Progress report: MODHIS polarimetry study
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June 25, 2024, rev 2

Change history:
Revision 1 of this report was discussed on June 28, revealing an important error in the orientation of NFIRAOS and some misconception about M3 motion. This report corrects these miscomceptions and the corresponding conclusions.

Executive summary:

This progress report describes an early assessment of the Mueller matrix for TMT and NFIRAOS. As-is, off-diagonal elements of the foreoptics' Mueller matrix exceed 0.7, values unlikely to be compatible with the desired polarimetric accuracy of 0.001. This report shows how some simple-ish compensation strategies can reduce these off-diagonal values to levels that seem consistent with the science requirements. At this point, three compensation strategies seem prudent and reasonable: 1) compensate for the NFIRAOS science beamsplitter with a second dichroic, of he same design and angle of incidence but opposite orientation, 2) compensate for the last fold mirror in NFIRAOS by making the initial fold mirror of MODHIS of the same design and angle of incidence but in the perpendicular plane, and 3) compensate for M3 by adding a liquid crystal tunable retarder.

Some requirements flowdown:

The CoDR-2 review[1] for MODHIS assessed that the polarization science case is compelling, and that further study was warranted. Delivering this polarization science requires incorporating a polarimeter into the optical train for MODHIS. A polarimeter measures light's polarization state, often described phenomenologically by a Stokes vector, *S*.

$$S = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} I_H + I_V \\ I_H - I_V \\ I_{45} - I_{135} \\ I_R - I_L \end{bmatrix}, \qquad 1$$

where *I* represents total intensity, and I_H, I_V, I_{45}, I_{135}, I_R, and I_L represent the intensity passed through ideal horizontal, vertical, +45, -45, right, and left polarizers, respectively. The CoDR2 presentation summarizes science cases, which generally have a "polarization signal" of a few percent; this conclusion is consistent with *Q*, *U*, & *V* << 1 for a Stokes vector normalized to unit intensity.

Measurement of the object's polarization properties is complicated by the polarization properties of the optics upstream of the polarimeter, called "foreoptics." This complication often described using a Mueller matrix, $M_{foreoptics}$,

$$S_{measured} = M_{foreoptics} * S_{object} \qquad 2$$

The ideal Mueller matrix for the foreoptics is the identity matrix.

The optical designer's task, then, is to make the foreoptics have very weak polarization effects. Mathematically, the task is to reduce the off-diagonal elements of the Mueller matrix to an acceptably low level. This task, and the interpretation of the results, is aided by a result from Chipman[2]: a weak polarization element can be approximated by:

$$M_{foreoptics} = \begin{bmatrix} 1 & d_H & d_{45} & d_R \\ d_H & 1 & \delta_R & -\delta_{45} \\ d_{45} & -\delta_R & 1 & \delta_H \\ d_R & \delta_{45} & \delta_H & 1 \end{bmatrix}, \qquad 3$$

where the *d* values are diattenuation, the δ values are retardance in radians, and the *H*, *45*, and *R* subscripts refer to horizontal linear polarization, 45deg linear polarization, and right circular polarization, respectively. The approximation is for diattenuation and retardance both much less than 1. The measured Stokes vector is then:

$$S_{measured} = S_{object} + S_{error} \cong S_{object} + \begin{bmatrix} Q\ d_H + U\ d_{45} \\ d_H - V\ \delta_{45} \\ d_{45} + V\ \delta_H \\ Q\ \delta_{45} + U\ \delta_H \end{bmatrix} = M_{foreoptics} * S_{object}, \qquad 4$$

where circular diattenuation and circular retardance are set to zero. This simplification is justified because all the optical surfaces in the foreoptics have linear eigenstates and because they are have weak polarization effects. The top element in the S_{error} shows that linear diattenuation in the optical path causes errors in the radiometry of the measurement; a well-known effect that is probably already in some TMT error budgets. The other terms in S_{error} show that linear diattenuation causes measured polarization in proportion 1) to the foreoptics' diattenuation and 2) to the product of the incident light's degree of polarization and the foreoptics' retardance.

The science requirements provide guidance for the allowable magnitudes for the diattenuation and retardance of the foreoptics. REQ-2-MODHIS-0450 calls for a goal of 0.1% polarimetric accuracy. This requirement seems consistent with requiring that all elements of the measured Stokes vector be within about 0.001 of the true Stokes vector. Simplistically assuming sources with Q, U, and V about 0.1, this requirement translates to a diattenuation $d < 0.001$ and a retardance $\delta < 0.01$ - a requirement we're unlikely to meet. More realistically, polarimeter calibration can ease these requirements somewhat. For now, let's leniently set benchmark goals at 10X the simpleminded values, yielding a benchmark of diattenuation $d < 0.01$ and retardance $\delta < 0.1$rad. Naturally, lower values are better, but these values are helpful for now.

Note that these error terms are a function of wavelength, telescope pointing, field coordinate, and pupil coordinate. For simplicity, and to gain insight, these complications will be neglected for now. This simplification is also justified by aberration theory, which can be considered a Taylor series expansion of errors about the axial ray; the first term in any such expansion is the DC term - the axial ray. Analysis in this report will be primarily for the midband (1.6um), telescope pointing at zenith, and the axial ray path.

Foreoptics' optical design

For many reasons, the polarimeter is unlikely to be placed in front of NFIRAOS. Therefore, the telescope and NFIRAOS will be part of the foreoptics for MODHIS. Calculation of the foreoptics' diattenuation and retardance starts with the optical design for these subsystems. For the optical design of these subsystems, I assumed the Zemax file 20220725A_MODHIS_FEI.zmx, which includes corrections identified in discussions regarding rev 1 of this report. Figure 1 shows this optical design at zenith, with the axial image point traced.

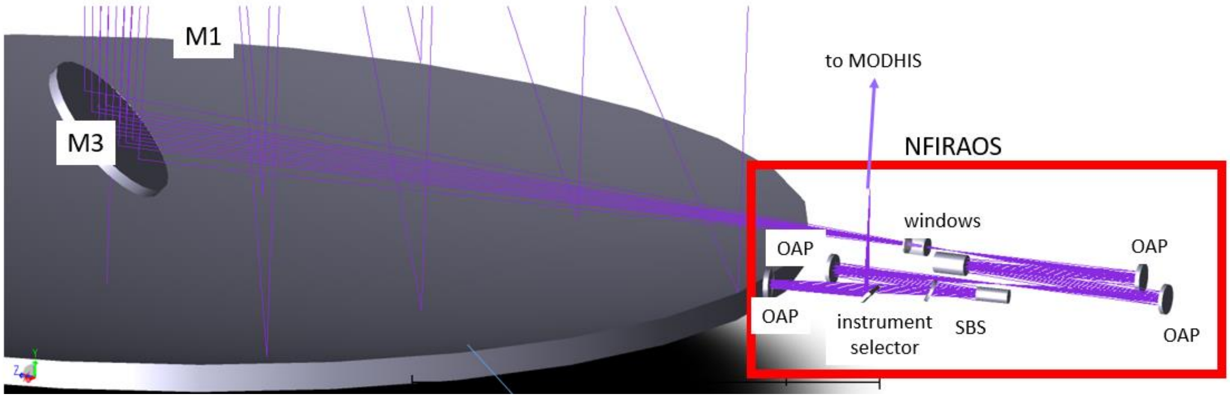


Figure 1: Raytrace of TMT with NFIRAOS, showing rays for an axial image point, with the telescope pointed at zenith. The telescope's primary (M1) and tertiary (M3) mirrors are shown. Within NFIRAOS, the off-axis parabolas (OAPs), deformable mirrors (DMs), instrument selector mirror (IS) and Science Beamsplitter (SBS) are shown. Note that the axial ray is in the same plane for all reflections; this symmetry causes all mirrors' polarization effects to add with one another.

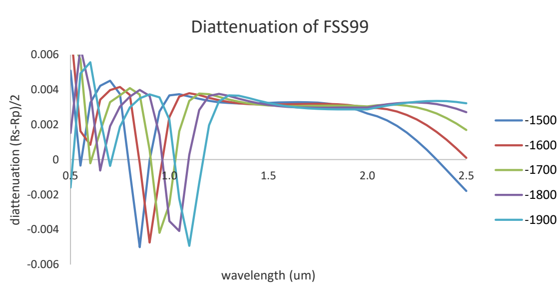
The optical design of MODHIS isn't included at this point because it's assumed to have some flexibility to help with polarization considerations. Furthermore, the location of the polarimeter within MODHIS has not yet been fixed.

Optical elements with near-zero angles of incidence for the axial ray are also omitted from this report because the isotropic materials used have no polarization effect at normal incidence.

Coating design: mirrors

The next step in polarization analysis is to generate coating designs. Unfortunately, these coating designs, which consist of a listing of coating materials and their thicknesses, generally not shared by coating vendors. This uncertainty needn't prevent CoDR-level analysis.

TMT seems to have settled on protected silver coating for the mirrors in the telescope and NFIRAOS. Specifically, Materion's Front Surface Silver coatings (FSS99) seem to the nominal choice. Several versions of this coating are available. Materion sent reflection coefficient spectra at 45deg for several of these variants. Figure 2 shows plots of the diattenuation spectra. In every case, the diattenuation is well under our 0.01 benchmark, so differences between variants will be ignored for now.



<u>Figure 2:</u> Diattenuation spectra of several variants of Materion's FSS99 optical coatings. Note that the magnitude of all values is well under our 0.01 target. Also notice that there is little variation in diattenuation for the various variants.

Materion didn't share the nominal retardance values for their coatings. These values can be readily calculated from the coating's optical design, but Materion didn't share that, either. Luckily, others' work allows us to make some good assumptions. This work will follow the coating design enumerated by Anche[3] for TMT, as shown in Figure 3 This assumption nicely matches other sources' description of a protected silver coating. Optical constants for the layers's materials are readily available, allowing calculation of diattenuation and retardance as a function of angle and wavelength. These published optical constants have some variation, due to contaminants and process variation. Such complications will be ignored for now, but should be considered in later design phases.

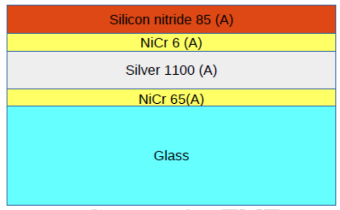


Figure 3: Nominal design for protected silver mirrors in TMT and NFIRAOS, from Anche.

The diattenuation and retardance calculations of the coating from Anche agree fairly well with the values shared by Materion, as shown in Table 1. Although some difference is evident, the agreement is good enough to drive design decisions at the CoDR level.

values (1.6um, 45 deg)	Materion	model
R_s	0.995	0.997
R_p	0.989	0.994
diattenuation	0.003	0.002
retardance (rad)	not shared	0.30

Table 1: Diattenuation and retardance of enhanced silver, at 45deg and 1.6um, showing both the data shared by Materion and the values calculated from Anche's strawman model.

Many considerations will be important as the design progresses to later design phases. Protected silver is known to change reflectivity during long environmental exposure; it's polarization properties are sure to change, too. On exposure to the air on-summit, M3 will be exposed to dust; such contamination is known to cause polarization effects, especially depolarization. Furthermore, design of the polarimeter must stay abreast of design fluidity in the telescope and NFIRAOS; design changes in the mirror coatings should be considered in polarization mitigation features of MODHIS polarimetry.

Coating design: dichroics

The coating design for the science beamsplitter in NFIRAOS is even more mysterious. Dichroic filters like this one often contain hundreds of layers, with their optical properties and tolerances tightly linked to the vendor's manufacturing capabilities. Materion has kindly shown some design and test data, collected in an Excel file: Materion NRCC NFIRAOS Dichroic BS Data Pack Rev1.xls This data included enough information to calculate the diattenuation spectrum at a 22deg angle of incidence, shown in Figure 4.

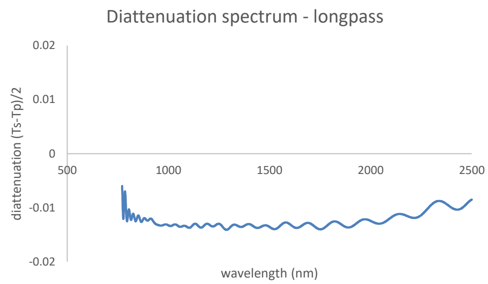


Figure 4: Diattenuation spectrum of the science beamsplitter, as shared by Materion. Diattenuation is near our benchmark of 0.01, and is spectrally flat.

Materion didn't share the optical design for their dichroic coating. However, this lack of information isn't too important now; the nominal design is likely to be something different for the as-delivered coating, thanks to changes in process capabilities. Even with a purchase, the vendor may not be willing to share nominal values for the data we need to do a proper polarization analysis. This lack of information is unlikely to be a problem for diattenuation; diattenuation naturally limited by the tight specs on throughput of the coating. Retardance of the science beamsplitter is unlikely to be well-controlled in the delivered part because throughput is of paramount consideration. We can probably specify a low-retardance dichroic coating with negligible effect on the coating's transmission properties; such a coating, in my experience, might reduce retardance ~3X the unconstrained values. To get an idea of what these unconstrained values might be, I looked at a generic longpass filter design that's included in Zemax, COLD_MIRROR_45DEG_B.

The performance of this generic longpass cold filter is shown in Figure 5. Its design is clearly inferior to Materion's: its average throughput is several percent lower and includes undesirable ripple.

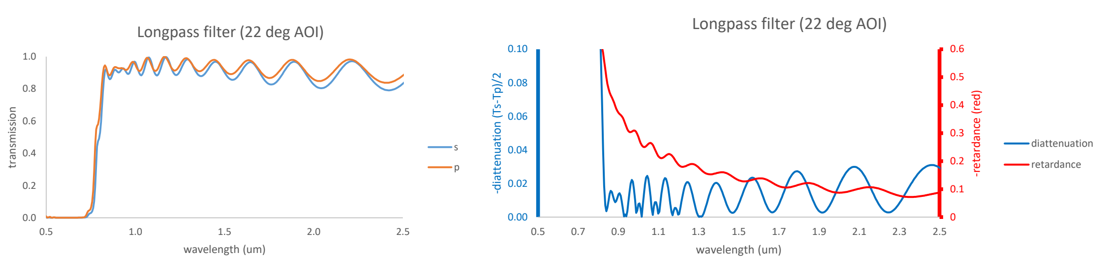


Figure 5: Polarization properties of a generic longpass filter at 22deg. Both diattenuation and retardance exceed our targets.

Calculation of the polarization properties for this generic longpass cold filter can inform our expectations for the polarization properties of the NFIRAOS science beamsplitter. In-band diattenuation is about 0.01 - similar to that of the NFIRAOS science beamsplitter. In-band retardance, which was certainly unconstrained during design optimization optimization, compares poorly to our benchmark of 0.1 - rising above 0.5 near the transition wavelength and never dipping much below 0.1. A 3X reduction in retardance may be achievable by adding it into the coating's merit function, but retardance at shorter wavelengths would still be well over 0.1 - clearly worrisome, although probably not catastrophic.

Calculating the Mueller matrix

With well-defined values for angle of incidence on each surface and reasonable models for coatings on each surface, values for polarization properties on each surface can be calculated. These values are summarized in Table 2. Diattenuation and retardance both exceeds our benchmark of 0.1.

Optical surface	coating	Angle of incidence (deg)	diattenuation	retardance (rad)	$\hat{s}$ orientation (deg)
M3	FSS99 protected silver	45	0.0015	0.3	0
NFIRAOS OAPs & DMs	FSS99 protected silver	8-12	0.0004	0.07	90
NFIRAOS science BS	COLD_MIRROR_45DEG_B	22	-0.02	-0.15	90
Instrument selector	FSS99 protected silver	45	0.0015	0.3	0
MODHIS optics	not considered yet		??	??	??

Table 2: Polarization properties of the polarimeter's foreoptics on-axis, at zenith, and a midband wavelength. $\hat{s}$ is the unit vector parallel to s-polarized light at each interface; note that $\hat{s}$ is parallel to the cross product between the incdident ray vector and the exiting ray vector. The values clearly exceed our lenient benchmark of diattenuation < 0.01 and retardance < 0.1rad.

Simple matrix multiplication yields the Mueller matrix for the foreoptics:

$$M_{foreoptics} = \begin{bmatrix} 1 & 0.02 & 0 & 0 \\ 0.02 & 1 & 0 & 0 \\ 0 & 0 & 0.69 & 0.70 \\ 0 & 0 & -0.70 & 0.69 \end{bmatrix} \quad 5$$

These values, simplemindedly considered as d = 0.02 and δ = 0.76, compare poorly with our lenient benchmark of d < 0.01 and δ < 0.1. Polarimetry through such foreoptics would be difficult. Measuring Stokes vectors the required accuracy of 0.001 seems unlikely. Therefore, mitigation and compensation options should be considered.

Note that this troublesome result is greatly simplified, in that it is for only the axial ray, only at a wavelength of 1.6um, and only with the telescope pointed at zenith, and with nominal coatings. The true Mueller matrix will be a function of field position, wavelength, pointing, and even time.

Such complexity doesn't change the fundamental result: the off-diagonal elements of the Mueller matrix are troublingly large. The approach used to achieve this result also provides insight for towards mitigating or compensating for these polarization errors.

<u>Mitigation strategies - very limited options</u>

Mitigation of the optical components' diattenuation is unlikely. Diattenuation is naturally driven to small values as throughput is optimized.

Mitigation of the mirror's retardance is unlikely. Such mitigation is generally accomplished via complex multilayer coatings - an approach that's usually inconsistent with TMT's wide spectral coverage.

Mitigation of the retardance of the science beamsplitter in NFIRAOS might offer some relief because retardance was unlikely to be part of the development of the nominal coating design. In my experience, adding retardance requirements to a coating design can reduce retardance by about 3X from its baseline performance when retardance is ignored; further reduction might be possible by releasing transmission requirements. Such reduction is likely to include a lot of spectral variation.

<u>Some compensation strategies - much more promising</u>

NFIRAOS' science beamsplitter's polarization effects are readily compensated by introducing an additional filter of the same design and same angle, but opposite orientation, as shown in Figure 6. Light that is s-polarized on the first, required, filter is p-polarized on the second, compensating filter. This arrangement perfectly cancels the polarization effects of the first filter, including variation with wavelength, marginal ray, chief ray, temperature, etc. Note that the compensating filter need not be placed near the required filter in the optical train; however, the Mueller matrix of intervening optics should be close to the identity matrix.

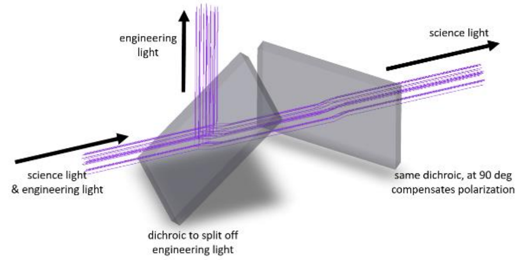


Figure 6: Compensating the polarization effects of a dichroic filter used in transmission. The filters have the same design and angle of incidence, but opposite rotation. Note that the compensating filter need not be paced immediately downstream of the required filter.

NFIRAOS' instrument selector's polarization properties can be compensated in a similar way. Figure 7 shows that, in the current concept for MODHIS, first fold mirror in MODHIS bends light in an plane orthogonal to the bend of the instrument selector. In this arrangement, light that is s-polarized on the NFIRAOS instrument selector mirror is p-polarized on the MODHIS fold mirror. If the MODHIS fold mirror is of the same design as the NFIRAOS instrument selector mirror, and if the two mirrors have the same angle of incidence, then all polarization effects of the NFIRAOS instrument selector mirror are compensated by the MODHIS fold mirror. This ideal situation is no longer met as MODHIS is rotated to maintain field orientation; more work is warranted to quantify the implications of this imperfection.

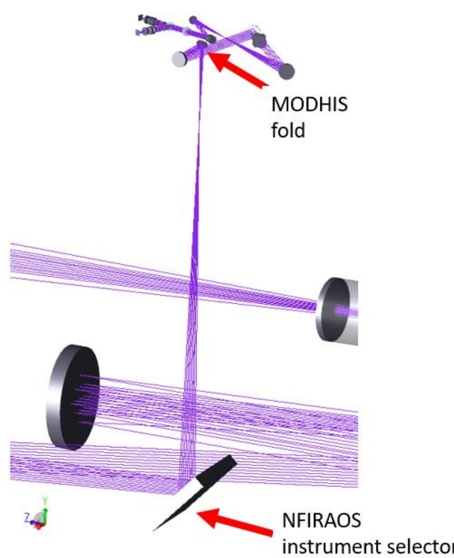


Figure 7: Front end instrument for MODHIS. If the fold mirror is rotated to be out of plane with the NFIRAOS instrument selector mirror, and if the two mirrors have the same coating and angle of incidence, then their polarization effects cancel one another.

M3's polarization properties are much more difficult to compensate. The 0.3 rad retardance shown in Table 2, for a wavelength of 1.6um and zenith pointing, could be readily compensated with a retarder of the same value but opposite orientation. Unfortunately, simple retarders are unlikely to match the dispersion of the mirror's retardance; this challenge can be alleviated with multi-material retarders, as shown in Figure 8.

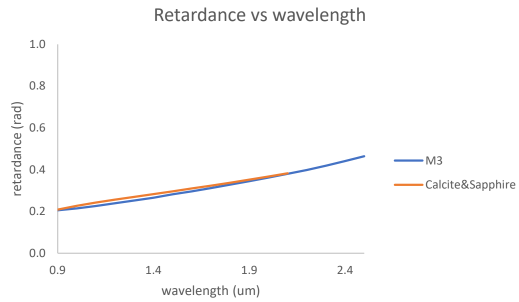


Figure 8: Retardance spectrum of M3, when TMT is pointed at zenith. A two-material retarder can cancel this retardance across the MODHIS waveband, with an RMS error of 0.01 rad.

A final complication to consider for now: the orientation of M3's retardance changes as the telescope pointing changes, as shown in Figure 9. Compensation of this changing orientation is probably best accomplished by rotating the compensating retarder about the optical axis. As with the dichroic compensator, the M3 compensator need not be placed near M3; however, the Mueller matrix of intervening optics should be close to the identity matrix.

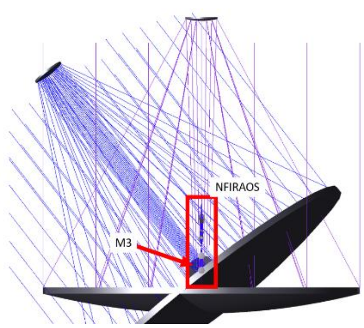


Figure 9: Rotation of the telescope out-of-plane with NFIRAOS. The orientation of M3's fast axis rotates with this out-of-plane telescope rotation. In the recommended compensation, the compensating retarder rotates to match the M3 fast axis.

Omitted considerations for now

Taken together, these compensation strategies are likely to reduce the foreoptics' diattenuation and retardance to a value that's within our benchmark values. As these polarization properties are reduced, other effects may become important contributors. These effects include:

- Stress birefringence in vacuum windows
- Residual birefringence in glass
- Dust & aging of mirrors
- Marginal ray effects, chief ray effects, Mueller Matrix PSF

Proposed next steps

- Where to put the polarimeter?
- What polarimeter designs are most appropriate?
- Where to put the compensators?
- How to calibrate the polarimeter?

References

1. TMT_MODHIS_CoDR-2_Review_Presentation.
2. Chipman, "Polarized Light and Optical Systems," Taylor & Francis, 2018
3. Anche et. al., "Analytical Modelling of Thirty Meter Telescope Optics Polarization," Proc. of SPIE Vol. 9654, 2015