Linear Mixed Effects Models in R

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This tutorial serves as a quick boot camp to jump-start your own analyses with linear mixed effects models. This text is different from other introductions by being decidedly *conceptual*; I will focus on *why* you want to use mixed models and *how* you should use them. While many introductions to this topic can be very daunting to readers who lake the appropriate statistical background, this text is going to be a softer kind of introduction… so, don't panic!

The tutorial requires R – so if you haven't installed it yet, go and get it! I also recommend reading tutorial 1 in this series before you go further. You can find it here:

http://www.bodowinter.com/tutorial/bw_LME_tutorial1.pdf

This tutorial will take you about 1 hour (possibly a bit more).

Introduction: Fixed and random effects

In tutorial 1, we talked about how we could use the linear model to express the relationships in our data in terms of a function. In one example, we modeled pitch as a function of age.

$$\text{pitch} \sim \text{age} + \varepsilon$$

We called "age" a fixed effect, and ε was our "error term" to represent the deviations from our predictions due to "random" factors that we cannot control experimentally. You could call this part the "probabilistic" or "stochastic" part of the model. Now, we'll unpack this "ε" and add complexity to it. That is, we change the random aspect of our model, essentially leaving the systematic part unchanged. In mixed models, everything in the "systematic" part of your model works just like with linear models in tutorial 1.

[1] For updates and other tutorials, check my webpage www.bodowinter.com. If you have any suggestions, please write me an email: bodo@bodowinter.com

In one of my studies, we have been interested in the relationship between pitch and politeness (Winter & Grawunder, 2012). So, essentially we're aiming for a relationship that looks like something like this:

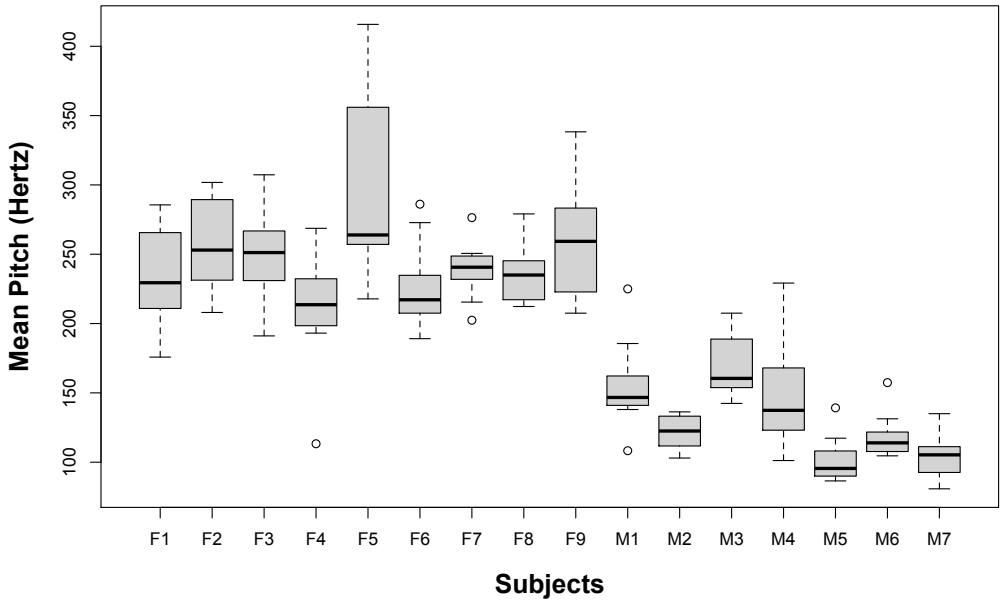
$$\text{pitch} \sim \text{politeness} + \varepsilon$$

Politeness was a categorical factor with two levels… a formal register and an informal register. On top of that, we also had an additional fixed effect, sex, and so our formula looks more like this:

$$\text{pitch} \sim \text{politeness} + \text{sex} + \varepsilon$$

So far so good. Now things get a little more complicated. Our design was so that we took multiple measures per subject. That is, each subject gave multiple polite responses and multiple informal responses. If we go back to the discussion of the assumptions of the linear model in tutorial 1, we can immediately see that this would violate the independence assumption: Multiple responses from the same subject cannot be regarded as independent from each other. Every person has a slightly different voice pitch, and this is going to be an idiosyncratic factor that you cannot control in your experiment. Multiple responses from the same subject will all have similar pitch, and thus these different responses cannot be regarded as independent.

The way we're going to deal with this is to add a ***random effect*** for subject. This allows us to resolve this non-independence by assuming a different "baseline" pitch value for each subject. So, subject 1 may have a mean voice pitch of 233 Hz across different utterances, and subject 2 may have a mean voice pitch of 210 Hz per subject. Here's a visual depiction of how this looks like:



Subjects F1 to F9 are female subjects. Subjects M1 to M7 are male subjects. You immediately see that males have lower voices than females (as is to be expected). But on top of that, within the male and the female groups, you see lots of individual variation, with some people having relatively higher values for their sex and others having relatively lower values.

We can model these individual differences by assuming different ***random intercepts*** for each subject. That is, each subject is assigned a different intercept value, and the mixed model estimates these intercepts for you.

Now you begin to see why the mixed model is called a "mixed" model. The linear models that we considered so far have been "fixed-effects-only" models that had one or more fixed effects and a general error term "ε". In the mixed model, we add to this one or more random effects (in this case: intercepts for subjects) – and that mixture of fixed and random effects makes the mixed model a mixed model.

Our updated formula looks like this:

$$\text{pitch} \sim \text{politeness} + \text{sex} + (1|\text{subject}) + \varepsilon$$

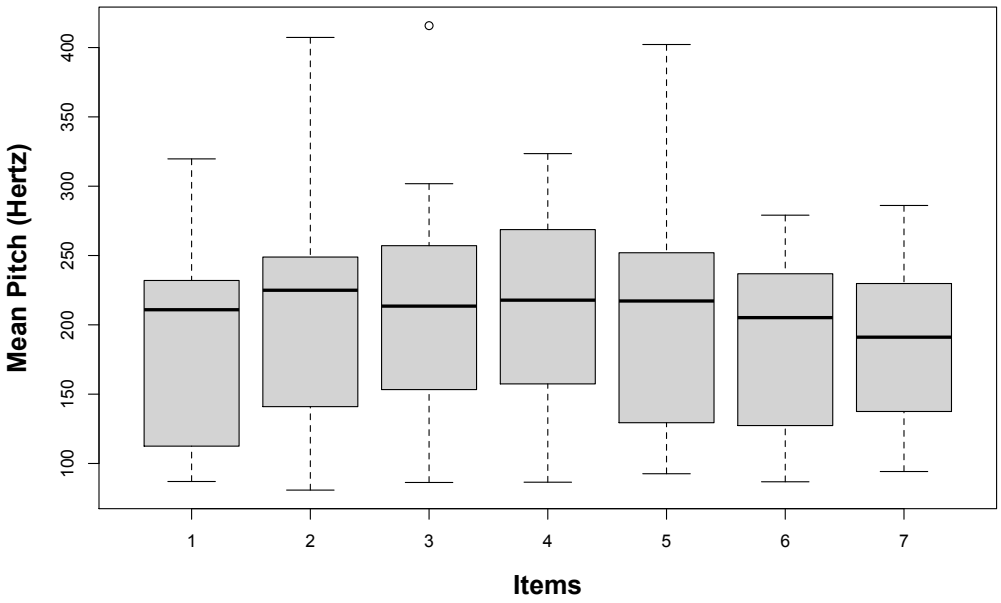
"(1|subject)" looks a little enigmatic. I'm already using the R-typical notation format here. What this is saying is "assume an intercept that's different for each subject" … and "1" stands for the intercept here. You can think of this formula as telling your model that it should expect that there's going to be multiple responses per subject, and these responses will depend on each subject's baseline level.

Note that the formula still contains a general error term "ε". This is necessary because even if we accounted for individual by-subject variation, there's still going to be "random" differences between different utterances from the same subject.

O.k., so far so good. But we're not done yet. In the design that we used in Winter and Grawunder (2012), there's an additional source of non-independence that needs to be accounted for: We had different items. One item, for example, was a "asking for a favor context", where we asked people to imagine asking a professor for a favor (polite condition), or asking a peer for a favor (informal condition). Another item was a "excusing for coming to late" context, which was similarly divided between polite and informal. In total, there were 7 such different items.

Similar to the case of by-subject variation, we also expect by-item variation. For example, there might be something special about "excusing for coming to late" which leads to overall higher pitch (maybe because it's more embarrassing than asking for a favor), regardless of the influence of politeness. And whatever it is that makes one item different from another, the responses of the different subjects in our experiment might similarly be affected by this random factor that is due to item-specific idiosyncrasies. That is, if "excusing for coming to late" leads to high pitch (for whatever reason), it's going to do so for subject 1, subject 2, subject 3 and so on. Thus, the different responses to one item cannot be regarded as independent, or, in other words, there's something similar to multiple responses to the same item – even if they come from different people.

Here's a visual representation of the by-item variability:



The variation between items isn't as big as the variation between subjects – but there are still noticeable differences, and we better account for them in our model!

We do this by adding an additional random effect:

$$\text{pitch} \sim \text{politeness} + \text{sex} + (1|\text{subject}) + (1|\text{item}) + \varepsilon$$

So, on top of different intercepts for different subjects, we now also have different intercepts for different items. We now "resolved" those non-independencies (our model knows that there are multiple responses per subject and per item), and we accounted for by-subject and by-item variation.

Note the efficiency and elegance of this model. Before, people used to do a lot of averaging. For example, in psycholinguistics, people would average over items for a subjects-analysis (each data point comes from one subject, assuring independence), and then they would also average over subjects for an items-analysis (each data point comes from one item). There's a whole literature on the advantages and disadvantages of this approach (Clark, 1973; Forster & Dickinson, 1976; Wike & Church, 1976; Raaijmakers, Schrijnemakers, & Gremmen, 1999; Raaijmakers, 2003; Locker, Hoffman, & Bovaird, 2007; Baayen, Davidson, & Bates, 2008; Bates, Barr, Levy, Scheepers, & Tilly, under review).

The upshot is: while traditional analyses that do averaging are in principle legit, mixed models give you much more flexibility … and they take the full data into account. If you do a subjects-analysis (averaging over items), you're essentially *disregarding* by-item variation. Conversely, in the items-analysis, you're disregarding by-subject variation. Mixed models account for both sources of variation *in a single model*. Neat, init?

Let's move on to R and apply our current understanding of the linear mixed effects model!!

Mixed models in R

For a start, we need to install the R package *lme4* (Bates, Maechler & Bolker, 2012). While being connected to the internet, open R and type in:

```
install.packages( )
```

Select a server close to you and choose the package *lme4* to install. Then, load it into R with the following command:

```
library(lme4)
```

Now, you have the function `lmer()` available to you, which is the mixed model equivalent of the function `lm()` in tutorial 1. This function is going to construct mixed models for us.

But first, we need some data! Download this dataset:

http://www.bodowinter.com/tutorial/politeness_data.csv

This is a shortened version of a dataset that was used for Winter and Grawunder (2012). Load the data into R:

```
politeness = read.csv(file.choose( ))
```

Now, you have a data frame called `politeness` in your R environment. You can familiarize yourself with the data by using `head()`, `tail()`, `summary()`, `str()`, `colnames()`… or whatever commands you commonly use to get an overview of a dataset. Also, it is always good to check for missing values:

```
which(is.na(politeness)==T)
```

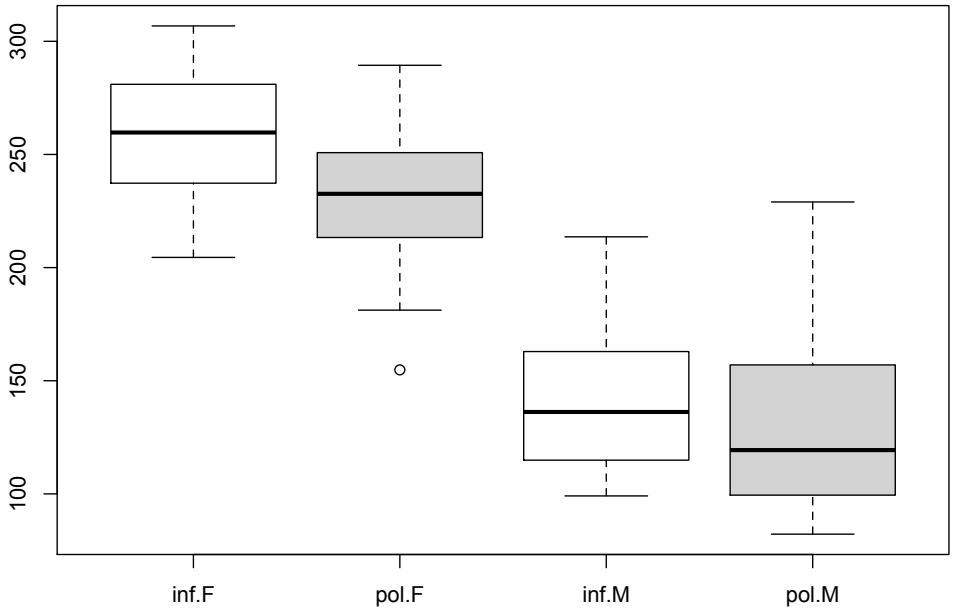
Apparently, there is a missing value in row 263. This is important to know but fortunately, a few missing values provide no problems for our mixed model analyses.

The difference in politeness level is represented in the column called "attitude". In that column, "pol" stands for polite and "inf" for informal. Sex is represented as "F" and "M" in the column "gender". The dependent measure is "frequency", which is the voice pitch measured in Hertz (Hz).

The interesting random effects for us are in the column "subject" and "scenario", the latter being the name of the item column.

Let's look at the relationship between politeness and pitch by means of a boxplot:

```
boxplot(frequency ~ attitude*gender,
     col=c("white","lightgray"),politeness)
```



What do we see? In both cases, the median line (the thick line in the middle of the boxplot) is lower for the polite than for the informal condition. However, there may be a bit more overlap between the two politeness categories for males than for females.

Let's start with constructing our model!

Type in the command below …

```
lmer(frequency ~ attitude, data=politeness)
```

… and you will retrieve an error that should look like this:

```
Error in lmerFactorList(formula, fr, rmInt = FALSE, drop = FALSE) :
  No random effects terms specified in formula
```

The model *needs* a random effect (after all, "mixing" fixed and random effects is the point of mixed models). We just specified a single fixed effect, attitude, and that was not enough.

So, let's add random intercepts for subjects and items (remember that items are called "scenarios" here):

```
politeness.model = lmer(frequency ~ attitude +
    (1|subject) + (1|scenario), data=politeness)
```

The last command created a model that used the fixed effect "attitude" (polite vs. informal) to predict voice pitch, controlling for by-subject and by-item variability. We saved this model in the object `politeness.model`. To see what the result is, simply type in `politeness.model` to print the output (in contrast to `lm()` you don't need to use `summary()` to get this output).

This is the full output:

```
Linear mixed model fit by REML
Formula: frequency ~ attitude + (1 | subject) + (1 | scenario)
   Data: politeness
   AIC   BIC logLik deviance REMLdev
 803.5 815.5 -396.7    807.1   793.5
Random effects:
 Groups   Name        Variance Std.Dev.
 scenario (Intercept)  218.98  14.798
 subject  (Intercept) 4014.54  63.360
 Residual              646.02  25.417
Number of obs: 83, groups: scenario, 7; subject, 6

Fixed effects:
            Estimate Std. Error t value
(Intercept)  202.588     26.750   7.573
attitudepol  -19.695      5.585  -3.527

Correlation of Fixed Effects:
            (Intr)
attitudepol -0.103
```

Again, let's work through this: First, the output reminds you of the model that you fit. Then, there's some general summary statistics such as Akaike's Information Criterion, the log-Likelihood etc. We won't go into the meaning of these different values in this tutorial because these are conceptually a little bit more involved. Let's focus on the output for the random effects first. Here it is again:

```
Random effects:
 Groups   Name        Variance Std.Dev.
 scenario (Intercept)  218.98  14.798
 subject  (Intercept) 4014.54  63.360
 Residual              646.02  25.417
```

Have a look at the column standard deviation. This is a measure of the variability for each random effect that you added to the model. You can see that scenario ("item") has much less variability than subject. Based on our boxplots from above, where we saw more idiosyncratic differences between subjects than between items, this is to be expected. Then, you see "Residual" which stands for the variability that's not due to either scenario or subject. This is our "ε" again, the

"random" deviations from the predicted values. Here, this reflects the fact that each and every utterance has some factors that affect pitch that are outside of the purview of our experiment.

The fixed effects output mirrors the coefficient table that we considered in tutorial 1 when we talked about the results of our linear model analysis.

```
Fixed effects:
            Estimate Std. Error t value
(Intercept)  202.588     26.750   7.573
attitudepol  -19.695      5.585  -3.527
```

The coefficient "attitudepol" is the slope for the effect of politeness. -19.695 means that to go from "informal" to "polite", you have to go down -19.695 Hz. In other words: pitch is lower in polite speech than in informal speech. Then, there's a standard error associated with this slope, and a t-value, which is simply the estimate divided by the standard error (check this by performing the calculation by hand).

Note that the `lmer()` function (just like the `lm()` function in tutorial 1) took whatever comes first in the alphabet to be the reference level. "inf" comes before "pol", so the slope represents the change from "inf" to "pol". If the reference category would be "pol" rather than "inf", the only thing that would change would be that the sign of the coefficient 19.695 would be positive. Standard errors, significance etc. would remain the same.

Now, let's consider the intercept. In tutorial 1, we already talked about the fact that oftentimes, model intercepts are not particularly meaningful. But this intercept is especially weird. It's 202.588 Hz … where does that value come from?

If you look back at the boxplot that we constructed earlier, you can see that the value 202.588 Hz seems to fall halfway between males and females – and this is indeed what this intercept represents. It's the average of our data (for the informal condition). As we didn't inform our model that there's two sexes in our dataset, the intercept is particularly off, in between the voice pitch of males and females. This is just like the classic example of a farm with a dozen hens and a dozen cows … where the mean legs of all farm animals considered together is three, not a particularly informative representation of what's going on at the farm.

Let's add gender as an additional fixed effect:

```
politeness.model = lmer(frequency ~ attitude +
     gender + (1|subject) +
     (1|scenario), data=politeness)
```

We overwrote our model object `politeness.model` with this new model. Note that we added gender as a fixed effect because the relationship between sex and pitch is systematic and predictable (i.e., we expect females to have higher pitch). This is different from the random effects subject and item, where the relationship between these and pitch is much more unpredictable and "random". We'll talk more about the distinction between fixed and random effects later.

Let's print the model output again. Let's have a look at the residuals first:

```
Random effects:
 Groups   Name        Variance Std.Dev.
 scenario (Intercept) 219.45   14.814
 subject  (Intercept) 615.57   24.811
 Residual             645.90   25.414
```

Note that compared to our earlier model without the fixed effect gender, the variation that's associated with the random effect subject dropped considerably. This is because the variation that's due to gender was confounded with the residual variation. The model didn't know about males and females, and so it's predictions were relatively more off, creating relatively larger residuals.

Let's look at the coefficient table now:

```
Fixed effects:
            Estimate Std. Error t value
(Intercept)  256.846     16.114  15.940
attitudepol  -19.721      5.584  -3.532
genderM     -108.516     21.010  -5.165
```

We see that males and females differ by about 109 Hz. And the intercept is now much higher (256.846 Hz), as it now represents the female category. The coefficient for the effect of attitude didn't change much.

Statistical significance

So far, we haven't talked about significance yet. But, if you want to publish this, you'll most likely need to report some kind of p-value. Unfortunately, p-values for mixed models aren't as straightforward as they are for the linear model. There are multiple approaches, and there's a discussion surrounding these, with sometimes wildly differing opinions about which approach is the best. Here, I focus on the Likelihood Ratio Test as a means to attain p-values.

Likelihood is the probability of seeing the data you collected given your model. The logic of the likelihood ratio test is to compare the likelihood of two models with each other. First, the model *without* the factor that you're interested in (the null model), then the model *with* the factor that you're interested in. Here's how you would do this in R. First, you need to construct the null model:

```
politeness.null = lmer(frequency ~ gender +
     (1|subject) + (1|scenario), data=politeness)
```

Then, you compare the null model to the model that we constructed above. The function you need for this is the `anova()` function:

```
anova(politeness.null,politeness.model)
```

This is the resulting output:

```
Data: politeness
Models:
politeness.null: frequency ~ gender + (1 | subject) + (1 | scenario)
politeness.model: frequency ~ attitude + gender + (1 | subject) + (1 | scenario)
                 Df    AIC    BIC  logLik  Chisq Chi Df Pr(>Chisq)
politeness.null   5 817.04 829.14 -403.52
politeness.model  6 807.40 821.91 -397.70 11.647      1   0.000643
```

Again, you're being reminded of the formula of the two models that you're comparing. Then, you find a Chi-Square value, the associated degrees of freedom and the p-value[2].

[2] You might wonder why we're doing a Chi-Square test here. There's a lot of technical detail here, but the main thing is that there's a theorem, called Wilk's Theorem, which states that the log likelihood ratio of two models approaches a Chi-Square distribution with degrees of freedom of the number of parameters that differ between the models (in this case, only "attitude"). So, somebody has done a proof of this and you're good to go!
Do note, also, that some people don't like "straight-jacketing" likelihood into the classical null-hypothesis significance testing framework that we're following here, and so they would disagree with the interpretation of likelihood the way we used it in the likelihood ratio test.

You would report this result the following way:

> "… politeness affected pitch ($\chi^2(1)=11.647$, $p=0.000643$), lowering it by about 19.7 Hz ± 5.6 (standard errors) …"

If you're used to t-tests, ANOVAs and linear model stuff, then this likelihood-based approach might seem weird to you. Rather than getting a p-value straightforwardly from your model, you get a p-value from a comparison of two models. To help you get used to the logic that is involved in this, here's an analogy: Say, you're running along a track and you have a bag with you. To really know whether this bag has an effect on your running speed, the best way is to put the bag away and to run … and to compare this to running with the bag. So, putting the bag away is analogous to the null model … and running with the bag is analogous to the model with the factor that you're interested in.

Note that we kept the predictor gender in the model. The only change between the full model and the null model that we compared in the likelihood ratio test was the factor of interest, politeness. In this particular test, you can think of "gender" as a control variable and of "attitude" as your test variable.

Super-crucial: Random slopes versus random intercepts

We're not done yet. One of the coolest things about mixed models is coming up now, so hang on!!

Let's have a look at the coefficients of the model by subject and by item:

```
coef(politeness.model)
```

Here is the output:

```
$scenario
  (Intercept) attitudepol   genderM
1    243.3398   -19.72111 -108.5164
2    263.4292   -19.72111 -108.5164
3    268.2541   -19.72111 -108.5164
4    277.4757   -19.72111 -108.5164
5    254.9102   -19.72111 -108.5164
6    244.6724   -19.72111 -108.5164
7    245.8426   -19.72111 -108.5164

$subject
   (Intercept) attitudepol   genderM
F1    242.9367   -19.72111 -108.5164
F2    267.2668   -19.72111 -108.5164
F3    260.3353   -19.72111 -108.5164
M3    285.2322   -19.72111 -108.5164
M4    262.2255   -19.72111 -108.5164
M7    223.0811   -19.72111 -108.5164
```

You see that each scenario and each subject is assigned a different intercept. However, the fixed effects (attitude and gender) are all the same for all subjects and items. Our model is what is called a ***random intercept model***. In this model, we account for baseline-differences in pitch, but we assume that whatever the effect of politeness is, it's going to be the same for all subjects and items.

But is that a valid assumption? In fact, it's not – it is quite expected that some items would elicit more or less politeness. That is, the effect of politeness might be different for different items. Likewise, the effect of politeness might be different for different subjects, as it is to be expected that some people are more or less polite. So, what we need is a ***random slope*** model, where subjects and items are not only allowed to have differing intercepts, but where they are also allowed to have different slopes for the effect of politeness. This is how we would do this in R:

```
politeness.model = lmer(frequency ~ attitude +
     gender + (1+attitude|subject) +
     (1+attitude|scenario), data=politeness)
```

Note that the only thing that we changed is the random effects, which now look a little more complicated. The notation "(1+attitude|subject)" means that you tell the model to expect differing baseline-levels of frequency (the intercept, represented by 1) as well as differing responses to the main factor in question – attitude. You then do the same for items.

Have a look at the coefficients of this updated model by typing in the following:

```
coef(politeness.model)
```

Here's a reprint of the output that I got:

```
$scenario
  (Intercept) attitudepol   genderM
1    244.4725   -19.00303 -111.1032
2    261.9439   -12.87457 -111.1032
3    270.9277   -23.46232 -111.1032
4    277.0645   -15.90576 -111.1032
5    255.8264   -18.72596 -111.1032
6    247.0404   -22.37927 -111.1032
7    249.7023   -25.93020 -111.1032

$subject
   (Intercept) attitudepol   genderM
F1    243.2783   -20.49943 -111.1032
F2    267.1184   -19.30435 -111.1032
F3    260.2852   -19.64689 -111.1032
M3    287.1039   -18.30249 -111.1032
M4    264.6681   -19.42718 -111.1032
M7    226.3843   -21.34632 -111.1032
```

Now, the column with the by-subject and by-item coefficients for the effect of politeness ("attitudepol") is different for each subject and item. Note, however, that it's always negative. This means that despite individual variation, there is also consistency in how politeness affects the voice: for all of our speakers, the voice tends to go down when speaking politely, but for some people it goes down more than for others.

Have a look at the column for gender. Here, the coefficients do no change. That is because we didn't specify random slopes for the by-subject or by-item effect of gender. A moment of thought reveals that a random slope component such as "(1+gender|subject)" does not make much sense … as each subject can be either only male or only female… so the effect of gender cannot vary for a particular subject. "(1+gender|scenario)" would be a possible component of our model, as each item is presented to both male and female speakers (and potentially, the effect of gender might differ for certain items). However, I chose not to include it in this model to keep my model lean. The main effect in question is politeness, and it's much more important to control for by-subject and by-item individual variability in the effect of politeness than for by-item variability for the control variable gender.

O.k., let's try to obtain a p-value. We keep our model from above (`politeness.model`) and compare it to a new null model in a likelihood ratio test. Let's construct the null model first:

```
politeness.null = lmer(frequency ~ gender +
     (1+attitude|subject) + (1+attitude|scenario),
     data=politeness)
```

Note that the null model needs to have the same random effects structure. So, if your full model is a random slope model, your null model also needs to be a random slope model.

Let's now do the likelihood ratio test:

```
anova(politeness.null,politeness.model)
```

This is, again, significant.

There are a few important things to say here: You might ask yourself "Which random slopes should I specify?" … or even "Are random slopes necessary at all?"

A lot of people construct random intercept-only models but conceptually, it makes a hella sense to include random slopes. People differ with how they react to an experimental manipulation – that's to be expected! And an experimental manipulation might also have differential effects for different items.

Moreover, researchers in ecology (Schielzeth & Forstmeier, 2009) and psycholinguistics (Barr, Levy, Scheepers, & Tilly, under review) have shown via simulations that mixed models without random slopes are anti-conservative or, in other words, they have a relatively high Type I error rate (they tend to find a lot of significant results which are actually due to chance).

Barr et al. (under review) recommend that you should "keep it maximal" with respect to your random effects structure. This means that you include all random slopes that are justified by your design … and you do this for all fixed effects that are important. This was my reason to not have random slopes for the effect of gender by item in the model. The effect of gender on pitch is something that's well established already and we were not interested in this. We were, however, interested in the effect of politeness on pitch (which was one of the main aspects of our study) … and therefore, random slopes for by-subject and by-item variability in how politeness affects pitch should be included.

Interactions

You might want to test the interaction between the effect of politeness and gender. In R, you specify interactions with a "*" rather than a "+". If you're interested specifically in the interaction, you would compare the model with "*" against the model with "+":

(1) Comparison
full model: pitch ~ politeness*gender
reduced model: pitch ~ politeness+gender

Alternatively, you can directly compare these two models with each other:

(2) Comparison
full model: pitch ~ politeness*gender
reduced model: pitch ~ gender

The difference is that in comparison (1), you're specifically testing for the interaction. In (2), you're testing the effect of politeness… regardless of whether it interacts with gender or not. However, you loose information as to whether there is an interaction or whether there is an effect of politeness without this effect depending on gender.

If there's a significant interaction, the interpretation of the main effects (i.e., the effect of politeness ignoring gender-dependent differences in politeness) isn't straightforward any more. In such a case, I would only report the significance of the interaction and then say that the effect of politeness came in the form of an interaction with gender.

You should spend a few minutes implementing the two comparisons above with the data that we played with so far.

Assumptions

In tutorial 1, we talked a lot about the many different assumptions of the linear model. The good news is: Everything that we discussed in the context of the linear model applies straightforwardly to mixed models. So, you also have to worry about colinearity and influential data points. And you have to worry about homoscedasticity (and potentially lack of normality). But you don't have to learn much new stuff. The way you check these assumptions in R is exactly the same as in the case of the linear model.

Independence, being the most important assumption, requires a special word: One of the main reasons we moved to mixed models rather than just working with linear models was to resolve non-independencies in our data. However, mixed models can still violate independence … if you're missing important fixed or random effects. So, for example, if we analyzed our data with a model that didn't include the random effect "subject", then our model would not "know" that there are multiple responses per subject. This amounts to a violation of the independence assumption. So choose your fixed effects and random effects carefully, and always try to resolve non-independencies.

Then, a word on influential data points. You will find that the function `dfbeta()` that we used in the context of linear models doesn't work for mixed models. There's two things you can do to still to influence diagnostics. There's a package called *influence.ME* (Nieuwenhuis, te Grotenhuis, & Pelzer, 2012) that you might consider. Or, you can write a loop for successively leaving one data point out and writing down the coefficients. In case you want to do this by hand, the following code gives you an outline of the general structure of how you might want to do this (you can check my "doodling" tutorials on loops and programming structures in R to get a better grasp of this):

```
all.res=numeric(nrow(mydataframe))
for(i in 1:nrow(mydataframe)){
     myfullmodel=lmer(response~predictor+
          (1+predictor|randomeffect),POP[-i,])
     all.res[i]=fixef(myfullmodel)[some number]
}
```
[3]

[3] The basic idea of this code snippet is this: Pre-define a vector that has as many elements as you have rows in your dataset. Then, cycle through each row. For each iteration, make a new mixed model *without that row* (this is achieved by `POP[-i,]`). Then, the function `fixef()`, extract whatever coefficient interests you.
With this code, you will need to fill in a lot of stuff. Besides the names of your data frame and your variables, you need to run `fixef()` on your model once so you know which position the relevant coefficient is. In our case, I would put a "2" in there because the effect of "attitudepol" appears second in the list of coefficients.

Go ahead and play with checking the assumptions. You can go back to tutorial 1 and apply the code in there to the new objects in this tutorial.

A final note on random versus fixed effects

I have evaded a precise definition of the difference between fixed and random effects. I deliberately did this because I wanted you to get some experience with linear mixed effects models in R before we finally take a step back and sharpen our concepts.

So, a random effect is generally something that can be expected to have a non-systematic, idiosyncratic, unpredictable, or "random" influence on your data. In experiments, that's often "subject" and "item", and you generally want to generalize over the idiosyncrasies of individual subjects and items.

Fixed effects on the other hand are expected to have a systematic and predictable influence on your data.

But there's more to it. One definition of fixed effects says that fixed effects "exhaust the population of interest", or they exhaust "the levels of a factor". Think back of sex. There's only "male" or "female", so these are the only two levels of this factor. Our experiment includes both categories and thus exhausts the category sex. With our factor "politeness" it's a bit trickier. You could imagine that there are more politeness levels than just the two that we tested. But in the context of our experiment, we *operationally defined* politeness as the difference between these two categories – and because we tested both, we fully "exhaust" the factor politeness (as defined by us).

In contrast, random effects generally sample from the population of interest. That means that they are far away from "exhausting the population" … because there's usually many many more subjects or items that you could have tested. The levels of the factor in your experiment is a tiny subset of the levels "out there" in the world.

The write-up

A lot of tutorials don't cover how to write up your results. And that's a pity, because this is a crucial part of your study!!!

The most important thing: You need to describe the model to such an extent that people can ***reproduce the analysis***. So, a useful heuristic for writing up your results is to ask yourself the question "Would I be able to re-create the analysis given the information that I provided?" If the answer is "yes" your write-up is good.

Another important thing is to give enough credit to the people who put so much of their free time into making *lme4* and R work so efficiently. So let's cite them! It's also a good idea to cite exactly the version that you used for your analysis. You can find out your version and who to cite by typing in…

```
citation()
```

… for your R-version … and …

```
citation("lme4")
```

… for the lme4 package.

Finally, it's important that you mention that you checked assumptions, and that the assumptions are satisfied. So here's what I would have written for the analysis that we performed in this tutorial:

> "We used R (R Core Teamn, 2012) and *lme4* (Bates, Maechler & Bolker, 2012) to perform a linear mixed effects analysis of the relationship between pitch and politeness. As fixed effects, we entered politeness and gender (without interaction term) into the model. As random effects, we had intercepts for subjects and items, as well as by-subject and by-item random slopes for the effect of politeness. Visual inspection of residual plots did not reveal any obvious deviations from homoscedasticity or normality. P-values were obtained by likelihood ratio tests of the full model with the effect in question against the model without the effect in question."

Yay, we're done!! I hope this tutorial was of help to you. I want to thank those people who taught me stats (in particular Roger Mundry), and the many readers of this tutorial who gave feedback to earlier versions. Finally, I want to thank you for joining me on this quick tour through mixed models.

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