Progress report: MODHIS polarimetry study
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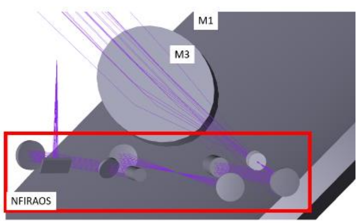
<u>Executive summary:</u>

This progress report concludes that 1) the presented conceptual design includes a level of polarization errors that seems likely to achieve the required precision of 0.001, given good calibration, although this calibration is not yet devised, and the analysis omits important perturbations, and 2) simple compensation measures can further reduce the polarization errors with low cost and design impact. This compensation should be implemented.

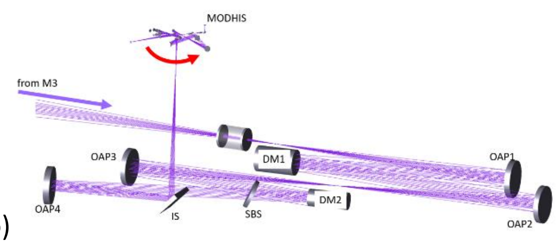
<u>MODHIS Mueller matrix</u>

Polarimetrically, it makes sense to split the optical path for MODHIS into three pieces:
- telescope
- NFIRAOS
- MODHIS FEI up to polarizer

This split makes sense because, within each of these pieces, the path for the axial ray lies in a plane. This symmetry, illustrated in Figure 1, allows the Mueller matrices of the subcomponents within each piece to be gathered into a Mueller matrix for each piece. The separation also makes sense because the pieces are rotated relative to one another during operation.



a)



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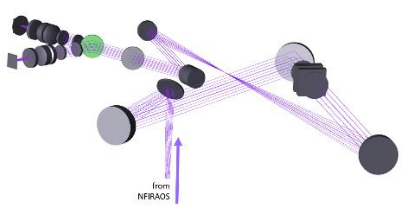


c)

<u>Figure 1</u>: Showing the orientation of the polarization properties of the MODHIS optical path. Figure 1a shows that, at an altitude of 45deg, the s-vector of M3 is at 45 deg to that of NFIRAOS. Figure 1b shoes that the s-vector for MODHIS rotates relative to that of NFIRAOS. Figure 1c shows that the s or p vectors for all surfaces lie in a plane.

As before, this analysis will generally address the axial ray only, midband only. The axial ray approximation is valid because, generally, the axial angles of incidence in this system are much larger than chief or marginal ray slopes. The midband approximation is less valid numerically, but it gives useful insight and scale for the tasks at hand.

Before calculating the Mueller matrix for each surface in the optical system, the surface's diattenuation and retardance must be calculated from the angle of incidence and the coating definitions for each surface. Coating definitions have some uncertainty, which are described in a previous report. The same assumptions are maintained here: mirrors are protected silver as described by Anche, with published optical constants for the coating materials. Dichroic filters are assumed to perform as the coating COLD_MIRROR_45DEG_B in Zemax. As the design for TMT, NFIRAOS, and MODHIS advance, the design team should update these assumptions with new understanding. Appendix 1 summarizes the angles of incidence, coating definitions, retardance, and diattenuation, for each surface. The resultant Mueller matrices are:

$$M_{TMT} = \begin{bmatrix} 0.998 & 0 & 0.002 & 0 \\ 0 & 0.954 & 0 & -0.295 \\ 0.002 & 0 & 0.998 & 0 \\ 0 & 0.295 & 0 & 0.954 \end{bmatrix} \quad (1)$$

$$M_{NFIRAOS} = \begin{bmatrix} 0.98 & 0.02 & 0 & 0 \\ 0.02 & 0.98 & 0 & 0 \\ 0 & 0 & 0.98 & -0.03 \\ 0 & 0 & 0.03 & 0.98 \end{bmatrix} \quad (2)$$

$$M_{MODHIS} = \begin{bmatrix} 0.999 & 0.001 & 0 & 0 \\ 0.001 & 0.999 & 0 & 0 \\ 0 & 0 & 0.9816 & -0.1857 \\ 0 & 0 & 0.0857 & 0.9816 \end{bmatrix} \quad (3)$$

Analysis later in this note will show that the most important elements of these matrices are nonzero elements in the top row. By far, the largest top-row value, in the NFIRAOS matrix, is 0.2; this value comes from the diattenuation in the dichroic science beamsplitter.

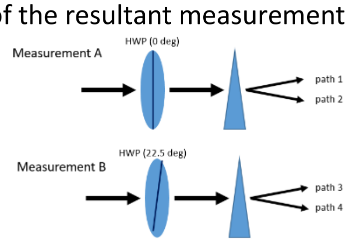
Two details should be noted. First, the ADC in MODHIS is neglected in this analysis. This omission is reasonable because 1) the angles of incidence for the axial ray are small (<0.1 rad for all air-glass interfaces) and because 2) the AR coatings in transmission rarely have meaningful amounts of polarization errors. Next, the Mueller matrix for TMT is for an altitude of 45 deg - the worst case, polarimetrically. This simplification is reasonable because an altitude of 45 deg is a reasonable use case.

The Mueller matrix for the optical system can be readily calculated, including the rotation between the different pieces. The rotation for the telescope relative to NFIRAOS can be quite large, to accommodate the full range of altitude angles accessible by TMT. The rotation for MODHIS relative to NFIRAOS is assumed to be small for each exposure.

Background: generic polarimetric data reduction

Polarimetric analysis is necessary to determine how these instrumental polarization effects propagate to the measured Stokes values. An earlier report justified the choice of a polarimeter: a rotating retarder followed by a polarizing beamsplitter. The report also noted a good location for the polarizing beamsplitter: in the MODHIS FEI after the last off-axis parabola. Two design decisions are left open: the location of the rotating retarder and the angles at which the retarder is set. The analysis in this section assumes that the retarder is placed immediately before NFIRAOS, noting that building such a large achromatic retarder is a daunting task. The analysis in this section also assumes using retarder angles of 0, 22.5 deg, the minimum measurement set for determining the linear Stokes parameters, I, *q*, & *u*. The well-established "double-differencing technique will be addressed later.

A schematic of the resultant measurement set is shown in Figure 2.



<u>Figure 2</u>: Minimum measurement set to measure the linear polarization using the proposed polarimeter.

Some simple Mueller matrix algebra can illuminate the function of this polarimeter. Each path has its own Mueller matrix. Let's say that in each measurement, the odd paths are for a horizontal (90 deg) polarizer and the even paths are for a vertical (0deg) polarizer, with Mueller matrices:

$$M_{path\,x} = M_{pol(\varphi)} * M_{HWP(\theta)} \tag{4}$$

Assuming no instrumental polarization and perfect polarization elements, the Mueller matrices are pretty simple:

$$M_{path\,1} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad M_{path\,2} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_{path\,3} = \frac{1}{2}\begin{bmatrix} 1 & 0 & -1 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad M_{path\,4} = \frac{1}{2}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

Assuming an input polarization state that's linear, of unit intensity, of unknown orientation and degree of polarization, the input Stokes vector is S_{in} = [1 q u 0]. The output Stokes vector for each path:

$$S_1 = \frac{1-q}{2}\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \qquad S_2 = \frac{1+q}{2}\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad S_3 = \frac{1-u}{2}\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \qquad S_4 = \frac{1+u}{2}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \tag{6}$$

The measured power is the first element of the Stokes vector:

$$P_1 = \frac{1-q}{2} \qquad P_2 = \frac{1+q}{2} \qquad P_3 = \frac{1-u}{2} \qquad P_4 = \frac{1+u}{2} \tag{7}$$

For this simple case, it's not too hard to see the solution to find the incoming polarization state, fully defined by q and u.

For more-realistic cases, it's helpful to take another step in the mathematics of the model. Note that, for each path, only power is measured, which is represented by the first element in the Stokes vector. Therefore, only the first row of each path's Mueller matrix is of interest. These rows can be collected into a matrix called a "polarization measurement matrix." This matrix is usually represented by W, and is 4xn, where n is the number of measurements. For our single-difference 4-path polarimeter, the polarization measurement matrix is:

$$W = \frac{1}{2}\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \tag{8}$$

A polarization measurement matrix converts a 1x4 incoming Stokes vector, S_{in}, into a 1xn vector, P, which consists of the power measurements in the polarimeter,

$$P = W * S_{in} \tag{9}$$

This relationship is really backwards from the measurement process, in which n power measurements are made to determine a Stokes vector. The pseudoinverse of the polarization measurement matrix converts the 1xn power measurements into a 1x4 Stokes vector:

$$W^{-1} * P = W^{-1} * W * S_{in} = S_{in} \tag{10}$$

W^{-1} is called the "polarimetric data reduction matrix." This pseudoinverse (also called the Moore-Penrose inverse) is a handy mathematical tool because it calculates the least squares best fit for overdetermined systems, such as when more than four measurements are used to measure the four components of the Stokes vector. For our example of the 4-path single-difference polarimeter, the polarimetric data reduction matrix is:

$$W^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ -2 & 2 & 0 & 0 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad 11$$

For this simple case of no instrumental polarization effects, this method correctly gives the four measurements that define the (assumed) incoming linear polarization state:

$$S_{in} = \begin{bmatrix} (P_1 + P_2 + P_3 + P_4)/2 \\ P_2 - P_1 \\ P_4 - P_3 \\ - \end{bmatrix} \quad 12$$

Note that the last element of the Stokes vector, v, isn't considered here; it's not sampled in this polarimeter design.

Astronomy typically uses a more-complex measurement method, called double-differencing, as shown schematically in Figure 3

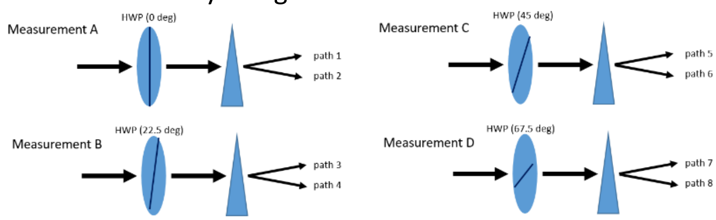


Figure 3: Schematic of a double-differencing polarimetric measurement.

Following the same methods as shown in equations 8-10 yields the expected results for the nominal case of perfect polarization elements and no instrumental polarization:

$$S_{in} = \begin{bmatrix} \sum P_i/4 \\ (P_2 - P_1 + P_5 - P_6)/2 \\ (P_4 - P_3 + P_7 - P_8)/2 \\ - \end{bmatrix} \quad 13$$

Applying polarimetric data reduction to MODHIS

This method can be further extended to include the effects of instrumental polarization and foreoptics, as shown schematically in Figure 4:

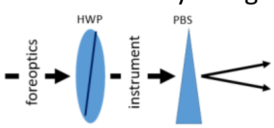


Figure 4: Minimum measurement set to measure the linear polarization using the proposed polarimeter.

Two methods will be used to calculate the effects of foreoptics and instrumental polarization. Numerical solutions will be calculated from the nominal MODHIS Mueller matrices shown in equations 1-3. Parametric estimates will be estimated for the limit of weak polarizing elements, showing symbolically how the errors depend on various polarization errors in the foreoptics and instrument.

For the symbolic estimates, both the foreoptics and the instrument are characterized by their depolarization, retardance, and diattenuation:

$$M = R(\theta) M_{dep} M_{ret} M_{diat} R(-\theta) \quad 14$$

where simple model is used for depolarization:

$$M_{dep} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-D & 0 & 0 \\ 0 & 0 & 1-D & 0 \\ 0 & 0 & 0 & 1-D \end{bmatrix} \tag{15}$$

A small-diattenuation approximation is used for diattenuation:

$$M_{diat} = \begin{bmatrix} 1-d & d & 0 & 0 \\ d & 1-d & 0 & 0 \\ 0 & 0 & 1-d & 0 \\ 0 & 0 & 0 & 1-d \end{bmatrix} \tag{16}$$

Retardance is assumed to be linear and aligned with the diattenuation:

$$M_{ret} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(\delta) & \sin(\delta) \\ 0 & 0 & -\sin(\delta) & \cos(\delta) \end{bmatrix} \tag{17}$$

The orientation of any of these optics is represented by:

$$R(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\theta) & -\sin(2\theta) & 0 \\ 0 & \sin(2\theta) & \cos(2\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

Note that this model assumes that all instrumental polarization is oriented in the same direction, and is all linear. The assumption of orientation is valid because this orientation is well controlled in design and fabrication. Finally, all cases will calculate an error Stokes vector, S_{err},

$$S_{meas} = S_0 + S_{err} \tag{19}$$

where S_{meas} is the measured Stokes vector, S_0 is the Stokes vector calculated by the nominal data reduction described in Equation 13 and S_{err} is the measured error, which must be addressed through calibration.

These calculations are prohibitively complex if attempted by hand. I use Mathematica for the calculations, but I'm sure other solutions exist.

The following results share some characteristics. Retardance of the foreoptics and instrument generally drop out. Errors are generally linear in diattenuation and depolarization; they do not drop out.

In the first special case, numerically, there is no TMT, and the rotating retarder is placed immediately in front of NFIRAOS. The resultant error is:

$$S_{err} = \begin{bmatrix} 0.01(P_2 + P_4 + P_6 + P_7) \\ 0.02\,(P_2 - P_6) \\ 0.02\,(P_4 - P_8) \\ - \end{bmatrix} \tag{20}$$

Symbolically, there is no foreoptics, and the polarimeter is aligned with instrumental polarization. This case is the motivating case for the double-difference method. For this case, the measured error is:

$$S_{err} = \begin{bmatrix} \frac{d}{2}(P_2 + P_3 + P_6 + P_7) \\ 2d\,(P_2 - P_6) - D(P_1 - P_2 + P_6 - P_5) \\ 2d\,(P_4 - P_8) - D(P_3 - P_4 + P_8 - P_7) \\ - \end{bmatrix} \tag{21}$$

In the limit of a weakly-polarized source, power differences are small, so the errors in *u* & *v* are also small. However, the normalization term is not small; it's proportional to both diattenuation and the sum of power measurements. Note that the result is independent of retardance in the instrumental optics, when retardance is small (δ<< 1 rad). Depolarization is likely to arise from nonuniformities in a large retarder placed in front of NFIRAOS; some depolarization is also introduced by polarization defocus, an effect not yet discussed in this work. Part of the attractiveness of this double-difference technique is that, nominally, diattenuation is measured in each exposure set, by properly combining the P_1, P_2, P_5, and P_6.

This mathematical method can be used to inform the decision of the ideal placement of the retarder - before NFIRAOS or immediately before the polarizing beamsplitter. Placing the retarder immediately before polarizing beamsplitter yields an error term of:

$$S_{err} \cong \begin{bmatrix} 0.01(P_1 + P_3 + P_6 + P_8) \\ -0.02(P_2 + P_5) + 0.03(P_3 + P_8) - 0.04(P_4 + P_7) \\ 0 \\ - \end{bmatrix} \qquad 22$$

In the limit of weakly polarized light all power measurements are about the same, and the S_{err} =~ [0.02, -.02, -0.001, -]. This seems reasonable value to calibrate through to achieve the required 0.001 precision. Placing the retarder immediately before NFIRAOS yields an error term of:

$$S_{err} \cong \begin{bmatrix} 0.01(P_2 + P_3 + P_6 - P_7) \\ 0.02(P_1 - P_5) \\ 0.02\ (P_4 - P_8) \\ - \end{bmatrix} \qquad 23$$

In the limit of weakly polarized light all power measurements are about the same, and the S_{err} =~ [0.01, 0, -0.002, -]. This also seems to be reasonable value to calibrate through to achieve the required 0.001 precision.

Using the same method symbolically shows that the leading term in the error components can is the diattenuation of the dichroic science beamsplitter in NFIRAOS. This diattenuation can be readily compensated by adding a matching dichroic coating at the proper orientation in NFIRAOS. This additional coating should be deposited with the NFIRAOS dichroic; as such, the coating should have zero marginal cost. Such a simple improvement should be implemented.

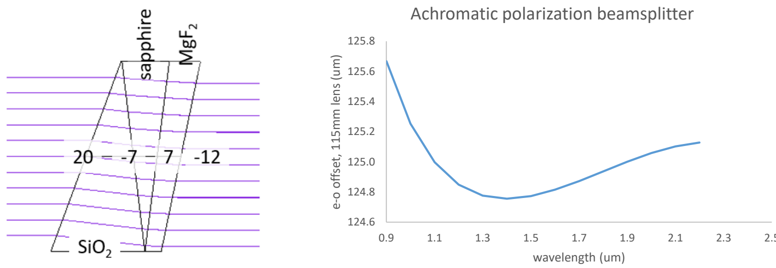
<u>Polarimeter design and compensation:</u>

In all cases, the dominant term is the diattenuation of NFIRAOS, which is dominated by the diattenuation of the dichroic science beamsplitter. As noted in an earlier report, this diattenuation is readily compensated using the same coatings at the same angles, but in the opposite orientation. Such a compensator should have zero marginal cost for the optics; it would be coated as a witness piece as the NFIRAOS beamsplitter is coated. Such a compensation approach has the added benefit of exact cancellation of the NFIRAOS optic, including spectral dependence.

As described in a previous report, the nominal polarimeter uses a birefringent wedge to split light into two orthogonal polarization states. The angle of the split must generate useful spot spacing when focused by MODHIS onto the fibers. A previous report further showed that achromatization would be necessary and that a small deflection for the nominal MODHIS path is desirable.

Figure 4 shows a conceptual design for the MODHIS polarizing beamsplitter. The beams are split symmetrically about the optical axis, so the design make no demands on the design of the focusing lenses. The two birefringent materials generate an achromatic image separation across the MODHIS spectral band. The design is also attractive because the angle of incidence on the front surface nearly matches that of the NFIRAOS science dichroic beamsplitter; this feature entices the possibility of using a dichroic coating on this surface to compensate for the NFIRAOS diattenuation. Figure 4b shows the dispersion of the offset between the two polarizations, assuming a paraxial lens of the same focal length as the MODHIS focusing lens; the range of +-0.4um is likely to achieve reasonably achromatic coupling.

The design should be refined to include dispersion of the focusing lenses. Fine-tuning of the fiber position and offset can likely be accomplished via tilting the assembly.



<u>Figure 4</u>: The nominal polarizing beamsplitter. Figure 4a is a schematic, showing materials and angles. Figure 4b shows that the assembly is achromatic over the MODHIS waveband.

<u>Interesting topics not covered to-date:</u>

- Calibration
- Additional perturbations, including post-split diattenuation and retarder imperfections
- TMT depolarization
- Circular components of input polarization

Appendix 1: Elements for calculating the Mueller matrices for each component

Elements of TMT

	axial AOI (rad)	coating	diattenuation	retardance (rad)	orientation (deg)
M1	0	-	0	0	-
M2	0	-	0	0	-
M3	0.785	FSS99	0.0015	0.300	=alt

when alt = 45deg (worst case), the Mueller matrix for TMT is

$$M_{TMT} = \begin{bmatrix} 0.998 & 0 & 0.002 & 0 \\ 0 & 0.954 & 0 & -0.295 \\ 0.002 & 0 & 0.998 & 0 \\ 0 & 0.295 & 0 & 0.954 \end{bmatrix} \quad 24$$

Elements of NFIRAOS

	AOI (rad)	coating	diattenuation	retardance	orientation (deg)
window 1	0	-	0	0	-
window 2	0	-	0	0	-
OAP1	0.140	FSS99	0.00005	0.008	0
DM1	0.175	FSS99	0.00007	0.012	0
OAP2	0.144	FSS99	0.00005	0.008	0
OAP3	0.144	FSS99	0.00005	0.008	0
DM0	0.240	FSS99	0.0001	0.025	0
dichroic	0.436	ZMX cold 45	-0.02	-0.200	90
OAP4	0.140	FSS99	0.00005	0.008	0
instr sel.	0.785	FSS99	0.0015	0.300	90

The Mueller matrix for NFIRAOS is:

$$M_{NFIRAOS} = \begin{bmatrix} 0.98 & 0.02 & 0 & 0 \\ 0.02 & 0.98 & 0 & 0 \\ 0 & 0 & 0.98 & -0.03 \\ 0 & 0 & 0.03 & 0.98 \end{bmatrix} \quad 25$$

Elements of MODHIS before the polarizing beamsplitter:

	AOI (rad)	coating	diattenuation	retardance	orientation (deg)
Fold 1	0.785	FSS99	0.0015	0.300	90+-δ
OAP 1	0.112	FSS99	0.00003	0.006	0
Fold 2	0.384	FSS99	0.000352	0.065	0
ADC-A	0	-	~0	~0	-
ADC-B	0.23	-	~0	~0	-
ADC-C	0.05	-	~0	~0	-
ADC-C'	0.07	-	~0	~0	-
ADC-C	0.07	-	~0	~0	-
ADC-B	0.05	-	~0	~0	-
ADC-A	0.23	-	~0	~0	-
ADC-A'	0	-	~0	~0	-

OAP2	0.112	FSS99	0.00003	0.006	0
OAP3	0.112	FSS99	0.00003	0.006	0
TTM	0.262	FSS99	0.00016	0.030	0
~~dichroic~~	~~0.785~~	~~ZMX cold 45~~	~~-0.05~~	~~-0.419~~	~~0~~

Note that the dichroic is not included here. Earlier reports suggested placing the birefringent wedge adter the first dichroic in MODHIS, but later discussions and the results of this reprot suggest that it's better to place it before this element.

The Mueller matrix for MODHIS is:

$$M_{MODHIS} = \begin{bmatrix} 0.999 & 0.001 & 0 & 0 \\ 0.001 & 0.999 & 0 & 0 \\ 0 & 0 & 0.9816 & -0.1857 \\ 0 & 0 & 0.0857 & 0.9816 \end{bmatrix} \tag{26}$$