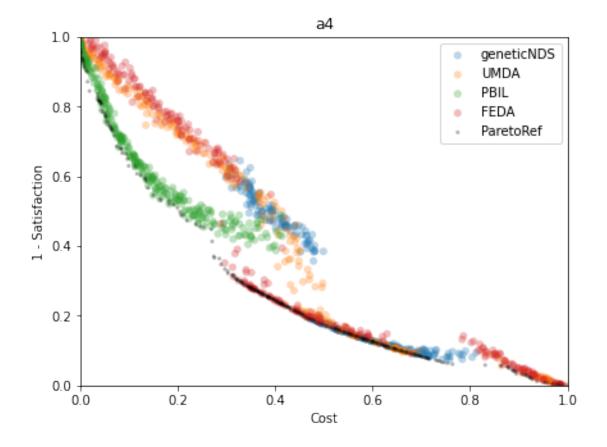
        3
        FEDA
        0.9435
        0.0143
        0.0187
        0.6313
        662.6650
        50.9667
```

Pareto Reference has 306 points Maximum UNFR possible is 10/306=0.0327



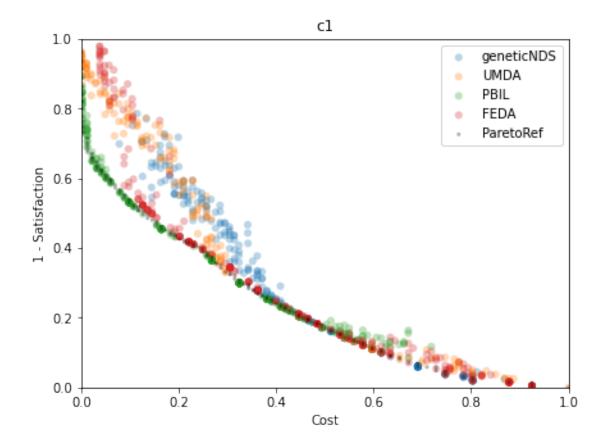
```
Method
                  {\tt HV}
                       UNFR
                               gd+
                                    spread
                                             time(s)
                                                         |NDS|
  geneticNDS 0.7022 0.0020 0.0805
                                    0.7520 1504.1177
                                                       86.9333
        UMDA 0.8031 0.0032 0.0663
                                    0.5809
                                            344.3830 126.4667
2
         PBIL 0.6558 0.0022 0.0275
                                    0.5703 828.3615 62.8667
         FEDA 0.8238 0.0041 0.0454
                                    0.5921 2268.7561 153.9000
```

Pareto Reference has 305 points Maximum UNFR possible is 10/305=0.0328



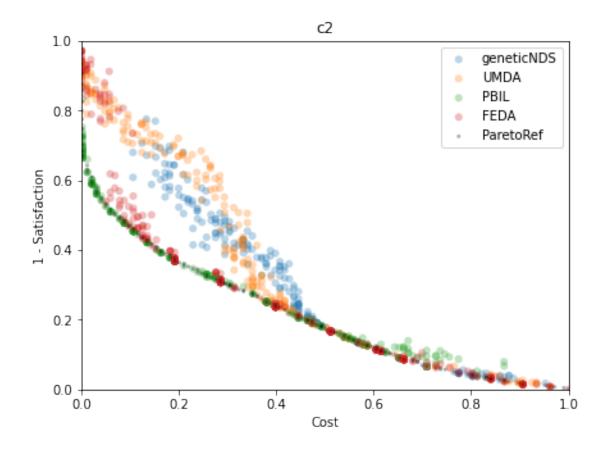
```
Method
                 HV
                      UNFR
                              gd+
                                   spread
                                            time(s)
                                                        |NDS|
  geneticNDS 0.6978 0.0049 0.0637
                                   0.7291 1761.2607 99.8333
        UMDA 0.7923 0.0023 0.0636
                                   0.5875
                                           375.8887 143.3667
2
        PBIL 0.6576 0.0013 0.0237
                                   0.5685 874.9545
                                                    64.6333
        FEDA 0.8022 0.0035 0.0493
                                   0.6169 2775.3945 170.3000
```

Pareto Reference has 155 points Maximum UNFR possible is 10/155=0.0645



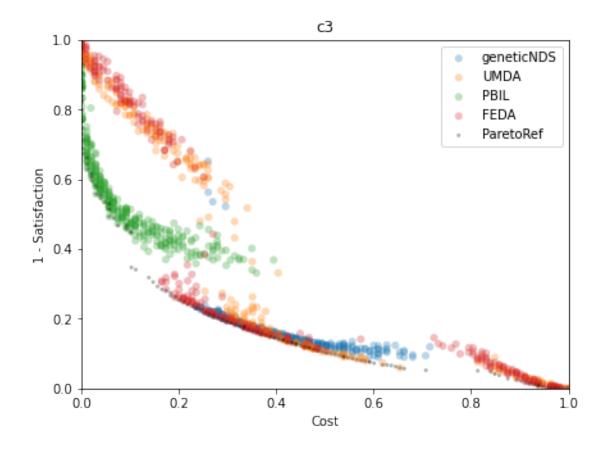
```
time(s)
       Method
                  {\tt HV}
                       UNFR
                               gd+
                                     spread
                                                         |NDS|
  geneticNDS 0.8468 0.0318 0.0338
                                     0.6418 1369.9759
                                                       86.2667
         UMDA 0.8817 0.0273 0.0301
                                     0.6068
                                             223.7486
                                                       79.3000
2
         PBIL 0.9030 0.0443 0.0037
                                     0.6150 743.3746 105.7333
         FEDA 0.8967 0.0157 0.0170
                                    0.6054 1202.8190
                                                       90.1667
```

Pareto Reference has 162 points Maximum UNFR possible is 10/162=0.0617



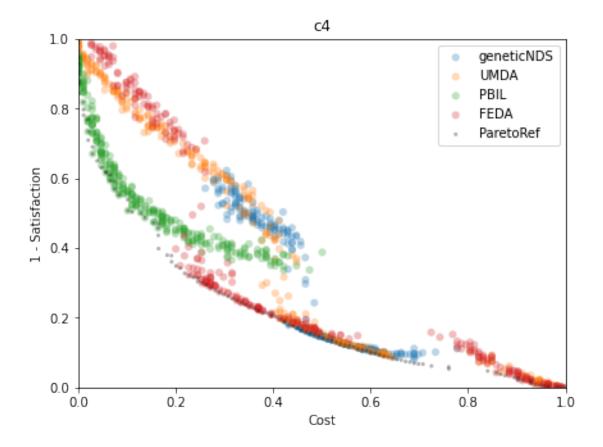
```
time(s)
       Method
                  {\tt HV}
                       UNFR
                               gd+
                                     spread
                                                         |NDS|
  geneticNDS 0.8252 0.0249 0.0509
                                     0.6707 1099.2675
                                                       81.3000
         UMDA 0.8584 0.0198 0.0505
                                     0.5627
                                            185.9247
                                                       69.6333
2
         PBIL 0.9421 0.0465 0.0030
                                     0.6031
                                            716.0132 103.4667
         FEDA 0.9320 0.0383 0.0072
                                    0.5564 1342.6053
                                                      98.3000
```

Pareto Reference has 126 points Maximum UNFR possible is 10/126=0.0794



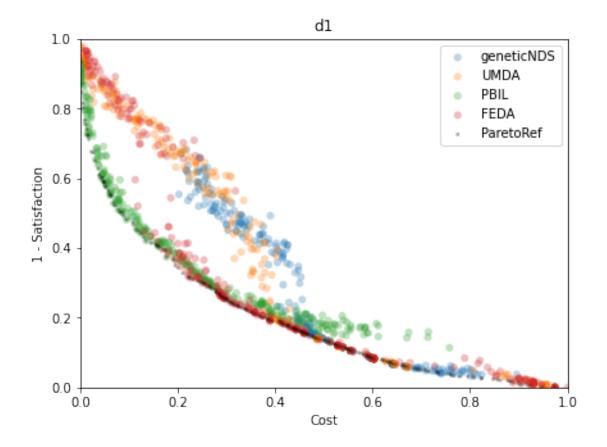
```
Method
                  HV
                       UNFR
                               gd+
                                    spread
                                             time(s)
                                                        |NDS|
  geneticNDS 0.8080 0.0016 0.0173
                                    0.7739
                                            874.2606 38.1333
         UMDA 0.8779 0.0037 0.0608
                                    0.5699
                                            197.7094 65.7333
2
         PBIL 0.7414 0.0042 0.0391
                                    0.6520
                                            531.1763 36.9000
         FEDA 0.9061 0.0050 0.0403
                                    0.6381 1115.9604 57.4333
```

Pareto Reference has 120 points Maximum UNFR possible is 10/120=0.0833



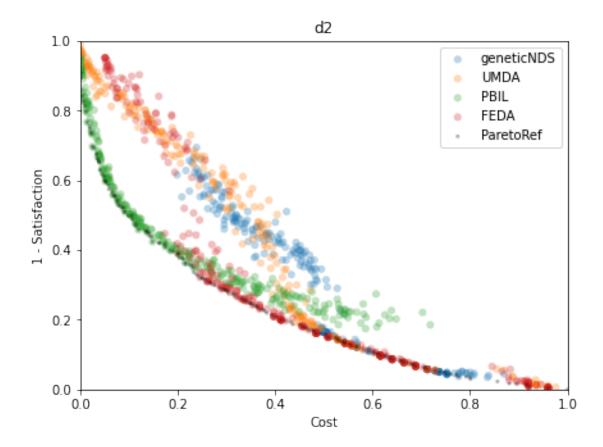
```
time(s)
       Method
                  {\tt HV}
                       UNFR
                                gd+
                                     spread
                                                         |NDS|
  geneticNDS 0.7231 0.0025 0.0771
                                     0.7743
                                             820.1618 32.4667
         UMDA 0.8204 0.0014 0.0751
                                     0.5832
                                             204.6724 61.0000
2
         PBIL 0.7336 0.0047 0.0408
                                     0.6020
                                             656.5295 48.4333
         FEDA 0.8497 0.0058 0.0474
                                    0.6148 1172.4405 59.4667
```

Pareto Reference has 204 points Maximum UNFR possible is 10/204=0.0490



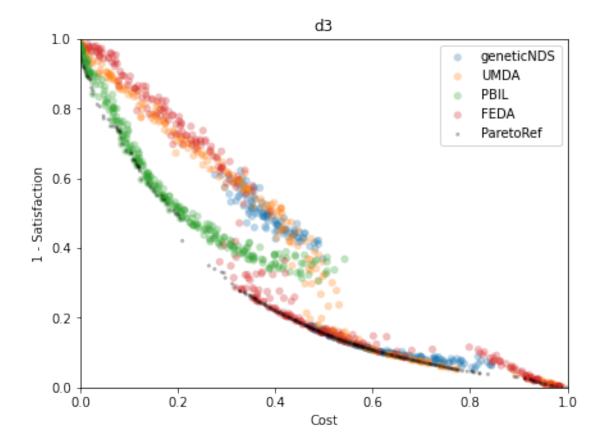
```
time(s)
       Method
                  {\tt HV}
                       UNFR
                                gd+
                                     spread
                                                          |NDS|
0
   geneticNDS 0.7906 0.0132 0.0697
                                     0.7176 1346.1592
                                                        74.6667
         UMDA 0.8612 0.0101 0.0665
                                     0.5579
                                             239.4590
                                                        90.2667
2
         PBIL 0.9033 0.0026 0.0201
                                     0.6027
                                             662.4540
                                                       72.0667
         FEDA 0.9060 0.0132 0.0270
                                     0.6000 1907.2975 121.5333
```

Pareto Reference has 167 points Maximum UNFR possible is 10/167=0.0599



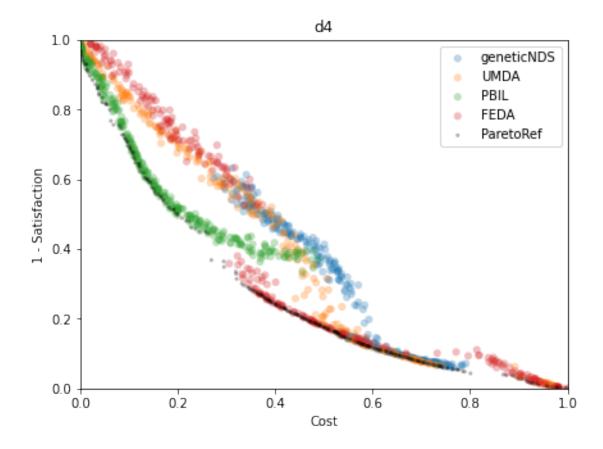
```
time(s)
      Method
                  HV
                       UNFR
                               gd+
                                    spread
                                                        |NDS|
  geneticNDS 0.7608 0.0220 0.0757
                                    0.7147
                                            939.1621 65.0000
        UMDA 0.8304 0.0140 0.0732
                                    0.5560
                                            209.9717 71.2333
2
        PBIL 0.8559 0.0044 0.0268
                                    0.5932
                                            671.5176 70.5667
         FEDA 0.8702 0.0234 0.0337
                                    0.6145 1414.7278 86.5000
```

Pareto Reference has 304 points Maximum UNFR possible is 10/304=0.0329



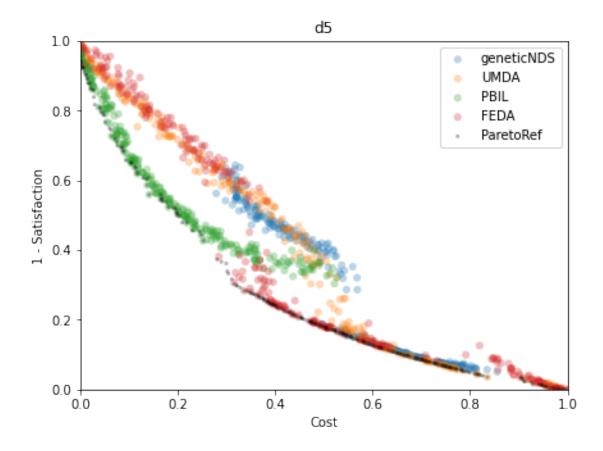
```
time(s)
      Method
                  {\tt HV}
                       UNFR
                               gd+
                                    spread
                                                         |NDS|
  geneticNDS 0.7073 0.0065 0.0629
                                    0.7581 1813.7526 86.8000
        UMDA 0.7945 0.0035 0.0717
                                    0.5627
                                            338.0281 123.7667
2
        PBIL 0.7197 0.0021 0.0388
                                    0.5822 979.6848 71.1667
         FEDA 0.8098 0.0041 0.0514
                                    0.5935 2108.9927 141.5000
```

Pareto Reference has 294 points Maximum UNFR possible is 10/294=0.0340



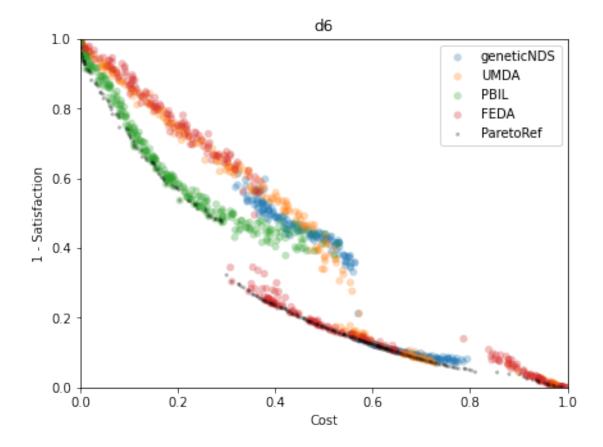
```
Method
                  {\tt HV}
                       UNFR
                               gd+
                                    spread
                                             time(s)
                                                         |NDS|
  geneticNDS 0.6969 0.0022 0.0820
                                    0.7401 1775.7885
                                                     89.2667
        UMDA 0.8006 0.0045 0.0558
                                    0.5709 397.6439 147.8667
2
         PBIL 0.6934 0.0015 0.0263
                                    0.5751 1184.9404 79.5667
         FEDA 0.8080 0.0027 0.0461
                                    0.5963 3208.9304 200.3000
```

Pareto Reference has 263 points Maximum UNFR possible is 10/263=0.0380



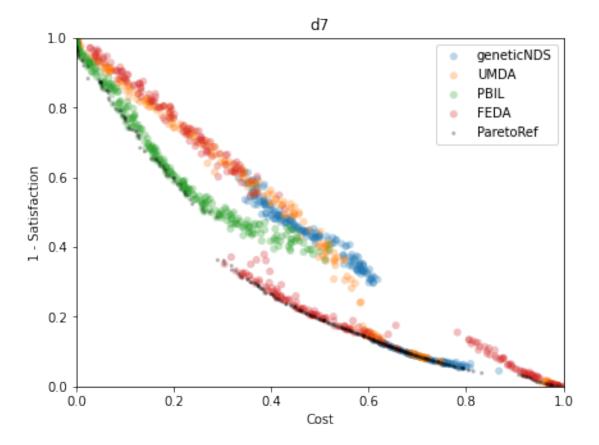
```
Method
                 HV
                      UNFR
                              gd+
                                   spread
                                            time(s)
                                                        |NDS|
  geneticNDS 0.7019 0.0004 0.0721
                                   0.7259 1916.8350 94.4667
        UMDA 0.7842 0.0035 0.0667
                                   0.5800
                                           328.0819 122.6667
2
        PBIL 0.7197 0.0028 0.0307
                                   0.5789 885.8085 67.7000
        FEDA 0.8090 0.0038 0.0487
                                   0.5953 2356.9579 149.2667
```

Pareto Reference has 277 points Maximum UNFR possible is 10/277=0.0361



```
Method
                  {\tt HV}
                       UNFR
                               gd+
                                    spread
                                             time(s)
                                                         |NDS|
  geneticNDS 0.6686 0.0030 0.0786
                                    0.7611 1568.3957
                                                      78.3333
        UMDA 0.7632 0.0014 0.0690
                                    0.5703 359.8537 124.0667
2
         PBIL 0.6441 0.0029 0.0327
                                    0.5626 1200.1904 70.9667
         FEDA 0.7980 0.0026 0.0429
                                    0.6004 2437.1066 166.1000
```

Pareto Reference has 324 points Maximum UNFR possible is 10/324=0.0309



```
Method
                  HV
                        UNFR
                                      spread
                                               time(s)
                                                           | NDS |
                                gd+
0
   geneticNDS 0.6545 0.0063 0.0896
                                      0.7401 2144.0206 104.4333
1
         UMDA 0.7503 0.0012 0.0657
                                      0.5694
                                              369.0416 148.0333
2
         PBIL 0.6451 0.0024 0.0322
                                     0.5723 1221.0485
3
         FEDA 0.7791 0.0019 0.0431
                                     0.6063 3178.8110 192.3667
```

```
Wins Counts:
```

```
{'geneticNDS': 0, 'UMDA': 0, 'PBIL': 5, 'FEDA': 12}
Wins in datasets:
```

```
{'geneticNDS': [], 'UMDA': [], 'PBIL': ['p1', 'a1', 'a2', 'c1', 'c2'], 'FEDA': ['p2', 'a3', 'a4', 'c3', 'c4', 'd1', 'd2', 'd3', 'd4', 'd5', 'd6', 'd7']}
```

Given the results, we see that PBIL behaves really well in datasets wich not a large number of requirements (aX and cX datasets). In the case dX datasets, with hundreds of requirements, FEDA obtains greater Hypervolumes than PBIL and the rest of algorithms. In some cases, FEDA obtains a very similar HV value than PBIL or UMDA; in such cases, in order to be sure that FEDA performs better we can take into account the UNFR value, which is pareto compliant, and when FEDA's HV is just slightly better, UNFR is clearly better than the other algorithm. \

A withdraw FEDA presents is that its execution time is much worse than the other algorithms. This is mostly due to the large number of Non Dominated Solutions it finds. \

Respect to gd+, FEDA is usually the second algorithm with best (lowest) mean general distance to the Pareto Reference, while PBIL commonly finds the closer solutions to the PR, although its HV covered is lower, as said, when the project presents hundreds of requirements.

0.4.1 5. Statistical tests of quality indicators

Antes de meterme en esto, a ver si veis alguna laguna en la experimentación. Por ejemplo me preocupa: - Ausencia de nsga-ii - qué hacer con GRASP - tiempos tan grandes por |NDS| y que ensombrece el tiempo real de learning+sampling - quitar algún dataset dX