https://github.com/UCR-HPC/cs211-hw2-solving-large-linear-system-pri

vate-MoonlyTower

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Q1

$$A = \begin{pmatrix} 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 \\ 4 & 13 & 18 & = \begin{pmatrix} 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} 4 & 5 & 6 \end{pmatrix} \\ 7 & 54 & 78 & 7 & 40 & 57 & 7 & 8 & 9 \end{pmatrix}$$

M is the lower triangular,

$$M = \begin{pmatrix} 0 & 0 & 0 & & & 1 & 0 & 0 \\ 4 & 0 & 0 & 0 & & L = M + I = \begin{pmatrix} 4 & 1 & 0 \\ 7 & 8 & 0 & & 7 & 8 & 1 \end{pmatrix}$$

U is the upper triangular,

$$U = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{pmatrix}$$

so that, A = LU

$$L = \begin{pmatrix} 1 & 0 & 0 \\ (4 & 1 & 0), & U = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \end{pmatrix} \\ 7 & 8 & 1 & 0 & 0 & 9 \end{pmatrix}$$

 $\mathbf{Q2}$

step1:log into the hpc-001

scp -r /Users/ruiliu/Downloads/lapack-3.12.0 ssh -J rui
001@bolt.cs.ucr.edu rui001@hpc-001

scp -r /Users/ruiliu/Downloads/lapack-3.12.0 <u>rui001@hpc-001.cs.ucr.edu</u> /home/rui001/CS211-hw/l_BaseKit_p_2024.2.1.100.sh ssh -J rui001@bolt.cs.ucr.edu rui001@hpc-001

step2:download the MKL library form the Intel on hpc-001

```
The following tools have been installed successfully:

Intel® oneAPI Math Kernel Library

Installation location: /home/rui001/intel/oneapi
```

```
Intel® oneAPI Math Kernel Library | 2024.2.2
[[rui001@cluster-001-login-node ~]$ source /home/rui001/intel/oneapi/setvars.sh
:: initializing oneAPI environment ...
   -bash: BASH_VERSION = 4.4.20(1)-release
   args: Using "$0" for setvars.sh arguments:
```

step3:code/compiler and test on file main2.

:: compiler -- latest
:: mkl -- latest
:: tbb -- latest

:: oneAPI environment initialized ::

I give the detailed comments process when using file main2 to debugging and verify the correctness of the code.



```
    RUI001 [SSH: HPC-001.cs...
    0.129645 - 0.798261 0.3779764 - 0.058655 - 1.439199 - 0.810646 - 1.299551 - 1.155876 0.099189 - 0.397477
    b. vscoode-server
    CS211-hw
    0.653179 0.875634 0.371728 0.133576 1.788122 0.098349 0.724988 - 0.683310 0.123207 0.975634
    0.65753 - 0.068753 - 0.042380 - 0.023184 1.218011 0.062617 - 0.0919928 - 1.633641 - 0.061653

                                                                       Step 4: Current max = 1.184990 at row 4
Swapping rows 4 and 8
Row 5 after division: 0.129645 -0.798361 0.377964 -0.058655 -0.804866 -0.810646 -1.299551 -1.155876 0.009189 -0.397472
Row 6 after division: 0.895295 0.513499 -0.196381 -0.148092 0.034006 -0.505655 -0.228909 -0.492904 -2.352308 -0.384535
Row 7 after division: -0.156482 0.620882 -0.733388 -0.151293 0.662255 -0.110929 -0.731664 -0.741725 0.063221 0.043322
Row 8 after division: -0.204266 -0.661267 0.580822 -0.280882 -0.628081 -0.662791 0.038236 -0.67479 -0.468823 0.455319 -0.984350
Row 9 after division: -0.168867 0.637587 0.068753 -0.042380 -0.012966 1.218011 0.062617 -0.910928 -1.633641 -0.061053
        ∨ hw2
         > extern

    aitianore

           ■ .nfs00000000... U
        C func_call.c
C include.h
C lapack.c
    main
C main.c

main.
         M makefile
                                                                       Step 5: Current max = 0.810366 at row 5
Swapping rows 5 and 9
Row 6 after division: 0.895295 0.513499 -0.196381 -0.148092 0.034006 -0.415157 -0.253562 -0.469668 -2.356498 -0.417611
Row 7 after division: -0.156482 0.620882 -0.733388 -0.151293 0.662255 -0.091264 -1.211791 -0.289200 -0.018374 -0.600809
Row 8 after division: -0.204266 -0.661267 0.589822 -0.280163 -0.662761 0.031582 -0.557047 -0.921683 0.553168 -0.339785
Row 9 after division: 0.129645 -0.798361 0.377964 -0.058655 -0.804866 -0.665317 -0.716033 -1.705848 0.108354 0.385568
          C my_block.c
                                           M
U
          C mv.c
         ≡ mylu
           ≡ pad.txt
                                                                      Row 9 after division: 0.129645 -0.798361 0.377964 -0.058655 -0.884866 -0.665317 -0.716033 -1.7058 Matrix A after step 5: 
-0.865367 0.324562 -0.571642 -0.492492 0.546081 -0.143504 0.536445 0.567530 0.988226 -0.115777 -0.522515 1.071072 0.034204 -1.042182 -0.692322 -0.592811 -0.407617 0.442408 1.29887 -0.600616 -0.777394 -0.478120 -1.070278 -1.644396 0.360607 -0.703301 -0.255174 0.936692 1.094184 -0.275433 -0.040643 -0.030346 0.793164 2.070068 -0.181930 3.736359 -0.212329 -1.629902 -1.104095 0.368456 -0.853179 0.875534 0.371728 0.133575 1.788122 0.000349 0.724988 -0.6583310 0.123207 0.972633 -0.188867 0.637587 0.068753 -0.042380 -0.012966 1.218815 0.072217 -0.919788 -1.652044 -0.048442 0.895295 0.513499 -0.196331 -0.118092 0.034006 -0.415157 -0.223664 -0.851524 -3.034051 -0.437722 -0.156482 0.620882 -0.733388 -0.151293 0.662255 -0.901264 -1.205219 -0.373143 -0.167320 -0.065236 -0.204266 -0.661267 0.589922 -0.280163 -0.662701 0.031582 -0.569322 -0.895250 -0.895255 0.899326 -0.569322 -0.280163 -0.667701 0.031582 -0.569322 -0.895263 -0.895255 0.80163 -0.667701 0.031582 -0.569321 -0.895256 0.895250 -0.393138
          starter.py
        > lapack-3.12.0
        $ I_BaseKit_p_2024.2....
       $ I_onemkl_p_2024.2....
        ≡ lapack-3.12.0.tar
         ≡ test mkl
      C test_mkl.c
                                   Step 6: Current max = 0.223664 at row 6
Swapping rows 6 and 7
Row 7 after division: 0.895295 0.513499 -0.196381 -0.148092 0.034006 -0.415157 0.185580 -0.851524 -3.034051 -0.437722
  〉大纲
 > 时间线
  资源管理器 ··· C my.c M C main.c C main2.c U X 를 mylu U C test_mkl.c

∨ RUI001 [SSH: HPC-001.CS... CS211-hw > hw2 > C main2.c > 分 main()

                                                                         179
      > .vscode-server
      ∨ CS211-hw
                                                                                180
                                                                                                      int main() {
                                                                                                        int n = 3;
       > hw1
                                                                              181
         ∨ hw2
                                                                          182
                                                                                                                     double *A = (double *)malloc(n * n * sizeof(double));
         > extern
                                                      183
184
                                                                                                                    double *B = (double *)malloc(n * sizeof(double));
           gitignore
           ≣ .nfs00000000... ∪
                                                                           185
           C func_call.c
                                                                                                                srand(time(NULL));
                                                                               186
                                                                           187
           C include.h
                                                                                                                     for (int i = 0; i < n; i++) {
          C lapack.c
                                                                             188
                                                                                                                                     for (int j = 0; j < n; j++) {
           ≡ main
                                                                           189
                                                                                                                                                 A[i * n + j] = (rand() % 10) + 1;
           C main.c
        C main.c 190

E main2 U 191

C main2.c U 192
                                                                                                                                  B[i] = (rand() % 10) + 1;
                                                                               192
           M makefile
           C my_block.c
                                                                                193
          C my.c M 195
                                                                           194
                                                                                                               printf("Matrix A:\n");
                                                                                                                     for (int i = 0; i < n; i++) {
                                                                                196
                                                                                                                                     for (int j = 0; j < n; j++) {
          ≡ pad.txt
                                                                                                                                              printf("%f ", A[i * n + j]);
                                                                               197
          starter.py
                                                                             198
         > lapack-3.12.0
        $ I_BaseKit_p_2024.2....
                                                                               199
                                                                                                                                     printf("\n");
        $ I_onemkl_p_2024.2....
                                                                              200
```

test on a main2 as shown below: gcc main2.c -o main2 -lm ./main2

```
[rui001@cluster-001-login-node hw2]$ gcc main2.c -o main2 -lm[rui001@cluster-001-login-node hw2]$ ./main2
    Matrix A:
    6.000000 3.000000 1.000000
    1.000000 3.000000 4.000000
    3.000000 7.000000 5.000000
    Vector B:
    2.000000 10.000000 5.000000
    Initial Matrix A:
    6.000000 3.000000 1.000000
    1.000000 3.000000 4.000000
    3.000000 7.000000 5.000000
    Initial Vector B:
    2.000000 10.000000 5.000000
    Step 0: Current max = 6.000000 at row 0
    Row 1 after division: 0.166667 3.000000 4.000000
Row 2 after division: 0.500000 7.000000 5.000000
   Matrix A after step 0:
6.000000 3.000000 1.000000
0.166667 2.500000 3.833333
0.500000 5.500000 4.500000
    Step 1: Current max = 2.500000 at row 1
    Swapping rows 1 and 2
    Row 2 after division: 0.166667 0.454545 3.833333
    Matrix A after step 1: 6.000000 3.000000 1.000000
    0.500000 5.500000 4.500000
    0.166667 0.454545 1.787879
    Matrix A after LU factorization:
    6.000000 3.000000 1.000000
0.500000 5.500000 4.500000
    0.166667 0.454545 1.787879
    Pivot indices:
    0 2 1
tempB[0] after pivoting: 2.000000
tempB[1] after pivoting: 5.000000
tempB[2] after pivoting: 10.000000
B[0] before forward substitution (after pivoting): 2.000000
B[1] before forward substitution (after pivoting): 5.000000
B[2] before forward substitution (after pivoting): 10.000000
B[0] after forward substitution: 2.000000
B[i]: 4.000000
A[i*n+j]]: 0.500000
B[1] after forward substitution: 4.000000
B[i]: 9.666667
A[i * n + j]]: 0.166667
B[i]: 7.848485
D[1]: 7.846465
A[i * n + j]]: 0.454545
B[2] after forward substitution: 7.848485
Vector B after forward substitution:
2.000000 4.000000 7.848485
B[i]: 7.848485
A[i*n+i]]: 1.787879
#########B[2] after backward substitution: 4.389831
B[i]: 4.000000
A[i*n+i]]: 5.500000 ########B[1] after backward substitution: -2.864407
B[i]: 2.000000
A[i*n+i]]: 6.000000 ########B[0] after backward substitution: 1.033898 Vector B after backward substitution (Solution X):
```

1.033898 -2.864407 4.389831 Solution X: 1.033898 -2.864407 4.389831

step4:code/compiler and test on file main.

complier:

cd /path/to/your/project CURDIR=\$(pwd) export CURDIR

LD_LIBRARY_PATH=\$CURDIR/extern:/act/opt/intel/composer_xe_2013.3.1/mkl/lib/intel64:\$LD_LIBRARY_PATHexport LD_LIBRARY_PATH make

I have shown the final experiment outcome as above.

./main function num

- [rui001@cluster-001-login-node hw2]\$./main my 1000 n=1000, pad=1 time=0.146002s
- [rui001@cluster-001-login-node hw2]\$./main my 2000 n=2000, pad=1 time=1.642781s
- [rui001@cluster-001-login-node hw2]\$./main my 3000 n=3000, pad=1 time=6.559644s
- [rui001@cluster-001-login-node hw2]\$./main my 4000 n=4000, pad=1 time=15.823135s
- [rui001@cluster-001-login-node hw2]\$./main my 5000 n=5000, pad=1 time=30.574100s
- [rui001@cluster-001-login-node hw2]\$./main lapack 100 n=100, pad=1 time=0.040017s
- [rui001@cluster-001-login-node hw2]\$./main lapack 1000 n=1000, pad=1 time=0.073281s
- [rui001@cluster-001-login-node hw2]\$./main lapack 1000 n=1000, pad=1 time=0.074497s
- [rui001@cluster-001-login-node hw2]\$./main lapack 2000 n=2000, pad=1 time=0.239922s
- [rui001@cluster-001-login-node hw2]\$./main lapack 3000 n=3000, pad=1 time=0.707063s
- [rui001@cluster-001-login-node hw2]\$./main lapack 4000 n=4000, pad=1 time=1.424908s
- [rui001@cluster-001-login-node hw2]\$./main lapack 5000 n=5000, pad=1 time=2.850201s
- [rui001@cluster-001-login-node hw2]\$

srun main function num

- [rui001@cluster-001-login-node hw2]\$ export LD_LIBRARY_PATH=/home/act-software/opt/intel/cor b/intel64:/home/rui001/CS211-hw/hw2/extern:\$LD_LIBRARY_PATH
- [rui001@cluster-001-login-node hw2]\$ srun main my 1000 n=1000, pad=1 time=0.133002s

- [rui001@cluster-001-login-node hw2]\$ srun main my 2000 n=2000, pad=1 time=1.292453s
- [rui001@cluster-001-login-node hw2]\$ srun main my 3000 n=3000, pad=1 time=5.619809s
- [rui001@cluster-001-login-node hw2]\$ srun main my 4000 n=4000, pad=1 time=14.073875s
- [rui001@cluster-001-login-node hw2]\$ srun main my 5000 n=5000, pad=1 time=28.704089s
- [rui001@cluster-001-login-node hw2]\$ srun main lapack 1000 n=1000, pad=1 time=0.039766s
- [rui001@cluster-001-login-node hw2]\$ srun main lapack 2000 n=2000, pad=1 time=0.203340s
- [rui001@cluster-001-login-node hw2]\$ srun main lapack 3000 n=3000, pad=1 time=0.640516s
- [rui001@cluster-001-login-node hw2]\$ srun main lapack 4000 n=4000, pad=1 time=1.304958s
- [rui001@cluster-001-login-node hw2]\$ srun main lapack 5000 n=5000, pad=1 time=2.614344s

Compare the outcome of my function and the lapack:

We firstly calculate the Number of Floating-Point Operations (FLOPs).

For an LU decomposition of an n×n matrix, the number of operations can be approximated by $\frac{n^3+6n^2-4n}{3}$.

So Gflops= $\frac{n^3+6n^2-4n}{3}$ / time(seconds)* 10^9.

Matrix Size 1000

FLOPs=3.35332*10^8

my function: Time: 0.133002s

Gflops= $3.35332*10^8 / 0.133002 \times 10^9 \approx 2.5212$ Gflops

LAPACK:Time: 0.039766s

Gflops=3.35332*10^8 / 0.039766×109≈8.43051 Gflops

Matrix Size 2000

FLOPs=2.674664*10^9

my function: Time: 1.242953s

Gflops=2.674664*10^9 / 1.242953×10^9≈2.151826 Gflops

LAPACK:Time: 0.203340s

Gflops=2.674664*10^9 / 0.203340×10^9=13.153654 Gflops

Matrix Size 3000

FLOPs=9.017996*10^9

my function: Time: 5.619809s

Gflops=9.017996*10^9 / 5.619809×10^9≈1.60468Gflops

LAPACK:Time: 0.640516s

Gflops=9.017996*10^9 / 0.640516×10^9≈14.0793 Gflops

Matrix Size 4000

FLOPs=2.1365328*10^10 my function:Time: 17.044483s

Gflops= $2.1365328*10^10 / 14.073875 \times 10^9 \approx 1.518$ Gflops

LAPACK:Time: 1.424908s

Gflops=2.1365328*10^10 / 1.304958×10^9≈16.3724Gflops

Matrix Size 5000

FLOPs=4.171666*10^10

my function:Time: 33.782017s

Gflops= $4.171666*10^10 / 28.704089 \times 10^9 \approx 1.4534$ Gflops

LAPACK:Time: 2.850201s

Gflops=4.171666*10^10 / 2.614344×10^9≈18.0412 Gflops

Q3

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 & 4 & 1 & 2 & 3 & 4 \\ 2 & 9 & 12 & 15 \\ 3 & 26 & 41 & 49 \end{pmatrix} = \begin{pmatrix} 2 & 9 & 12 & 15 \\ 3 & 26 & 41 & 49 \end{pmatrix}$$

$$5 \quad 40 \quad 107 \quad 135 \quad 5 \quad 40 \quad 107 \quad 135$$

we calculate
$$\begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ 6 & 7 \end{pmatrix} = \begin{pmatrix} 33 & 44 \\ 51 & 62 \end{pmatrix}$$

so A=
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 1 & 2 & 3 & 4 \\ 2 & 5 & 6 & 7 & 9 & = \begin{pmatrix} 2 & 5 & 6 & 7 \\ 3 & 4 & 8 & 9 & 9 & = \begin{pmatrix} 2 & 5 & 6 & 7 \\ 3 & 4 & 8 & 9 & 9 & = \end{pmatrix}$$

L is the lower triangular, which is $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 3 & 4 & 1 & 0 & 0 \\ 5 & 6 & 7 & 1 & = 1 \end{pmatrix}$

U is the upper triangular, which is $\begin{pmatrix} 0 & 5 & 6 & 7 \\ 0 & 0 & 8 & 9 \\ 0 & 0 & 0 & 10 & = 1 \end{pmatrix}$

Q4

```
void mydgetrf_block(int n, double *A) {
     for (int k = 0; k < n; k += BLOCK_SIZE) {
         int end = k + BLOCK_SIZE < n ? k + BLOCK_SIZE : n;</pre>
         // Factorize the diagonal block
         for (int i = k; i < end; ++i) {
             for (int j = k; j < i; ++j) {
                 A[i * n + j] /= A[j * n + j];
                 for (int l = j + 1; l < end; ++l) {
                     A[i * n + l] -= A[i * n + j] * A[j * n + l];
         // Update the trailing submatrix
         #pragma omp parallel for collapse(2)
         for (int i = end; i < n; ++i) {</pre>
             for (int j = k; j < end; ++j) {
                 A[i * n + j] /= A[j * n + j];
                 for (int l = end; l < n; ++l) {
                     A[i * n + l] = A[i * n + j] * A[j * n + l];
        // Perform matrix multiplication for the trailing submatrix
        #pragma omp parallel for collapse(2)
        for (int i = end; i < n; i += BLOCK_SIZE) {</pre>
            for (int j = end; j < n; j += BLOCK_SIZE) {</pre>
                for (int ii = i; ii < i + BLOCK_SIZE && ii < n; ++ii) {
                   for (int jj = j; jj < j + BLOCK_SIZE \&\& <math>jj < n; ++jj) {
                       double sum = 0.0;
                       for (int kk = k; kk < end; ++kk) {
                           sum += A[ii * n + kk] * A[kk * n + jj];
                       A[ii * n + jj] -= sum;
test result:
on block size=32
n=1000, pad=1
Time taken: 0.068316 seconds
n=2000, pad=1
Time taken: 0.537873 seconds
 n=3000, pad=1
 Time taken: 1.862734 seconds
n=4000, pad=1
Time taken: 4.064807 seconds
 n=5000, pad=1
Time taken: 8.126369 seconds
```

on block size=64:

n=1000, pad=1

Time taken: 0.074267 seconds

n=2000, pad=1

Time taken: 0.535702 seconds

n=3000, pad=1

Time taken: 1.738807 seconds

n=4000, pad=1

Time taken: 4.140256 seconds

n=5000, pad=1

Time taken: 8.497961 seconds

on block size=128

n=1000, pad=1

Time taken: 0.099635 seconds

n=2000, pad=1

Time taken: 0.577570 seconds

n=3000, pad=1

Time taken: 1.811601 seconds

n=4000, pad=1

Time taken: 4.347364 seconds

n=5000, pad=1

Time taken: 8.669026 seconds

We can see that compare to non-block algorithm is much quicker, especially as the n size increase, the block pattern get a better time cost than non-block one.

- [rui001@cluster-001-login-node hw2]\$ export LD_LIBRARY_PATH=/home/act-software/opt/intel/cor b/intel64:/home/rui001/CS211-hw/hw2/extern:\$LD_LIBRARY_PATH
- [rui001@cluster-001-login-node hw2]\$ srun main my 1000 n=1000, pad=1 time=0.133002s

- [rui001@cluster-001-login-node hw2]\$ srun main my 2000 n=2000, pad=1 time=1.292453s
- [rui001@cluster-001-login-node hw2]\$ srun main my 3000 n=3000, pad=1
- time=5.619809s [rui001@cluster-001-login-node hw2]\$ srun main my 4000 n=4000, pad=1
 - time=14.073875s
- [rui001@cluster-001-login-node hw2]\$ srun main my 5000 n=5000, pad=1 time=28.704089s

We get the conclusion that maybe as the block size is too large, the performance is began to lose, so the approciate size like 32 or 64 would be much better.