GEOG0125

ADVANCED TOPICS IN SOCIAL AND GEOGRAPHIC DATA SCIENCE

INTRODUCTION TO BAYESIAN INFERENCE

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About the course

Description of the GEOG0125 Course

- This module will introduce students to advanced topics and methods that are essential for social and geographic data science, i.e., advanced or specialised techniques e.g., applied **Bayesian Inference and Machine Learning** applied to this domain.

- This module covers two areas:
    - How to use **Bayesian Inference** for both evidence-based research in making spatial predictions as well as studying the associations between social-risk factors and outcomes
    - How to use **Machine Learning** for various image processing and network sciences for tackling future social geographical challenges

Module Outline & Assessment

Bayesian Statistics | Anwar [AM]
Machine Learning | Stephen [SL]

- **WK01**: Introduction to Bayesian Inference [AM]
- **WK02**: Bayesian Generalised Linear Modelling (GLMs) [AM]
- **WK03**: Bayesian Hierarchical Models [AM]
- **WK04**: Spatial Risk Models (Part I) [AM]
- **WK05**: Spatiotemporal Risk Model & Bayesian Updating (Part II) [AM]
- **WK06**: Introduction to Deep Learning [SL]
- **WK07**: Image Analytics [SL]
- **WK08**: Introduction to GeoAI (Part I) [SL]
- **WK09**: Introduction to GeoAI (Part II) [SL]
- **WK10**: Future Outlook and Summary [SL]

All are lecture and teaching materials are posted on external websites to Moodle. Bayesian content is on a dedicated GitBook.

Machine Learning content is hosted on a Jupyter Notebook.

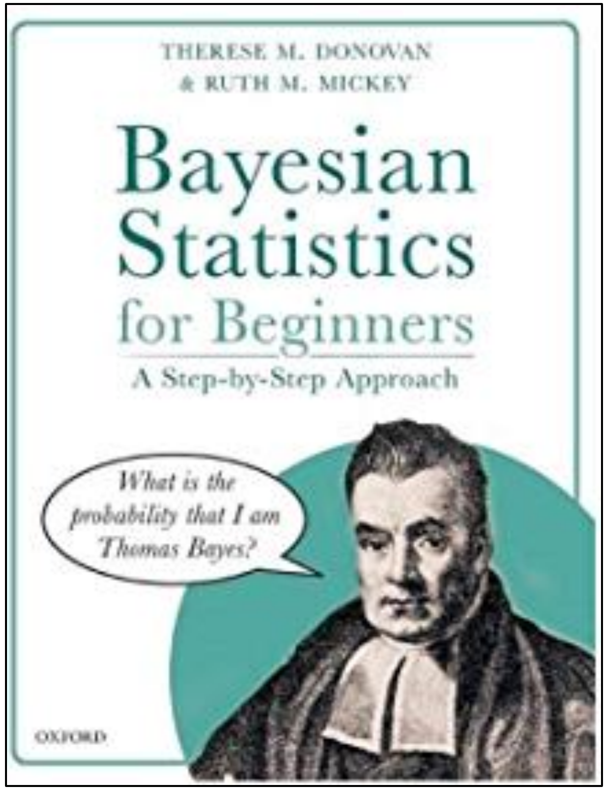
Details about assessment

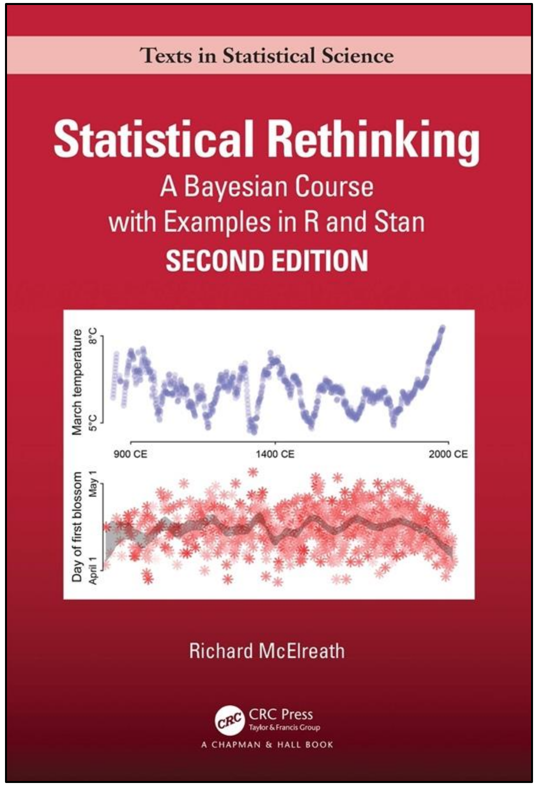
100% coursework with 2 x reports (each 1,500 words)

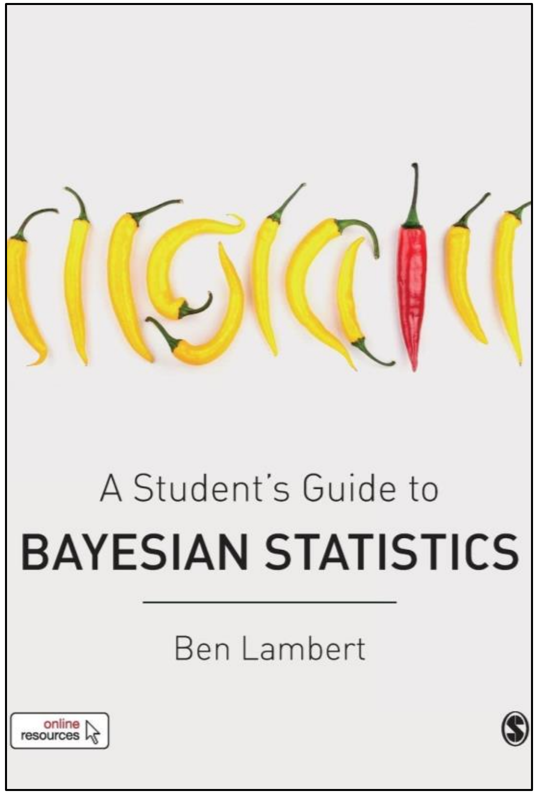
1 report focuses uses applied Bayesian methods for spatial analysis.

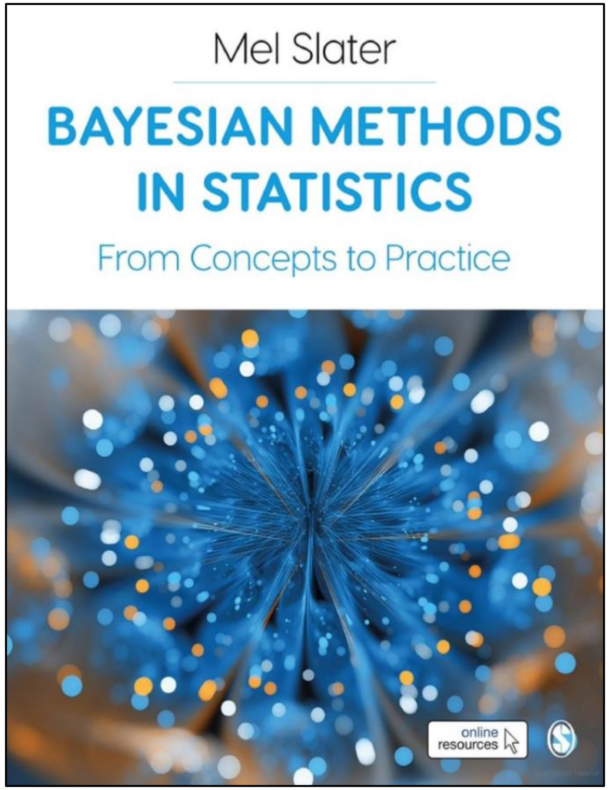
1 report focuses on Machine Learning for spatial analysis and image classification

Book recommendations









High recommendation for the mastery of the basic theory and principles of Bayesian Statistics

High recommendation for the coding experience and execution of statistical analysis in RStudio and Stan

General format of the course

- **1-hour lecture**
  - Weekly sessions every Thursday at 10:00am – 11:00am
  - Delivered by the module tutors
- **2-hour computer practical seminar**
  - Takes place every Friday 11:00am – 01:00pm
  - These sessions are facilitated by both the module and seminar tutors

Location(s):

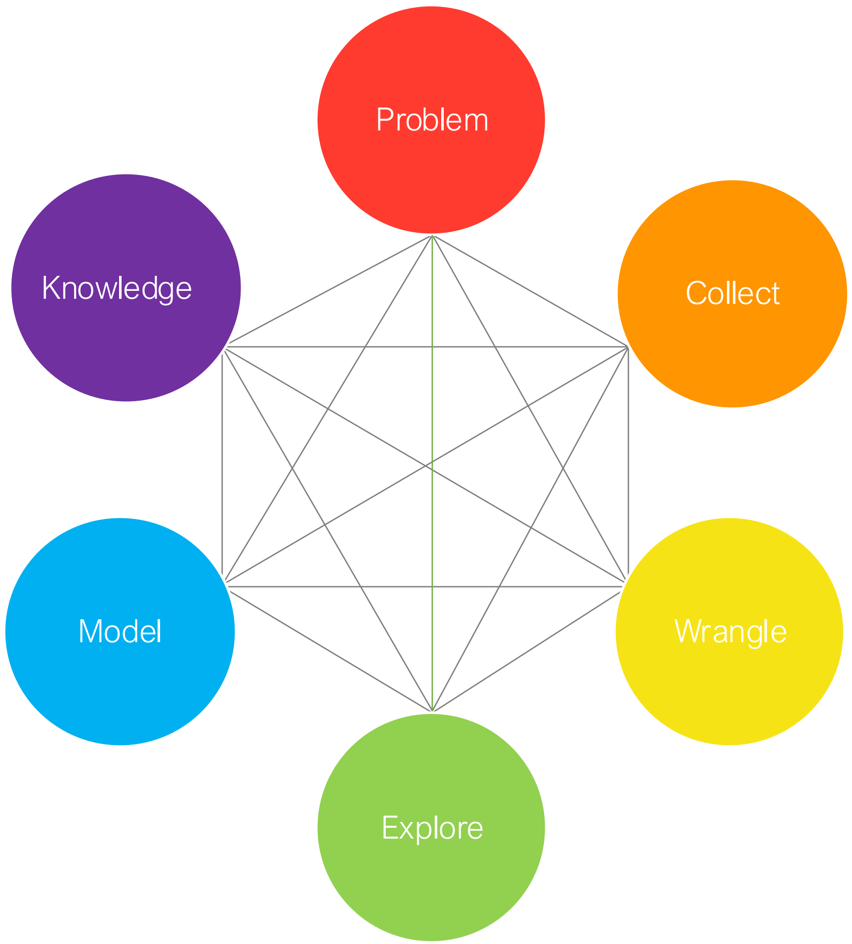
Lectures:	10:00am-11:00am UCL GOSICH's Wolfson Centre in Room A (Ground Floor)
Computer practicals:	11:00am-01:00pm North-West Wing Building Room G07

Please bring your own laptops to the computer practicals as NWW G07 is not a PC Cluster room

NOTE: All lectures and tutorials sessions are compulsory & will be delivered in-person.

Contents

- What is Bayesian Statistics?
- What are the types of probabilities?
- Bayes' theorem
- What are the various Probability Distributions
- The basics of Bayesian Inference



What is a Bayesian Statistics?

Definition:

Bayesian statistics is all about using **probability** as a tool for quantifying uncertainty in statistical problems.

Probabilities

Probability Distribution

Bayes' Rule

Bayesian Inference

What is a Probability?

Definition:

Probability is the language of uncertainty. It quantifies the uncertainty of a defined event, taking a value between 0 and 1. An event with a probability near zero implies that it is very unlikely to occur, while a probability near one implies that it is very likely.

- The usual notation for a probability is: Pr( ) or P( ) or Prob( ) (we will use the notation for Pr( ) throughout this lecture)

- A probability, Pr( ) is always defined in relation to an event 'E' which has a set number of outcomes. The outcomes are calculated within a sample space (or population size), which is the total number of all possible outcomes that event 'E'

- We can write this and say Pr(E), which is read as "the probability that event E will occur"

Note 1: When we say uncertainty, we are referring to lack of complete sureness (or knowledge) about an event.

Note 2: There are three key terms here, which will be defined in the next slide – An event, an outcome, and a sample space.

Basic terminology: event, outcome & sample space [1]

In probability, the following means:

- An **outcome** simply refers to a single result that can emerge from a study.
- An **event** is referred to as the set of outcomes that share a common characteristic in a study.
- A **sample space** is the set of all possible outcomes observed in a study.

- Example 1: We are conducting a mosquito infestation survey in 7 households in a village, where a home is either **infested** or **not infested.** The dataset collected was as follows = **Infested, Not Infested, Not Infested, Not Infested, Infested, Not Infested, Infested**

Outcome: Each home "Infested" or "Not infested" is an individual outcome, so in our dataset, the first entry "Infested" is one outcome and so on.

Event: The event "Infested" consists of all 3 homes (i.e., 1st, 5th and 7th) that are infested i.e., the number of outcomes is 3 infested homes in this event
Suppose the event is "Not infested", then it will consist of all 4 homes (i.e., 2nd, 3rd, 4th and 6th) that not infested. In this instances, its 4 outcomes in this event.

Sample space: The possible outcomes that can emerge: {Infested, Not infested}
All observed sample space that include 7 outcomes = {Infested, Not Infested, Not Infested, Not Infested, Infested, Not Infested, Infested}

Remember, **Pr(E), the probability that an event 'E' will occur.** This is what the notation for the above example will look like:

- The probability that a house is infested with mosquitoes = **Pr(Infested)**
- The probability that a house is not infested with mosquitoes = **Pr(Not infested)**

Basic terminology: event, outcome & sample space [2]

In probability, the following means:

- An **outcome** simply refers to a single result that can emerge from a study.
- An **event** is referred to as the set of outcomes that share a common characteristic in a study.
- A **sample space** is the set of all possible outcomes observed in a study.

- Example 2: We are performing a survey to know the burden of Obesity by measuring BMI status of 10 people whose BMI can range anywhere between 15.0kg/m^2 to 66.0kg/m^2. The data collected was as follows = **18.9, 24.7, 32.4, 40.1, 21.4, 29.2, 24.5, 16.3, 19.7, 30.3**

  Outcome: Each BMI value, such as 18.9, 24.7, or 32.4 and so on, is an individual outcome. The 8^{th} entry 16.3 is one outcome.

  Event: We are interested Obesity (BMI $\geq$ 30) therefore the observed values meet this criteria {32.4, 40.1, 30.3}. This event contains 3 outcomes.

  Sample space: The possible outcome that can emerge can be any continuous BMI measurements within this range 15.0kg/m^2 to 66.0kg/m^2
  All observed sample space that includes 10 BMI outcomes = {18.9, 24.7, **32.4, 40.1**, 21.4, 29.2, 24.5, 16.3, 19.7, **30.3**}

Remember, **Pr(E), the probability that an event 'E' will occur.** This is what the notation for the above example will look like:

- Probability that a person from this cohort is obese = **Pr(Obesity)**

Basic Calculation of a Probability

Example 1: We are conducting a mosquito infestation survey in 7 households in a village. We want to know the probability that a house is infested with mosquitoes:

Event: The event "<u>Infested</u>" consists of all 3 outcomes (i.e., 1st, 5th and 7th home) that are infested
Sample space: All observed 7 outcomes = {**Infested**, Not Infested, Not Infested, Not Infested, **Infested**, Not Infested, **Infested**}

Pr(Infested) = (the number of outcomes in E / total in sample space) = 3/7 = 0.4285 = 42.85%

Example 2: We are performing a survey to know the burden of Obesity by measuring BMI status of 10 people. We want to know the probability that a person's obese:

Event: This event contains 3 outcomes that meet the criteria of obesity status (BMI ≥ 30) i.e., {32.4, 40.1, 30.3}.
Sample space: All observed 10 BMI outcomes = {18.9, 24.7, **32.4, 40.1**, 21.4, 29.2, 24.5, 16.3, 19.7, **30.3**}

Pr(Obesity) = (the number of outcomes in E / total in sample space) = 3/10 = 0.30 = 30.00%

- The above instance dealing with 'single events' are typical examples of an **Unconditional Probability.**
- There are three major types of probabilities: **Unconditional**, **Joint** and **Conditional probability**.

Unconditional Probabilities

An unconditional probability is the chance (or likelihood) that a particular event will occur, without regard to external circumstances such as past, present, or future events.

- Usually, the set of outcomes in a single event can be affected by any number of factors; however, with unconditional probabilities, the likelihood of an event ending with a specific results does not account for other conditions that may affect it.
- It is also called a **Marginal Probability**
- When there's data - unconditional (or marginal) probabilities are calculated accordingly as follows:

Probability that E will occur

$$\Pr(\mathrm{E}) = \frac{\text{Number of observed outcomes (n)}}{\text{Total sample space (N)}} = \frac{\mathrm{n}}{\mathrm{N}}$$

Probability that E will NOT occur

$$\Pr(\mathrm{E}') = 1 - \left(\frac{\text{Number of trials (n)}}{\text{Total sample size (N)}}\right) = 1 - \frac{\mathrm{n}}{\mathrm{N}}$$

We call this **complement** of E (i.e., E')

Note 1: P(E) + P(E') = 1

Note 2: It is a basic prevalence or proportion value

Note 3: Union (U) is also referred to as the total sample space

Joint Probabilities

A joint probability refers to the likelihood of more than one event occurring at the same time. For example, if there are two events i.e., E_1 and E_2, the joint probability is the chance that both events will occur at the same time.

- To simply put this – it is basically the probability of E_1 and E_2 when they happen at the same time. Best seen specifically in contingency tables.

- There are several notation for representing joint probabilities:

  $\Pr(\mathrm{E}_1$ & $\mathrm{E}_2)$ is joint probability of events E_1 & E_2; where E_1 and E_2 are two different type events that intersect or occur together

- When there's data particularly from a contingency table – joint probabilities are calculated as follows:

$$\Pr(\mathrm{E}_1 \,\&\, \mathrm{E}_2) = \frac{\text{Number of observed outcomes in both events}}{\text{Grand total in that sample space}}$$

Conditional Probabilities

A **conditional probability** of an event E_1 is the probability that E_1 will happen given that the event E_2 has already occurred.

- Simply put – it's the likelihood of an event E_1 occurring, based on the occurrence of a previous event E_2.
- We say: "Probability of 'this' given 'that'" – where the probability of event E_1 depends on the occurrence of event E_2
- The notation for representing conditional probabilities using this symbol "|" to represent 'given'. It is written as: $Pr(E_1 \mid E_2)$ which means the probability of E_1 given E_2
- Conditional probabilities are computed accordingly as follows:

Conditional probabilities:

$$\Pr(E_1 \mid E_2) = \frac{\Pr(E_1 \,\&\, E_2)}{\Pr(E_2)}$$

Notes: Calculating the conditional probabilities are quite involved. You will need to calculated the joint probabilities of E1 and E2, and unconditional probability of E2 and divide them together.

Example: Study on measuring abundance of Adult mosquitoes in Location A [1]

	Breeding sites: Aedes	Breeding sites: No Aedes	Sum (Row)
Urban	182	1008	1190
Rural	132	861	993
Sum (Column)	314	1869	Grand total: 2183

- We are interested detected presence of adult mosquitoes from breeding sites within an urban or rural setting

Let the B represent the event Breeding sites: Aedes
Let the B' represent the event Breeding sites: No Aedes

Let the U represent the event urban area (i.e., total number of breeding sites found in an urban setting)
Let the U' represent the event rural area (i.e., total number of breeding sites found in a rural setting)

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Example: Study on measuring abundance of Adult mosquitoes in Location A [2]

	Breeding sites: Aedes	Breeding sites: No Aedes	Sum (Row)
Urban	182	1008	1190 (/2183)
Rural	132	861	993 (/2183)
Sum (Column)	314 (/2183)	1869 (/2183)	Grand total: 2183

Here, we can compute the probabilities by simply dividing the number of events observed by the overall total sample space which is 2183

The second table, we have simply converted the raw values to probabilities. The light blue shaded cells are the unconditional (or marginal) probabilities

	Breeding sites: Aedes (B)	Breeding sites: No Aedes (B')	Sum (Row)
Urban (U)			0.55 P(U)
Rural (U')			0.45 P(U')
Sum (Column)	0.14 P(B)	0.86 P(B')	Grand total: 1

Probability of finding breeding site in Urban location = 0.55

Probability of finding breeding site in Rural location = 0.45

Probability of finding breeding site with Aedes mosquito = 0.14

Probability of finding breeding site with no Aedes mosquito = 0.86

Unconditional Probabilities

Example: Study on measuring abundance of Adult mosquitoes in Location A [2]

	Breeding sites: Aedes	Breeding sites: No Aedes	Sum (Row)
Urban	182	1008	1190 (/2183)
Rural	132	861	993 (/2183)
Sum (Column)	314 (/2183)	1869 (/2183)	Grand total: 2183

	Breeding sites: Aedes (B)	Breeding sites: No Aedes (B')	Sum (Row)
Urban (U)	B & U	B' & U	0.55 P(U)
Rural (U')	B & U'	B' & U'	0.45 P(U')
Sum (Column)	0.14 P(B)	0.86 P(B')	Grand total: 1

Example: Study on measuring abundance of Adult mosquitoes in Location A [3]

	Breeding sites: Aedes	Breeding sites: No Aedes	Sum (Row)
Urban	182 (/2183)	1008 (/2183)	1190
Rural	132 (/2183)	861(/2183)	993
Sum (Column)	314	1869	Grand total: 2183

	Breeding sites: Aedes (B)	Breeding sites: No Aedes (B')	Sum (Row)
Urban (U)	0.0834 Pr(B & U)	0.4617 Pr(B' & U)	0.55 Pr(U)
Rural (U')	0.0605 Pr(B & U')	0.3944 Pr(B' & U')	0.45 Pr(U')
Sum (Column)	0.14 Pr(B)	0.86 Pr(B')	Grand total: 1

In the second table, we can convert these raw values to joint probabilities. The dark blue shaded cells are the joints probabilities computed by using the formula shown in slide #16

Probability of finding a breeding site with Aedes mosquitoes in rural areas = 0.0605 (6.05%)

Probability of finding a breeding site with no Aedes mosquitoes in rural areas = 0.3944 (39.44%)

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Example: Study on measuring abundance of Adult mosquitoes in Location A [3]

	Breeding sites: Aedes	Breeding sites: No Aedes	Sum (Row)
Urban	182	1008	1190
Rural	132	861	993
Sum (Column)	314	1869	Grand total: 2183

	Breeding sites: Aedes (B)	Breeding sites: No Aedes (B')	Sum (Row)
Urban (U)	0.0834 Pr(B & U)	0.4617 Pr(B' & U)	0.55 Pr(U)
Rural (U')	0.0605 Pr(B & U')	0.3944 Pr(B' & U')	0.45 Pr(U')
Sum (Column)	0.14 Pr(B)	0.86 Pr(B')	Grand total: 1

Suppose, we want to know what the probability that that Aedes mosquito are present in breeding sites given the setting is urban i.e., Pr(B|U), a conditional probability.

We will need the joint probability Pr(B & U)
We will need the unconditional probability for Pr(U)

Conditional probabilities:

$$\Pr(\mathrm{E}_1 \mid \mathrm{E}_2) = \frac{\Pr(\mathrm{E}_1 \,\&\, \mathrm{E}_2)}{\Pr(\mathrm{E}_2)}$$

$$\Pr(B|U) = \frac{\Pr(B \,\&\, U)}{\Pr(U)} = \frac{0.0834}{0.55} = 0.1529 = 15.29\%$$

Probabilities

Probability Distribution

Bayes’ Rule

Bayesian Inference

Probability Distributions

Definition:

Probability distribution is a mathematical function estimates the **plausibility** of observing a particular value (or range of values) for a variable. It can be used to estimate **cumulative probability up to a certain value**.

There are broadly two groups for probability distributions with several subtypes:

Probability Mass Function (PMFs) (Discrete)

- Discrete uniform distribution
- Bernoulli distribution
- Binomial distribution
- Poisson distribution

Note: These distributions can handle variables that counts, or a discrete (or distinct) in nature.

Probability Density Function (PDFs) (Continuous)

- Uniform distribution
- Gaussian (or Normal) distribution
- Gamma distribution (flexible)
- Beta distribution (flexible)

Note: These distributions can handle variables that are continuous in nature

Important note:

In Bayesian inference, probability distributions are applied to the components of Bayes' rule, including the likelihood function and prior probability, to derive the posterior probability.

We assign a probability distribution to model the plausibility of an outcome (likelihood) and subjectively choose another probability distribution (the prior) to represent our beliefs about the parameters we want to estimate. The prior reflects our assumptions about the plausibility (or shape) of the parameter values before observing the data.

How does Probability Distribution work? [1]

Using a simple probability distribution i.e., Normal Distribution for computing **plausibility** of an observed value

$$\Pr(BMI) = \frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\frac{1}{2}\left(\frac{BMI - BMI_{\mu}}{\sigma}\right)^{2}} \longleftrightarrow \mathrm{norm}(BMI_{\mu}, \sigma)$$

FIXED INPUTS

- BMI_{μ} = 23.0 (i.e., mean is assumed to be the most plausible BMI value)
- σ = ±1.2 (i.e., assumed standard deviation for BMI)
- π = 3.141593

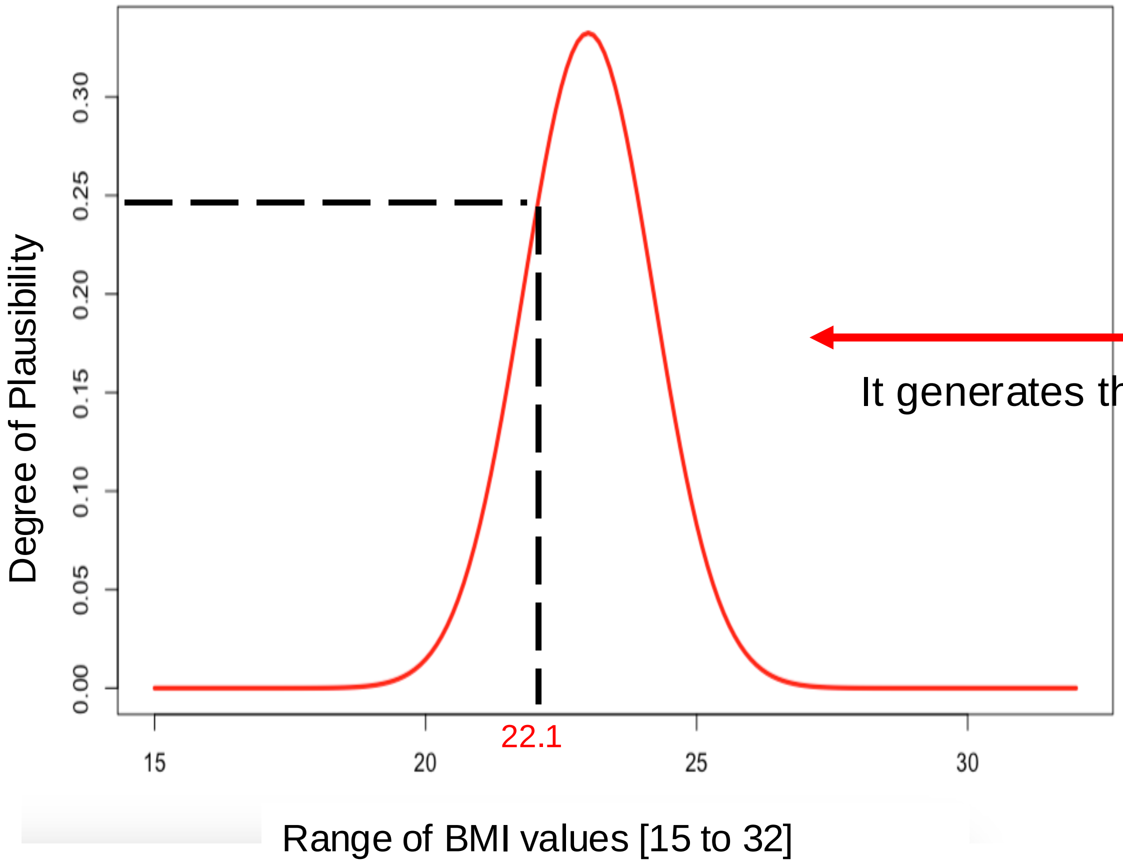
INPUTS

y = BMI data ranging from 15.0 to 32.0 (at increments of 0.1, so 15.0, 15.1, 15.2… 31.8, 31.9 and 32.0)

Each data point is inserted into this formula to get a plausibility value for range of BMI outcomes.

$$\Pr(\mathrm{BMI}) = \frac{1}{1.2\sqrt{2(3.1415193)}} \mathrm{e}^{-\frac{1}{2}\left(\frac{\mathrm{BMI} - 23.0}{1.2}\right)^{2}}$$

It generates this curve



For instance, we can estimate how plausible it is for someone have a BMI of exactly 0.2509. This value of 0.2509 represents the likelihood of BMI being 22.1 relative to all other BMI values on the distribution

This is not a probability nor percentage!

How does Probability Distribution work? [2]

Using a simple probability distribution i.e., Normal Distribution for computing **probability** of an observed value

$$\Pr(BMI) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{BMI-BMI_{\mu}}{\sigma}\right)^2} \longleftrightarrow \mathrm{norm}(BMI_{\mu}, \sigma)$$

FIXED INPUTS

- $BMI_{\mu} = 23.0$ (i.e., mean is assumed to be the most plausible BMI value)
- $\sigma = \pm 1.2$ (i.e., assumed standard deviation for BMI)
- $\pi = 3.141593$

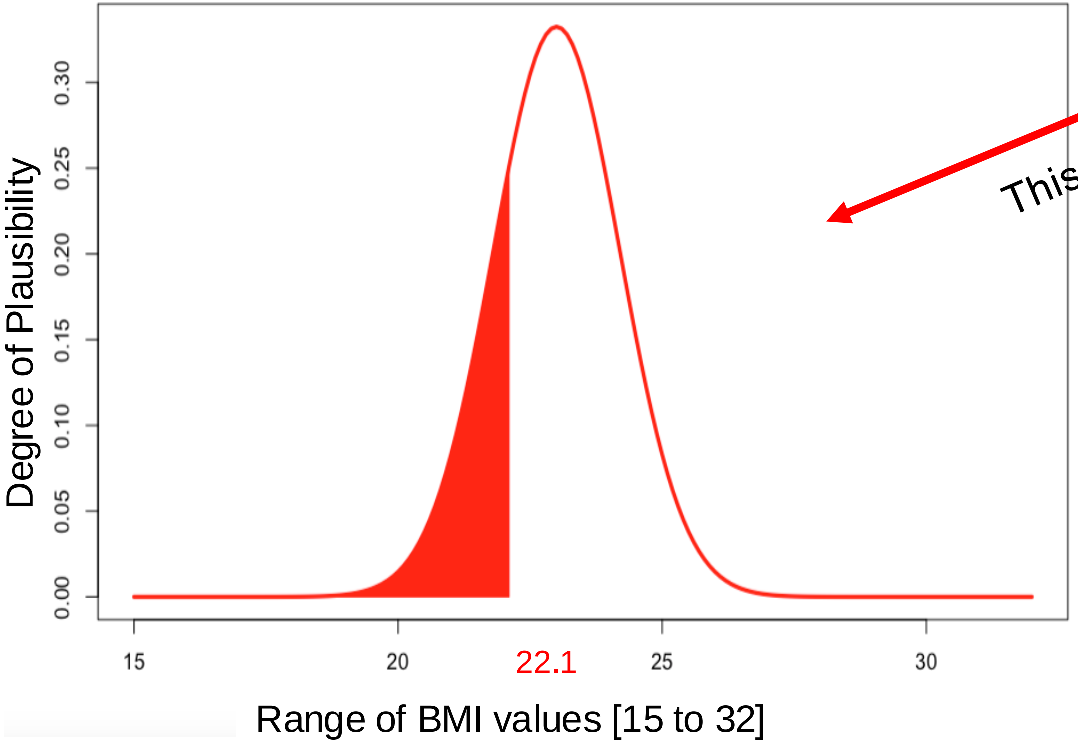
INPUTS

y = BMI data ranging from 15.0 to 32.0 (at increments of 0.1, so 15.0, 15.1, 15.2… 31.8, 31.9 and 32.0)

Each data point is inserted into this formula to get a plausibility value for range of BMI outcomes.

$$\Pr(\mathrm{BMI}) = \frac{1}{1.2\sqrt{2(3.1415193)}} e^{-\frac{1}{2}\left(\frac{\mathrm{BMI}-23.0}{1.2}\right)^2}$$

This generates this curve



Find the area under the curve from this BMI distribution for values up to 22.1 calculates the probability of BMI being a up to a value.

$$\Pr(15.0 \leq \mathrm{BMI} \leq 22.1) = \int \frac{1}{1.2\sqrt{2(3.1415193)}} e^{-\frac{1}{2}\left(\frac{\mathrm{BMI}-23.0}{1.2}\right)^2}$$

We estimate the probability of someone's BMI being up to 22.1 is $0.2266 \approx 22.7\%$.

Probabilities

Probability Distribution

Bayes' Rule

Bayesian Inference

Bayes' Theorem

Definition:

Bayes' theorem (or law) determines the posterior probability of the parameters we estimate given the observed dataset.

- It's a mathematical formula comprise of conditional and marginal probabilities
- It expresses how a subjective degree of belief should rationally change to account for new evidence.
- The basic formulation for Bayes' Theorem (or Law):

$$\Pr(\text{Parameter} \mid \text{Data}) \propto \Pr(\text{Data} \mid \text{Parameter}) \; \Pr(\text{Parameter})$$

Probability of our new parameter being true after we've observed dataset

Plausibility of the observed data we have, given the parameter has a specific value(s)

Our belief about what the parameter is, before observing the data

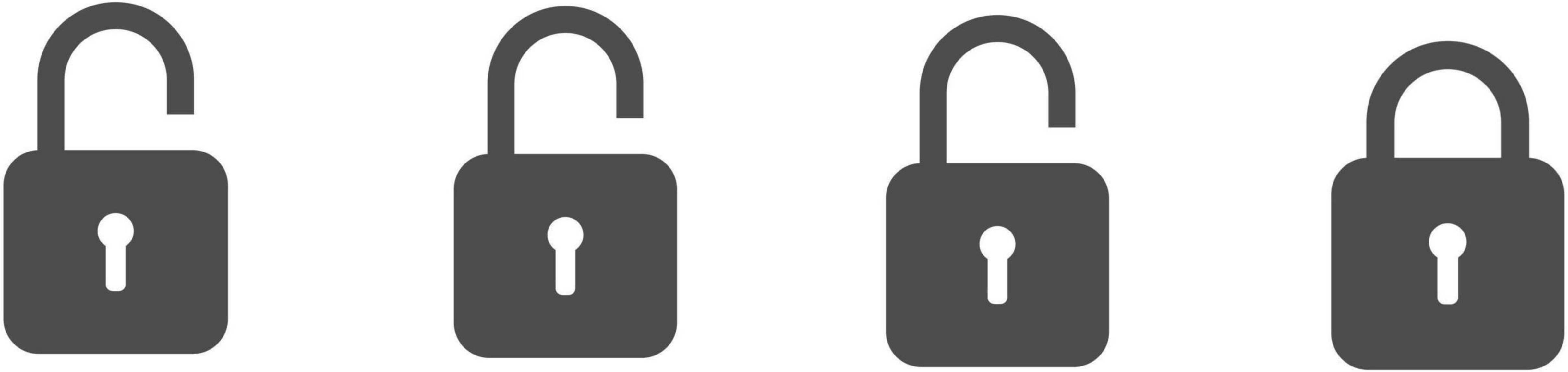
Posterior probability $\propto$ Likelihood x Prior Probability

This equation is the backbone of Bayesian Statistics

Thomas Bayes (1701 – 1761)

Selection of priors for Likelihood $\mathbf{P}(\boldsymbol{Y}|\boldsymbol{\theta})$ or Priors $\mathbf{P}(\boldsymbol{\theta})$

Characteristic	Check shape	Function	Distribution	Statistical Notation	Usage
• Continuous measure (scale, interval)	Bell shape (centre, or shifted)	PDF	Gaussian	norm(mu, sigma)	• Set it as a likelihood • Use as prior
• Proportion (with information on the numerator and denominator)	Does not matter	PMF	Binomial	bin(n, p)	• Set it as a likelihood (only)
• Counts	Does not matter	PMF	Poisson	poisson(rate)	• Set it as a likelihood (only)
• Binary measure (Yes, No)	Does not matter	PMF	Bernoulli	bern(p)	• Set it as a likelihood (only)
• Continuous measure (scale, interval) • Proportion (with information on the numerator and denominator) • Counts	Flat shape	PDF/PMF	Uniform	uniform(a, b)	• Use ONLY as a prior • Use it if you don’t have any prior information • This is an uninformative prior
• Continuous measure (scale, interval) • Counts	Skewed shape	PDF/PMF	Gamma	Gamma(rate, shape)	• Use ONLY as a prior • It’s flexible
• Proportion (with information on the numerator and denominator)	Does not matter	PDF	Beta	Beta(theta, shape1, shape2)	• Use ONLY as a prior • It’s flexible



Probabilities

Probability Distribution

Bayes’ Rule

Bayesian Inference

Bayesian Inference

Performing Bayesian Inference

$\Pr(\theta \mid y) \propto \Pr(\theta)\Pr(y \mid \theta)$

- For point or interval estimation of a parameter θ, the inference is based off:

$$\Pr(\text{Parameter} \mid \text{Data}) \propto \Pr(\text{Data} \mid \text{Parameter})\Pr(\text{Parameter})$$

Posterior probability ∝ Likelihood x Prior Probability

Where:

- $\Pr(\text{Parameter})$ is the, which must be given a prior probability function
- $\Pr(\text{Data} \mid \text{Parameter})$ is the statistical model (or likelihood), which must be given a probability function depending y
- $\Pr(\text{Parameter} \mid \text{Data})$ is the posterior density for the parameter θ

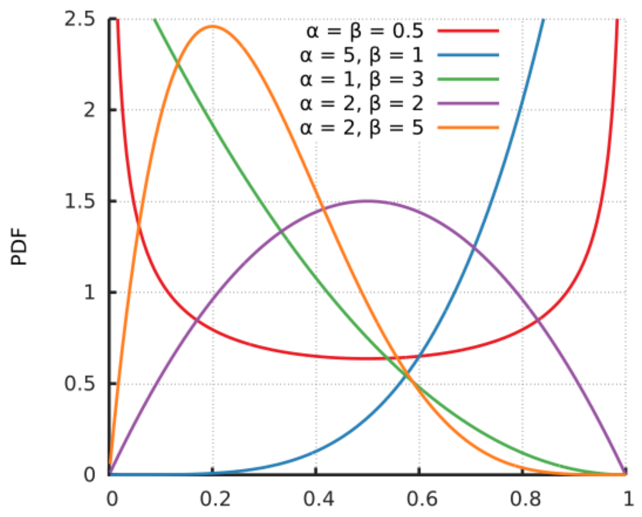
- Specifying the likelihood's probability function is very easy. Often, you will be using normal, Poisson or binomial distribution to deal continuous, counts, or binary outcomes respectively.

- The difficulty lies in the specification of the prior probability function. Here its entirely subjective.

Motivating example

- What is the plausible prevalence for infestation in Recife right now?

Last survey year: 09/2024	Number properties detected with Aedes	Overall number of properties surveyed
* Most recent data collection effort.	428	976



Beta distribution

Important Information:
- Number of infested properties (428) [Data]
- Total number of properties surveyed (976) [Data]
- The prevalence of infestation is unknown, but we want to estimate the plausible value for prevalence of infestation [Parameter]
- Prior information for prevalence (i.e., our knowledge or belief) is assumed 0.20 (in most cases, the prevalence from past research around this time is often this value of 20-25%. [Parameter]

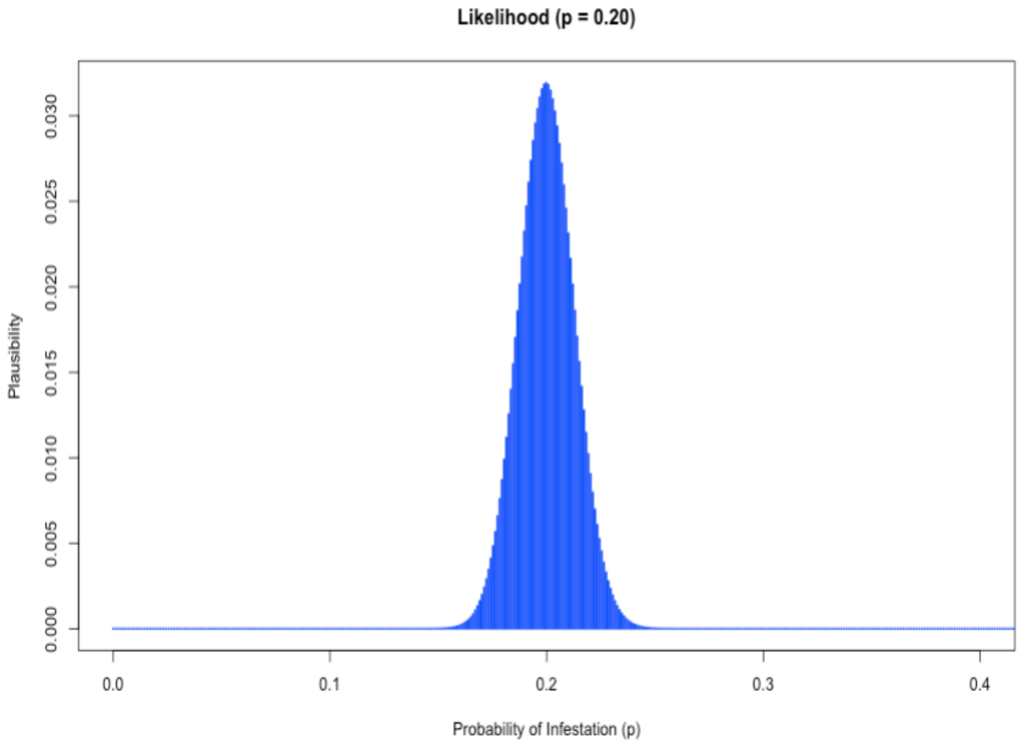
Likelihood: $\mathrm{Pr(Infested \mid Prevalence)} \Rightarrow \mathrm{Infested} \sim \mathrm{Bin(Total, Prevalence)}$

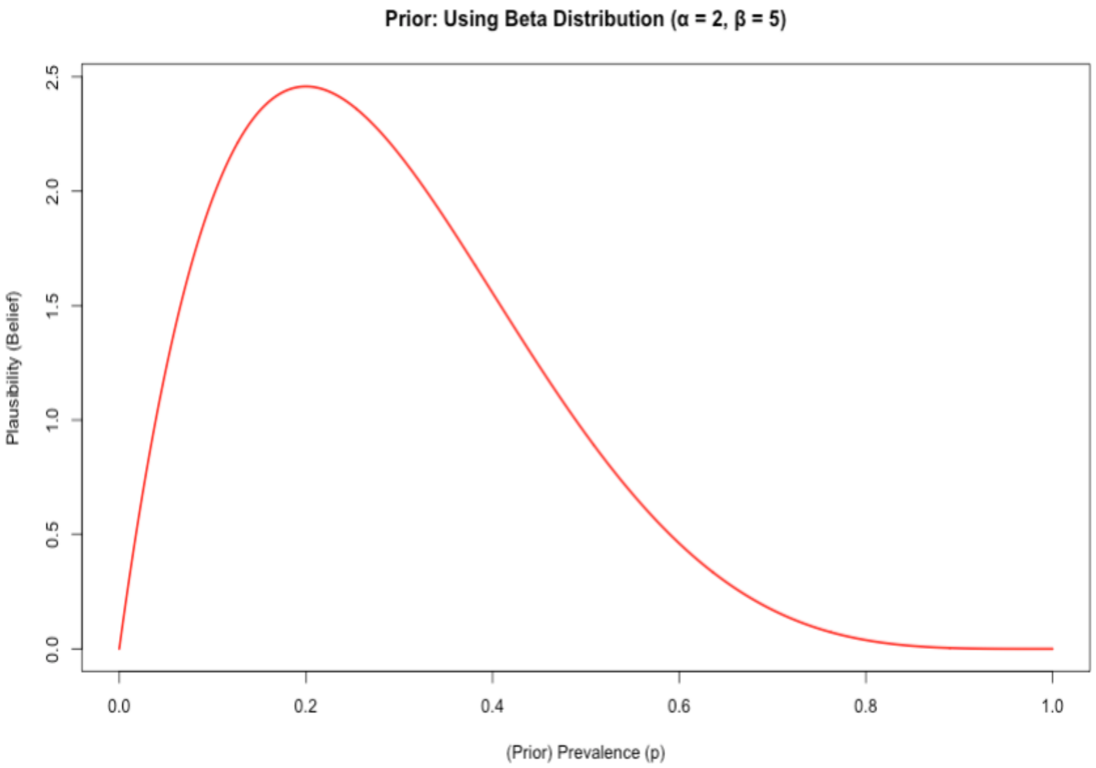
Prior: $\mathrm{Pr(Prevalence)} \Rightarrow \mathrm{Beta(Prevalence} \mid \alpha - 1, \beta - 1)$, where $\alpha = 2, \beta = 5$

Posterior: $\mathrm{Pr(Prevalence \mid Infested)} \Rightarrow \mathrm{Bin(Total, Prevalence)}$ x $\mathrm{Beta(Prevalence} \mid 1, 4)$

Motivating example (cont.)

$$\mathrm{Bin}(\mathrm{Total}, \mathrm{Prevalence}) \quad \text{x} \quad \mathrm{Beta}(\mathrm{Prevalence} \mid a - 1, \mathrm{b} - 1)$$

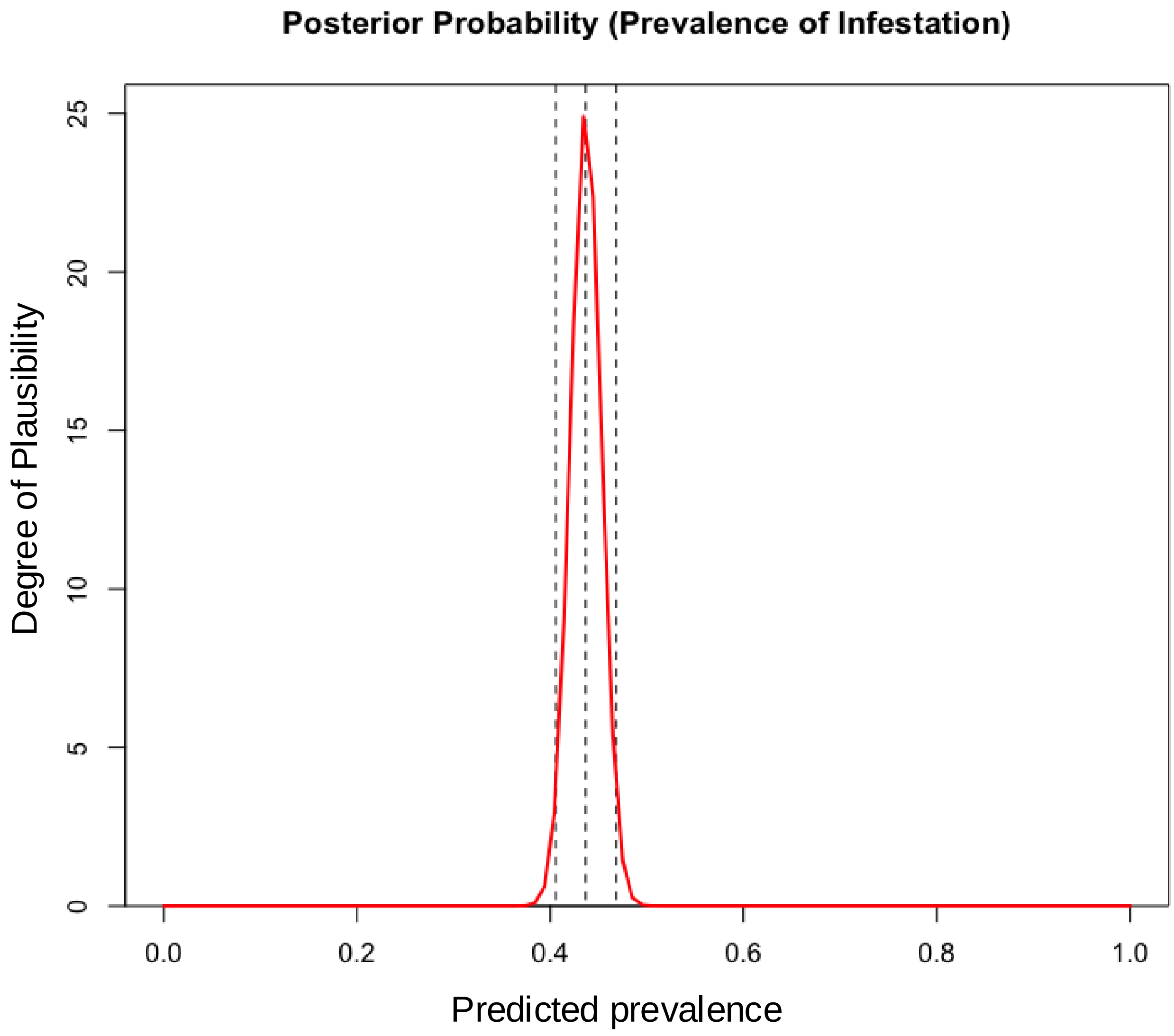




Combining the likelihood function with prior distribution based on our assumptions/knowledge, we get a new posterior distribution with an updated prevalence values

Updated result

$$\text{Bin}(\text{Total}, \text{Prevalence}) \times \text{Beta}(\text{Prevalence} \mid a - 1, b - 1)$$



Interpretation:

- The predicted prevalence is approximately 44% with 95% credible intervals (40% to 47%)
- The most plausible value for prevalence of infestation is 44% because it has a highest degree of plausibility
- The credibility limits show the degree of uncertainty around prevalence which could be anywhere between 40-47%

Probabilities

Bayes’ Rule

Bayesian Inference

Probability Functions

Any questions?