Parallel Level Set Methods on Adaptive Tree-Based Grids

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Abstract

We present scalable parallel algorithms for the level-set technology on adaptive Quadtree and Octree Cartesian grids. The algorithms are based on a domain decomposition technique and implemented using MPI and the open-source p4est library. An important contribution is a scalable parallel semi-Lagrangian method, which, similar to its serial implementation, is free of any time-step restrictions. This is achieved by introducing a scalable global interpolation scheme on adaptive Quadtree and Octree grids. Moreover, we present a simple parallel reinitialization scheme using the pseudo-time transient formulation. Both parallel algorithms scale on the Stampede supercomputer, where we are limited to 4096 cores at most. Finally a relevant application of the algorithms is presented in modeling the crystallization phenomena by solving a Stefan problem, illustrating a level of detail that would be impossible without a parallel adaptive strategy. We believe that the algorithms presented in this article will be of interest and useful to researchers working with the level-set framework and modeling multi-scale physics in general.

Keywords: Quadtree/Octree Grids, Parallel Computing, Space Filling Curves, Semi-Lagrangian Method, Level-set Method

1. Introduction

The level-set method, originally proposed by Sethian and Osher [31], is a popular and powerful framework for tracking arbitrary interfaces that undergo complicated topological changes. As a result, the level-set method has been used to a wide range of applications such as multiphase flows, image segmentation, and computer graphics [29, 38]. An important feature of this method is that the location of the interface is defined implicitly on an underlying grid. This convenience, however, comes at a price. First, compared to an explicit method, e.g. front tracking [24, 44], the level-set method is typically less accurate and mass conservation could be a problem, although progress has been made in resolving this issue [18]. Second, the level-set function has to be defined in a higher dimensional space that that of the interface. If only the location of the interface is needed, the added dimension greatly increases the overall computational cost. One way to avoid this problem is by computing the level-set only close to the interface, e.g. as in the narrow-band level-set method [2] or, more recently, by using a hash table to restrict both computation and storage requirements [11].

Another approach that can address both problems is the use of local grid refinement. In [39] the idea of using tree-based grids for level-set calculations was first introduced and later extended in [32, 26] for fluid simulations. More recently, authors in [28] proposed second-order accurate level-set methods on Quadtree (two spatial dimensions) and Octree (three spatial dimensions) grids. The use of adaptive tree-base grids in the context of the level-set method is quite advantageous because (i) it gives fine-grain control over errors, which typically occur close to the interface and (ii) it can effectively reduce the dimensionality of the problem by focusing most of the grid cells close to the interface. Fortunately, constructing the tree is quite simple in the presence of an interface that naturally defines an ideal metric for refinement. However,

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