Progress report - August 30, 2024
rev 0: Please expect updates soon, regarding mechanical engineering and program management
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**Executive summary**

Results are presented for previously-unexplored polarimetric errors: depolarization from TMT and imperfections in the polarimetric components. A blind calibration method is presented and used to show superiority of highly-polarized calibration states, relative to states that are similar to the anticipated values for astronomical sources. An modification to the NFIRAOS calibration module is proposed to enable MODHIS polarimetric calibration. Volume requirements for MOSHIS polarimetry are evaluated, and seem modest. A preliminary project plan is presented, estimating a marginal cost of ??? to add polarimetry to MODHIS.

**Reminder of the approach to-date**

Light's polarization state is often described phenomenologically by a 1x4 Stokes vector, *S*.

$$S = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} I_H + I_V \\ I_H - I_V \\ I_{45} - I_{135} \\ I_R - I_L \end{bmatrix}, \qquad 1$$

where $I$ represents total intensity, and $I_H$, $I_V$, $I_{45}$, $I_{135}$, $I_R$, and $I_L$ represent the intensity passed through ideal horizontal, vertical, +45, -45, right, and left polarizers, respectively. A 4x4 matrix, called a Mueller Matrix and represented by $M$, describes the effect of an optical path on light's polarization state,

$$S_{out} = M * S_{in} \qquad 2$$

A polarimeter passes incoming light through $m$ polarizing paths, making a series of $m$ optical power measurements. (In the commonly-used double-differencing method, light is split into 2 orthogonal states for four different retarder positions, so $m = 8$.) These power measurements can be helpfully collected into a 1x$m$ vector, $P$. Each of the $m$ measurements can be represented by a Mueller matrix, but only the first row of these measurements contributes to the calculation of the power measurement. These $m$ first rows of the paths' Mueller matrices can be helpfully collected into a 4x$m$ matrix $W$, called the "polarization measurement matrix," which maps the 1x4 Stokes vector of the incoming light onto a set of $m$ power measurements:

$$P = W * S_{in} \qquad 3$$

A more-helpful construction for polarimetry is the inverse of this relationship, mapping the m power measurements onto the incoming Stokes vector:

$$W^{-1} * P = W^{-1} * W * S_{in} = S_{in} \qquad 4$$

where the $m$x4 matrix $W^{-1}$ is called the "polarimetric data reduction matrix."

**Depolarization:**

MODHIS work to-date has not discussed depolarization

A known source for depolarization is imaging through curved surfaces[1]. Because the angle of incidence and plane of incidence vary across the pupil, every pupil point has a difference Mueller matrix. An unresolved PSF averages over the pupil, so the retardance and diattenuation of a surface cause depolarization. For an axial point on TMT, the pupil-averaged Mueller matrix is:

$$M_{TMT,marg} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 - r^2/12 & 0 & 0 \\ 0 & 0 & 1 - r^2/12 & 0 \\ 0 & 0 & 0 & 1 - r^2/12 \end{bmatrix} \quad 5$$

where *r* is the retardance of the marginal ray. (Diattenuation of the mirrors is neglected here because it's 10X smaller than retardance for the FSS99 mirrors anticipated on TMT.) The angle of incidence of the marginal ray is 14 & 15 deg on the primary & secondary, respectively. The total retardance of the FSS99 coating is then r = 0.06rad, and depolarization is 0.0003 - about 1/3 the desired measurement precision. Effects of such a size merit tracking.

Other sources of depolarization include pupil-averaging of polarization variation and surface defects. I've found no basis for quantifying these effects in design, so I'll ignore them for now.

**<u>Summary of the results to-date:</u>**

This approach described in equations 1-4 led to an understanding of the scale of effects to be calibrated. To get an idea of the scale of errors to be calibrated, this approach was used to calculate the error in measured Stokes parameters, assuming the data reduction of a system without errors.

Errors are proportional to the uncorrected diattenuation & depolarization. Similar analysis further shows that errors are also proportional to the square of uncorrected retardance. The magnitude of these errors is shown in Table 1.

| | diattenuation | retardance (rad) | depolarization |
|---|---|---|---|
| TMT | 0.002 | 0.3 | 0.0003 |
| NFIRAOS | 0.02 | 0.03 | - |
| MODHIS | 0.001 | 0.09 | - |

*<u>Table 1</u>: Instrumental polarization in MODHOIS*

TMT errors are dominated by M3, and can be compensated with a custom retarder oriented parallel to M3. Errors in NFIRAOS are dominated by the dichroic, and should be compensated by a crossed dichroic of the same design and angle of incidence. Errors in MODHIS are dominated by the fold mirror. M3 errors can be compensated by an external retarder, if desired. Errors from the NFIRAOS dichroic should be compensated by an identical dichroic, nominally located within MODHIS, rotated 90 about the optical axis relative to the MODHIS beamsplitter. Errors from MODHIS are dominated by the fold mirror and are perfectly compensated by the NFIRAOS instrument selector mirror at the proper MODHIS rotation.

In general, the effects of the three subsystems do not add, because they are meaningfully rotated relative to one another. TMT rotates by large angles relative to NFIRAOS. MODHIS rotates relative to NFIRAOS, too; but the magnitude of the rotation is determined by the need to match pupil alignment - an open question.

A similar approach shows that errors caused by the polarimeter are linear with retardance errors and with the transmission of the nominally-blocked polarization state. Retardance errors for a superachromatic retarder are about 3deg = 0.05. For the anticipated Wollaston-type polarizer in collimated space, anticipated transmission of the nominally-blocked state is anticipated to be under 0.001. The retardance errors are much larger than the 0.001 measurement accuracy for MODHIS, so they should be considered in analysis.

In total, errors from each subsystem greatly exceed the 0.001 targeted accuracy for polarimetry. Therefore compensation is suggested and calibration is required.

**Parametric calibration vs. blind calibration**

Two broad methods exist for calibrating polarimeters. In a parametric calibration, models are assumed for all optical elements, modeled by Mueller matrices; the calibration task is then to find the values for these modeled parameters. Blind calibration uses the linear algebra to solve for the polarimetric data reduction matrix, $W^{-1}$, while making no assumptions about the form of the polarization errors. Note that these two approaches aren't mutually exclusive; a successful polarimetry scheme is likely to combine the two approaches.

Parametric calibration[3] assumes a parameterization for each optical element. For example, for a retarder, one will assume it is fully characterized by a retardance and an orientation. A regression analysis is then performed on a set of calibration data. This approach has the drawback that it neglects many non-nominal characteristics of the optical elements. Such non-nominal characteristics can become particularly important in high-quality optics. For example, a superachromatic retarder can have retardance within a few degrees of nominal across a multidecade waveband; I suspect, but haven't yet shown, that this retardance also has nonlinear eigenstates of a scale similar to its retardance error. Parametric calibration generally omits such subtleties.

Blind calibration[2] gracefully considers these subtleties; however, it seems to be less widespread, so it merits more explanation. Blind calibration uses the properties of the polarimetric data reduction matrix ($W^{-1}$). It also uses a set of $L$ polarization states, each represented by 1x4 Stokes vector. These $L$ Stokes vectors are collected into a $Lx4$ matrix represented by $SS$.

$$SS = \begin{bmatrix} I_1 & I_2 & I_3 & \cdots I_L \\ Q_1 & Q_2 & Q_3 & \cdots Q_L \\ U_1 & U_2 & U_3 & \cdots U_L \\ V_1 & V_2 & V_3 & \cdots V_L \end{bmatrix} \qquad 6$$

As in equation 3, each of these $L$ Stokes vectors is measured via m power measurements. In the double-differencing method, $m = 8$. These power measurements can be helpfully collected into a $L$x$m$ matrix $PP$:

$$PP = \begin{bmatrix} P_{1,1} & P_{2,1} & P_{3,1} & \cdots P_{L,1} \\ P_{1,2} & P_{2,3} & P_{3,2} & \cdots P_{L,2} \\ P_{1,3} & P_{2,3} & P_{3.3} & \cdots P_{L,3} \\ \vdots & \vdots & \vdots & \vdots \\ P_{1,m} & P_{2,m} & P_{3,m} & \cdots P_{L,m} \end{bmatrix} \qquad 7$$

As with equation 4, the $m$x4 polarimetric data reduction matrix ($W^{-1}$) converts between the measurement vectors and the Stokes vectors:

$$W^{-1} * PP = SS \qquad 8$$

To calculate the polarimetric data reduction matrix ($W^{-1}$), right-multiply both sides by the $m$x$L$ pseudoinverse of the power measurement matrix:

$$W^{-1} = W^{-1} * PP * PP^{-1} = SS * PP^{-1} \qquad 9$$

Via the properties of the pseudoinverse, this blind calibration method offers the RMS minimization as used in the parametric calibration. The blind calibration is superior to the parametric calibration in that no assumptions are made about the polarization elements. Note that the pseudoinverse operation represents a least-squares fit to overdetermined datasets, as is typically the case in polarimetric calibration.[4]

This clean result from linear algebra hides a complication - the power measurement matrix is ill-conditioned, in part because the problem is overdetermined, with nonunique solutions, and in part because the power measurements include noise. This ill-conditioning can cause errors to propagate unhelpfully into the pseudoinverse operation and therefore into the polarimetric data reduction. Linear algebra, via singular value decomposition, offers insight into this problem; and physics offers a mitigation it. In singular value decomposition, a matrix *a* is decomposed as:

$$a = u\ s\ v \qquad 10$$

where *u* & *v* are orthogonal to one another and *s* is a diagonal matrix. (Beware a notational confusion here. Upper case *U*, *S*, & *V* are used to define Stokes parameters. Linear algebra texts seem to use the uppercase *U* & *V* to denote the orthogonal matrixes. This note uses lower case *u* & *v* for these orthogonal matrices.) The values along the diagonal of *s* are called the "singular values" and represent the eigenmodes of the system. In the theoretical approach for the double-difference polarimeter, there are only three nonzero singular values because there are only three Stokes vector elements probed. In a real calibration, more terms will be present in the singular value decomposition, but they should be discarded as nonphysical. This removal of nonphysical singular values offers useful filtering of the polarimetric signal, helping to reduce noise in the measurement while maintaining the generality of the data reduction.

In polarimeter design, this singular value decomposition can offer insight into the calibration process via the "condition number", which is equal to the largest singular value divided by the smallest nonzero singular value.[5] Because the power measurement matrix, *PP*, must be inverted to find the polarimetric data reduction matrix, $W^{-1}$, the value of the condition number is a measure of the desirability of the calibration method. Early discussions of the MODHIS calibration process offered two opposing views of the optimum calibration source: a polarizer with a high extinction ration or glass plates with a well-defined extinction ratio. Singular value decomposition offers insight into this question; results are summarized in Table 2. The table shows that calibration with idealized polarization elements is nearly as attractive as calibration with a real polarizer and reasonable polarization elements. The table also shows that calibration with a low degree of polarization is inferior to calibration with weakly polarized sources. Consideration of other errors, such as imperfect knowledge of the calibration states, might change this conclusion. For now, though only highly polarized calibration sources will be considered.

| **DOP** | **other errors** | **condition number** |
|---|---|---|
| 1<br>(perfect polarizer) | none | 2 |
| 0.999<br>(good polarizer) | achievable with difficulty | 2 |
| 0.8<br>(bad polarizer) | readily achieved | 2.6 |
| 0.1<br>(tilted plates) | readily achieved | 22 |

*Table 2: Condition number for various calibration schemes. (DOP represents the degree of polarization of the calibration polarization states.) A good polarizer with good polarization elements is not meaningfully worse than the ideal case. The use of tilted plates would cause a degradation in calibration under this analysis*

**Potential calibration sources:**

Polarimetric calibration sources should be placed upstream of any optics with any meaningful polarization effects. Table 1 summarizes the elements with meaningful polarization effects for MODHIS. Special difficulties are presented when considering polarimetric calibration sources in front of the atmosphere and in front of TMT.

Several potential calibration sources were considered and discarded. Standard stars are an attractive source because such a calibration would include effects from the atmosphere and the telescope; although useful in a full polarimetric program, such objects are too sparse to be considered as calibration sources. The sky itself has some attraction as a calibration source because it includes the effects of the telescope; this approach was discarded because the objects of interest are generally in parts of the sky with weak polarization signatures. Sources within the TMT dome would also be convenient polarimetrically, but are unreasonable because of space and size constraints.

Therefore I recommend that the nominal calibration sources be within the NFIRAOS calibration subsystem. Figure 1 shows a schematic of this concept, in which highly depolarized light is provided by an integrating sphere, and rotating polarizing elements are added in front of the NFIRAOS entrance port. A rotating polarizer is used for calibration of linear polarization states. This polarizer and rotator need to be stowed unobtrusively when not used for polarimeter calibration.

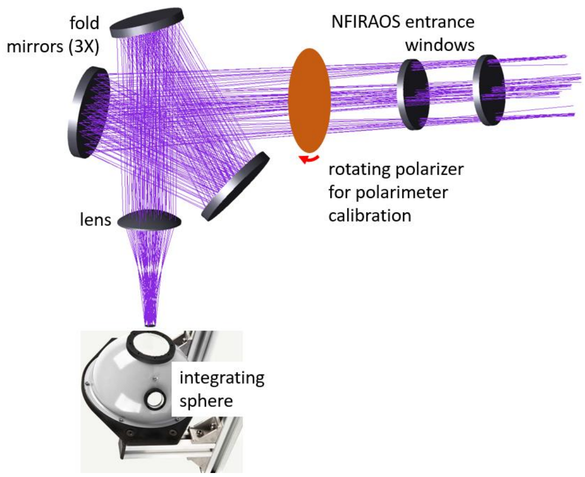


*Figure 1: Schematic of the NFIRAOS calibration module, showing the addition of a rotating polarizer for calibration of linear polarimetry.*

A design tweak to the NFIRAOS calibrator would be helpful. As designed, the effects of the fold mirrors are advantageously similar to those of M3. The effects could, nominally, match the effects of M3 precisely if the design was tweaked so that the sum of the squares of the angles of incidence match the square of the angles of incidence on M3. The angles of incidence are now 15, 19, and 15 deg. If the design is tweaked so that the angles of incidence are all, for example, 26 deg, then the s & p transmission of the calibrator would match that of M3. With such a change, the fold mirrors in the NFIRAOS calibrator would nominally have the same polarization effects as M3, which dominates the polarization effects of RMR. With such a change, the rotating polarizer should be moved to upstream of the fold mirrors in the NFIRAOS calibrator. The rotation of M3 with altitude would be simulated by rotation of a half wave plate downstream of the fold mirrors. Such a plate offers its own complications, which aren't pursued here. Such a change might be advantageous for radiometric calibration, too.

**<u>Polarizers to generate calibration states:</u>**

The polarizers used in the nominal calibration scheme shown in Figure 1 must be of sufficient size (~400mm dia to fill NFIRAOS entrance aperture), compact (thickness of a few millimeters to place the calibrator output in its nominal position axially), and have excellent extinction ratio across the entire aperture and angular range.

Dichroic polarizers best meet these requirements. Figure 2 shows performance of various dichroic polarizers, published on the Thorlabs website. Wire grid polarizers are quite appealing[6], although it's not clear that they can be procured with sufficient aperture size. Traditional polaroid film is somewhat less attractive, although long-term stability is unknown; if this traditional polaroid film is chosen, lamination in glass is probably best because it offers greater stability and fewer parasitic polarization effects.  Many manufacturers provide such parts, but I suspect all use the same small set of film providers.  Note in Figure 2 that, across the MODHIS spectral range, the materials generall meet the headline extinction ratios of 1000:1.  Wire grid is appealing for its broadband performance, but its oscillations in the pass state transmission look worrisome.  (A brief note from Thorlab's online help suggests, unconvincingly, that these oscillations are real, not a measurement artifact.)  Performance of LPNIRA looks acceptable, although it also contains oscillations.

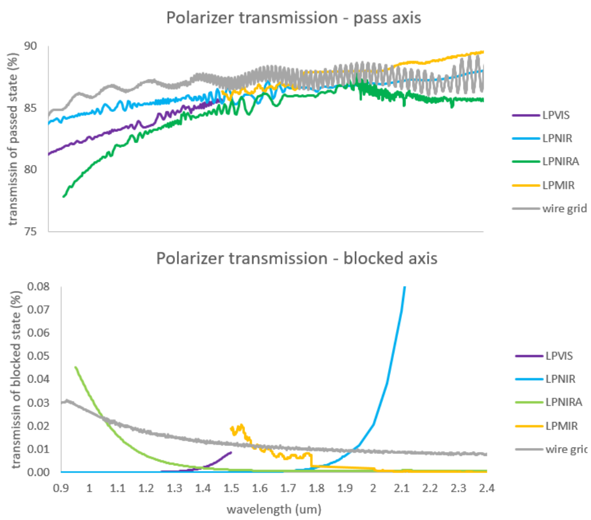


*Figure 2: Thorlabs data for various dichroic polarizers. Imperfections are all large compared to our polarimetric accuracy requirement of 0.001. Therefore, the calibrator will have to be well-characterized to provide well-known polarization states for MODHIS calibration*

These polarization errors can hide effects that can be important for polarimeter performance. For example, imperfect transmission of the blocked axis can be caused by several underlying issues, including: imperfect absorption of the blocked state, variation in the orientation of the blocked state, or birefringence in the transmissive elements of the element.  Each of these imperfections would affect the calibration differently.  Note that these effects would generally not be captured by parametric calibration; they would be captured by blind calibration.  An enticing option, cascading LPNIRA and LPNIR to achieve excellent broadband blocking, must be considered only with special care to such fabrication errors; for example, misalignment between the cascaded elements will cause the eigenstates to be non-orthogonal.

On commissioning of the polarimeter, many performance metrics should be validated, including pass & block transmission spectra, variation of pass axis orientation, and perpendicularity of block & pass axes.

In service, degradation of the polarizer is a concern because the polaroid material is a polymer. All the validation measurements should be monitored periodically, using an instrument that's not susceptible to the same degradation as the polarizer itself.

**<u>Space claim</u>**
details coming soon

**<u>Project plans</u>**:
details coming soon

**References:**

1. Inoue S, Hyde W. “Studies on depolarization of light at microscope lens surfaces” J Biophys Biochem Cytol. 1957 Nov 25;3(6):831-8
2. Boulbry, Ramella-Roman, and Germer, "Improved method for calibrating a Stokes polarimeter," Appl. Opt. 46, 8533-8541 (2007)
3. van Holstein et. al., “Polarimetric imaging mode of VLT/SPHERE/IRDIS II. Characterization and correction of instrumental polarization effects,” Astronomy & Astrophysics, vol 633, paper A64 (2020)
4. Many sources, but I like https://see.stanford.edu/materials/lsoeldsee263/05-ls.pdf
5. Many sources, but I like pillowlab.princeton.edu/teaching/statneuro2018/slides/notes02_SVD.pdf
6. https://www.meadowlark.com/product/versalight-polarizer/